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ON THE CONTRIBUTION OF QUANTUM ELECTRODYNAMICS
TO THE ANOMALY OF THE MUON

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ABSTRACT

Quantum electrodynamics contributions to the anomalous magnetic moment of the muon are summarized using the latest available theoretical results. At sixth order they amount to:

$$(a_{\mu} - a_e)_{\text{QED}}^{(6)} = (21.73 \pm 0.16) \left(\frac{\alpha}{\pi}\right)^3.$$

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Time has now come, owing to the fast increase in the precision of the experimental value of the muon anomaly ¹⁾ to give a short summary of the quantum electrodynamics contribution to the theoretical value. We divide this contribution into two parts, according to the power in α .

1) α^2 contribution

This one is well known and we refer to two review papers for references ^{2),3)}. Taking the Josephson value ⁴⁾ for α^{-1} , i.e., $\alpha^{-1} = 137.03608(26)$ it yields

$$(a_\mu - a_e)^{(4)} = 5904.1 \times 10^{-9}$$

2) α^3 contribution

Following Ref. 2), we divide into three classes the 24 graphs contributing to this order (see Figure).

a) Class I: 6 graphs containing scattering of light-by-light subgraphs.

Their contribution is :

$$\begin{aligned} (a_\mu - a_e)_I^{(6)} &= (19.79 \pm 0.16) \left(\frac{\alpha}{\pi}\right)^3 \\ &= (247 \pm 2) \times 10^{-9} \end{aligned}$$

This result is slightly different from the previously published one ⁵⁾. The numerical integrations are performed with the routine RIWIAD ⁶⁾. In some cases it is possible to show that this routine underestimates the integral calculated (powers of logarithms for instance ^{*}). However, no corrections are done here on the results given by RIWIAD. Several runs were made up to 2×10^6 hypercubes, and the weighted value and error taken. An improved estimation of the coefficient of $\log(m_e^2/m_\mu^2)$ has also been performed. It gives

$$C = -3.145 \pm 0.028$$

*) This type of correction was taken into account to get the value quoted in Ref. 3).

- b) Class II : 14 graphs containing a second order electron-positron loop insertion. The contribution for this class is known analytically ⁷⁾

$$\begin{aligned} (a_\mu - a_e)_{\text{II}}^{(6)} &= -2.30 \left(\frac{\alpha}{\pi}\right)^3 \\ &= -29 \times 10^{-9} \end{aligned}$$

- c) Class III : 4 graphs containing fourth-order electron vacuum polarization subgraphs. Their contribution has been known exactly for a long time ^{2),3)}

$$(a_\mu - a_e)_{\text{III}}^{(6)} = 4.24 \left(\frac{\alpha}{\pi}\right)^3 = 53 \times 10^{-9}$$

The total sixth-order QED contribution to the difference $(a_\mu - a_e)$ is :

$$(a_\mu - a_e)_{\text{Q.E.D.}}^{(6)} = (21.73 \pm 0.16) \left(\frac{\alpha}{\pi}\right)^3 = (271 \pm 2) \times 10^{-9}$$

If one considers the following value for the electron anomaly ⁸⁾

$$a_e^{(6)} = (1.19 \pm 0.03) \left(\frac{\alpha}{\pi}\right)^3$$

one gets finally for the QED contribution to the muon anomaly :

$$a_\mu = (1165827 \pm 3) \times 10^{-9} \text{ Q.E.D.}$$

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FIGURE CAPTION

Graphs contributing to the difference $(a_{\mu} - a_e)$ at sixth order.

*) Notice that in this Ref. α^{-1} has been chosen as $\alpha^{-1} = 13703604(11)$.