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A B S T R A C T

In models of weak and electromagnetic interactions with six or more quarks, universality and, in some cases, the V-A structure of leptonic and semi-leptonic interactions have to be put in by hand. We show how both properties re-emerge naturally if replacing the usual $SU(2)$ weak isospin group by a larger $SU(4)$ group, which also lends itself quite naturally to grand unifications (including strong interactions). The model needs eight quarks and two charged heavy leptons, and features, besides universality and V-A, non-equality of neutral current scattering of ν 's on leptons and on hadrons, and very reasonable relations for (bare) fermion masses.

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1. The first part of the report is a general introduction to the subject.

2. The second part is a detailed description of the area.

3. The third part is a summary of the findings.

4. The fourth part is a conclusion and recommendations.

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The most recent fashion in quark models¹⁻³⁾ introduces heavy leptons and at least six quarks plus right-handed currents. Apart from other reasons, one of the attractions of such schemes is that they are able to explain the $\Delta I = 1/2$ rule and even the relative orders of magnitude of $\Delta I = 1/2$ and $\Delta I = 3/2$ processes for non-leptonic weak decays. Alas, you cannot have the cake and eat it, and this new understanding is only gained by losing something that you had; namely, universality and the V-A structure for leptonic and semileptonic weak interactions. Indeed, in the old four-quark, left-handed theory⁴⁾ you could show [cf. Gaillard et al.⁵⁾ for a simple discussion] that V-A was an automatic consequence of the zero mass of (Weyl) neutrinos, and that the interplay of weak, electromagnetic, and strong interactions allowed you only to Cabibbo-rotate the s- and d-quarks so that universality was automatic. However, if you have six quarks and if the L^0 neutrino is massless (the situation is worse for more quarks and/or massive L^0) the most general weak multiplets are

$$\begin{pmatrix} \nu_e \\ e_L^- \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu_L^- \end{pmatrix}, \begin{pmatrix} L^0 \\ L_L^- \end{pmatrix}; \begin{pmatrix} u_L \\ d_L' \end{pmatrix}, \begin{pmatrix} s_L \\ s_L' \end{pmatrix}, \begin{pmatrix} t_L \\ b_L' \end{pmatrix}, \quad (1)$$

where, in terms of physical particles (unprimed ones),

$$\begin{aligned} d_L' &= d_L (\cos \theta \cos \psi - \cos \varphi \sin \theta \sin \psi) - s_L (\sin \theta \cos \psi + \cos \theta \cos \varphi \sin \psi) \\ &\quad + b_L \sin \psi \sin \varphi, \\ s_L' &= d_L (\cos \theta \sin \psi + \cos \varphi \sin \theta \cos \psi) + s_L (-\sin \theta \sin \psi + \cos \theta \cos \varphi \cos \psi) \\ &\quad - b_L \cos \psi \sin \varphi, \\ b_L' &= d_L \sin \varphi \sin \theta + s_L \cos \theta \sin \varphi + b_L \cos \varphi, \end{aligned} \quad (2)$$

so universality of u, d, s, v, e, μ weak interactions is a mere accident that occurs when $|\phi|, |\psi| \ll |\theta|$. If you take into account also the right-handed multiplets, then the V-A structure of leptonic and semileptonic weak interactions is an even more unlikely coincidence which happens because, out of twelve possible angles, all but one are arbitrarily set equal to either 0 or $\pi/2$.

It is not difficult to see that this state of affairs has come to pass because the weak-plus-electromagnetic (wem, henceforth) group, $SU(2) \times U(1)$ is much too small so that models with more than two quarks, and two lepton multiplets are hardly constrained at all. The remark points to a possible remedy: one has to consider larger wem groups.

Larger wem groups than $SU(2) \times U(1)$, namely, $SU(3) \times U(1)$, have been discussed in the literature⁶⁾, mostly to avoid charm. As now there seems to exist some experimental evidence in favour of this quantum number, we should use groups that include it. In this note, we will propose a scheme based on an $SU(4) \times U(1)$ wem group^{*}). This will allow us to restore universality and V-A in a natural way, producing, as side effects, different neutral-current scattering of ν 's on leptons and on hadrons, and interesting relations among bare fermion masses.

With an eye on eventual unifications of weak, electromagnetic, and strong interactions (grand unification), we will write the fermion multiplets as

$$\psi_{i\pm}^{/a} = \begin{pmatrix} \nu_{e\pm}^{/r} & u_{\pm}^{/r} & u_{\pm}^{/y} & u_{\pm}^{/v} \\ E_{\pm}^{/o} & c_{\pm}^{/r} & c_{\pm}^{/y} & c_{\pm}^{/v} \\ e_{\pm}^{/r} & d_{\pm}^{/r} & d_{\pm}^{/y} & d_{\pm}^{/v} \\ E_{\pm}^{/r} & b_{\pm}^{/r} & b_{\pm}^{/y} & b_{\pm}^{/v} \end{pmatrix}, \quad \Psi_{i\pm}^{/a} = \begin{pmatrix} \nu_{\mu\pm}^{/r} & h_{\pm}^{/r} & h_{\pm}^{/y} & h_{\pm}^{/v} \\ M_{\pm}^{/o} & t_{\pm}^{/r} & t_{\pm}^{/y} & t_{\pm}^{/v} \\ \mu_{\pm}^{/r} & s_{\pm}^{/r} & s_{\pm}^{/y} & s_{\pm}^{/v} \\ M_{\pm}^{/r} & l_{\pm}^{/r} & l_{\pm}^{/y} & l_{\pm}^{/v} \end{pmatrix}, \quad (3)$$

where ν , E , e , μ , M are leptons, the rest quarks; r (red), y (yellow), and v (violet) are the colour labels, the lower $\pm$ indices refer to R (right)/L (left) handedness, the upper a indices label columns, and the lower i indices label rows.

We could start by discussing the $SU(4) \times U(1)$ wem group by itself; instead, we will go directly to a partial unification with strong interactions, based upon an $SU(4) \times SU(4)'$ group [$SU(4)'$ containing the "strong interactions" colour $SU(3)'$ ⁷⁾]. We assign ψ' to the representation $4 \times 4'$, and also Ψ' will be in a $4 \times 4'$. The vector mesons will be W_k' , belonging to $15 \times 1'$, and $V_{\mu}^{'c}$ in $1 \times 15'$. The matter-gauge fields piece of the interaction Lagrangian is then

$$g \sum_a \sum_{ijk} \lambda_{ijk} \left(\sum_{\pm} \bar{\psi}_{i\pm}^{/a} \gamma_{\mu} \psi_{j\pm}^{/a} + \sum_{\pm} \bar{\Psi}_{i\pm}^{/a} \gamma_{\mu} \Psi_{j\pm}^{/a} \right) W_k^{/'\mu} + g' \sum_i \sum_{abc} \lambda^{abc} \left(\sum_{\pm} \bar{\psi}_{i\pm}^a \gamma^{\mu} \psi_{i\pm}^b + \sum_{\pm} \bar{\Psi}_{i\pm}^a \gamma^{\mu} \Psi_{i\pm}^b \right) V_{\mu}^{'c}, \quad (4)$$

where λ are the Clebsch-Gordan coefficients that relate $\bar{4}, 4$ to 15 in the decomposition $\bar{4} \otimes 4 = 15 \oplus 1$. Equation (4) is invariant under an extra $SU(2)^{\mu}$ (muonic isospin) that rotates ψ into Ψ , but this extra invariance shall not be relevant for us here. The relation between primed fields ("weak" multiplets) and unprimed fields (physical, i.e. with well-defined mass) can be written as

^{*}) $SU(3) \times U(1)$ could also be used; we prefer $SU(4) \times U(1)$ for aesthetical reasons.

$$\begin{aligned}\psi_{i\pm}^{'a} &= \psi_{i\pm}^a \cos \theta_{i\pm}^a + \Psi_{i\pm}^a \sin \theta_{i\pm}^a, \\ \Psi_{i\pm}^{'a} &= -\psi_{i\pm}^a \sin \theta_{i\pm}^a + \Psi_{i\pm}^a \cos \theta_{i\pm}^a.\end{aligned}\tag{5}$$

As a matter of fact, (5) is not the most general mixing possible. If we use the same Higgs fields to give masses to leptons and quarks [which shall be done here; cf. expression (14) below], then we still have at our disposal an SU(4) rotation R which would replace $\psi_{i\pm}^a, \Psi_{i\pm}^a$ in the right-hand side of (5) by^{*})

$$\psi_i^a \rightarrow \sum_j R_{ij} \psi_j^a, \quad \Psi_i^a \rightarrow \sum_j R_{ij} \Psi_{j\pm}^a.$$

However, as R belongs to the invariance group we can always un-rotate the ψ, Ψ by R^{-1} thereby bringing the R rotation to bear on the W, so we can always assume that Eq. (5) is the most general relation between primed and unprimed fields. (The corresponding relations for W, V will be discussed later.)

We want parity invariance to be broken also spontaneously. This means that we must have

$$\begin{aligned}U(P) \psi_{i\pm}^{'a} U(P)^{-1} &= \epsilon_{\pm} \sum_{k,b} P_{ik}^{ab} \psi_{k\mp}^{'b}, \\ U(P) \Psi_{i\pm}^{'a} U(P)^{-1} &= \epsilon_{\pm} \sum_{k,b} Q_{ik}^{ab} \Psi_{k\mp}^{'b},\end{aligned}$$

where U(P) is the (generalized) parity operator⁹⁾, and P, Q are orthogonal matrices that verify $(P)^2 = \pm 1, (Q)^2 = \pm 1$. Requiring P, Q to commute with those quantum numbers which we want to remain absolutely conserved (charge, lepton number, and colour) thus implies that you can write

$$P_{ik}^{ab} = \delta_{ab} P_{ik}^{(a)}, \quad Q_{ik}^{ab} = \delta_{ab} Q_{ik}^{(a)}\tag{6a}$$

with

$$P^{(a)}, Q^{(a)} = \begin{pmatrix} C_{1P,Q}^{(a)} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & C_{2P,Q}^{(a)} \end{pmatrix}, \quad C^2 = \pm 1.\tag{6b}$$

Apart from trivial phases we thus have an eightfold choice for each a (all combinations of $C = 1$ and $C = i\sigma_2$). Parity violations arise (after breaking the

*) It should be obvious that you can always devise suitably perverse Higgs multiplets such that, for example, R depended on a; or, indeed, that you can have any relation you wish between primed and unprimed fields. A model, however, is not only the group, but also the breaking, and we choose the last to be the same for all fermion multiplets, as stated. In this context, see also Georgi and Pais⁸⁾.

symmetry) when some of the C equal $i\sigma_2$, and the V-A structure for leptonic and semileptonic weak interactions obtain for a particular combination of $1, i\sigma_2$. This is the only choice we make, and it is a Yes-No choice. Reabsorbing an unimportant phase, we take specifically

$$P_{ik}^{(1)} = \begin{pmatrix} 1 & 0 \\ 0 & i\sigma_2 \end{pmatrix}, Q_{ik}^{(1)} = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & 1 \end{pmatrix}, P_{ik}^{(c)} = Q_{ik}^{(c)} = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix}, \quad (6c)$$

$c = \text{colour} \neq 1$.

Next, we can get rid of some of the angles in Eq. (5). First of all, by a global rotation we can always arrange that the lepton multiplets coincide with the physical leptons. Second, if we keep ν_e, ν_μ massless we can (by eventually re-defining them) get that $u' = u, h' = h$. Lastly, all physical particles being colourless, it is not difficult to realize that relative rotations between quarks of different colours are physically unobservable. With all of this, it follows that the most general structure can always be re-cast into the form

$$\psi_{i-}^{/a} = \begin{pmatrix} \gamma_{eL} & \vec{d}_L \\ E_L^0 & \vec{c}_L \cos \varphi + \vec{f}_L \sin \varphi \\ e_L^- & \vec{d}_L \cos \theta + \vec{f}_L \sin \theta \\ E_L^- & \vec{b}_L \cos \psi + \vec{f}_L \sin \psi \end{pmatrix}, \psi_{i+}^{/a} = \begin{pmatrix} \gamma_{eR} & -(\vec{c}_R \cos \varphi + \vec{f}_R \sin \varphi) \\ E_R^0 & \vec{d}_R \\ -e_R^- & -(\vec{b}_R \cos \psi + \vec{f}_R \sin \psi) \\ e_R^- & \vec{d}_R \cos \theta + \vec{f}_R \sin \theta \end{pmatrix}, \quad (7a)$$

$$\Psi_{i-}^{/a} = \begin{pmatrix} \gamma_{\mu L} & \vec{h}_L \\ M_L^0 & -\vec{c}_L \sin \varphi + \vec{f}_L \cos \varphi \\ \mu_L^- & -\vec{d}_L \sin \theta + \vec{f}_L \cos \theta \\ M_L^- & -\vec{b}_L \sin \psi + \vec{f}_L \cos \psi \end{pmatrix}, \Psi_{i+}^{/a} = \begin{pmatrix} M_R^0 & \vec{c}_R \sin \varphi - \vec{f}_R \cos \varphi \\ \gamma_{\mu R} & \vec{h}_R \\ \mu_R^- & \vec{b}_R \sin \psi - \vec{f}_R \cos \psi \\ M_R^- & -\vec{d}_R \sin \theta + \vec{f}_R \cos \theta \end{pmatrix}, \quad (7b)$$

where, for any quark q , $\vec{q}$ stands for q^x, q^y, q^z .

Later on we shall give our specific choice of Higgs fields; before doing so, we will discuss the masses of the W 's and V 's from a phenomenological point of view, changing the notation slightly in that we now write V_μ^{ab} for the field that couples to the current

$$\sum_{i,\pm} (\bar{\psi}_{i\pm}^a \gamma^\mu \psi_{i\pm}^b + \bar{\Psi}_{i\pm}^a \gamma^\mu \Psi_{i\pm}^b), \quad b \neq a$$

and W_{ik}^{μ} for the field that couples to

$$\sum_{a,\pm} (\bar{\psi}_{i\pm}^{/a} \gamma_\mu \psi_{k\pm}^{/a} + \bar{\Psi}_{i\pm}^{/a} \gamma_\mu \Psi_{k\pm}^{/a}), \quad k \neq i.$$

As discussed before, the W' will in general not be physical particles, but rather we have

$$W'_{ij} = \sum_{i',j'} C_{ij,i'j'} W_{i'j'}, \quad i \neq j, \quad i' \neq j',$$

the W being the physical fields.

We will preserve the colour $SU(3)'$ subgroup as an exact symmetry and therefore leave the corresponding V^{ab} , $a \neq 1$, $b \neq 1$, $a \neq b$ massless [gluons⁷⁾]. We can fix g so that

$$\frac{g^2}{m^2(W_{13})} \simeq g^2 \sum_{i'j'} |C_{13,i'j'}|^2 \frac{1}{m^2(W_{i'j'})} = \frac{1}{\sqrt{2}} G_F \quad (8a)$$

where G_F is the Fermi coupling constant, and arrange also that

$$m(V^{1b}) \gg m(W), \quad b \neq 1. \quad (8b)$$

We shall choose Higgs fields^{*)} for the W so that they contribute equally to all W_{ij} . This means that we shall have mass equality of all the W_{ij} , up to the breaking induced by the mechanism we use to get fermion masses. If q is the heaviest fermion, we will then have

$$m^2(W_{ij}) = m^2(W_{13}) \text{ independent of } i \neq j, \quad (8c)$$

to order $m^2(q)/m^2(W_{13})$.

For comparison with experiment (at low energies) we first consider the effective interactions among the lightest fermions, $\nu_e, \nu_\mu, e, \mu, u, d, s$. From Eqs. (4), (7), (8) they are

$$\frac{G_F}{\sqrt{2}} \left| \bar{\nu}_e \gamma^\mu (1-\gamma_5) e^- + \bar{\nu}_\mu \gamma^\mu (1-\gamma_5) \mu^- + \sum_{\text{colour}} \bar{u} \gamma^\mu (1-\gamma_5) [d \cos \theta + s \sin \theta] \right|^2 \quad (9)$$

universality and the V-A structure obtain to order $m^2(q)/m^2(W_{13})$. Only the Cabibbo rotation θ appears in formula (9); all other angles ϕ, ψ [Eqs. (7)] have been either swept under the rug of currents involving heavy particles, or dumped into the dustbin of non-leptonic interactions. As for the last, besides the terms entering in formula (9) which contain universal V-A, $\Delta I = 1/2$ as well as $\Delta I = 3/2$ interactions, there are altogether seven more terms with couplings

*) More details about the whole scheme shall be presented elsewhere.

$(1/\sqrt{2})G_{ij} \sim g^2/m^2(W_{ij})$, $i \neq 1,2$, $j \neq 3,4$, $i \neq j$, $V + A$ as well as $V-A$, that contribute to $\Delta I = 1/2$ non-leptonic interactions. Thus the explanation of the enhancement of $\Delta I = 1/2$ non-leptonic processes is maintained and indeed strengthened as it now depends much less on dubious mass-term arguments.

We next turn to the neutral, colourless piece of formula (4). By appropriate re-labelling of the neutral W , V , and suppressing γ matrices, we can write it as

$$\begin{aligned} & g \left\{ \frac{1}{2\sqrt{2}} [\bar{\nu}_e \nu_e + \bar{\nu}_\mu \nu_\mu - \bar{E}^0 E^0 - \bar{M}^0 M^0 + \sum_{\text{colour}} (\bar{u}u + \bar{h}h - \bar{c}c - \bar{t}t)] X \right. \\ & + \frac{1}{2\sqrt{2}} [\bar{e}^- e^- + \bar{\mu}^- \mu^- - \bar{E}^- E^- - \bar{M}^- M^- + \sum_{\text{colour}} (\bar{d}d + \bar{s}s - \bar{b}b - \bar{l}l)] Y \\ & + \frac{1}{4} [\bar{\nu}_e \nu_e + \bar{\nu}_\mu \nu_\mu + \bar{E}^0 E^0 + \bar{M}^0 M^0 - \bar{e}^- e^- - \bar{\mu}^- \mu^- - \bar{E}^- E^- - \bar{M}^- M^- \\ & \quad + \sum_{\text{colour}} (\bar{u}u + \bar{h}h + \bar{c}c + \bar{t}t - \bar{d}d - \bar{s}s - \bar{b}b - \bar{l}l)] W_3 \\ & + g' \frac{1}{4\sqrt{3}} \left\{ \sum_{\text{colour}} (\bar{u}u + \bar{h}h) - 3 \bar{\nu}_e \nu_e - 3 \bar{\nu}_\mu \nu_\mu + \dots \right. \\ & \quad \left. + \sum_{\text{colour}} (\bar{b}b + \bar{l}l) - 3 \bar{E}^- E^- - 3 \bar{M}^- M^- \right\} V_3, \end{aligned} \quad (10)$$

where X , Y are already massive physical fields and we mix W_3 , V_3 to get the physical photon field A and the physical massive Z field:

$$\begin{aligned} W_3 &= A \sin \alpha + Z \cos \alpha \\ V_3 &= A \cos \alpha - Z \sin \alpha. \end{aligned} \quad (11)$$

It is to be noted that X and Y couple mixed vector and axially*), Z and A purely vector-like. Hence, in our model, the relative strength of vector and axial couplings of neutral currents is essentially free. Just as in standard models¹⁻³), formulae (10) and (11) yield the relations

$$g \sin \alpha = \sqrt{3} g' \cos \alpha, \quad g \sin \alpha = 2e \quad (12a)$$

where e is the charge of the proton and, denoting by e_q to the charge of quarks q ,

$$e_u = e_c = e_t = e_h = \frac{2}{3}e, \quad e_d = e_s = e_b = e_l = -\frac{1}{3}e. \quad (12b)$$

In a grand unified scheme we would take $SU(4) \times SU(4)'$ to be part of a larger simple group; then we would get the extra relationships, for bare quantities

$$g = g', \quad \sin^2 \alpha = 3/4, \quad g^2/4\pi = (16/3) e^2/4\pi. \quad (13)$$

*) To be precise, X and Y couple vector to ν_e , E^0 , μ^- , M^- , axial to ν_μ , M^0 , e^- , E^- , and all quarks couple axially to both X and Y .

Before finishing this note by considering Higgs couplings, and fermion masses, we would like to comment on other phenomenological features of the model (besides restoration of universality and V-A). First of all, we predict eight quarks and two charged heavy leptons. This will give a value of 26/3 for $R(e^+e^-)$, somewhat higher than the experimental¹⁰⁾ 6 to 7; but then $R(e^+e^-)$ seems to be still increasing at the highest available energies, which suggests, invoking asymptotic freedom arguments¹¹⁾, that not all quarks and/or leptons are fully excited yet. On the other hand, it now seems that there is some experimental evidence^{10,12)} for no less than four, and perhaps six, reasonably narrow heavy e^+e^- resonances: so naive counting would favour eight rather than four quarks.

Secondly, neutral-current interactions between neutrinos and leptons are only mediated by the Z, and are in particular purely vector. However, neutral interactions of ν_μ 's with hadrons may also go through X [cf. expression (10)], axially. This may very well be a way out of the present inconsistencies (not too decisive, it is true) between theory, and experiment, for purely lepton or semileptonic neutral currents¹⁰⁾, in particular why less events are found for $\nu_\mu e$ than for ν_μ hadron processes.

At long last we now come to the mass terms for fermions. It is quite obvious that by suitably introducing many Higgs fields we can do (almost) whatever we wish. However, we want to introduce a minimal set of Higgs multiplets. We take the set ξ_{ik}, η_{ik} of $SU(4)'$ scalar, $4 \times \bar{4}$ $SU(4)$ fields; and the set ζ^{ab}, ω^{ab} of $SU(4)$ scalar, $4 \times \bar{4}$ $SU(4)'$ fields. We will also formally preserve muon isospin, which is spontaneously broken owing to the different vacuum expectation values (VEV) of the ξ and η , and of the ζ and ω . Then we have the couplings

$$\begin{aligned} & f \sum_a \sum_{ik} (\bar{\psi}_{it}^a \psi_{k-}^a \xi_{ik} + \bar{\psi}_{i-}^a \psi_{k+}^a \xi_{ki}^* + \bar{\psi}_{it}^a \psi_{k-}^a \eta_{ik} + \bar{\psi}_{i-}^a \psi_{k+}^a \eta_{ki}^* \\ & + f' \sum_{i,ab} (\bar{\psi}_{it}^a \psi_{i-}^b \zeta^{ab} + \bar{\psi}_{i-}^a \psi_{it}^b \zeta^{ba*} + \bar{\psi}_{it}^a \psi_{i-}^b \omega^{ab} + \bar{\psi}_{i-}^a \psi_{it}^b \omega^{ba*}) \\ & + p(\xi, \eta, \zeta, \omega), \end{aligned} \quad (14)$$

where p is a polynomial, and we have written formula (14) directly in terms of unprimed fields as we are now only interested in mass terms. [To (14) one should add Higgs couplings with the W, V that yield the vector-field masses already discussed.] We then let the VEV matrices be

$$\begin{aligned} \xi_{ik} & \rightarrow \delta_{ik} m(i, \xi), \quad \eta_{ik} \rightarrow \delta_{ik} m(i, \eta), \\ \zeta^{ab} & \rightarrow \delta^{1a} \delta^{1b} m, \quad \omega^{ab} \rightarrow \delta^{1a} \delta^{1b} m', \end{aligned}$$

so that we get at once the relations

$$\begin{aligned}
 m(u) &= m(\nu_e) + m_1 & m(h) &= m(\nu_\mu) + m_2 \\
 m(c) &= m(E^0) + m_1 & m(t) &= m(M^0) + m_2 \\
 m(d) &= m(e) + m_1 & m(s) &= m(\mu) + m_2 \\
 m(b) &= m(E^-) + m_1 & m(l) &= m(M^-) + m_2,
 \end{aligned} \tag{15}$$

where m_1, m_2 are unknown constants. Of course, the masses that enter in formulae (15) are bare, so renormalization will alter these relations which, nevertheless, appear very attractive (except for the h quark).

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