Present Status of Our Knowledge of $|V_{cb}|$

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Abstract

The Cabibbo-Kobayashi-Maskawa parameter $|V_{cb}|$ plays an important role among the experimental constraints of the Yukawa sector of the Standard Model. The present status of our knowledge will be summarized with particular emphasis to the interplay between theoretical and experimental advances needed to improve upon present uncertainties.

1 Introduction

In the framework of the Standard Model, the quark sector is characterized by a rich pattern of flavor-changing transitions, described by the Cabibbo-Kobayashi-Maskawa (CKM) matrix:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \quad (1)$$

Since the CKM matrix must be unitary, it is determined by only four independent parameters. Wolfenstein proposed an approximate parameterization [2] that reflects the hierarchy between the magnitude of matrix elements belonging to different generations. Very frequently it is quoted in the approximation valid only to $\lambda^3$. We need to carry out this expansion further in order to incorporate CP violation in neutral $K$ decays. This expression, accurate to $\lambda^3$ for the real part and $\lambda^5$ for the imaginary part, is given by:

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\[
\begin{pmatrix}
1 - \lambda^2/2 & \lambda
\end{pmatrix}
\begin{pmatrix}
A\lambda^3(\rho - i\eta(1 - \lambda^2/2))
\end{pmatrix}
\begin{pmatrix}
1 - \lambda^2/2 - A\lambda^2
\end{pmatrix}
\begin{pmatrix}
A\lambda^3(1 + i\eta)^2
\end{pmatrix}
\begin{pmatrix}
\lambda^2
\end{pmatrix}
\begin{pmatrix}
1
\end{pmatrix}
\]  

(2)

The parameter $\lambda$ is well measured as $0.2196 \pm 0.0023$ [1], constraints exist on $\rho$ and $\eta$ from measurements of $V_{ub}$ and $B^0\bar{B}^0$ mixing. This paper focuses on the magnitude of the CKM element $|V_{cb}|$, related to the Wolfenstein parameter $A$ [2].

2  

$|V_{cb}|$ from the exclusive decay $B \to D^*\ell\bar{\nu}$.

HQET predicts that the differential partial decay width for this process, $d\Gamma/dw$, is related to $|V_{cb}|$ through:

\[
\frac{d\Gamma}{dw}(B \to D^*\ell\bar{\nu}) = \frac{G_F^2|V_{cb}|^2}{48\pi^3} \mathcal{K}(w)\mathcal{F}(w)^2,
\]

(3)

where $w$ is the inner product of the $B$ and $D^*$ meson 4-velocities, $\mathcal{K}(w)$ is a known phase space factor and the form factor $\mathcal{F}(w)$ is generally expressed as the product of a normalization factor $\mathcal{F}(1)$ and $g(w)$, the Isgur-Wise function, whose shape is constrained by dispersion relations [3]. The analytical expression of $g(w)$ is not known a-priori, and this introduces an additional uncertainty in the determination of $\mathcal{F}(1)|V_{cb}|$. First measurements of $|V_{cb}|$ were performed assuming a linear approximation for $\mathcal{F}(w)$. It has been shown [10] that this assumption is not justified, and that linear fits systematically underestimate the extrapolation at zero recoil ($w = 1$) by about 3%. Most of this effect is related to the curvature of the form factor, and does not depend strongly upon the details of the non-linear shape chosen [10]. All recent results use a non-linear shape for $g(w)$, approximated with an expansion near $w = 1$ [11], and is parameterized in terms of the variable $\rho^2$, which is the slope of the form factor at zero recoil given in [11].

Considerable theoretical work has been devoted to the parameter $\mathcal{F}(1)$. Ultimately a precise value for it may be determined by lattice gauge calculations. Presently only a quenched lattice evaluation is available and gives $0.913^{+0.024}_{-0.017} \pm 0.016 \pm 0.003 + 0.014 + 0.006 - 0.006$. The errors reflect the statistical accuracy, the matching error, the finite lattice size, the uncertainty in the quark masses and an estimate of the error induced by the quenched approximation, respectively. The central value obtained with OPE sum rules is similar, with an error of $\pm 0.04$ [4]. Consequently, I will use $\mathcal{F}(1) = 0.91 \pm 0.04$ [4].
Table 1: Experimental results for $\mathcal{F}(1)|V_{cb}|$ and $\rho^2$ rescaled to common inputs [5]

| Experiment                  | $\mathcal{F}(1)|V_{cb}|(\times10^3)$ | $\rho^2$ | Corr$_{\text{stat}}$ |
|------------------------------|---------------------------------------|----------|----------------------|
| ALEPH update                 | 33.6±2.1±1.6                          | 0.75±0.25±0.37 | 94% |
| OPAL (partial reconstruction)| 38.4±1.2±2.4                          | 1.25±0.14±0.39 | 77% |
| OPAL (excl)                  | 39.1±1.6±1.8                          | 1.49±0.21±0.26 | 95% |
| DELPHI (partial reco.)       | 36.8±1.4±2.5                          | 1.52±0.14±0.37 | 94% |
| DELPHI (excl. prelim.)      | 38.5±1.8±2.1                          | 1.32±0.15±0.34 | 89% |
| Belle                        | 36.7±1.9±1.9                          | 1.45±0.16±0.20 | 91% |
| CLEO                         | 43.6±1.3±1.8                          | 1.61±0.09±0.21 | 87% |
| BaBar                        | 34.1±0.2±1.3                          | 1.23±0.02±0.28 | 92% |
| World average                | 36.7±0.8                              | 1.44±0.14     | 91% |

The main contributions to the $\mathcal{F}(1)|V_{cb}|$ systematic error in the LEP results come from the uncertainty on the $B \to D^{*}\ell\nu$ shape and $B(b \to B_d)$, $(0.57 \times 10^{-3})$, fully correlated among these experiments, the branching fraction of $D$ and $D^*$ decays, $(0.4 \times 10^{-3})$, fully correlated among all the experiments, and the slow pion reconstruction from Belle, CLEO, and BaBar which are uncorrelated. The main contribution to the $\rho^2$ systematic error is from the uncertainties in the measured values of $R_1$ and $R_2$ $(0.12)$, fully correlated among all the experiments. Because of the large contribution of this uncertainty to the non-diagonal terms of the covariance matrix, the averaged $\rho^2$ is higher than one would naively expect.

Using $\mathcal{F}(1) = 0.91 \pm 0.04$ [4], this method gives $|V_{cb}| = (40.2 \pm 0.9_{\text{exp}} \pm 1.8_{\text{theo}}) \times 10^{-3}$. The dominant error is theoretical, but there are good prospects that lattice gauge calculations will significantly improve their accuracy.

### 3 $|V_{cb}|$ from the exclusive decay $B \to D\ell\bar{\nu}$.

The study of the decay $B \to D\ell\bar{\nu}$ poses new challenges both from the theoretical and experimental point of view.

The differential decay rate for $B \to D\ell\nu$ can be expressed as:

$$\frac{d\Gamma_D}{dw}(B \to D\ell\nu) = \frac{G_F^2|V_{cb}|^2}{48\pi^3}\mathcal{K}_D(w)\mathcal{G}(w)^2,$$

(4)

where $w$ is the inner product of the B and D meson 4-velocities, $\mathcal{K}_D(w)$ is the
\begin{table}[h]
\centering
\begin{tabular}{|l|c|c|}
\hline
Experiment & $|G(1)|V_{cb}|(\times10^3)$ & $\rho_D^2$ \\
\hline
ALEPH & 40.0$\pm$ 10.0$\pm$ 6.4 & 1.02 $\pm$ 0.98$\pm$ 0.37 \\
Belle & 41.8$\pm$ 4.4$\pm$ 5.2 & 1.12 $\pm$ 0.22$\pm$ 0.14 \\
CLEO & 44.9$\pm$ 5.8$\pm$ 3.5 & 1.27 $\pm$ 0.25$\pm$ 0.14 \\
\hline
World average & 42.1 $\pm$ 3.7 & 1.15 $\pm$ 0.16 \\
\hline
\end{tabular}
\caption{Experimental results corrected to common inputs b and world average \cite{5}. $\rho_D^2$ is the slope of the form-factor given in \cite{11} at zero recoil.}
\end{table}

phase space and the form factor $G(w)$ is generally expressed as the product of a normalization factor $G(1)$ and a function, $g_D(w)$, constrained by dispersion relations \cite{3}.

The strategy to extract $G(1)|V_{cb}|$ is identical to that used for the $B \to D^*\ell\nu$ decay. However both theory and experiments have additional difficulties in dealing with this channel. From the theoretical standpoint, the non-perturbative expansion includes $1/m_b$ and $1/m_c$ terms, as there is no suppression mechanism. Moreover, this is a decay that is experimentally challenging as it is difficult to isolate from the larger $D^*\ell\bar{\nu}$ final state.

Belle \cite{6} and ALEPH \cite{7} studied the $\bar{B}^0 \to D^+\ell^-\bar{\nu}$ channel, while CLEO \cite{8} studied both $B^+ \to D^0\ell^+\bar{\nu}$ and $\bar{B}^0 \to D^+\ell^-\bar{\nu}$ decays. The results scaled to common inputs are shown in Table 2. Averaging \cite{9} the data in Table 2, using the procedure of \cite{9}, we get $G(1)|V_{cb}| = (41.3 \pm 4.0) \times 10^{-3}$ and $\rho_D^2 = 1.19 \pm 0.19$, where $\rho_D^2$ is the slope of the form-factor at zero recoil given in \cite{11}.

The theoretical predictions for $G(1)$ are consistent: a quark model evaluation gives $1.03 \pm 0.07$\cite{12}, and a recent heavy quark sum rule calculation \cite{13} gives $1.04 \pm 0.02 \pm \delta_{exp}$, where $\delta_{exp}$ represents the error in $\mu_\pi^2(1\text{GeV})$, defined in the next section. A quenched lattice calculation gives $G(1) = 1.058^{+0.021}_{-0.017}$\cite{14}, where the errors do not include the uncertainties induced by the quenching approximation and lattice spacing. Using $G(1) = 1.04 \pm 0.07$, we get $|V_{cb}| = (40.5 \pm 3.6_{\text{exp}} \pm 2.7_{\text{theo}}) \times 10^{-3}$, consistent with the value extracted from $B \to D^*\ell\nu$ decay, but with a larger uncertainty.

4 \hspace{1cm} |V_{cb}| \hspace{1cm} \text{from the inclusive decay } B \to X_c \ell\bar{\nu}.

The decay $B \to X_c \ell\bar{\nu}$ is an alternative experimental approach to extract $|V_{cb}|$. In this case, the Operator Product Expansion (OPE) is the theoretical
tool used. It yields the heavy quark inclusive decay rates as an asymptotic series in inverse powers of the heavy quark mass. More precisely, several mass scales are relevant: the b quark mass $m_b$, the c quark mass $m_c$ and the energy release $E_r \equiv m_b - m_c$ [15]. The uncertainties in the predicted $\Gamma_{sl}/|V_{cb}|$ have been discussed in numerous theoretical papers [16]. However, the theory needs to provide predictions on independent observables that can be used to validate its accuracy. Experimental input include the semileptonic width, as well as the determination of the theoretical parameters governing the hadronic matrix element, discussed below.

The key parameter in the theoretical expression for the semileptonic width is $m_b$. As the bare quark mass is affected by perturbative and non-perturbative contributions, considerable attention has been devoted to its proper definition [17], [18]. Similarly, $m_c$ is a parameter in the hadronic matrix element and, recently, it has been argued [16] that extracting it from the relationship between $(m_b - m_c)$ and the spin averaged meson mass difference $(\bar{M}_B - \bar{M}_D)$ [19] may be inadequate.

The leading non-perturbative corrections arise only to order $1/m_b^2$ and are parameterized by the quantities $\mu_2, \pi$ (or $-\lambda_1$) [19], [20] related to the expectation value of the kinetic energy of the b quark inside the b hadron, and $\mu_2, \zeta$ (or $\lambda_2$) [19], [20] related to the expectation value of the chromomagnetic operator. Quark-hadron duality is an important ab initio assumption in these calculations. While several authors [21] argue that this ansatz does not introduce appreciable errors as they expect that duality violations affect the semileptonic width only in high powers of the non-perturbative expansion, other authors recognize that an unknown correction may be associated with this assumption [22]. Arguments supporting a possible sizeable source of errors related to the assumption of quark-hadron duality have been proposed [23].

I will start the discussion with the experimental studies of the moments of inclusive distributions. Most of the experimental studies have focused on the lepton energy and the invariant mass $M_X$ of the hadronic system recoiling against the lepton-$\bar{\nu}$ pair. CLEO published the first measurement of the moments of the $M^2_X$ distributions. This analysis includes a 1.5 GeV/c lepton momentum cut, that allows them to single out the desired $b \to c\ell^-\bar{\nu}$ signal from the “cascade” $b \to c \to s\ell^+\nu$ background process. The hermeticity of the CLEO detector is exploited to reconstruct the $\nu$ 4-momentum vector. Moreover, the $B\bar{B}$ pair is produced nearly at rest and thus it allows a determination of $M_X$ from the $\nu$ and $\ell$ momenta. They obtain $<M^2_X - \bar{M}_D^2> = \ldots$
0.251±0.066 GeV² and <(M₂₆−M₂₉)>² = 0.576±0.170 GeV⁴, where \( \bar{M}_D \) is the spin-averaged mass of the D and \( D^* \) mesons. The lepton momentum cut may reduce the accuracy of the OPE predictions, because restricting the kinematic domain may increase quark-hadron duality violations. The shape of the lepton spectrum provides further constraints on OPE. Moments of the lepton momentum with a cut \( p_{CM}^L \geq 1.5 \) GeV/c have been measured by the CLEO collaboration [24]. The two approaches give consistent results, although the technique used to extract the OPE parameters has still relatively large uncertainties associated with the \( 1/m_b^3 \) form factors. The sensitivity to \( 1/m_b^3 \) corrections depends upon which moments are considered. Bauer and Trott [25] have performed an extensive study of the sensitivity of lepton energy moments to non-perturbative effects. In particular, they have proposed “duality moments,” very insensitive to neglected higher order terms. The comparison between the CLEO measurement of these moments [24] and the predicted values shows a very impressive agreement:

\[
D_3 \equiv \frac{\int_{1.6 \text{ GeV}}^{2.7} \frac{dE}{dE_{\ell}} dE_{\ell}}{\int_{1.5 \text{ GeV}}^{2.7} \frac{dE}{dE_{\ell}} dE_{\ell}} = \begin{cases} 0.5190 \pm 0.0007 & \text{(T)} \\ 0.5193 \pm 0.0008 & \text{(E)} \end{cases}
\]

\[
D_4 \equiv \frac{\int_{1.6 \text{ GeV}}^{2.3} \frac{dE}{dE_{\ell}} dE_{\ell}}{\int_{1.5 \text{ GeV}}^{2.3} \frac{dE}{dE_{\ell}} dE_{\ell}} = \begin{cases} 0.6034 \pm 0.0008 & \text{(T)} \\ 0.6036 \pm 0.0006 & \text{(E)} \end{cases}
\]

(5)

(where “T” and “E” denote theory and experiment, respectively).

More recently, both CLEO and BaBar explored the moments of the hadronic mass \( M_X^2 \) with lower lepton momentum cuts. In order to identify the desired semileptonic decay from background processes including cascade decays, continuum leptons and fake leptons, CLEO performs a fit for the contributions of signal and backgrounds to the full three-dimensional differential decay rate distribution as a function of the reconstructed quantities \( q^2, M_X^2, \cos \theta_{WR} \). The signal includes the components \( B \to D \ell \bar{\nu}, B \to D^* \ell \bar{\nu}, B \to D^{**} \ell \bar{\nu}, B \to X_c \ell \bar{\nu} \) non-resonant and \( B \to X_u \ell \bar{\nu} \). The backgrounds considered are: secondary leptons, continuum leptons and fake leptons. BaBar uses a sample where the hadronic decay of one \( B \) is fully reconstructed and the charged lepton from the other \( B \) is identified. In this case the main sources of systematic errors are the uncertainties related to the detector modeling and reconstruction.

Fig. 1 shows the extracted \( <M_X^2 - \bar{M}_D^2> \) moments as a function of the minimum lepton momentum cut from these two measurements, as well as the
original measurement with $p_\ell \geq 1.5$ GeV/c. The results are compared with theory bands that reflect experimental errors, $1/m_b^3$ correction uncertainties and uncertainties in the higher order QCD radiative corrections [26]. The CLEO and BaBar results are consistent and show an improved agreement with theoretical predictions with respect to earlier preliminary results [27]. Moments of the $M_X$ distribution without an explicit lepton momentum cut have been extracted from preliminary DELPHI data [28] and give consistent results.

The second element needed to extract $|V_{cb}|$ with this method is the measured semileptonic width. Experiments operating at the $\Upsilon(4S)$ center-of-mass energy use a dilepton sample to separate the decay process $b \rightarrow c\ell^-\bar{\nu}$ (primary leptons) from the $b \rightarrow c \rightarrow s\ell^+\nu$ (cascade leptons). This technique allows a direct determination of the primary lepton spectrum over almost all the kinematically allowed range. Thus, the semileptonic branching fraction extracted from this measurement has almost no model dependence. Fig. 2 shows a summary of the $\Upsilon(4S)$ measurements of inclusive semileptonic branching fractions. The overall experimental error is of the order of 2%. Different extractions of the HQE non-perturbative parameters cannot be combined in a straightforward manner because they use different methods to estimate the theoretical uncertainties and they do not fully agree [31]. I will choose a representative set of parameters [24] and obtain:

$$|V_{cb}| = (41.5 \pm 0.4 |\Gamma_{sl} \pm 0.4 | \lambda_1, \bar{\Lambda} \pm 0.9 | \lambda_{th} \times 10^{-3}, \quad (6)$$

where the first uncertainty is from the experimental value of the semileptonic width, the second uncertainty is from the HQE parameters ($\lambda_1$ and $\bar{\Lambda}$) and the third error is the theoretical uncertainty in the hadronic matrix element. No quantitative account is given for possible quark hadron duality violation. The present difference between the two values of $|V_{cb}|$ obtained from $B \rightarrow D^*\ell\bar{\nu}$ and from inclusive semileptonic branching fraction measurements may be used to make a very rough estimate of the non quantified errors in the inclusive determination of $|V_{cb}|$ at a level of about 6%.
5 Conclusions

The values of $|V_{cb}|$ obtained both from the inclusive and exclusive method agree within errors. The value of $|V_{cb}|$ obtained from the analysis of the $B \to D^* \ell \nu$ decay is:

$$|V_{cb}|_{\text{exclusive}} = (40.2 \pm 0.9_{\exp} \pm 1.8_{\text{theo}}) \times 10^{-3}$$  \hspace{1cm} (7)

where the first error is experimental and the second error is from the $1/m_b^2$ corrections to $\mathcal{F}(1)$. The value of $|V_{cb}|$, obtained from inclusive semileptonic branching fractions is:

$$|V_{cb}|_{\text{incl}} = (41.4 \pm 0.5_{\exp} \pm 0.4_{\lambda_1, \pi} \pm 0.9_{\text{theo}}) \times 10^{-3},$$  \hspace{1cm} (8)

where the first error is experimental, the second error is from the measured values of $\lambda_1$, and $\pi$, assumed to be universal up to higher orders, and the last from $1/m_b^3$ corrections and $\alpha_s$. Non-quantified uncertainties are associated with a possible quark-hadron duality violation. An estimate through a comparison between these two results implies an additional uncertainty of the order of 6%. For this reason I choose not to average these two numbers, but quote a conservative estimate based on $|V_{cb}|$ exclusive.

High precision tests of HQET and more precise assessment of quark-hadron duality in inclusive semileptonic decays are needed to achieve the ultimate accuracy in this measurement.

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References


Figure 1: The results of the recent CLEO analysis [29] compared to previous measurements [30, 31] and the HQET prediction. The theory bands shown in the figure reflect the variation of the experimental errors on the two constraints, the variation of the third-order HQET parameters by the scale $(0.5 \text{ GeV})^3$, and variation of the size of the higher order QCD radiative corrections [26].
Figure 2: Summary of model independent semileptonic branching fraction measurements performed at the $\Upsilon(4S)$ center-of-mass energy.