Ref.TH.1997-CERN

NUCLEON STRUCTURE EFFECTS IN PION-NUCLEON BREMSSTRAHLUNG

Ch. Picciotto *)
CERN -- Geneva

A B S T R A C T

Cross-sections for the process $\pi^- p \rightarrow \pi^- p \gamma$ are studied in a region which should be quite sensitive to off-shell effects of the nucleon electromagnetic vertex. Comparison with a recent experiment at 294 MeV shows that, while good agreement is obtained in general for photons of energy up to 50 MeV, for higher photon energies the assumption of threshold dominance in a dispersion relation calculation of the form factors yields results which are incompatible with the data. As with NN bremsstrahlung, it is found that the data favour abandoning the threshold dominance assumption for one of the form factors.

*) On leave from the University of Victoria, British Columbia, Canada.

Ref.TH.1997-CERN

20 March 1975

1. - INTRODUCTION

The past few years have seen several attempts to use bremsstrahlung in the study of the properties of hadron systems. For example, the process $NN \rightarrow NN\gamma$ has been used in an effort to narrow down the choice of potentials which describe the NN interaction ¹⁾, and the process $\pi N \rightarrow \pi N\gamma$ has been proposed as a way to study the electromagnetic properties of the $\Delta(\frac{3}{2}, \frac{3}{2})$ resonance ^{2),3)}. These processes in general have a nucleon (or a pion) emitting a photon before and after the strong interaction, and thus one of the particles involved in the strong process is off the mass shell. An opportunity is thus gained to study the strong amplitude off shell, provided the electromagnetic interaction with one nucleon off shell is understood properly. Calculations performed for NN bremsstrahlung with the on-shell electromagnetic vertex have given good agreement with experimental data ¹⁾. However, there is no reason to expect that the on-shell vertex will give correct results for large photon energies (of the order of one pion mass), and it is at large photon energies that one can hope to extract useful information about the strong interaction.

A study of the electromagnetic vertex with one nucleon off shell was carried out by Nyman ⁴⁾ using dispersion relations in the nucleon mass. Numerical results were obtained for the form factors. The results were applied to proton-proton bremsstrahlung ⁵⁾, where the protons had a laboratory energy of the order of 100 MeV and the intermediate protons could get up to 50 MeV off the mass shell. It was found that the assumption of threshold dominance in the dispersion relations for both form factors gave results which were incompatible with the data, except at very low photon energies. Abandoning the threshold dominance assumption, agreement with the data could be obtained by letting that form factor assume a static value of the order of the proton anomalous magnetic moment.

This additional uncertainty is clearly a potential source of difficulty in future study of radiative processes. It is thus useful to carry out a similar calculation with other bremsstrahlung processes to see whether results similar to Nyman's are obtained. A recent experiment ⁶⁾ has looked at the process $\pi N \rightarrow \pi N\gamma$ at a pion laboratory energy of 294 MeV. Photons of up to 120 MeV were detected. The nucleons were thus approximately 100 MeV off shell, and this region should be quite sensitive to off-shell effects of the electromagnetic vertex. Comparison with theoretical models

for the process $\pi^+ p \rightarrow \pi^+ p \gamma$ has so far been rather problematic. There are various reasons for this. First, the contribution to the amplitude from the graphs in which the external protons and pions radiate is far from being gauge invariant, and there is no obvious unambiguous way of finding gauge counter-terms ⁷⁾. Second, radiation from the internal structure, dominated by the $\Delta(\frac{3}{2}, \frac{3}{2})$ resonance, must be taken into account properly ^{2), 8)}. Third, the geometry in the experiment has been chosen in such a way that graphs in which external particles radiate interfere destructively ⁶⁾, reducing their contribution by about a factor of ten. Thus, any variation among the various strong amplitudes involved causes serious fluctuations in the resulting total amplitude. Finally, we must include uncertainties in off shell behaviour. The process $\pi^- p \rightarrow \pi^- p \gamma$ does not suffer from the first three problems. The contribution from external graphs is nearly gauge invariant and it is straightforward to find a gauge-invariant expression. The internal resonance Δ^0 presumably does not radiate because it carries no charge and [SU(6) prediction] no magnetic moment. Also, interference among the various graphs for this particular geometry is constructive. This process is thus an ideal one to test possible off shell contributions. Any insight gained will be very useful in attempts to resolve the $\pi^+ p \rightarrow \pi^+ p \gamma$ problem.

2. - RADIATIVE AMPLITUDE FOR $\pi^- p \rightarrow \pi^- p \gamma$

As previously mentioned, the intermediate Δ^0 is not expected to contribute meaningfully to the radiative amplitude. The process is thus dominated by the graphs shown in Fig. 1. The radiative matrix element for the graphs is given by

$$\begin{aligned} \frac{1}{e} M_\mu \epsilon^\mu = & - \frac{q' \cdot \epsilon}{q' \cdot k} T_a + \frac{q \cdot \epsilon}{q \cdot k} T_b + \Gamma_\mu \epsilon^\mu \frac{\cancel{p} + \cancel{k} + m}{2p' \cdot k} T_c \\ & - T_d \frac{\cancel{p} - \cancel{k} + m}{2p \cdot k} \Gamma_\mu \epsilon^\mu \end{aligned} \quad (2.1)$$

where q, q', p, p' are the momenta of the initial and final pions and protons, respectively, k, ϵ are the photon momentum and polarization, respectively, m is the nucleon mass, and

$$\begin{aligned} s_f &= (q' + p')^2 \\ s_i &= (q + p)^2 \\ t_1 &= (q' - q)^2 \\ t_2 &= (p' - p)^2 \end{aligned}$$

The strong πN amplitudes T are given by

$$\begin{aligned} T_a &= A(s_f, t_1) + \frac{1}{2} (q' + q - k) B(s_f, t_1) \\ T_b &= A(s_f, t_2) + \frac{1}{2} (q' + q + k) B(s_f, t_2) \\ T_c &= A(s_i, t_2) + \frac{1}{2} (q' + q) B(s_i, t_2) \\ T_d &= A(s_i, t_1) + \frac{1}{2} (q' - q) B(s_i, t_1) \end{aligned} \quad (2.2)$$

$A(s, t)$ and $B(s, t)$ being the invariant amplitudes. On shell the nucleon electromagnetic vertex is given by

$$\Gamma_\mu = \gamma_\mu - \frac{i}{2m} \lambda^+ \sigma_{\mu\nu} k^\nu \quad (2.3)$$

where λ^+ is the nucleon anomalous magnetic moment. The strong amplitudes have been taken on the mass shell. This approximation is justified since the mass dependence is expected to be small⁹⁾, and gauge invariance forces any such contribution to be of second order¹⁰⁾. To see that this expression is nearly gauge invariant, it is possible to rewrite Eq. (2.1) in such a way that charge radiation is exhibited explicitly (the magnetic moment contributions are independently gauge invariant) :

$$\begin{aligned} \frac{1}{c} M_\mu &= -\frac{q'_\mu}{q' \cdot k} T_a + \frac{q_\mu}{q \cdot k} T_b + \frac{p'_\mu}{p' \cdot k} T_c - \frac{p_\mu}{p \cdot k} T_d \\ &+ \frac{\gamma_\mu k}{2 p' \cdot k} T_c + T_d \frac{k}{2 p \cdot k} \gamma_\mu \end{aligned} \quad (2.4)$$

Gauge invariance can be obtained simply by adding the counter-term $\frac{1}{2} \kappa [B(s_f, t_2) + B(s_i, t_2)]$ and by evaluating the strong amplitudes at the same $\bar{t} = (t_1 + t_2)/2$. Although the last step does change the value of the strong amplitudes, the final cross-section changes only for the largest values of photon energy and never by more than ~25%. The amplitude thus becomes

$$\begin{aligned}
 \frac{1}{e} M_\mu \varepsilon^\mu = & - \frac{q' \cdot \varepsilon}{q' \cdot k} [A(s_i, \bar{t}) + \gamma_2 (\not{q} + \not{q}' - \not{k}) B(s_i, \bar{t})] + \\
 & + \frac{q \cdot \varepsilon}{q \cdot k} [A(s_f, \bar{t}) + \gamma_2 (\not{q} + \not{q}' + \not{k}) B(s_f, \bar{t})] + \\
 & + \Gamma_\mu \varepsilon^\mu \frac{p' + k + m}{2 p' \cdot k} [A(s_i, \bar{t}) + \gamma_2 (\not{q} + \not{q}') B(s_i, \bar{t})] + \\
 & + [A(s_f, \bar{t}) + \gamma_2 (\not{q} + \not{q}') B(s_f, \bar{t})] \frac{\not{p} - \not{k} + m}{2 p \cdot k} \Gamma_\mu \varepsilon^\mu + \\
 & + \gamma_2 \not{\varepsilon} [B(s_f, \bar{t}) + B(s_i, \bar{t})] .
 \end{aligned} \tag{2.5}$$

It is now clear that $M_\mu k^\mu = 0$. Further details of the evaluation of the cross-section will be found in the Appendix.

3. - OFF-SHELL ELECTROMAGNETIC VERTEX

For the case where the initial proton is virtual, the vertex Γ_μ can be taken to have the following form ¹¹⁾:

$$\begin{aligned}
 \bar{u}(p, s) \Gamma_\mu = & \bar{u}(p, s) \left\{ \gamma_\mu - i \frac{\sigma_{\mu\nu} k^\nu}{2m} \left[F_2^+(W^2) \frac{\not{p} + m}{2m} \right. \right. \\
 & \left. \left. + F_2^-(W^2) \frac{-\not{p} + m}{2m} \right] \right\}
 \end{aligned} \tag{3.1}$$

where P is the momentum of the off-shell proton, and $W^2 = P^2$ is the square of its mass. In the limit $W \rightarrow m$ we have

$$\begin{aligned} F_2^+(m^2) &= \lambda^+ \\ F_2^-(m^2) &= \lambda^- \end{aligned} \quad (3.2)$$

Since the coefficient of F_2^- vanishes on-shell through the Dirac equation, λ^- is an unknown constant. As a consequence of the Ward identity¹²⁾, charge coupling has no form factor for the nucleon or pion off-shell and real photon.

F_2^+ and F_2^- have the expansions

$$F_2^\pm(W^2) = \lambda^\pm [1 + E/\epsilon_0 + O(\epsilon^2)] \quad (3.3)$$

where $W = m + E$. They satisfy a dispersion relation in W^2 , with a cut starting at $W^2 = (m + \mu)^2$, where μ is the pion mass¹¹⁾. Assuming threshold dominance, Nyman⁴⁾ has found numerical values for F_2^+ . Assuming threshold dominance also in the dispersion relation for F_2^- , a simple scaling law is found between the two form factors :

$$F_2^-(W^2) \simeq \frac{2m}{\mu} F_2^+(W^2) \quad (3.4)$$

Without this assumption of threshold dominance, the relation between them is

$$F_2^-(W^2) = \lambda^- + \frac{2m}{\mu} [F_2^+(W^2) - \lambda^+]. \quad (3.5)$$

The value $\lambda^- = 24$ yields the simple scaling law of Eq. (3.4). It was found⁵⁾ that the value $\lambda^- = 24$ applied to pp bremsstrahlung at 99 MeV yields results which are incompatible with the experimental data. Instead, the value $\lambda^- \approx \lambda^+$ gives results approximately equal to the cross-section with the on-shell vertex, and any value of λ^- in the range $-6 \lesssim \lambda^- \lesssim 8$ is consistent with the data. In evaluating these results, one must keep in mind that the assumption of threshold dominance has been used with reasonable results in various calculations¹³⁾⁻¹⁶⁾.

4. - CALCULATION AND DISCUSSION

The calculations for the $\pi^- p \rightarrow \pi^- p \gamma$ cross-section were performed for the geometry of the experiment described in Ref. 6). The energy of the incident pions was 294 MeV. The quantity $d\sigma/d\Omega_\pi d\Omega_\gamma dk$ was measured for photon energies up to 120 MeV. The measurements were averaged over a range of photon and pion angles given by $-43.5^\circ \geq \theta_\pi \geq -56.5^\circ$, $-10^\circ \leq \theta_\gamma \leq 22^\circ$, $180^\circ \geq \phi_\gamma \geq 120^\circ$, $-36^\circ \leq \phi_\pi \leq 0^\circ$. Details of the calculation are given in the Appendix.

The data and the results of the calculations are given in Fig. 2. All results agree well with the data and with a calculation based on the soft photon theorem ¹⁷⁾ up to photon energies of about 50 MeV. At higher photon energies, it is clear that the value $\lambda^- = 24$, corresponding to threshold dominance in the F_2^- dispersion relation, gives results incompatible with the data. The calculation with the nucleon on shell at the electromagnetic vertex gives a much better fit to the data, and still better fits can be obtained with negative values of λ^- . Figure 2 shows a fit with $\lambda^- = -14$, but a large range $\lambda^+ \gtrsim \lambda^- \gtrsim -20$ give reasonable fits. The figure also shows the curve given by $d\sigma/d\tilde{\Omega}_\gamma d\tilde{\Omega}_\pi d\tilde{k} = \sigma_0/\tilde{k}$, where the tilde denotes centre-of-mass parameters and the constant σ_0 is given by the limit $\tilde{k} \rightarrow 0$. This curve was taken from Ref. 6).

From the results we can conclude that efforts to extract the Δ magnetic moment from a fit to the $\pi^+ p$ radiative scattering data cannot ignore the off-shell effects in the nucleon electromagnetic vertex. Although exchange currents and off-shell effects in the πN amplitude have been ignored here, it is very unlikely that they would contribute sufficiently to correct the threshold dominance fit.

The author is grateful to Dr. G. Preparata for a critical reading of the manuscript.

APPENDIX

The parametrization used for the invariant πN amplitudes is

$$\begin{aligned}\frac{1}{4\pi} A &= \frac{\tilde{W}+m}{\tilde{E}+m} f_1 - \frac{\tilde{W}+m}{\tilde{E}-m} f_2 \\ \frac{1}{4\pi} B &= \frac{1}{\tilde{E}+m} f_1 + \frac{1}{\tilde{E}-m} f_2 \\ f_1(\theta) &= \sum_{l=0}^{\infty} [f_{l+} P'_{l+}(\cos\theta) - f_{l-} P'_{l-}(\cos\theta)] \\ f_2(\theta) &= \sum_{l=1}^{\infty} [f_{l-} - f_{l+}] P'_l(\cos\theta) \\ f_{l\pm} &= \frac{e^{2i\delta_{l\pm}} - 1}{2i\tilde{q}}\end{aligned}\tag{A.1}$$

$\tilde{W}$, $\tilde{E}$ and $\tilde{q}$ are the total energy and the nucleon energy and momentum, respectively, in the centre-of-mass system. S, P and D phase shifts¹⁸⁾ were taken into account. The usual isotopic spin combinations were used :

$$\begin{aligned}A_{\pi^-p} &= \frac{1}{3} A^{3/2} + \frac{2}{3} A^{1/2} \\ B_{\pi^-p} &= \frac{1}{3} B^{3/2} + \frac{2}{3} B^{1/2}\end{aligned}\tag{A.2}$$

The radiation cross-section in the laboratory frame is given by

$$\begin{aligned}\frac{d\sigma}{d\Omega_\pi d\Omega_\gamma dk} &= \frac{1}{8(2\pi)^5} \frac{m}{q} \frac{q'^3 k}{q' E - \omega' \vec{q}' \cdot \vec{p}'} \\ &\times \frac{1}{2} \sum |\langle \bar{u}(p', s') | M_\mu \epsilon^\mu | u(p, s) \rangle|^2\end{aligned}\tag{A.3}$$

where ω' is the final pion energy. The last term represents the average over initial and the sum over final nucleon spins and photon polarization. In the cross-section calculations, Eq. (A.3) was averaged over the geometry of Ref. 6).

REFERENCES

- 1) See :
E.M. Nyman - Physics Reports 3, 179 (1974), for a review of NN bremsstrahlung.
- 2) W.E. Fischer and P. Minkowski - Nuclear Phys. B36, 519 (1972).
- 3) L.A. Kondratyuk and L.A. Ponomarev - Yad.Fiz 7, 111 (1968) ; Soviet J. Nuclear Phys. 7, 82 (1968) ;
V.L. Zakharov, L.A. Kondratyuk and L.A. Ponomarev - Yad.Fiz. 8, 783 (1968) ; Soviet J.Nuclear Phys. 8, 456 (1968).
- 4) E.M. Nyman - Nuclear Phys. A154, 97 (1970).
- 5) E.M. Nyman - Nuclear Phys. A160, 517 (1971).
- 6) M. Arman et al. - Phys.Rev.Letters 29, 962 (1972) ; Phys.Rev.D., to be published.
- 7) C. Picciotto - CERN Preprint TH. 1945 (1974) ; Nuclear Phys.B., to be published.
- 8) D. Beder - Nuclear Phys. B84, 362 (1975).
- 9) F. Selleri - Nuovo Cimento 39, 1122 (1965) ; 40, 236 (1965).
- 10) H. Feshbach and D.R. Yennie - Nuclear Phys. 37, 150 (1962).
- 11) A.M. Bincer - Phys.Rev. 112, 855 (1958).
- 12) J.C. Ward - Phys.Rev. 77, 293 (1950) ; 78, 182 (1950) ;
Y. Takahashi - Nuovo Cimento 6, 371 (1957).
- 13) S.D. Drell and H. Pagels - Phys.Rev. 140, B397 (1965).
- 14) S.D. Drell and D. Silvermann - Phys.Rev.Letters 20, 1325 (1968).
- 15) H. Pagels - Phys.Rev. 144, 1261 (1966).
- 16) M.G. Hare and G. Papini - Phys.Rev. D4, 684 (1971) ; Nuclear Phys. B34, 200 (1971).
- 17) F.E. Low - Phys.Rev. 110, 974 (1958).
- 18) D.J. Hendon et al. - CERN-Kirsopp solution, UCRL-20030 πN, Berkeley, California (1970).

FIGURE CAPTIONS

Figure 1 Graphs corresponding to radiation by external particles.

Figure 2 π^-p bremsstrahlung for 294 MeV pions as a function of photon energy. The geometry and the data are those of Ref. 6). The curves show predictions for the off-shell electromagnetic vertex of Ref. 4) (dashed line), the on-shell vertex (dash-dot line), and an off-shell vertex with $\lambda^- = -14$ (solid line). The open-circles curve corresponds to $d\sigma/d\tilde{\Omega}_\pi d\tilde{\Omega}_\gamma d\tilde{k} = \sigma_0/\tilde{k}$, and was taken from Ref. 6).

- Figure 1 -

- Figure 2 -