TRAJECTORIES OF COUPLING CONSTANTS IN UNIFIED GAUGE FIELD THEORIES

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ABSTRACT

We use the renormalization group techniques to compute the energy behaviour of coupling constants in a variety of models unifying weak and electromagnetic, strong and electromagnetic, or even all three interactions. Two terms in the development of the relevant Callan-Symanzik β function are taken into account.

Three main conclusions are drawn. First, it is noted that, since the effective invariance groups are energy-dependent, we may have different behaviours at different energies. The trajectories of the various coupling constants are displayed for a variety of fashionable models. Second, following up these trajectories, we can determine the (minimum) masses of the vector bosons U that would eventually close the grand theory (all interactions unified, and hence all coupling constants equal up to Clebsch-Gordan coefficients). If the group effective at energy $\mu = 10^2$ GeV (say) is SUN , and you have n_f fermion multiplets, then (neglecting the variation of the fine structure constant), you have typically $M_U \sim 10^{36}/N^{1/2}$, $N = N_f - 2n_f/11$, for the U mass in GeV. This means that the mass of the U are strongly dependent on the specific breaking mechanisms we use [like $SU(3)$ colour all the way, or $SU(3)$ colour at low energies, and for example, $SU(3)$ colour $\times$ $SU(3)$ flavour at medium energies]. Again here we investigate this question for a cross-section of models. Finally, we note that if more flavours are excited at PETRA energies, then there exists models with non-zero fixed points, and hence the medium energy coupling constants can be obtained on purely group-theoretic grounds. Two specific cases are shown where the predicted electromagnetic coupling constant is $e^2(10 \text{ GeV})/4\pi \sim 10^{-2}$, and the gluon coupling constant is $g_c(10^2 \text{ GeV})/4\pi \sim 10^{-1}$.

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INTRODUCTION

It is well known that in renormalizable field theories the effective coupling constants are energy dependent; and, at least in principle, this behaviour with energy can be computed with the help of the differential equations of the renormalization group. In particular, we can use the so-called Callan-Symanzik β function.

On the other hand, there are now models, based upon non-Abelian gauge field theories (hopefully renormalizable) that unify all interactions -- weak, electromagnetic, and strong. These grand unifications force relations among the various coupling constants, which seems to pose a major problem as, at low energies, the electromagnetic coupling constant is very different from the strong one. The problem is, however, only apparent: the relations among coupling constants only hold at very high energies, when the breaking of the grand symmetry becomes negligible and, therefore, do not bear directly on low-energy quantities. The thing to do, then, is to take the ultra-short distance coupling constants given by grand unifications, and deduce the effective coupling constants at reasonable energies using the techniques of the renormalization group.

This has been done, in the context of a very specific model, by Georgi, Quinn and Weinberg (1974); in the present paper we extend their methods for a more systematic study of different unified theories. The energy scale that determines when the relations following from unifications should hold is given by the masses of the heaviest particles in the theory (usually vector bosons). So we will, in particular, be able to give estimates on such masses: estimates that vary enormously from one model to the other. For example, in the case studied by Georgi, Quinn and Weinberg, the masses were estimated to be of 10^{11} to 10^{17} GeV. It will be seen that this was due to the brutal way of breaking the grand symmetry, $SU(5) \rightarrow SU(3) \text{ (colour)} \times SU(2) \times U_1$, and that if the symmetry is broken in several steps we can do with more reasonable masses.

In the calculation of Georgi, Quinn and Weinberg or, indeed, in almost all existing work, only the lowest order value for the Callan-Symanzik β function was used. However, now we have available the first two terms for β . In non-Abelian gauge theories the signs of these two terms are not fixed but depend upon the fermion content of the theory. In particular, it is in some cases possible to obtain zeros of β . As we will show here, these points (referred to as fixed points of the renormalization equations) are infrared attraction points; that is to say, no matter from what values (in a certain range) for the coupling constants you start at high-energy, the low-energy coupling constant is necessarily given by the fixed point. This opens up the exciting possibility that we can actually calculate the effective, low-energy values of the strong coupling constant, and of the

fine structure constant, working to lowest non-trivial order in perturbation theory^{*)}. Alternatively, if you require that fixed points exist, and coincide with experimental coupling constants, then this will imply very severe limitations on the gauge groups -- so severe that there are very few models verifying them. Non-Abelian gauge theories start with massless fields, and masses are usually introduced via the so-called Higgs mechanism. In most of this paper we will forget the complications caused by the Higgs fields. However, a curious fact shall be noted: namely, that the limitations imposed if we want to have Higgs fields, and keep asymptotic freedom, lead to models with reasonable fixed points.

Section 1.1 is devoted to presenting the results from the renormalization group techniques relevant for us here, and in Section 1.2 we review briefly the philosophy behind unification schemes. In Sections 2.1, 2.2, and 2.3 we give the β -functions and the trajectories of the coupling constants for a few gauge theories. Sections 3.1 and 3.2 contain examples of models with non-zero fixed points, while a few comments on Higgs fields are made in Section 3.3. The article is finished in Section 4 with a brief recapitulation and comments, in particular on phenomenological implications of the analysis.

1. GENERAL CONSIDERATIONS

1.1 Renormalization group equations and high- and low-energy behaviour of coupling constants

It is known since a very long time (Stückelberg and Petermann, 1953; Gell-Mann and Low, 1954) that one can calculate (under certain assumptions) both the infrared (IR) and ultraviolet (UV) behaviour of renormalizable field theories using techniques of the so-called renormalization group. The idea behind this is actually very simple. Suppose we consider a field theory described by a Lagrangian with some couplings like $g\bar{\psi}\gamma_{\mu}\psi V^{\mu}$..., characterized by a dimensionless constant g . [The case of several couplings is similar. See Crewther (1976) for an up-to-date exposition.] Suppose also that we take all particles to be massless; masses shall be discussed later. If the Green's functions are

$$\Gamma(p_1, \dots, p_n),$$

then it is known that, in order that they be defined, one needs to specify their values at a given point. If the $p_1, \dots, p_n$ are non-exceptional (i.e. essentially linearly independent) Euclidean momenta, we may write $p_1 = q\hat{p}_1, \dots, p_n = q\hat{p}_n$, where the $\hat{p}_i$ are dimensionless (and therefore q has dimensions of energy) and, for example, restricted to lie on the surface of the Euclidean sphere in $4n$ dimensions. Fixing $\hat{p}_1, \dots, \hat{p}_n$, which we denote collectively by $\vec{u}$, we can specify the value of Γ at $q = \mu$ fixed. Then, for arbitrary $p_1, \dots, p_n$ we may write Γ as

*) This possibility was first suggested for non-Abelian gauge theories by Gross and Wilczek (1973).

$$\Gamma(q; g(\mu); \mu; \vec{u}),$$

where the dependence of Γ and of the coupling constant on μ is explicitly written. We may consider changing the normalization point, $\mu \rightarrow \mu'$; renormalization theory then tells us that we can re-scale the fields, in particular altering the coupling constants,

$$g(\mu) \rightarrow [Z_g(\mu')/Z_g(\mu)] g(\mu), \dots$$

in such a way that Γ does not vary. This set of transformations constitute the renormalization group. Now, dimensional considerations show that it is always possible to re-absorb the change in μ , $\mu \rightarrow \mu'$, by re-scaling q , $q \rightarrow q'$: it is then to be expected that information on how Γ behaves as we let q vary could be obtained from knowledge of the variation of the renormalization constants Z . This is indeed the case under (reasonable?) assumptions about the nature of the perturbative expansions. The net result is that the behaviour of Γ is controlled by a new coupling constant, $g(q)$, whose dependence on q may be deduced from knowledge of, for example, the Callan (1970)-Symanzik (1970) β -function: which, in turn, can be calculated perturbatively. To be precise, $g(q)$ is related to β through a differential equation with a boundary condition:

$$\frac{\partial}{\partial \log(q/\mu)} g(q; g(\mu)) = \beta[g(q; g(\mu))], \quad g(\mu; g(\mu)) = g(\mu). \quad (1.1)$$

Any discussion about how β controls the behaviour of coupling constants would be impossible without some hypotheses which, by their very nature, cannot be checked using perturbation theory alone. In particular, one has to suppose that $\beta(x)$ is continuous, does not possess an infinite number of zeros as $x \rightarrow 0$, and can be approximated by its perturbative expansion, at least when x is small enough^{*)}. In this paper we will assume that β may be approximated by the first two terms in its expansion, at least when (and this will always be the case in all practical applications here) the estimated contribution of the third term is small. Borrowing the notation conventions from Caswell (1974) (apart for a change in the sign of b_0), we then write

$$\beta(x) \simeq x \left\{ \frac{x^2}{16\pi^2} b_0 + \left(\frac{x^2}{16\pi^2} \right)^2 b_1 \right\}. \quad (1.2)$$

*) The counter examples $\beta(x) = x^3 \sin(x^{-2}) \phi(x) + x^3 f(x^2)$, any f, ϕ entire, show that, even if β is analytic, such assumptions are indeed assumptions. From a rigorous standpoint we know since recently that the Schwinger functions for massive ϕ^4 in two dimensions are approximated asymptotically by their perturbative expansions, so pathologies like that above cannot occur for them. This result certainly goes in the right direction, but is still way off of what we would need.

With the help of this, it is easy to solve (1.1) obtaining the implicit expression

$$\log(q/\mu) = \frac{2\pi}{b_0} \left\{ \alpha_g^{-1}(\mu) - \alpha_g^{-1}(q) + \frac{b_1}{4\pi b_0} \log \frac{4\pi b_0 \alpha_g^{-1}(q) + b_1}{4\pi b_0 \alpha_g^{-1}(\mu) + b_1} \right\}, \quad (1.3)$$

$$\alpha_g(q) \equiv g^2(q; g(\mu))/4\pi.$$

Several possibilities are now open, depending on the signs of b_0 , b_1 . Skipping the trivial cases when b_0 and/or b_1 vanish, which the reader can work by himself, we may have:

Class I: $b_0 > 0$, $b_1 > 0$, any $\alpha_g(\mu)$. This is the case in ordinary quantum electrodynamics (QED) [Landau et al., (1954); Jost and Luttinger (1950). See also de Rafael and Rosner, 1974]. Then, as $q \rightarrow 0$, $\alpha_g(q)$ goes to zero logarithmically (IR stable) and, as q tends to the critical energy q_{UV} ,

$$\log(q_{UV}/\mu) = \frac{2\pi}{b_0} \left\{ \alpha_g^{-1}(\mu) - \frac{b_1}{4\pi b_0} \log \left(1 + \frac{4\pi b_0}{b_1} \alpha_g^{-1}(\mu) \right) \right\}, \quad (1.4)$$

$\alpha_g(q)$ tends for infinity (UV unstable). In fact, the solution provided by (1.3) breaks down near critical energies as the effective coupling constant has become so large that expression (1.2) for β can no longer be considered to be valid. This is indeed a general property of critical energies [see, for example, Bogolyubov and Shirkov (1960), Crewther (1976) for discussions on this].

Class II: $b_0 > 0$, $b_1 < 0$. β then possesses a non-trivial zero (β always vanishes at $g = 0$) at

$$\hat{\alpha}_g = 4\pi \left| \frac{b_0}{b_1} \right|. \quad (1.5)$$

There are two possibilities:

- a) $\alpha_g(\mu) < \hat{\alpha}_g$. Then, as $q \rightarrow 0$, $\alpha_g(q)$ vanishes logarithmically (IR stable) and, as $q \rightarrow \infty$, $\alpha_g(q) \rightarrow \hat{\alpha}_g$ (UV fixed point). This is the situation hoped for (but with β to all orders!) in the Baker-Johnson (1964, 1967, 1969, 1971) - Adler (1971) program as this would mean in, for example, the particular case of QED, that one could calculate the fine structure constant α at high energies [it need not coincide with the low-energy constant, $\alpha \approx 1/137$].
- b) $\alpha(\mu) > \hat{\alpha}_g$. As $q \rightarrow \infty$, $\alpha_g(q)$ tends to $\hat{\alpha}_g$ (UV fixed point) but, as q tends to the critical energy q_{IR} ,

$$\log(q_{IR}/\mu) = \frac{2\pi}{b_0} \left\{ \alpha_g^{-1}(\mu) + \frac{|b_1|}{4\pi b_0} \log \left(1 - \frac{4\pi b_0}{|b_1|} \alpha_g^{-1}(\mu) \right) \right\}, \quad (1.6)$$

$\alpha_g(q)$ diverges logarithmically (IR unstable). This phenomenon has been suggested as a basis for quark confinement [Gross and Wilczek (1973; 1974)] in colour-gluon theories [Bardeen, Fritzsche and Gell-Mann (1973)].

Class III: $b_0 < 0$, $b_1 \leq 0$, any $\alpha_g(\mu)$. This occurs in non-Abelian gauge theories without too many fermions or scalars [Gross and Wilczek (1973); Politzer (1973), Caswell (1974)]. Now, as $q \rightarrow \infty$, $\alpha_g(q)$ tends to zero logarithmically (UV stable): The theory enjoys the celebrated property of asymptotic freedom, i.e. up to logarithmic corrections it behaves like free field theory. As $q \rightarrow q_{IR}$ with

$$\log(q_{IR}/\mu) = \frac{2\pi}{|b_0|} \left\{ -\alpha_g^{-1}(\mu) + \frac{|b_1|}{4\pi|b_0|} \log \left(1 + \frac{4\pi|b_0|}{|b_1|} \alpha_g^{-1}(\mu) \right) \right\}, \quad (1.7)$$

$\alpha_g(q)$ diverges (IR unstable) so we here again have confinement.

Class IV: $b_0 < 0$, $b_1 > 0$. A non-trivial zero of β exists at $\hat{\alpha}_g$ given by (1.5). This occurs for some gauge theories with specific fermion content.

- a) $\alpha_g(\mu) < \hat{\alpha}_g$. As $q \rightarrow \infty$, $\alpha_g(q)$ vanishes logarithmically (UV stable) so the theory is asymptotically free while, for $q \rightarrow 0$, $\alpha_g(q) \rightarrow \hat{\alpha}_g$ (IR fixed point).
- b) $\alpha_g(\mu) > \hat{\alpha}_g$. When $q \rightarrow 0$, $\alpha_g(q) \rightarrow \hat{\alpha}_g$ (IR fixed point); when $q \rightarrow q_{UV}$,

$$\log(q_{UV}/\mu) = \frac{2\pi}{|b_0|} \left\{ \alpha_g^{-1}(\mu) + \frac{|b_1|}{4\pi|b_0|} \log \left(1 - \frac{4\pi|b_0|}{b_1} \alpha_g^{-1}(\mu) \right) \right\}, \quad (1.8)$$

$\alpha_g(q)$ diverges (UV unstable).

In Fig. 1 we have plotted the different β against α_g . Of course all these considerations are well known, but we have collected them here for our particular approximation to β , Eq. (1.2), for ease of reference when applying them to unified gauge theories, to which we now turn.

1.2 Unified gauge theories

Gauge theories of weak and electromagnetic (WEM, henceforth) interactions have been constructed as a means of unifying electromagnetic and weak interactions and of making the last renormalizable [Weinberg (1967); Salam (1968); 't Hooft (1971)]. It is also possible to unify strong and electromagnetic (SEM, henceforth) interactions either to try and calculate electromagnetic mass differences [Georgi and Goldman (1973), Ynduráin, unpublished preprint (1973)] or -- and this is the point of view we shall adopt here -- to incorporate leptons into the colour scheme and achieve eventual unification of all interactions [Pati and Salam (1973); Georgi and Glashow (1974); Fritzsche, Gell-Mann and Minkowski (1975); Fritzsche and Minkowski (1975); Mohapatra (1976); Gürsey and Sikivie (1976); Ynduráin (1976); Fritzsche and Minkowski (1976), etc.]

We shall refer to these schemes as GU (grand unified) schemes. In general, all these unifications are broken by mass terms at different levels, giving a hierarchy of symmetries [see Fritzsch and Minkowski (1975) and Georgi, Quinn and Weinberg (1974) for discussion of the underlying philosophy]. A typical situation is as follows:

At energies that are high compared with the masses of the intervening particles, the full symmetry holds. Thus, when

$$q \gg \text{all masses}, \quad (1.9a)$$

we have GU symmetry with a certain group,

$$G_{GU} \quad (1.9b)$$

In some models [Georgi and Glashow (1974); Gürsey and Sikivie (1976)] such energies are truly fantastic, $q \sim 10^{12}$ to 10^{24} GeV, but in other cases "reasonable" energies (10^4 to 10^8 GeV) are sufficient. Typically, the vector bosons^{*)} in the adjoint representation of G_{GU} fall into three sets: a set, to be denoted by W_i^{μ} that mediates the weak and (part of) the electromagnetic interactions, generating a WEM group G_{WEM} ; a set, to be denoted V_{μ}^a mediating strong interactions (gluons), (part of) the electromagnetic interactions and extra interactions necessary to unify them within a group G_{SEM} ; and, finally, the rest, denoted by U. Usually, we have

$$M(W), M(V) \ll M(U),$$

giving a transition region

$$M(W), M(V) \ll q \ll M(U). \quad (1.10a)$$

Here the U decouple and the effective invariance group has degenerated [Symanzik (1973); see also Appelquist and Carrazzone (1975)] into two independent groups,

$$G_{SEM} \times G_{WEM}. \quad (1.10b)$$

Finally, when in the energy region

$$q \ll M(W_h), M(V_h) \quad (1.11a)$$

where W_h, V_h are the massive among the W, V fields, the only vector bosons that contribute effectively are the photon that generates

$$G_{QED} = U(1), \quad (1.11b)$$

*) We consider, in this discussion, breakings through vector boson masses for definiteness.

and the gluons, generating

$$G_C = SU(3) \quad (1.11c)$$

(C for colour).

We see that the effective gauge group is energy dependent, and we therefore expect different β -functions in different energy ranges, leading to different behaviours with energy for the effective coupling constants. This has been argued by Georgi, Quinn and Weinberg (1974) and by Gürsey and Sikivie (1976) to provide an "explanation" for the existence of super heavy bosons with masses $\sim 10^{11}$ to 10^{23} GeV, but, as discussed in the Introduction, this is merely a consequence of the very specific model (and breaking) used by these authors. What happens is that in the energy region (1.9) there is only one coupling constant so the fine structure constant α_{QED} and the gluon coupling constant α_C are simply related through Clebsch-Gordon coefficients. In the models quoted, this Clebsch-Gordan is close to unity; furthermore, the symmetry was supposed to be $SU_C(3) \times [SU(2) \times U(1)]_{\text{WEM}}$ all the way from low energies to region (1.9) when it suddenly became $SU(5)_{\text{GU}}$ [for the Georgi and Glashow (1974) model]. The variation with energy of both α_{QED} and α_C are then so slow that enormous energies are needed to bridge the gap between $1/137$ and $\sim 1/3$. This is not the case in other models, as the energy variation is much sharper.

We have been talking about symmetry breaking in gauge theories: this brings immediately to mind Higgs fields. We shall, however, forget entirely about Higgs fields for the time being, postponing a discussion of their effects until Section 3.3.

2. β -FUNCTIONS AND COUPLING CONSTANTS FOR UNIFIED GAUGE THEORIES

In this section we give the coefficients b_0 and b_1 of the function β for a cross-section of gauge theories, including for convenience also QED. This permits us to determine the energy dependence of the corresponding coupling constants in the approximation (1.2). It is noteworthy that, by coupling the results of Coleman and Gross (1973) and of Caswell (1974) we can easily check that Situation II may never occur. This means that there is little hope of getting reasonable UV fixed points in perturbation theory and, therefore, within our approximation, all theories are either asymptotically free or UV unstable.

2.1 The β -function in QED

We have [Landau, Abrikosov and Khalatnikov (1954), Jost and Luttinger (1950), Bogolyubov and Shirkov (1960)]

$$b_0 = \frac{2}{3} \sum_{i=1}^n Q_i^2, \quad b_1 = 2 \sum_{i=1}^n Q_i^4, \quad (2.1)$$

if we have n fermions with charges Q_i (measured in units of the proton charge e). In this case, the following term in the development of β has also been calculated by de Rafael and Rosner (1974), but we shall not use it here, except to say that it causes no sizeable alteration in our results.

2.2 The β -function for non-Abelian gauge theories

Let us write the fermion vector boson piece of the interaction Lagrangian as

$$g \sum_f \sum_{ika} M_{ik}^a(R_f) \bar{\psi}_i^f \gamma_\mu \psi_k^f B_a^\mu, \quad (2.2)$$

where the fermions f are in the representation R_f of the group. Then [Gross and Wilczek (1973); Politzer (1973); Caswell (1974)]

$$b_0 = \frac{1}{3} \left\{ -11 C_2(A) + 4 \sum_f T(R_f) \right\}, \quad (2.3a)$$

$$b_1 = \frac{1}{3} \left\{ -34 [C_2(A)]^2 + 20 C_2(A) \sum_f T(R_f) + 12r \sum_f [T(R_f)]^2 / d(R_f) \right\}, \quad (2.3b)$$

where C_2 is the Casimir operator, A the adjoint representation of the group, $d(R_f)$ the dimension of representation R_f , r the number of generators of the group and, finally,

$$\sum_{ik} M_{ik}^a(R_f) M_{ki}^b(R_f) = \delta_{ab} T(R_f). \quad (2.3c)$$

If the group is $SU(N)$ and all fermions belong to either vector (fundamental; dimension N) (n_V of these) or adjoint representations (n_A of these), then (2.3) can be written very explicitly as

$$b_0 = \frac{1}{3} \left\{ (-11 + 4n_A)N + 2n_V \right\},$$

$$b_1 = \frac{N}{3} \left\{ (-34 + 32n_A)N + 13n_V - 3n_V/N^2 \right\}. \quad (2.4)$$

The values of b_0 , b_1 for some specific theories may be found in Tables 1, 2, and 3.

2.3 Models and trajectories

It is of course impossible, when considering explicit theories, to take into account all models that have been proposed. We shall restrict ourselves to theories

based upon unitary groups and of the vector type; that is to say, left-handed and right-handed fermions occur in the same representations (the individual fermions may appear permuted). Even within these limitations, only a small cross-section of models shall be discussed: we hope that the general formulas previously given will allow the reader to draw the pertinent conclusions for his favourite model in the case that it did not fall into the categories explicitly discussed.

In this section we will also restrict ourselves to "minimal" models, by which we mean models where as few new particles as possible are introduced. This somewhat obscure definition should become clearer in Section 3 when we consider non-minimal theories.

Let us start by considering WEM models. A WEM model will be denoted by

$$WEM \{SU(N); n_V; n_A; \text{Yes/No}\} \quad (2.5)$$

if it is based on $SU(N)$, contains n_V vector multiplets (each in both left- and right-handed versions), n_A adjoint multiplets (*idem*), and the generator of electric charge is/is not fully contained into the group. For $N = 2$: No, we have the classical de Rújula, Georgi and Glashow (1975), Fritzsche, Gell-Mann and Minkowski (1975), Pakvasa, Simmons and Tuan (1975), etc. models. For $N = 3$: No, we have the Weinberg (1972) model (slightly modified). For $N = 3$: Yes, we have the Fritzsche and Minkowski (1976) model, etc. For $N = 4$: No, we have the models discussed in Fritzsche and Minkowski (1975), Mohapatra (1976), Ynduráin (1976), etc. The author is not aware of the existence of WEM models with N larger than four.

SEM models shall be denoted

$$SEM \{SU(N); n_V\}, \quad (2.6)$$

because the only models that shall be considered are pure $SU(3)$ colour or $SU(4)$ colour-plus-lepton number, and in these the fermions always come in in vector representations. $N = 3$ corresponds to the Fritzsche-Minkowski (1976) model, while the rest of the above-quoted authors employ $N = 4$.

If $\mathcal{G}_{WEM} \times \mathcal{G}_{SEM}$ are part of a grand group, $\mathcal{G}_{GU}$, you get asymptotic relations among the five coupling constants

$$\alpha_{WEM}, \alpha_{SEM}, \alpha_c, \alpha_{QED}, \alpha_{GU};$$

$$\alpha_x \equiv g_x^2/4\pi;$$

indeed, you have

$$\begin{aligned} \alpha_{WEM} &= C_{WEM}^2 \alpha_{GU}, \quad \alpha_{SEM} = C_{SEM}^2 \alpha_{GU}, \\ \alpha_C &= C_C^2 \alpha_{GU}, \quad \alpha_{QED} = C_{QED}^2 \alpha_{GU}, \quad q \gg M(U), \end{aligned} \quad (2.7)$$

where the C_x are Clebsch-Gordan (and hence group-dependent) coefficients. The information we have is that, at low energies ($q \sim 3$ GeV),

$$\alpha_{QED}^{-1}(q) \simeq 137, \quad \alpha_C^{-1}(q) \simeq 3 \text{ to } 10. \quad (2.8)$$

Therefore, as relations (2.7) have to hold for $q \gg M(U)$, we can, by following up the energy trajectories of α_{WEM} , α_{SEM} , find out how large q has to be to have (2.7) satisfied, starting from (2.8). This, in turn, gives an indication of how heavy we have to take the U that complete the grand unification. The results are summarized in Fig. 2 for "minimal" models. If one takes the criterion for grand unification that relations (2.7) be satisfied to

$$O(\alpha_{GU}^2),$$

then we can read off from Fig. 2 the values of q , and hence the masses of the U vector bosons, for each particular theory. The result is

$$\begin{aligned} 1: WEM \{SU(2); 12; 0; No\} \times SEM \{SU(3); 6\} : M(U) &\sim 10^6 \text{ GeV} \\ 2: WEM \{SU(3); 6; 1; Yes\} \times SEM \{SU(3); 6\} : M(U) &\sim 10^{12} \text{ GeV} \\ 2': WEM \{SU(3); 6; 1; Yes\} \times [SEM \{SU(3); 6\} \\ &\quad \subset SEM \{SU(6); 3\}] : M(U) \sim 10^4 \text{ GeV} \\ 3: WEM \{SU(4); 8; 0; No\} \times SEM \{SU(4); 8\} : M(U) &\sim 10^7 \text{ GeV} \end{aligned} \quad (2.9)$$

Model 2' corresponds to $SU(3)$ colour up to $q \sim 10^2$ GeV, $SU(6)$ colour and flavour thereafter. In model 3, the full $SU(4)$ symmetry holds above $\sim 10^2$ GeV, below only $SU(3)$ colour remains. It will be seen from the difference between 2 and 2' in (2.9) that unification is enormously accelerated if it is performed in two stages; that is to say, "strong" interactions become invariant under a larger group before unifying with "weak" interactions. This is true in general; the reason is obvious, as the coefficient in the logarithmic decrease of the "gluon" coupling constant is essentially the rank of the relevant group. Also, vector-like character of the WEM theory accelerates the unification. This is due to the fact that the Clebsch-Gordan coefficients that enter (2.7) are more favourable than in Weinberg (1967)-Salam (1968)-Glashow-Iliopoulos-Maiani (1970) theories.

We will close this section with three remarks. First, in the calculations here we have taken the masses of all quarks to be ~ 1 to 5 GeV, and this effect on the behaviour of α_g has been neglected. Similarly, the transition from one group regime to another has been made brusquely. We could easily take masses

[i.e. terms $O(M^2/q^2)$] into account, which would not change results in any significant way except by smoothing the junctions. This shall be done in the next section.

Secondly, if a sub-theory is Class IV or III, then a theory obtained from it by enlarging the group is also Class IV or III. The reason for this is clear for Eq. (2.3) as, on the one hand, increasing the rank of the group increases $C_2(A)$, and on the other hand, the number of fermion multiplets can only diminish (or stay constant) and hence $\Sigma T(R_f)$ never increases.

Finally, we have checked that, in all models discussed, the estimated relative contribution of next term in the expansion of β , $O(C_2(A)\alpha_g)$, is indeed much smaller than unity, thereby justifying our approximation for β , Eq. (1.2), with only two terms taken into account.

3. THEORIES WITH NON-TRIVIAL FIXED POINTS AND "CALCULABLE" COUPLING CONSTANTS. HIGGS FIELDS

3.1 General discussion

Of all the models discussed so far ("minimal" models), there is only one that possesses non-trivial fixed points: the Fritzsche-Minkowski (1976) model of WEM interactions,

$$WEM \{SU(3); 6; 1; Yes\}.$$

All other models are either Class I or III. However, the F-M model's fixed point is widely off the mark as for it $[\hat{\alpha}_{WEM}]^{-1} = 2.48$ and we would rather have $[\hat{\alpha}_{WEM}]^{-1} \sim \alpha_{QED}^{-1} \approx 137$.

It is easy to see that fixed points for SEM $\{SU(4); n_V\}$ models only occur when we have at least 9 quadruplets: and if we require the fixed point to be close to the experimental gluon coupling constant, then we need something near (or equal) to 16 quadruplets. On the other hand, the only possibility to have fixed points for WEM interactions of the order of α_{QED} is easily checked to occur only^{*)} if the group is $SU(3)$ and we have 16 triplets. Furthermore, it is necessary that either the WEM group be smaller than the SEM one, or that there be more WEM multiplets than SEM ones. This does not totally pin down a particular model, but makes a pattern emerge.

Now, quarks in sixteen flavours and leptons to match may look like an excessive number of elementary particles. This is also the author's opinion; but then, schemes that are now considered conservative use a minimum of 24 fermions,

*) We are implicitly assuming schemes of the types described in the previous section.

not to speak of the hundreds of vector bosons necessary to unify everything. If SPEAR showed that Frascati did not exhaust all physics, there is no reason to suppose that PETRA is not going to excite anything new (whether fermions or physicists). Indeed, perhaps the logical point of view, if the whole scheme makes sense, could be that there is no limit to the variety of flavours that will be encountered as we move closer and closer to the light cone.

As a matter of fact, we are not going to go as far as all that here. We will just assume that PETRA will be about as exciting as ADONE, and forget for the time being the possibility of higher excitations. The precise number of flavours, leptons, etc., to be found is of course unknown, as are the underlying group structures. Therefore, we will merely illustrate our point by considering two particular examples: no doubt industrious readers will be able to think up a variety of other possibilities.

3.2 Trajectories with fixed points in two typical models

A) We will take a total of 64 elementary fermions. Of these, 32 will be super heavy, with masses in the range 15 to 20 GeV; the remaining 32 shall be relatively light, weighing up to some 5 GeV. At very low energies ($q \ll 1$ GeV), the only remaining gauge fields will be the photon and the gluons and therefore (with the exception of the electron) no fermions will contribute. Hence, $\alpha_{\text{QED}}(q)$ stays practically constant near $1/137$, while $\alpha_{\text{colour}}(q)$ decreases sharply (see Table 3, entry {SU(3); 0}) to $\alpha_{\text{colour}}(1 \text{ GeV}) = 3/8 \times g_{\text{colour}}^2(1 \text{ GeV})/4\pi$, $g_{\text{colour}}^2/4\pi \sim 0.5$ ($3/8$ is the Clebsch-Gordan coefficient relevant to our symmetry). Between $q \rightarrow 1$ GeV and $q \sim 50$ GeV, the 32 "light" fermions are excited; $\alpha_{\text{QED}}(q)$ moves according to a β with coefficients given by Table 1, entry {4,8}; $\alpha_{\text{colour}}(q)$ moves according to entry {SU(3); 8} of Table 3. For $q > 500$ GeV, the W, Z, V, ... fully enter the picture, and the invariance group is enlarged on the one hand to $\mathcal{G}_{\text{WEM}}$, which we take to be SU(3), and on the other to $\mathcal{G}_{\text{SEM}}$, which we take to be SU(4)' (colour-plus-lepton number). But also the number of fermions has increased, as now all 64 light and super heavy fermions are excited. They are classified according to sixteen SU(4)_{SEM} quadruplets, sixteen SU(3)_{WEM} triplets and sixteen SU(3)_{WEM} singlets. $\alpha_{\text{SEM}}(q)$ moves with a β_{SEM} given by entry {SU(4); 16} of Table 3, $\alpha_{\text{WEM}}(q)$ with a β_{WEM} given by entry {SU(3); 16; 0; No} of Table 2. At higher energies, $q \sim 10^8$ GeV, the SEM group stays unchanged, but a new set of W_H are excited that extend the WEM group to SU(4), and the fermions lie in 16 quadruplets of it. $\alpha_{\text{WEM}}(q)$ is now governed by a new β_{WEM}^H , given by entry {SU(4); 16; 0; No} of Table 2. At this stage we have the choice of achieving grand unification of the existing symmetry SU(4)_{WEM} $\times$ SU(4)_{SEM} [Mohapatra (1976); Ynduráin (1976)] by just adding the discrete transformation WEM $\leftrightarrow$ SEM, or introducing extra vector bosons closing a larger group: SU(16)_{GU} or even SU(64)_{GU}. (More details about this model may be found in the Appendix.)

According to the discussion in Section 1, we have to start from $q \sim 10^9$ GeV, where all Clebsch-Gordan relations among different coupling constants [Eqs. (2.7)] hold. There we take $\alpha_{GU}^{-1}(q) \sim 40$.

This corresponds to $\alpha_C^{-1}(q) \sim 15$, $\alpha_{QED}^{-1}(q) \sim 80$. As we diminish q , the group decomposes into $SU(3)_{WEM} \times SU(4)_{SEM}$. For the first group, the β_{WEM} function has a zero at $\hat{\alpha}_{WEM}(q \rightarrow 0)$ corresponding to $\hat{\alpha}_{QED}^{-1}(q \rightarrow 0) \approx 90$. For the second group, the zero of β_{SEM} at $\hat{\alpha}_{SEM}(q \rightarrow 0)$ corresponds to $\hat{\alpha}_C^{-1} \approx 5$. Now, as we decrease q we are in Class IVb for the WEM sector; $\alpha_{WEM}(q)$ is attracted by the fixed point and decreases: while the SEM sector is in Class IVa, $\alpha_{SEM}(q)$ is attracted by the fixed point and hence increases (see Figs. 3 and 4). The bifurcation of α_{GU} into α_{WEM} and α_{SEM} is then qualitatively understood as the effect of the existence of two different fixed points, both IR attractive, and the fact that α_{GU} lies between them. At lower energies (q between 20 and 10^3 GeV) the heavy V of $SU(4)_{SEM}$ decouple, leaving only $SU(3)_C$ and the gluons, the $W, Z, \dots$ of $SU(3)_{WEM}$ also decouple, leaving $U(1)$ of QED, and the superheavy fermions also decouple, leaving the SPEAR leptons and quarks. Finally, at very low q , all quarks (except perhaps u, d ones) decouple so that only gluons remain in the colour sector until we eventually reach the IR unstable point, $q_{IR} \approx 0.4$ GeV (Table 3). Higgs fields in the SEM sector do not alter significantly $\hat{\alpha}_{SEM}$; one Higgs triplet in the WEM sector changes $\hat{\alpha}_{WEM}$ to give $\hat{\alpha}_{QED} = 1/180$, and two or more Higgs triplets destroy the fixed point. See Section 3.3 below for more comments on Higgs fields.

B) Our second example will be the Fritzsch-Minkowski (1976) model, with one extra quark triplet in three colours (i.e. a total of nine quark flavours). For this model,

$$b_0^{WEM} = -1, b_1^{WEM} = 108, b_0^{SEM} = -5, b_1^{SEM} = 12.$$

This gives fixed points, corresponding to

$$\hat{\alpha}_{QED}^{-1} \approx 8, \hat{\alpha}_C^{-1} \approx 0.2.$$

The situation here appears less favourable than in the previous case. However, we may improve it by taking Higgs fields into account. With two Higgs triplets and one Higgs octet (certainly necessary in the original model), the value of $\hat{\alpha}_C$ is almost unchanged, but now

$$\hat{\alpha}_{QED}^{-1} \approx 60.$$

More Higgs fields would destroy the WEM fixed point. The considerations in this case are essentially similar to those made for the previous one; neither these nor a graph will be presented.

To conclude this section, we may say that the error committed by using (1.2) for, for example, calculating the fixed points is small. For instance, in the WEM sector, inclusion of a correction of

$$\hat{g} \left(\frac{\hat{\alpha}_{\text{WEM}}}{\pi} \right)^3 \text{Max} \{ C_2(A), \sum_f T(R_f) \}^3,$$

moves $\hat{\alpha}_{\text{WEM}}$ by some 20%. This is of the order of the difference between the values obtained for $\hat{\alpha}_{\text{QED}}$ and $\alpha_{\text{QED}}(q \approx 0) \approx 1/137.06 \dots$.

3.3 Higgs fields

This is as good a moment as any to insert the long-postponed discussion about Higgs fields.

Higgs fields are a mess as far as renormalization group arguments are concerned. Any conclusion we can draw from a model containing only fermions and vector bosons is almost certainly upset by the Higgs fields necessary to give masses to all particles that require a mass: but then, you can always postpone such problems by making the Higgs particles sufficiently massive. For our purposes here, there are three points of view with respect to Higgs fields:

- i) Higgs fields are just a phenomenological device, used because we do not know how to introduce masses dynamically. Therefore, we should work as if they did not exist, as would be the case in the true theory. For a discussion of this point of view, together with references, see Gross and Wilczek (1973), Crewther (1976). This has been our attitude through most of the present paper.
- ii) Only the Higgs fields contributions to the equations for the gauge coupling constant should be considered, and we do not concern ourselves with the UV unstability of the quartic Higgs couplings. As Higgs fields contribute very little to b_0 , b_1 , we can easily keep the conclusions of our article even if adopting this point of view -- with two outstanding exceptions: for model B of Section 3.2 Higgs fields were necessary to get a reasonable $\hat{\alpha}_{\text{WEM}}$ fixed point; for model A there, more than one Higgs triplet would spoil the fixed point.
- iii) One may require asymptotic freedom for everybody (including Higgs quartic couplings). This may be used to limit the number of fermions. For example, in a model based upon $SU(3)_{\text{WEM}}$, $SU(4)_{\text{SEM}}$, we need sixteen fermion multiplets to be able to introduce one Higgs multiplet^{*)}. We do not take this very seriously, but it certainly is an indication that large numbers of fermion

*) This is not sufficient to give mechanical masses to everybody, but it is known that at least some masses can be calculated dynamically [Georgi and Glashow (1972)].

multiplets may be necessary: and it is at least an amusing coincidence that the number required by asymptotic freedom is exactly what we needed in Section 3.2.A to obtain IR fixed points in agreement with experiment.

4. CONCLUSIONS AND COMMENTS

In this article we have explored systematically the energy behaviour of coupling constants for renormalizable gauge theories. In the realm of general results, the most important conclusions are, perhaps, that one should realize that the invariance group is energy dependent, and that the coefficients of the β -function used to compute the trajectories of the coupling constants will, even if the group stays fixed, vary again with energy due to variation of the number of effective fermion multiplets which intervene at each energy. Therefore, any conclusion that may be drawn from low-energy groups and fermion content will almost certainly get destroyed by what one may encounter at higher energies. This was made transparent by showing how the energies required to reach grand unification depend upon the detailed structure of the breaking mechanism. As a striking example, see the two possible breaking schemes of the same grand symmetry [Eq. (2.9) and Fig. 2, models 2 and 2']. Roughly speaking, this result can be summarized as follows: assume that $\alpha_{\text{QED}}(q)$ varies little, so we can approximate

$$\alpha_{\text{QED}}^{-1}(q) \simeq \alpha^{-1} = 137.06... \quad (4.1)$$

If, at energy $q = \mu$ the effective SEM group is $S[N(\mu)]$, and we have excited $n_V(\mu)$ vector multiplets^{*)} of fermions, then for our discussion it follows that

$$\alpha_{\text{SEM}}^{-1}(q) \simeq \frac{11N(\mu) - 2n_V(\mu)}{6\pi} \log(q/\mu), \quad q \gg \mu. \quad (4.2)$$

Eqs. (1.3) and (2.4) have been used. As explained before, $N(\mu)$ can only increase, and $n_V(\mu)$ decrease, as μ gets large. Therefore, using (2.7), we see that we will have unification at least for q_{GU} given by

$$q_{\text{GU}}/\mu \simeq \exp \left\{ \frac{6\pi}{[11N(\mu) - 2n_V(\mu)]\alpha} \cdot \left(\frac{C_{\text{QED}}}{C_{\text{SEM}}} \right)^2 \right\}. \quad (4.3)$$

If, for example, at $\mu = 10^2$ GeV the fermion contribution was negligible and we take a vector model of the "no" type, then (4.3) gives the amazing formula [note that we can take $q_{\text{GU}} \sim 10 M(U)$]

*) It is trivial to extend the result to the case in which other representations occurred for the fermions, by using Eqs. (2.3) instead of Eq. (2.4).

$$M(U) \approx 10^{36/N_{eff} + 1} \text{ GeV}, \quad N_{eff} = N(\mu) - \frac{2}{11} n_v(\mu), \quad (4.4)$$

for the mass of the vector bosons necessary to complete the grand unification.

Particularly appealing are theories with IR fixed points. It should again be stressed that there is, at present, no compelling reason why a theory should possess IR fixed points, nor why a theory could not be UV unstable^{*)}. Nevertheless, if we require both IR fixed points coinciding with experimental coupling constants, we are led to a very narrow class of models, and it is an amusing coincidence (with or without deeper meaning it would be hard to say in the present state of the art) that the limitations obtained are essentially those that enable us to introduce Higgs fields at all, without spoiling asymptotic freedom.

Finally, we would like to finish this paper by devoting a few words to phenomenological implications of our analysis. It is clear that any conclusions we may draw will be strongly model dependent. However, we think that, if the whole scheme makes sense, we have three possible general patterns. Firstly, it may be that nothing new emerges at higher energies. The gluon coupling constant would then continue to decrease at the present rate; no new flavours would be excited. This would be very dull, but it is certainly possible. Secondly, the group may grow faster than the number of excited multiplets. This would mean that the corrections for scaling would become utterly negligible at, say, PETRA energies. The Zweig rule would be enormously enhanced, and, since $\alpha_C \sim \alpha_{QED}$, it would become impossible to differentiate between strong and electromagnetic interactions: the widths of "strong" resonances and of "weak" particles would be the same. Finally, we may have something like the models discussed in Section 3.2. The appearance of new flavours would go on for some time yet (perhaps forever) but would be compensated (partially) by increase in the rank of the groups. Corrections to scaling would then fluctuate.

*) With respect to UV unstability, however, see the arguments of Landau and collaborators (1954), (1955), but also their criticism by Bogolyubov and Shirkov (1960), Crewther (1976), etc.

APPENDIX: DESCRIPTION OF THE MODEL OF SECTION 3.2

The model is a variant of the Ynduráin (1976) model [see also Mohapatra (1976)]. We consider four matrices, $\psi(n; -)$

$$\begin{pmatrix} \nu_e & \vec{u} \\ E^0 & \vec{c} \\ e^- & \vec{d} \\ E^- & \vec{b} \end{pmatrix}_L, \begin{pmatrix} \nu_\mu & \vec{t} \\ M^0 & \vec{h} \\ \mu^- & \vec{s} \\ M^- & \vec{a} \end{pmatrix}, \begin{pmatrix} N_3^0 & \vec{q}_t \\ N_3'^0 & \vec{q}'_t \\ N_3^- & \vec{q}_b \\ N_3'^- & \vec{q}'_b \end{pmatrix}, \begin{pmatrix} N_4^0 & \vec{p}_t \\ N_4'^0 & \vec{p}'_t \\ N_4^- & \vec{p}_b \\ N_4'^- & \vec{p}'_b \end{pmatrix}, \quad (A.1)$$

$n = 1, 2, 3, 4$, respectively,

where the "vectors" are quarks and for any of them, $\vec{\chi} = \chi^r, \chi^y, \chi^v$, with r (red), y (yellow), v (violet) the colour labels. The right-handed multiplets are obtained from (A.1) by exchanging the first and second, and the third and fourth rows in each matrix $\psi(n; \cdot)$ ^{*}). After reduction to $SU(4)'_{SEM} \times SU(4)_{WEM}$, the fermion-gauge field piece of the Lagrangian is

$$g \sum_{\pm} \sum_{n=1}^4 \left\{ \sum_{a=1}^4 \sum_{ijk} \lambda_{ijk} \bar{\psi}_i^a(n; \pm) \gamma_\mu \psi_j^a(n; \pm) W_k^\mu + \sum_{i=1}^4 \sum_{abc} \lambda_{abc} \bar{\psi}_i^a(n; \pm) \gamma^\mu \psi_i^b(n; \pm) V_\mu^c \right\}. \quad (A.2)$$

The 16 rows of the four matrices (A.1) form therefore 16 quadruplets (left-handed, and the corresponding number of right-handed ones) under $SU(4)'_{SEM}$. If we alter the notation, writing W_{14}^μ for the W that couples to the current

$$\sum_{a,n,\pm} \bar{\psi}_i^a(n; \pm) \gamma_\mu \psi_j^a(n; \pm),$$

and V_μ^{ab} for the V that couples to

$$\sum_{i,n,\pm} \bar{\psi}_i^a(n; \pm) \gamma^\mu \psi_i^b(n; \pm),$$

then $SU(4)_{WEM}$ is broken by giving large masses ($\sim 10^7$ GeV) to the fields W_{14}^μ ; so, the symmetry becomes $SU(3)_{WEM}$, and the ψ break up into 16 left-handed triplets,

$$\begin{pmatrix} \nu_e \\ E^0 \\ e^- \end{pmatrix}_L, \begin{pmatrix} \vec{u} \\ \vec{c} \\ \vec{d} \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ M^0 \\ \mu^- \end{pmatrix}_L, \dots, \begin{pmatrix} \vec{p}_t \\ \vec{p}'_t \\ \vec{p}_b \end{pmatrix};$$

^{*}) We do not take into account possible Cabibbo-like notations. A possible justification is to assume that they will arise dynamically, à la Fritzsch and Minkowski (1976).

the rest, $E_L^-, \vec{b}_L, \dots, N'_{4L}, \vec{p}'_{bL}$ being singlets, and the same for the right-handed multiplets. Similarly, we give large masses ($\sim 10^2$ GeV) to the V_μ^{a1} , so $SU(4)'_{SEM}$ degenerates into $SU(3)'_{colour}$, with 16 triplets (quarks) $\vec{u}, \vec{c}, \vec{d}, \vec{b}, \dots, \vec{p}_t, \vec{p}'_t, \vec{p}_b, \vec{p}'_b$, and 16 singlets (leptons). Finally, the fermions sixteen-plet $\psi_1^a(1)$, $\psi_1^a(2)$ are "light" (masses ≤ 5 GeV), the $\psi_1^a(3)$, $\psi_1^a(4)$ are superheavy (masses ~ 10 GeV). If we identify u, d, s with the low-mass quarks, then it is easy to see that the scheme is consistent with all low-energy phenomenology [Ynduráin (1976); Mohapatra (1976)].

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Table 1

QED. q_{UV} measured in units of μ . n_ℓ : number of charged leptons. n_q : number of quark colour triplets. Quarks always in pairs, with charges $2/3$, $-1/3$. We take $\alpha_{QED}^{-1}(\mu) \approx 137$.

QED $n_\ell ; n_q$	b_0	b_1	q_{UV}
2 ; 4	$\frac{32}{9}$	$\frac{176}{27}$	$10^{104.5}$
3 ; 6	$16/3$	$\frac{88}{9}$	10^{70}
4 ; 8	$\frac{64}{9}$	$\frac{352}{27}$	10^{52}
8 ; 16	$\frac{128}{9}$	$\frac{704}{27}$	10^{26}

Table 2

WEM theories. Critical q in units of μ , $\mu \sim$ mass of W's.

WEM $SU(N); n_V; n_A; Q$ Yes/No	Class	b_0	b_1	Non-zero fixed point	Critical q
$SU(2); 8; 0; \text{No}$	IV a	-2	20	$[\hat{\alpha}_{WEM}]^{-1} = 2.12$	*
$SU(2); 12; 0; \text{No}$	I	2	$158/3$	*	$q_{UV} > 10^{100}$
$SU(3); 8; 0; \text{No}$	III	$-17/3$	-213	*	$q_{IR} = 6 \times 10^{-63}$
$SU(3); 16; 0; \text{No}$	IV a or b	$-1/3$	$422/3$	$[\hat{\alpha}_{WEM}]^{-1} = 33$	*
$SU(3); 6; 1; \text{Yes}$	IV a	-3	70	$[\hat{\alpha}_{WEM}]^{-1} = 2.48$	*
$SU(4); 8; 0; \text{No}$	III	$-28/3$	$-244/3$	*	$q_{IR} = 9 \times 10^{-41}$
$SU(4); 16; 0; \text{No}$	IV a	-4	92	$[\hat{\alpha}_{WEM}]^{-1} = 1.8$	*

*) Non-existent.

Table 3

SEM and colour theories

SEM, colour SU(N); n_V	Class	b_0	b_1	Non-zero fixed point	Critical q	$\frac{1}{4\pi} g^2$ colour (μ) (at μ)
SU(3);0	III	-11	-102	*	$q_{IR} = 0.4$ GeV	1/2 (at 1 GeV)
SU(3);3	III	-27/4	-64	*	$q_{IR} = 0.06$ GeV	1/5 (at 3 GeV)
SU(3);6	III	-21/3	-26/3	*	†	†
SU(3);8	III	-17/3	-2/3	*	†	†
SU(3);16	IV b	-1/3	422/3	$[\hat{\alpha}_{SEM}]^{-1} = 33$	*	$\frac{1}{4\pi} g^2$ colour (μ) > 1/33
SU(4);8	III	-28/3	-244/3	*	†	†
SU(4);16	IV a	-4	92	$[\hat{\alpha}_{SEM}]^{-1} = 1.8$	*	$\frac{1}{4\pi} g^2$ colour (μ) < 1/2

*) Non-existent.

†) Critical q and g^2 colour (μ)/ 4π given only when meaningful.

Figure captions

- Fig. 1 : Plots of β/g versus α_g .
- Fig. 2 : Trajectories of coupling constants; the models are described in (2.9) in the main text. The Clebsch-Gordan coefficients have been divided out.
- Fig. 3 : Trajectories of coupling constants in the model of Section 3.2.A. The Clebsch-Gordan coefficients have been divided out.
- Fig. 4 : Plots of β functions, and fixed points, at break-up energy for the model of Section 3.2.A.

Fig. 1

Fig. 2

Fig. 3

Fig. 4