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QUANTUM NUMBER TRANSFER AND OVERLAP CALCULATIONS

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A B S T R A C T

The relationship between Regge behaviour of two-body amplitudes and quantum number exchange in multiparticle production is analyzed. An exact bound on the elastic forward peak in terms of inclusive $p_{\perp}$ correlations is given. Another result is that one cannot obtain $\alpha'_p > \alpha'_p$ from overlap calculations, using only the present inclusive data as input.

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1. - INTRODUCTION

An outstanding old problem in high energy strong interaction theory is the connection between two-body and multi-body processes by unitarity ¹⁾. This has been most intensively studied in the framework of the multiperipheral model (MPM). The results obtained in different calculations disagree considerably, some versions yielding too broad an elastic forward peak ²⁾, while others predict it to shrink much too fast ³⁾. Various modifications of the original MPM have been proposed, but the situation seems to be far from settled.

In addition to searching for further models, it seems worth while to study model-independent methods to connect multi-body with two-body data. Indeed, some time ago several authors ⁴⁾ have studied rigorous upper bounds on elastic amplitudes in terms of inclusive spectra. Unfortunately, these bounds are not very strong. They would still allow a broadening of the elastic peak at $s \rightarrow \infty$.

More recently, stronger estimates have been given by Webber ⁵⁾ and by Weingarten ⁶⁾ and Krzywicki ⁷⁾. Webber's method, which seems to yield the tightest estimates, uses exclusive n body production cross-sections and is therefore not applicable to the bulk of the data. Weingarten and Krzywicki, on the other hand, use the intuitively very appealing notion of local quantum number compensation ⁸⁾⁻¹¹⁾. Roughly speaking, they argue that two-body amplitudes for exchange of any quantum number q (q = transverse momentum, charge, strangeness...) are suppressed relatively to the elastic forward amplitude since such an exchange implies a mismatch of the two states entering the unitarity integral (see Fig. 1). From Fig. 1, one sees that it is just the transfer of q (across any rapidity $\leq \frac{1}{2} \ln s$!) which do not match in the q exchange amplitude. Thus the effect will be the more important, the narrower the q transfer distribution, i.e., the more locally q is compensated. Assuming that transfers are uncorrelated when the rapidity distance between them is large, this leads to an exponential damping in rapidity

$$e^{-\text{const} \cdot q^2 Y} = s^{-\text{const} \cdot q},$$

where the constant is inversely proportional to the dispersion of the transfer of q and the length over which transfers are correlated.

The drawback of Refs. 6), 7) is that they use mathematically rather cumbersome derivations, and yet obtain closed formulae only for the limit $s \rightarrow \infty$. Also, it is not clear whether their proofs are rigorous. In the present paper, we shall show that a bound for the elastic forward slope very similar to the ones of Refs. 6), 7) can be obtained exactly, using elementary methods similar to the ones of Ref. 5). The derivation allows a simple geometrical interpretation: what we use is essentially the uncertainty relation between impact parameter distance and transverse momentum transfer, which become canonically conjugate variables at high energy¹²⁾.

The discussion of $p_{\perp}$ transfer is done in Section 2. In the following sections, we discuss charge exchange and the simultaneous exchange of charge and transverse momentum. The latter is, of course, connected to the slope α'_{ρ} of the ρ trajectory. The bounds on charge transfer - which were already found in Refs. 6), 10), by somewhat different methods - are unfortunately not very strong. We try to explain this in Section 3. Finally, in Section 4, we shall argue that α'_{ρ} cannot be estimated in this kind of overlap calculations, in contrast to suggestions made in Refs. 6), 10). In particular, it seems hard to derive $\alpha'_{\rho} > \alpha'_P$ model-independently on the basis of present multiparticle data.

2. - BOUNDS ON THE ELASTIC FORWARD SLOPE

Consider the elastic process $a + b \rightarrow a + b$. The forward slope is defined as

$$b = \left. \frac{d}{dt} \left(\ln \frac{d\sigma}{dt} \right) \right|_{t=0} . \quad (2.1)$$

As is usually done, we neglect real part and spin effects in the elastic amplitude, and shall therefore actually study the quantity

$$b = \frac{2}{A(s,0)} \left. \frac{d}{dt} A(s,t) \right|_{t=0} . \quad (2.2)$$

Let us use a centre-of-mass co-ordinate system with the beam along the z axis. By using unitarity and rotational invariance, we can write [see also Ref. 13]

$$\begin{aligned} A(s,t) &= \langle R_1(\theta) (a,b) | \frac{1}{i} (T - T^\dagger) | a,b \rangle \\ &= \langle a,b | T^\dagger e^{i\theta J_1} T | a,b \rangle, \end{aligned} \quad (2.3)$$

with $t = -2p_{in}^2 (1 - \cos \theta)$, and thus

$$\begin{aligned} b &= \frac{1}{p_{in}^2} \frac{\langle a,b | T^\dagger J_1^2 T | a,b \rangle}{\langle a,b | T^\dagger T | a,b \rangle} \\ &\equiv \frac{1}{p_{in}^2} \langle J_1^2 \rangle. \end{aligned} \quad (2.4)$$

In the following, all expectation values [such as in Eq. (2.4)] will be taken with the state

$$\frac{T | a,b \rangle}{\| T | a,b \rangle \|}, \quad (2.5)$$

the scalar product always being understood to include the correct energy momentum conservation δ functions.

Inclusive densities are obtained by keeping some particles of $T | a,b \rangle$ fixed, and integrating over all others. This means that we can write them in terms of creation and annihilation operators as ¹⁴⁾

$$\rho_c(\vec{p}) = \langle a_c^\dagger(\vec{p}), a_c(\vec{p}) \rangle \quad (2.6)$$

$$\rho_{cd}(\vec{p}, \vec{k}) = \langle a_c^\dagger(\vec{p}) a_d^\dagger(\vec{k}) a_c(\vec{p}) a_d(\vec{k}) \rangle, \text{ etc.}, \quad (2.7)$$

with

$$[a_c(\vec{p}), a_d^\dagger(\vec{p}')] = \delta_{cd} 2p_0 \delta^3(\vec{p} - \vec{p}'). \quad (2.8)$$

The same operator can, of course, be used to express the angular momentum operators in second quantized form ($\vec{d}^3 p = \vec{d}^3 p / 2p_0$)

$$J_1 = \frac{1}{i} \int d^3 p \sum_c a_c^\dagger(\vec{p}) \left(p_2 \frac{\partial}{\partial p_3} - p_3 \frac{\partial}{\partial p_2} \right) a_c(\vec{p}). \quad (2.9)$$

To obtain a bound on $\langle J_1^2 \rangle$, we follow Ref. 5) in using the uncertainty principle, which reads for any hermitean operator P

$$4 \langle J_1^2 \rangle (\langle P^2 \rangle - \langle P \rangle^2) \geq |\langle [J_1, P] \rangle|^2. \quad (2.10)$$

We choose the particular ansatz

$$P = \sum_c \int d^3 p \, w_c(\vec{p}) \, p_2 \, a_c^\dagger(\vec{p}) \, a_c(\vec{p}), \quad (2.11)$$

where $w_c(\vec{p})$ are real-valued functions to be determined later. The commutator of J_1 and P is straightforwardly computed. To bring $\langle P^2 \rangle$ into a suitable form, we use transverse momentum conservation. After some simple algebraic manipulations, we get

$$b \geq \frac{\left[\sum_c \int d^3 p \, \theta_c(\vec{p}) \left(w_c(\vec{p}) \cdot x - \frac{p_2}{p_{in}} \left(p_2 \frac{\partial}{\partial p_3} - p_3 \frac{\partial}{\partial p_2} \right) w_c(\vec{p}) \right) \right]^2}{-2 \sum_{c,d} \iint d^3 p \, d^3 k \, C_{cd}(\vec{p}, \vec{k}) (p_2 \cdot k_2) (w_c(\vec{p}) - w_d(\vec{k}))^2}. \quad (2.12)$$

Here, $x = p_3/p_{in}$ is the Feynman variable, and $C_{cd}(\vec{p}, \vec{k})$ is the inclusive two-body correlation.

Equation (2.12) must hold for any set of functions $w_c(\vec{p})$. Once the inclusive one- and two-body densities are sufficiently well known, one can use this to optimize the bound choosing w_c such that $\delta b / \delta w_c(\vec{p}) = 0$ ^{*}).

At present, the best one can do is to guess a good choice for w_c . If azimuthal correlations are short range, a good bound is asymptotically got with

$$w_c(\vec{p}) = y = \frac{1}{2} \ln \frac{p_0 + p_3}{p_0 - p_3} , \quad (2.13)$$

i.e., using the rapidity as a weight function. Defining the transverse momentum correlation at fixed rapidities as

$$\langle \vec{p}_\perp(y) \cdot \vec{p}_\perp(y') \rangle = \sum_{c,d} \iint d^2 p_\perp d^2 k_\perp \vec{p}_\perp \cdot \vec{k}_\perp C_{cd}(\vec{p}_\perp, y; \vec{k}_\perp, y') , \quad (2.14)$$

^{*}) In this connection, it is interesting to notice that Eq. (2.12) can be cast into the more symmetrical form

$$b \geq \frac{\sum_{c,d} \iint d^3 p d^3 k \tilde{g}_c(\vec{p}) \tilde{g}_d(\vec{k}) x \cdot x' [w_c(\vec{p}) - w_d(\vec{k})]^2}{4 \sum_{c,d} \iint d^3 p d^3 k C_{cd}(\vec{p}, \vec{k}) (p_2 \cdot k_2) [w_c(\vec{p}) - w_d(\vec{k})]^2} ,$$

with

$$\tilde{g}_c(\vec{p}) = g_c(\vec{p}) + \frac{p_2}{p_3} \left(p_2 \frac{\partial}{\partial p_3} - p_3 \frac{\partial}{\partial p_2} \right) g_c(\vec{p}) .$$

From this, one sees in particular that $w_c(\vec{p})$ is determined only up to a constant.

We might call Eq. (2.12) the "inclusive angular momentum conservation sum rule", as it is the closest possible analogue to the inclusive sum rules for energy momentum conservation. We get only an inequality since the vectors

$$a_{c_1}(\vec{p}_1) \dots a_{c_n}(\vec{p}_n) T |a, b\rangle$$

themselves are unobservable, while only their norms are observables. Thus the expectation value of any derivative operator squared can only be bounded from below.

we get in this case

$$b \geq \frac{\left\{ \sum_c \int d^3 p \, \theta_c(\vec{p}) \left(x \cdot y - \frac{1}{2} \frac{\vec{p}_\perp^2 p_0}{(\vec{p}_\perp^2 + m_c^2) p_{in}} \right) \right\}^2}{-\int_{-\infty}^{+\infty} dy dy' (y-y')^2 \langle \vec{p}_\perp(y) \cdot \vec{p}_\perp(y') \rangle} \quad (2.15)$$

This exact estimate is our central result, which we are now going to discuss in some detail. Let us start with the numerator. Using energy conservation, we get for its second term

$$\sum_c \int d^3 p \, \theta_c(\vec{p}) \frac{1}{2} \frac{\vec{p}_\perp^2 p_0}{(\vec{p}_\perp^2 + m_c^2) p_{in}} < \frac{\sqrt{s}}{2 p_{in}} \approx 1. \quad (2.16)$$

Due to the factor x , the first term in the numerator of Eq. (2.15) is dominated by fast particles, with rapidities

$$|y| \approx \frac{Y}{2} - \text{const} ; \quad Y = \ln s, \quad (2.17)$$

$$\text{const} \sim \frac{1}{2}$$

Inserting the last two equations into Eq. (2.15), we get the simple bound

$$b \geq \frac{(\frac{Y}{2} - \text{const})^2}{-\int \int dy dy' (y-y')^2 \langle \vec{p}_\perp(y) \cdot \vec{p}_\perp(y') \rangle} \quad (2.18)$$

with $\text{const} \approx 1-2$.

With the present data, the denominator is much harder to estimate. A natural guess is that azimuthal correlations are short range. This does not mean that all inclusive correlations are short range, as stressed by Krzywicki ^{7),*)}. In this case, only a domain $|y-y'| < \Delta$ contributes to the double integral, and moreover this contribution is, in the central region, independent of $y+y'$. Thus we expect

*) If long-range correlations were due to diffractive production by Pomeron pole exchange, azimuthal correlations would still be short range.

$$\int_{-\infty}^{+\infty} dy dy' (y-y')^2 \langle \vec{p}_\perp(y) \cdot \vec{p}_\perp(y') \rangle \sim \text{const} \cdot Y, \quad (2.19)$$

and consequently $b \sim \text{const} Y$, i.e., a logarithmic shrinkage corresponding to a simple Regge pole.

Unfortunately, present data on azimuthal correlations are still rather crude, and in particular one cannot conclude that they are indeed of short-range nature ¹⁵⁾. On the contrary, the data show a fairly long range component with a correlation length λ of at least ~ 3 units of rapidity.

A simple estimate can be made if one assumes an exponential fall-off of azimuthal correlations

$$\langle \vec{p}_\perp(y) \cdot \vec{p}_\perp(y') \rangle = - \frac{\langle n \rangle \langle \vec{p}_\perp^2 \rangle}{2Y\lambda} e^{-\frac{1}{\lambda} |y-y'|}, \quad (2.20)$$

where the normalization is fixed by the transverse momentum conservation sum rule. Inserting this into Eq. (2.18) and using

$$b \approx 2 \alpha'_P \cdot \ln s, \quad (2.21)$$

we get

$$\alpha'_P \gtrsim \frac{Y}{4 \lambda^2 \langle \vec{p}_\perp^2 \rangle \langle n \rangle}. \quad (2.22)$$

With $\langle n \rangle / Y \approx 3$ and $\langle \vec{p}_\perp^2 \rangle \approx 0.18$, we have thus

$$\alpha'_P \gtrsim \frac{0.5}{\lambda^2}. \quad (2.23)$$

If we insert the experimental value $\lambda \approx 3$, this is amply fulfilled. It is even far from being saturated, which indicates that either the distribution of P is far from being a Gaussian or that other effects, as, e.g., the phase of the wave function, are important.

On the other hand, in the simplest version of the MPM where longitudinal momentum transfer and clustering are both neglected, transverse momentum is essentially compensated between neighbouring particles. Thus the correlation length λ is of the order of the mean gap length. Inserting $\lambda \approx Y/\langle n \rangle \approx 1/3$ into Eq. (2.23), we recover the usual difficulty of this model ³⁾, namely α_P^1 turns out to be much too large.

A longer range of azimuthal correlations can result from various sources. One is longitudinal momentum transfer, another is emission of spinless clusters ¹⁶⁾. The very long range seen experimentally seems, however, to be hard to understand even in this way. In the multiperipheral cluster emission model, it would require large transverse momenta of the clusters, which were unexpected a priori. Most probably, both of the above effects are present ¹⁷⁾ simultaneously.

From Eq. (2.15) one could not only determine α_P^1 (i.e., the asymptotic value of b) but also the value of b at any finite energy. To do this actually, we would, however, need azimuthal correlations between particles in the fragmentation regions. It is hardly necessary to stress that better data are urgently needed.

In the above estimates we have assumed that the correlations between neutral and charged particles are the same as the ones between two charged particles. We could have avoided this assumption by taking $w_c(\vec{p}) = 0$ for neutral particles. The bound becomes then slightly weaker, but involves only charged particle densities.

The similarity between our bound and the result of Webber ⁵⁾ is obvious from our derivation. The main difference is that he, working in first quantization, gives separate bounds on the contributions to the forward slope from the various multiplicities.

To see the connection with Refs. 6), 7), we notice that - with the choice $w_c(\vec{p}) = y$ - we could have written

$$P = \int_{-\infty}^{+\infty} dy (K_2(y) - K_2(0)) \quad , \quad (2.24)$$

with

$$\vec{K}(y) = \int_{-\infty}^y d\eta \int d^2\vec{p}_\perp \vec{p} a_c^\dagger(\vec{p}_\perp, \eta) a_c(\vec{p}_\perp, \eta) - \vec{P}_a. \quad (2.25)$$

Obviously, $\vec{K}(y)$ is the momentum carried by the particles emitted at rapidity larger than y , minus the momentum "injected" at rapidity $+Y/2$. Due to momentum conservation, this is equal to the momentum transferred across rapidity y [in the language of Refs. 6)-8), $\vec{K}(y)$ is the operator corresponding to the momentum zone graphs].

Indeed, the denominator in Eq. (2.15) and in Eq. (2.18) is nothing else but

$$2 \iint dy dy' \langle \vec{K}_1(y) \cdot \vec{K}_1(y') \rangle, \quad (2.26)$$

i.e., the correlation integral for transverse momentum transfers. The virtue of our derivation is, of course, that we could also determine the numerator so that our result is exact, and that we obtained also the more general Eq. (2.12).

Finally, let us point out that the bound (2.18) can be derived heuristically in an extremely simple and intuitively appealing way. Equation (2.4) tells us that the forward slope is equal to the mean squared impact parameter between the incoming particles,

$$b = \frac{1}{2} \langle (\vec{B}_a - \vec{B}_b)^2 \rangle. \quad (2.27)$$

The variable canonically conjugate to the impact parameter is the transverse momentum. More precisely ¹²⁾, the distance $\vec{B}_a - \vec{B}_b$ is at high energy conjugate to the transverse momentum transfer across $x=0$, i.e., across any rapidity in the central plateau $-Y/2 + \text{const} < y < Y/2 - \text{const}$,

$$[(B_a - B_b)_i, K_j(y)] \cong \delta_{ij} \quad \text{for any } |y| \leq \frac{Y}{2} - \text{const}. \quad (2.28)$$

Thus, using the uncertainty principle we arrive immediately at

$$b \geq \frac{(Y - \text{const})^2}{2 \iint dy dy' \langle \vec{K}_1(y) \cdot \vec{K}_1(y') \rangle}, \quad (2.29)$$

which is the same as Eq. (2.18).

Without referring to impact parameter, we could have read off the same from Fig. 1 : to get the amplitude $A(s, t)$, we have to shift the momentum transfers for all rapidities by an amount q in the y direction, i.e.,

$$b \approx -\frac{1}{2} \left\langle \left(\int_{-Y/2}^{Y/2} dy \frac{\delta}{\delta K_2(y)} \right)^2 \right\rangle, \quad (2.30)$$

and again Eq. (2.29) follows immediately.

3. - CHARGE EXCHANGE

We now turn in our discussion to the problem of charge transfer. Compared to momentum transfer, there are two important differences. One is that charge is a discrete quantum number, thus there exists no infinitesimal charge exchange. The other difference becomes obvious if we consider a particular process like $\pi^- p \rightarrow \pi^0 n$. To get exact bounds, we would need data both for $\pi^- p$ and for $\pi^0 n$ scattering. As long as the latter are not available, we can only derive heuristic bounds which should be valid only at $s \rightarrow \infty$. Stated differently, we can estimate the energy dependence but not the absolute magnitude of $A^{\text{CEX}}(s, 0)$.

At the end of the previous section, we have shown that the suppression of the non-forward relative to the forward amplitude is due to the quantum number flow through the longitudinal phase space. The same should hold for charge exchange. Consider exchange of charge Δq . Define the charge transfer operator, analogously to $\vec{K}(y)$, by

$$Q(y) = \int_y^\infty dy' \int d^2 p_\perp \sum_c q_c a_c^\dagger(\vec{p}_\perp, y') a_c(\vec{p}_\perp, y') - Q_a, \quad (3.1)$$

where Q_b is the charge of the incoming particle moving to the right, i.e., the charge injected at $y=+Y/2$. Neglecting effects from the fragmentation regions (where $\pi^0 n \rightarrow \text{anything}$ might be different from $\pi^- p \rightarrow \text{anything}$), the charge exchange amplitude is suppressed by

$$\frac{A_{\Delta q}(s,0)}{A_0(s,0)} = \langle e^{\Delta q \int dy \frac{\delta}{\delta Q(y)}} \rangle \equiv \langle e^{i \Delta q \cdot P_Q} \rangle, \quad (3.2)$$

where P_Q is a "shifting operator" for charge transfer

$$P_Q = \frac{1}{i} \int dy \frac{\delta}{\delta Q(y)}. \quad (3.3)$$

In the central region, the charge transfer distribution is symmetric, so only even powers $\langle P_Q^n \rangle$ contribute, and

$$\begin{aligned} \frac{A_{\Delta q}(s,0)}{A_0(s,0)} &= 1 - \frac{(\Delta q)^2}{2} \langle P_Q^2 \rangle \pm \dots \\ &= e^{-\left[\frac{(\Delta q)^2}{2} \langle P_Q^2 \rangle \pm \dots \right]}, \end{aligned} \quad (3.4)$$

where the neglected terms are of order $(\Delta q)^4$ *).

For infinitesimal Δq , we could use the uncertainty principle between P_Q and $\int dy Q(y)$ to get immediately for the Regge intercepts

*) More generally, by the definition of the cumulants d_n of the P_Q distribution, one has ^{(6), (14)}

$$\langle e^{i \Delta q \cdot P_Q} \rangle = \exp \left(\sum_n \frac{(i \Delta q)^n}{n!} d_n \right).$$

$$\alpha_P(0) - \alpha_{\Delta q}(0) \geq \frac{(\Delta q)^2}{8} \frac{Y}{\iint dy dy' \langle Q(y) \cdot Q(y') \rangle} \quad (3.5)$$

which was also found in Refs. 6), 10). This inequality would be saturated only if the charge transfer distribution (integrated over rapidity) were Gaussian.

For finite Δq , we can obtain $\alpha_{\Delta q}(0) < \alpha_P(0)$ only if we know something about all higher moments $\langle [\int dy Q(y)]^n \rangle$ of the charge transfer distribution ^{*)}. Optimal bounds (i.e., bounds which in principle could be saturated) are, however, even then very difficult to obtain. In particular, the bounds of Refs. 6), 10) are not optimal except when all higher correlation integrals of the charge transfer distribution vanish, in which case Eq. (3.5) holds for any Δq and is optimal. We discuss this question in some detail since a very similar problem becomes relevant in the next section.

Let us assume that Eq. (3.5) holds for $\Delta q = 1$. When inserting experimental numbers, one sees that it is not very tight: at NAL energies ¹⁸⁾ $\langle Q(0)^2 \rangle \approx 0.9$. The correlation length can be estimated from inclusive charged-particle correlations. Using Eq. (3.1) and charge conservation, we get (assume $y < y'$):

$$\begin{aligned} \langle Q(y) Q(y') \rangle - \langle Q(y) \rangle \langle Q(y') \rangle &= \\ = \int_{-\infty}^y dz \int_{y'}^{\infty} dz' (C_{+-}(z, z') + C_{-+}(z, z') - C_{++}(z, z') - C_{--}(z, z')) \end{aligned} \quad (3.6)$$

For the particular combination of correlations which enters in the right-hand side we can make an ansatz

$$\frac{g^2}{\lambda} e^{-\frac{1}{\lambda} |z - z'|} \quad , \quad (3.7)$$

^{*)} If only the second moment $\langle [\int dy Q(y)]^2 \rangle$ is known, the best one can show exactly is $A_{\Delta q}/A_0 \lesssim (\ln s)^{\text{const}}$.

with $g^2 = \langle n_+ \rangle / Y \approx 1$. The normalization is given by charge conservation ; the correlation length λ is experimentally ~ 1 unit. With this ansatz we get in the central region, where $\langle Q(y) \rangle = 0$,

$$\langle Q(y) Q(y') \rangle = g^2 \lambda e^{-\frac{1}{\lambda} |y-y'|} \quad (3.8)$$

In particular, one finds

$$\langle Q(0)^2 \rangle = g^2 \lambda, \quad (3.9)$$

in good agreement with the experimental numbers quoted. Inserting this into Eq. (3.5), we find $\alpha_P(0) - \alpha_\rho(0) \geq 0.1$, compared to the true value $\alpha_P(0) - \alpha_\rho(0) \approx 0.5$. Essentially the same was found in Refs. 6), 8), 9), using different methods.

In Ref. 9), some reasons for the weakness of the bound were suggested. The present analysis shows that their importance was overestimated. To find other sources, let us study the simplest non-trivial model with short-range correlations, namely the multiperipheral model with equal amount of $I=0$ and $I=1$ exchange [the M model of Ref. 19) with $g_0 = g_1 = g$; $g^2 = \langle n_+ \rangle / Y$]. The output trajectories found in Ref. 19) are

$$\begin{aligned} I=0 & : \alpha_0 = \text{const} + 3g^2 ; & \alpha_0^{(2)} &= \text{const} - g^2 ; \\ I=1 & : \alpha_1 = \text{const} + g^2 ; & & \\ I=2 & : \alpha_2 = \text{const} - g^2 . & & \end{aligned} \quad (3.10)$$

In obtaining these, it was assumed that interference terms as shown in Fig. 2a, i.e., terms where on both sides of the ladder different isospins are exchanged, are absent. Including them, with no phase relative to the other contributions, one finds two additional $I=1$ trajectories (see Appendix)

$$I=1 : \alpha_1^{(2,3)} = \text{const} \pm \sqrt{5} g^2 . \quad (3.11)$$

The leading $I=1$ trajectory is then not given by Eq. (3.10) but by (3.11). It has, however, some paradoxical features. It decouples, e.g., from Mueller graphs, i.e., the inclusive correlations in the central region are determined only by Eq. (3.10) ^{*}). Usually, interference terms are assumed to be absent due to exchange degeneracy, and thus Eq. (3.11) is discarded. We cannot do so, however, when deriving exact bounds on α_1 . Therefore every bound - assuming transfers of charge $-1, 0$, and $+1$ only, with probabilities $1/4 : 1/2 : 1/4$ - will necessarily be such that

$$\alpha_{1,\max} \geq \text{const} + \sqrt{5} g^2 \quad (3.12)$$

Otherwise stated, the best we can hope to obtain is

$$\alpha_P(0) - \alpha_S(0) \geq (3-\sqrt{5}) \frac{\langle n_+ \rangle}{Y} = 0.76 \frac{\langle n_+ \rangle}{Y}, \quad (3.13)$$

compared to the actual value

$$\alpha_P(0) - \alpha_S(0) = 2 \frac{\langle n_+ \rangle}{Y}. \quad (3.14)$$

Even this is, however, not got, but instead we found in Ref. 9)

$$\alpha_P(0) - \alpha_S(0) \geq (2-\sqrt{2}) \frac{\langle n_+ \rangle}{Y} = 0.58 \frac{\langle n_+ \rangle}{Y}. \quad (3.15)$$

The reason for this is simply that in obtaining Eqs. (3.10) and (3.11), we have taken care of isospin conservation, while in getting (3.15), only conservation of the third component of isospin is guaranteed. Isospin fixes the sign of the couplings in the multiperipheral ladder and forces some contributions to the unitary sum to be negative. One cannot, of course, keep track of this, using the Schwarz inequality as in Ref. 9) or the uncertainty principle as in the present paper.

*) The reason is simply that inclusive correlations are related to elastic amplitudes via the generalized optical theorem, while the interference terms shown in Fig. 2a clearly correspond to isospin exchange.

The above model is certainly not realistic. Nevertheless it points out two reasons for the weakness of the bound on α_p which - though being hard to take into account in an exact way - seem fairly general. The first is that a model with local charge compensation will, in general, also have local isospin compensation. More specifically, one should expect the isospin transfer in the central region to be limited according to $\langle (\vec{I}\text{-transfer})^2 \rangle = 3 \langle Q^2 \rangle$, and this additional constraint may lead to additional mismatch in the charge exchange amplitude. The second point is that in most models there are certain types of graphs which do not contribute to charge exchange, by some dynamical reason. In the above model, these were the interference terms shown in Fig. 2a - and they indeed had the biggest effect. The situation is similar in models where resonances (or clusters) are produced along a multiperipheral chain, as shown in Fig. 2b. In the model of Chan et al.²⁰⁾, excellent results were obtained assuming that spill-over graphs as shown in Fig. 2c are absent. Of course, this assumption is very hard to justify a priori. Indeed, in models with neutral cluster production only, graphs of the type shown in Fig. 2c are the only ones contributing to charge exchange.

Even if the above considerations explain correctly the suppression of charge exchange, it seems very hard to formulate them as exact bounds.

4. - SIMULTANEOUS CHARGE AND MOMENTUM TRANSFER

The same methods that led to Eqs. (2.29) and (3.5) can of course be used to get bounds on simultaneous momentum and charge transfer. This means that one can get bounds on $\alpha_p(t)$ for $t \neq 0$. It would of course be extremely interesting if one could understand from inclusive data why $\alpha'_p > \alpha'_p$, as suggested from Weingarten⁶⁾ and Krzywicki¹⁰⁾.

Making the same assumptions as above, we have

$$\frac{A_{\Delta q}(s,t)}{A_0(s,0)} \approx \left\langle e^{i(\Delta q P_A + \sqrt{-t} \cdot B_2)} \right\rangle, \quad (4.1)$$

where P_Q is the charge transfer shifting operator defined in Eq. (3.3) and B_2 is the impact parameter distance in the y direction,

$$B_2 = B_{a,2} - B_{b,2} .$$

As seen before, B_2 is canonically conjugate to the momentum transfer, thus we can also write it

$$B_2 = \frac{1}{i} \int_{-y/2}^{y/2} dy \frac{\delta}{\delta K_2(y)} . \quad (4.2)$$

For small Δq and $|t|$, we can write Eq. (4.1) as

$$\begin{aligned} \frac{A_{\Delta q}(s,t)}{A_0(s,0)} = \exp \left\{ -\frac{(\Delta q)^2}{2} \langle P_Q^2 \rangle - \frac{|t|}{2} \langle B_2^2 \rangle + \right. \\ \left. + \frac{(\Delta q)^2 |t|}{4} (\langle P_Q^2 B_2^2 \rangle - \langle P_Q^2 \rangle \langle B_2^2 \rangle) \pm \dots \right\} . \end{aligned} \quad (4.3)$$

This time we cannot use the uncertainty relation to estimate $A_{\Delta q}(s,t)/A_0(s,0)$, as the resulting bound would be too weak. But it was shown in Refs. 6), 10) that

$$\frac{A_{\Delta q}(s,t)}{A_0(s,0)} \lesssim s^{-\frac{(\Delta q)^2}{8c_{20}} - \frac{|t|}{8c_{02}} + \frac{(\Delta q)^2 |t|}{64} \frac{c_{22}}{(c_{20}c_{02})^2} \pm \dots} \quad (4.4)$$

where c_{nm} are the cumulants of the joint transfer distribution, divided by $\ln s$:

$$\begin{aligned} c_{20} &= \langle [\int dy Q(y)]^2 \rangle / \ln s \\ c_{02} &= \langle [\int dy K_2(y)]^2 \rangle / \ln s \\ c_{22} &= \left\{ \langle [\int dy Q(y)]^2 [\int dy' K_2(y')]^2 \rangle \right. \\ &\quad \left. - \langle [\int dy Q(y)]^2 \rangle \langle [\int dy' K_2(y')]^2 \rangle \right\} / \ln s . \end{aligned} \quad (4.5)$$

Comparing this with a Regge ansatz

$$\frac{A_{\Delta q}(s,t)}{A_0(s,0)} = s^{-(\alpha_p(0)-\alpha_{\Delta q}(0)) - |t|\alpha'_p + |t|(\alpha'_p - \alpha'_{\Delta q})} \quad (4.6)$$

one might suggest (for $\Delta q = 1$)

$$\alpha'_p - \alpha'_g \leq \frac{C_{22}}{64 (C_{20} C_{02})^2} \quad (4.7)$$

So one would get $\alpha'_p > \alpha'_g$, if

$$C_{22} < 0, \quad (4.8)$$

i.e., if there is a negative correlation between charge and momentum transfer.

Under what conditions does Eq. (4.7) really follow? One sufficient set of assumptions would be that Eqs. (4.4) and (4.6) hold up to arbitrarily large $|t|$, while at the same time

$$\alpha'_p = \frac{1}{8 C_{02}} \quad (4.9)$$

The former means that all higher cumulants c_{nm} ($n \geq 4$ or $m \geq 4$) vanish and that Regge trajectories are linear, which seems already a rather strong assumption. Equation (4.9), on the other hand, is equivalent to saying that the inequality Eq. (2.18) is saturated, which seemed to be far from being the case.

Another sufficient set of assumptions would be Eq. (4.9) together with

$$\alpha_p(0) - \alpha_g(0) = \frac{1}{8 C_{20}} \quad (4.10)$$

The latter is equivalent to Eq. (3.5) being saturated, which also was not the case, when inserting experimental numbers. But there is also a deeper difficulty with these assumptions : as Eqs. (2.18) and (3.5) were essentially uncertainty relations between canonically conjugate quantities, they can only be saturated if the joint transfer distribution is Gaussian. This would then imply that all cumulants except c_{02} and c_{20} vanish (i.e., also c_{22}), and hence we could at best obtain the bound $\alpha'_p - \alpha'_p \leq 0$. Otherwise stated, the bound (4.4) is not optimal (unless $c_{22} = 0$). Therefore it is not possible to estimate the functional dependence of $A_{\Delta q}(s,t)/A_0(s,0)$ from the functional dependence of the bound.

In spite of these negative remarks, the correlations between charge and transverse momentum transfers is an interesting quantity by its own. Therefore we think it interesting that a rough estimate of c_{22} can be made with present data. While it seems premature to quote numbers, we shall show below that $c_{22} > 0$. This is a final indication that the difference between Pomeron and Reggeon slopes cannot be explained along the above lines. Its most probable explanation is that signature factors play a larger rôle in the charge exchange overlap, implying there a phase mismatch off the forward direction ²⁰⁾.

In detail, we can write

$$c_{22} = \frac{4}{Y} \iint_{y_1 < y_2} dy_1 dy_2 \iint_{y_3 < y_4} dy_3 dy_4 \left\{ \langle K_2(y_1) K_2(y_2) Q(y_3) \cdot \right. \\ \left. \cdot Q(y_4) \rangle - \langle K_2(y_1) K_2(y_2) \rangle \langle Q(y_3) Q(y_4) \rangle \right\}. \quad (4.11)$$

All operators are here defined in terms of integrals from y_i to $+\infty$, for instance [see Eq. (2.26)]

$$K_2(y_2) = \int_{y_2}^{\infty} dy \, p_2(y) \quad (4.12)$$

with

$$p_2(y) = \sum_c \int d^2 p_{\perp} \, a_c^\dagger(\bar{p}_{\perp}, y) \, p_2 \, a_c(\bar{p}_{\perp}, y) \quad (4.13)$$

Now we use the conservation laws and the fact that $K_2(y_2)$ and $Q(y_1)$ commute with each others. Since $T|ab\rangle$ is a momentum eigenstate with no transverse momentum, we can recast Eq. (4.12) into

$$\langle \dots K_2(y_2) \dots \rangle = - \int_{-\infty}^{y_2} dy \sum_c \langle \dots p_2(y) \dots \rangle \quad (4.14)$$

The same trick can be done with the charge transfer $Q(y_4)$, and after interchanging the orders of integration we get

$$c_{22} = \frac{1}{4Y} \int_{-\infty}^{+\infty} \int \int \int dy_1 dy_2 dy_3 dy_4 (y_1 - y_2)^2 (y_3 - y_4)^2 \cdot \left\{ \langle p_2(y_1) p_2(y_2) q(y_3) q(y_4) \rangle - \langle p_2(y_1) p_2(y_2) \rangle \langle q(y_3) q(y_4) \rangle \right\}, \quad (4.15)$$

where $q(y)$ is defined similarly to $p_2(y)$ as

$$q(y) = \sum_c \int d\vec{p}_\perp a_c^\dagger(\vec{p}_\perp, y) q_c a_c(\vec{p}_\perp, y) \quad (4.16)$$

When normal-ordering the creation and annihilation operators in Eq. (4.15) to get inclusive densities, we get a large number of terms, many of which, however, drop out due to the factors $(y_1 - y_2)^2 (y_3 - y_4)^2$. Still, there remain terms which involve two-, three- and four-body densities. To get our final result, we assume that all inclusive three- and four-body correlations can be neglected. This leads then to

$$c_{22} \cong \frac{1}{4Y} \int \int dy_1 dy_2 (y_1 - y_2)^4 \langle (q \vec{p}_\perp)_{y_1} (q \vec{p}_\perp)_{y_2} \rangle \quad (4.17)$$

with

$$\begin{aligned} \langle (q \vec{p}_\perp)_{y_1} (q \vec{p}_\perp)_{y_2} \rangle &= \iint d\vec{p}_\perp d\vec{p}'_\perp \sum_{c,d} q_c q_d \vec{p}_\perp \cdot \vec{p}'_\perp \cdot \\ \cdot C_{cd}(\vec{p}_\perp, y_1; \vec{p}'_\perp, y_2) &= \langle \vec{p}_\perp(y_1) \cdot \vec{p}_\perp(y_2) \rangle_{++} - \langle \vec{p}_\perp(y_1) \cdot \vec{p}_\perp(y_2) \rangle_{+-} \\ &\quad - \langle \vec{p}_\perp(y_1) \cdot \vec{p}_\perp(y_2) \rangle_{-+} + \langle \vec{p}_\perp(y_1) \cdot \vec{p}_\perp(y_2) \rangle_{--} \end{aligned} \quad (4.18)$$

Data on combined charge and azimuthal correlations exist from Dubna and FNAL ¹⁵⁾. Their most striking feature is that azimuthal correlations are charge-independent for large rapidity distance, while at small distance they are stronger for unequal charges than for equal ones :

$$\langle \vec{p}_1(y_1) \cdot \vec{p}_1(y_2) \rangle_{+-} < \langle \vec{p}_1(y_1) \cdot \vec{p}_1(y_2) \rangle_{++} < 0. \quad (4.19)$$

Consequently we find $c_{22} > 0$, as already mentioned.

Finally, we might point out that Eq. (4.18) describes just the $p_{\perp}$ correlations within the central $I=1$ exchange component in a Mueller inclusive description. The sign we got is what one would naively expect if $p_{\perp}$ were conserved separately within the $I=1$ and $I=0$ components.

5. - CONCLUSION

The main concern of the present paper was to give a simple and nevertheless exact derivation of a bound on the elastic forward slope in terms of inclusive scattering data. A very similar bound has previously been derived using the notion of local compensation of $p_{\perp}$, which is equivalent both to short-range azimuthal correlations and to limited $p_{\perp}$ transfer. In our derivation, this assumption was not at all necessary. But only if it holds, the bound requires an at least logarithmically shrinking elastic peak as, e.g., predicted by a simple Pomeron pole. The small slope of the Pomeron trajectory is directly related to the recently observed long range of azimuthal correlations. One should, however, be careful : present crude data on $p_{\perp}$ correlations seem to indicate that $p_{\perp}$ is not locally enough compensated even to explain the small shrinkage of the elastic peak. This would mean that other effects, as, e.g., phases of the production amplitudes, are important in realistic overlap calculations.

The latter seems definitely to be the case for charge exchange. The boundedness of charge transfer in multibody events accounts for part - but not all - of the suppression of charge exchange versus elastic amplitudes.

The remaining suppression is most probably due to the fact that there exist interference or spill-over graphs which do not contribute to charge exchange for dynamical reasons. In addition, not only the transfer of I_3 but also the one of $\vec{I}^2$ should be limited, which might imply a further suppression. Both of these effects are hard to take into account in a model-independent way.

It was recently suggested that overlap calculations for combined charge and momentum transfer could explain the difference between Pomeron and meson slopes directly from experimental data. This would be the case if certain inequalities such as the above bounds are saturated. We found that not only present preliminary data are in disagreement with this, but also that such saturation is also impossible in principle, if $\alpha'_p > \alpha'_P$. Therefore other explanations are needed for the latter.

The fact that limited quantum number transfer gives rise to Regge behaviour should not be too surprising, as it was more or less the basic idea in the multiperipheral model. The MPM with low particle density and non-exotic exchanges is the prototype of local compensation models. The large average multiplicity, the large momentum transfer and the presence of diffractive production, imply, however, that the MPM has to be modified in a way which is not entirely obvious. The virtue of the methods employed in the present paper is that they are still applicable in a model-independent way.

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where the indices are arranged in the above order. The solutions of

$$\det (\hat{K} + \text{const} - \alpha) = 0$$

are given in Eq. (3.10).

If we include interference terms, K is a 6×6 matrix with $(i,j) = (+,+), (-,-), (0_v, 0_v), (0_s, 0_s), (0_v, 0_s)$ and $(0_s, 0_v)$. The phases of the interference terms are not uniquely defined. Taking them real and ordering the indices in the above way, we get

$$K = g^2 \begin{pmatrix} 1 & 0 & 1 & 1 & 1 & -1 \\ 0 & 1 & 1 & 1 & -1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \quad (\text{A.6})$$

The solutions of Eq. (A.2) with this K are given by Eqs. (3.10) and (3.11). The same result would have been obtained by considering the process with total exchange of one unit of charge.

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FIGURE CAPTIONS

- Figure 1 Contribution to the overlap function for a process with quantum number q exchange. This contribution will be the stronger damped, the narrower the transfer distribution is.
- Figure 2a Multiperipheral graph with isospin $I=0$ and $I=1$ interference. Notice that such graphs do not contribute to elastic scattering but only to $I=1$ exchange.
- Figure 2b Multiperipheral cluster production.
- Figure 2c Contribution to the overlap function where particles spill over from one cluster to another.

FIG.1

FIG. 2a

FIG. 2b

FIG. 2c