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CONTRIBUTION TO THE MUON ANOMALY
FROM A SET OF EIGHTH ORDER DIAGRAMS

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A B S T R A C T

The value of the 18 diagrams of eighth order, containing second order vacuum polarization and scattering of light-by-light insertions is calculated, giving a value of $(111.1 \pm 8.1) \times (\alpha/\pi)^4$. It agrees with the estimate given formerly by Lautrup.

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Three years ago, Lautrup ¹⁾ has given an estimate of a dominant gauge invariant set of diagrams in eighth order radiative corrections to the anomaly of the muon. He used for this purpose an approximate formula given by Kinoshita ²⁾, who replaces the second order electron vacuum polarization in a virtual photon of momentum K^2 by its asymptotic value

$$VP_e^{as.} = \left(\frac{2}{3} \log \left(c \frac{m_\mu}{m_e} \right) - \frac{5}{3} \right) \frac{\alpha}{\pi}$$

where c depends on the diagram considered.

This set predominates for two reasons :

- 1) the sixth order correction $a^{(6)}$ to the anomaly is much larger than the other contributions [roughly $20 \times (\alpha/\pi)^6$ ³⁾];
- 2) all contributions coming from vacuum polarization insertions are of the same sign as the basic diagram in which they are inserted.

Depending on the value given to c , his estimate should be contained between $120 \times (\alpha/\pi)^4$ ($\log c = -5/4$) and $166(\alpha/\pi)^4$ ($\log c = 0$).

The first number $120(\alpha/\pi)^4$ corresponds to the value $\log c = -5/4$ which seems to give the best estimate for the majority of the fourth order diagrams in which an electron loop is inserted into a photon. This value is called by the author of Ref. 1) the "magic number".

A numerical calculation has been performed by the authors of this letter. The result is

$$(111. \pm 8.) \times \left(\frac{\alpha}{\pi} \right)^4 = (3.2 \pm 0.2) \times 10^{-9}$$

As already pointed out in 1), this gauge invariant contribution is of the same order of magnitude as many sixth order diagrams, as the uncertainty of the baryon's vacuum polarization contribution and as weak interaction correction. Additionally, we point out that we are at the limit of validity of an expansion in α/π for this process.

The numerical integration has been performed with the subroutine RIWIAD ⁴⁾ with a first preset of iterations with 200 000 hypercubes, then a second preset of 10 iterations with 1 000 000 hypercubes and finally the run with 8 iterations with one million hypercubes for which the adjustment of the intervals on each axes starts to stabilize. Of course, RIWIAD, though being the best method at hand nowadays has many drawbacks : systematic errors which tend to give too small a result in the integration of powers of logarithms, saturation in the log tails, etc. Nevertheless, one has now a calculated number, not any longer a guess for this set of diagrams.

Finally, one may observe that the value obtained is somewhat smaller than the one guessed. Physically, one can make the following comment : if the diagram in which the insertion is made contains only muons, it is clear that Kinoshita's approach sees really the asymptotic part of the electron vacuum polarization, the momenta K_{as} being of the order of the muon mass. These momenta dominate the whole effect as it is well known. Here, one has to deal with effects where already an electron loop (γ - γ scattering) is inserted. So the momenta of the order $K \sim m_e$ contribute as well and one does not see as clearly the asymptotic part of the electron vacuum polarization as in the previous case. One may thus expect the effect of the insertion to be less pronounced (analogy with Lenz's law for induction in classical electrodynamics). It is therefore not so surprising to find a depression of the contribution.

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