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PERTURBATION THEORY AND RENORMALIZATION OF SUPERSYMMETRIC
YANG-MILLS THEORIES

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A B S T R A C T

We solve the problem of introducing, in supersymmetric Yang-Mills Lagrangians, a supersymmetric gauge breaking term and a Faddeev-Popov ghost interaction term. The resulting Lagrangian turns out to be invariant under a global symmetry transformation which is the supersymmetric extension of the Slavnov symmetry. We show that the complete analysis of all primitively divergent supergraphs ensures, in conjunction with the Slavnov identities, the renormalizability of the theory, once a supersymmetric and gauge invariant regularizing procedure has been introduced. We find that the simplest regularizing procedure is a generalization of the higher covariant derivatives method. In the case of interaction with matter fields we prove that no mass counter term is needed, in exact analogy with the model without gauge fields. Finally we show that, in the Abelian situation, a supersymmetric mass term for the vector multiplet can be introduced without spoiling the renormalizability, thus providing the supersymmetric extension of massive vector bosons theories.

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1. INTRODUCTION

Recently it has been shown that a wide class of Lagrangian models exist which are invariant under supersymmetry transformations [1-5].

In the simplest of these models, the cubic self-interaction of a chiral multiplet, it has been shown not only that supersymmetry is preserved by the renormalization to all orders in perturbation theory, but also that only an infinite constant (a wave-function renormalization) is sufficient to renormalize the theory [6]. This latter remarkable property has been shown to arise from compensations of divergences among different Feynman diagrams [1,6]. Using the powerful technique of superfields [7,8], which is nothing but the tensor calculus for supersymmetry, it has been clarified later [9] why such a cancellation of divergences occurs. These compensations of divergences among different graphs of field components of the same multiplet turn out to be related to the fact that power-counting for supergraphs is rather different from the usual one for individual component diagrams.

Examples of Lagrangian models have been built consistently, in order to combine supersymmetry with local symmetries [2-4]. The Abelian case, which may be considered as an extension of Q.E.D., was constructed by Wess and Zumino [2]. Salam and Strathdee [4] and Zumino and one of the present authors [3] found the extension to the non-Abelian case, i.e. to a supersymmetric Yang-Mills theory. Such supersymmetric gauge theories look renormalizable by power-counting only if one considers a particular class of gauge transformations, namely those gauges where some of the unphysical components of the vector supermultiplet vanish [2]. With such a non-covariant gauge [Wess-Zumino gauge]^{*)}, consistency of supersymmetry with gauge invariance and renormalization has been shown up to one loop [2], but it would be extremely difficult to verify in higher orders. These classes of gauges fix a particular supersymmetry frame of reference, so it is not at all evident that such theories can be renormalized preserving the original symmetry,

*) We recall that in the Wess-Zumino gauge [2] the component fields C , χ , M , N of the vector multiplet vanish and one is still allowed to make a usual gauge transformation on the vector V_μ . The existence of such a gauge in the non-Abelian theory has been proved in ref. [3].

just because of the non-invariance of such a gauge-fixing term with respect to supersymmetry transformations. These complications are more dramatic in Yang-Mills theories, because of the complication of Faddeev-Popov ghost fields [10]. It is then clear that, in order to try to renormalize the gauge theories, preserving the Ward identities associated with supersymmetry [11], one has to introduce a supersymmetric invariant gauge-breaking term^{*)} and a supersymmetric Faddeev-Popov term in the Lagrangian.

It is the aim of the present paper to solve these problems and to show that supersymmetric gauge theories can be consistently renormalized, provided one assumes that an appropriate supersymmetric and gauge-invariant regularization procedure has been introduced. After having generalized the gauge-breaking term as well as the Faddeev-Popov trick to the supersymmetric Yang-Mills Lagrangians, we derive generalized Ward identities [12] (Slavnov identities) for the connected Green's function functional as well as for the 1PI functional. We show that the supersymmetric gauge-invariant Lagrangian, with the addition of the gauge-breaking term and of the Faddeev-Popov interaction part, is then invariant under a global symmetry transformation, with an anticommuting parameter, the so-called Slavnov symmetry [13]. The Faddeev-Popov multiplet is a chiral anticommuting complex superfield which therefore contains, beyond the conventional bosonic anticommuting fields, also commuting fermionic ghost fields. We derive a general power-counting formula, which allows us to classify all primitively divergent supergraphs of the Yang-Mills theory. In spite of the fact that, at any loop order, there are an infinite number of primitively divergent graphs, we show that, because of the Slavnov symmetry and of the associated Slavnov identities, they are all related to some basic primitively divergent graphs which correspond to a wave-function renormalization and charge renormalization of the Yang-Mills multiplet and to a wave-function renormalization of the ghost fields. In fact, following a parallel

*) In a recent preprint [11], Slavnov carried out a similar analysis for the Abelian model, using a different gauge-breaking term. However, the conclusions of this paper are not complete. In particular, he did not realize that as we shall see, no independent mass counter term δm is needed in such a theory.

argument which applies in conventional Yang-Mills theory [14], the introduction of counter terms which provide such basic diagrams to be finite automatically implies, because of the Slavnov identities, that all other divergent graphs are made finite.

In case of interaction with matter fields only an additional wave-function renormalization for the matter multiplet occurs. In contrast with conventional theories, no additional mass renormalization or self-interaction of the matter multiplet is generated in the supersymmetric Yang-Mills theory. In the Abelian gauge theory, where no charge renormalization occurs, the renormalization is purely multiplicative. We finally show that in the Abelian theory one can also add a supersymmetric mass term for the vector multiplet without spoiling the renormalizability of the theory. Such a model then provides the supersymmetric generalization of the massive vector theory.

The paper is organized as follows. In section 2 we derive the pure Yang-Mills Lagrangian, including the gauge-breaking term and possible interactions with matter fields, and write the corresponding action as an integral over the points of superspace. This will turn out particularly convenient in order to derive Feynman supergraphs with the functional formalism.

In section 3 we show how to introduce, given an arbitrary gauge-breaking term, the Faddeev-Popov ghosts. Then we derive their equations of motion, together with the Slavnov identities and the Slavnov transform. An adaptation of these Slavnov identities for the LPI functional, which is a generalization of the identities derived in ref. [13] is given. Such generalized Ward identities, together with the equations of motion of the Faddeev-Popov ghosts are the basic ingredient to prove [14], in a constructive way, the renormalizability of the supersymmetric Yang-Mills theory. Of course, we already assume from the beginning the existence of a supersymmetric and gauge-invariant regularization procedure. It turns out, as we shall see in section 6, that the simplest regularization procedure one can introduce in supersymmetric gauge theories is the higher covariant derivative method.

In section 4 we consider in detail the Abelian gauge theory, including a mass term for the vector multiplet and derive the selected Ward identities.

In section 5 we give the Feynman rules for the Abelian and non-Abelian gauge theory, using the perturbation expansion for the generating functional of the amputated Green's functions and the explicit form of all propagators of the superfield involved.

In section 6 we obtain a power-counting formula, which provides the superficial degree of divergence of any Feynman supergraph of the theory and discuss the renormalization problem. Finally, we discuss the problem of the regularization. Among the various possibilities, we examine in some detail the one which seems the simplest, even if it gives only a partial result, namely the higher covariant derivative method. We show how such a notion of covariant derivative can be extended to supersymmetric Yang-Mills theories and that, exactly as in conventional Yang-Mills theories, the higher covariant derivatives procedure makes all superdiagrams finite with the exception of one-loop superdiagrams.

Some basic formulae of supersymmetric tensor calculus, needed in the text, are collected in appendix A. In appendix B we give the group properties for the local gauge algebra, which are needed for the group theoretical derivation of the Slavnov identities. In appendix C we report some one-loop calculations and in appendix D we give the explanation in terms of superfields of some results obtained in previous works in the context of supersymmetric theories.

2. YANG-MILLS LAGRANGIAN

In the present section we derive the equations of motion of a self-interacting Yang-Mills multiplet. We strictly follow the conventions and notation of refs. [3] and [8].

We recall that a Yang-Mills supermultiplet is a real multiplet $V_1(x, \theta, \bar{\theta})$, which transforms according to the adjoint representation of the symmetry group G . We will always consider $G = SU(N)$. Under a local gauge transformation the finite transformation of V is given by

$$e^{gV} \rightarrow e^{-ig\Lambda^\dagger} e^{gV} e^{ig\Lambda} \quad (2.1)$$

where Λ is a chiral field ($\bar{D}_\alpha \Lambda = 0$) which also belongs to the adjoint representation and in formula (2.1) matrix multiplication is understood, being

$$V = V_i \lambda^i, \quad \Lambda = \Lambda_i \lambda^i, \quad \Lambda^\dagger = \bar{\Lambda}_i \lambda^i \quad (2.2)$$

where λ_i are the $N^2 - 1$ (Hermitian) generators in the vector representation of $SU(N)$. The $\dagger$ means Hermitian conjugation.

The gauge-invariant part of the Lagrangian (pure Yang-Mills) is the F component of the following chiral multiplet

$$\frac{1}{g^2} T_2 (W^\alpha W_\alpha) \quad (2.3)$$

where

$$W_\alpha = \bar{D} \bar{D} (e^{-gV} D_\alpha e^{gV}) \quad (\bar{D}_\alpha W_\alpha = 0)$$

Then it follows that the gauge-invariant part of the action can be written as

$$\int dS \frac{1}{g^2} T_2 (W^\alpha W_\alpha) + \int d\bar{S} \frac{1}{g^2} T_2 (\bar{W}_\alpha \bar{W}^\alpha) \quad (2.4)$$

where

$$\bar{W}_\alpha = -W_\alpha^\dagger = D D (e^{gV} \bar{D}_\alpha e^{-gV})$$

where the chiral measure is $dS = d^4x d^2\theta = d^4x d\theta^\alpha d\theta_\alpha$ and we used the spinor integration defined as follows [15]

$$\int d\theta_i \theta_j = \delta_{ij}, \quad \int d\theta_i = 0 \quad (2.5)$$

Note that eq. (2.5) implies that

$$\int d^4x d^2\theta \quad (2.6)$$

as can be easily checked.

The complete Yang-Mills Lagrangian will be the sum of several pieces. Namely, the pure gauge-invariant Yang-Mills part given by (2.4), to which one must add the gauge-invariant interaction of the matter fields, a gauge-breaking term, a ghost part, and source terms. For the moment we will consider only the piece (2.4). As in (2.4) V appears only in the form e^{gV} , it is simpler to make the variation with respect to $\Psi = e^{gV}$, in order to derive the equations of motion for the V field.

Using (2.5) and (2.6), we can rewrite (2.4) as

$$I = \frac{1}{g^2} \int dV T_2 (e^{-gV} D^\alpha e^{gV}) W_\alpha + T_2 (e^{gV} \bar{D}_\alpha e^{-gV} \bar{W}^\alpha) \quad (2.7)$$

where $dV = d^4x d^4\theta = dS d^2\bar{\theta} = d\bar{S} d^2\theta$.

Performing the variation one gets

$$\begin{aligned} \delta I &= \frac{2}{g^2} \int dV T_2 (\delta(\psi^{-1} D^\alpha \psi) W_\alpha) + h.c. = \\ &= \frac{1}{g^2} \int dV T_2 (-2 \psi^{-1} \delta \psi [\psi^{-1} D^\alpha \psi W_\alpha + \\ &+ D^\alpha W_\alpha - W^\alpha \psi^{-1} D_\alpha \psi]) + T_2 (2 \delta \psi \psi^{-1} \cdot \\ &\cdot [-\bar{D}_\alpha \psi \psi^{-1} \bar{W}^\alpha + \bar{W}_\alpha \bar{D}^\alpha \psi \psi^{-1} + \bar{D}_\alpha \bar{W}^\alpha]) = \quad (2.8) \\ &= -\frac{2}{g^2} \int dV T_2 \delta \psi \psi^{-1} [D^\alpha (\psi W_\alpha \psi^{-1}) - \\ &- \psi \bar{D}_\alpha (\psi^{-1} \bar{W}^\alpha \psi) \psi^{-1}] = \\ &= -\frac{4}{g^2} \int dV T_2 \delta \psi \psi^{-1} D^\alpha (\psi W_\alpha \psi^{-1}) \end{aligned}$$

because of the dual identity

$$D^\alpha (\psi W_\alpha \psi^{-1}) = -\psi \bar{D}_\alpha (\psi^{-1} \bar{W}^\alpha \psi) \psi^{-1} \quad (2.9)$$

which can be easily verified writing the explicit form of W_α , $\bar{W}_\alpha$ and performing the derivatives. It follows that the Euler-Lagrange equations of motion are simply

$$\frac{1}{g} D^\alpha (e^{gV} W_\alpha e^{-gV}) = 0 \quad (2.10)$$

which are manifestly supersymmetric and gauge-invariant. We now add to the gauge-invariant Lagrangian a gauge-breaking term. In order to quantize the theory, we add as a gauge-breaking term the supersymmetric extension of the usual term $(1/2\alpha)(\partial_\mu v^\mu)^2$ of conventional Yang-Mills theories. It is

$$\frac{2}{\alpha} T_2 \bar{D} \bar{D} V D D V \quad (2.11)$$

In fact the D component of this vector multiplet contains just the usual $(\partial_\mu v^\mu)^2$ term as can be easily checked.

Because of the identity

$$D^\alpha \bar{D} \bar{D} D_\alpha V = \bar{D}_\alpha D D \bar{D}^\alpha V \quad (2.12)$$

one gets for the free part of the Lagrangian the following expression

$$T_2 \left(2 D^\alpha V \bar{D} \bar{D} D_\alpha V + \frac{2}{\alpha} \bar{D} \bar{D} V D D V \right) \quad (2.13)$$

which leads to the following equation of motion

$$\left(D^\alpha \bar{D} \bar{D} D_\alpha - \frac{1}{2\alpha} (D D + \bar{D} \bar{D})^2 \right) V = \text{interaction} + \text{source terms} \quad (2.14)$$

The inverse of the left-hand side of eq. (2.14) will give the propagator of the gauge supermultiplet and will be explicitly worked out in section 3. The interaction with matter field can be introduced in a straightforward way, using the technique of refs. [3] and [4]. The matter field is assumed to be a chiral multiplet S_{+i} ($\bar{D}_\alpha S_{+i} = 0$) which transform according to some representation of $SU(N)$. Under a (finite) local gauge transformation it transforms as a column vector

$$S_{+i} \rightarrow (e^{-ig\Lambda})_{ij} S_{+j} \quad \bar{S}_{+i} \rightarrow \bar{S}_{+j} (e^{ig\Lambda^\dagger})_{ji} \quad (2.15)$$

where Λ is a matrix which corresponds to the generators τ^a of the group in the representation space of S_i : $\Lambda_{ij} = \Lambda^a \tau_{ij}^a$. The quantity

$$\bar{S}_{+i} (e^{igV})_{ij} S_{+j} \quad (2.16)$$

is gauge invariant.

However, as parity exchanges $V \rightarrow -V$ in order to preserve parity, one needs another chiral field S_{-i} , which transforms according to the complex conjugate representation of S_{+i} . Namely

$$\bar{S}_{-i} \rightarrow (e^{-ig\Lambda^\dagger})_{ij} \bar{S}_{-j} \quad S_{-i} \rightarrow S_{-j} (e^{ig\Lambda})_{ji} \quad (2.17)$$

In this way parity exchanges $S_+ \rightarrow \bar{S}_-$ and a gauge-invariant and parity-preserving Lagrangian is given by

$$\frac{1}{2} (\bar{S}_+ e^{gV} S_+ + S_- e^{-gV} \bar{S}_-) \quad (2.18)$$

Another invariant is, of course, the mass term, given by the following chiral field

$$-2m S_+ S_- + h.c. \quad (2.19)$$

As observed in ref. [3], if S belongs to a real representation of $SU(N)$ then $\Lambda^T = -\Lambda$, $V^T = -V$ and one can put $S_+ = S_-$. In this case (2.18) and (2.19) simplify into

$$\bar{S} e^{gV} S \quad (2.20)$$

$$-2m S^2 + h.c. \quad (2.21)$$

respectively.

Note that, in principle, one could add some other renormalizable couplings to the previous Lagrangian, which are both supersymmetric and gauge-invariant. For example, if S belongs to the adjoint representation of $SU(3)$ then

$$g' d^{ijk} S_i S_j S_k + h.c. \quad (2.22)$$

would be a further allowed dimensionless coupling with a new coupling constant g' . Note, however, that such a term cannot be generated by the Yang-Mills coupling because of the discrete symmetry $S \rightarrow -S$ of the Yang-Mills Lagrangian^{*)}.

In order to complete the Yang-Mills Lagrangian one must add source terms for the vector field as well as for the matter fields

$$T_\mu V^\mu + \frac{1}{2} (J_+ S_- + J_- S_+ + \bar{J}_+ \bar{S}_- + \bar{J}_- \bar{S}_+) \quad (2.23)$$

*) We thank Prof. B. Zumino for this observation.

the first term of (2.23) being of course a vector field, while the other terms are chiral.

It is always understood in the following that such superfields give a contribution to the action when integrated over the appropriate measure, namely dV , dS , or $d\bar{S}$.

The Faddeev-Popov part will be introduced in the next section. We will show that, for any gauge term of the form

$$\text{Tr } BB^\dagger \quad (2.24)$$

where B is an arbitrary chiral field of the form $\bar{D}\bar{D}V + \dots$, one can introduce a Faddeev-Popov ghost term in the Lagrangian and derive the related Slavnov-Ward identities and Slavnov transformations. For practical applications it is, of course, convenient to make the previous choice, i.e.

$$B = \bar{D}\bar{D}V \quad (2.25)$$

because in this case the Faddeev-Popov interaction term has a very simple local expression, but our method, which is an extended version of the conventional derivation of the Faddeev-Popov interaction term, can be equally applied starting from a more general gauge-breaking term of the form given by (2.24).

3. FADDEEV-POPOV SUPERGHOSTS, SLAVNOV IDENTITIES AND SLAVNOV TRANSFORMATIONS

We will prove, in this section, that the tree approximation Green's functions fulfil a generalized form of the Slavnov-Taylor identities [12] valid in conventional Yang-Mills theories. We shall introduce ghost superfields generalizing the Faddeev-Popov ghost fields; their integral spin (or half-integral spin) components will verify anticommutation (or commutation) rules. Then it will follow that the total Lagrangian (including the gauge-breaking term and the ghost term to be constructed) is invariant under global symmetry transformations, generalizing the so-called Slavnov transforms [13].

It is well known that the tree approximation connected Green's functions are generated by the classical solution of the action, where we have included external source terms [17]. Taking the Lagrangian of the preceding section, the external

source-dependent action reads

$$\begin{aligned}
 I(V, S_{\pm}, \bar{S}_{\pm}, J, J_{\pm}, \bar{J}_{\pm}) = & \int dV \left\{ \frac{1}{g_2} \text{Tr} [(e^{-gV} D_{\alpha} e^{gV}) W_{\alpha}] + \right. \\
 & + \frac{1}{g_2} \text{Tr} [(e^{gV} \bar{D}_{\alpha} e^{-gV}) \bar{W}^{\alpha}] + \frac{1}{2} \bar{S}_{+} e^{gV} S_{+} + \\
 & + \frac{1}{2} S_{-} e^{-gV} \bar{S}_{-} \left. \right\} - 2m \left\{ \int dS S_{+} S_{-} + \int d\bar{S} \bar{S}_{+} \bar{S}_{-} \right\} + \quad (3.1) \\
 & + \int dV \frac{g}{2} \text{Tr} B B^{\dagger} + \int dV \text{Tr} V J + \frac{1}{2} \int dS (J_{+} S_{-} + J_{-} S_{+}) + \\
 & + \frac{1}{2} \int d\bar{S} (\bar{J}_{+} \bar{S}_{-} + \bar{J}_{-} \bar{S}_{+}) = I_{\text{INV}} + I_{\text{BREAK}} + I_{\text{SOURCES}}
 \end{aligned}$$

Let us define

$$G^c\{\mathcal{Y}\} = i I\{V, S_{\pm}, \dots\}$$

with

$$\{\mathcal{Y}\} = \{J, J_{\pm}, \bar{J}_{\pm}\}$$

(3.2)

and where the fields $V, S_{\pm}, \bar{S}_{\pm}$ are taken as the solution of the equations of motion induced by (3.1)

$$V = V\{\mathcal{Y}\}, S_{\pm} = S_{\pm}\{\mathcal{Y}\}, \bar{S}_{\pm} = \bar{S}_{\pm}\{\mathcal{Y}\} \quad (3.3)$$

Functional differentiation of (3.2) yields

$$\begin{aligned}
 \delta G^c / \delta J \{\mathcal{Y}\} &= i V\{\mathcal{Y}\}; \quad \delta G^c / \delta J_{\pm} = \frac{i}{2} S_{\mp}\{\mathcal{Y}\} \\
 \delta G^c / \delta \bar{J}_{\mp} &= \frac{i}{2} \bar{S}_{\pm}\{\mathcal{Y}\}
 \end{aligned} \quad (3.4)$$

so that $G^c\{J\}$ is nothing else but the generating functional of the tree approximation connected Green's functions

$$G^c\{H\} = \sum_{k,l,m} \frac{i^{k+l+m}}{k!l!m!} \int dV_1 \dots dV_k dS_1 \dots dS_l d\bar{S}_1 \dots d\bar{S}_m \cdot$$

$$\cdot \langle T(V(1) \dots V(k) S_{\pm}(1) \dots S_{\pm}(l) \bar{S}_{\pm}(1) \dots \bar{S}_{\pm}(m)) \rangle_{\text{CONN TREE}} \cdot \quad (3.5)$$

$$\cdot 2^k J(1) \dots J(k) 2^l J_{\mp}(1) \dots J_{\mp}(l) 2^m \bar{J}_{\mp}(1) \dots \bar{J}_{\mp}(m)$$

In order to prove the tree approximation Slavnov identities, we closely follow the method of Rouet and Stora [16], adapted to supersymmetric theories. First the Ward identities are obtained by varying the fields in (3.1) around the solution (3.3) of the equations of motion, taking for the variations δV , $\delta S_{\pm}$, $\delta \bar{S}_{\pm}$ the infinitesimal gauge transformations defined by (2.1), (2.15) and (2.17)

$$\delta V_a = \Lambda^b C_{ba} + \bar{\Lambda}^b \bar{C}_{ba}$$

$$\delta S_{+i} = -ig \Lambda_{ij} S_{+j} \quad , \quad \delta \bar{S}_{+i} = ig \bar{S}_{+j} \Lambda_{ji}^+ \quad (3.6)$$

$$\delta S_{-i} = ig S_{-j} \Lambda_{ji} \quad , \quad \delta \bar{S}_{-i} = -ig \Lambda_{ij}^+ \bar{S}_{-j}$$

Before going further let us note that the variation of V^a can be computed as a multiple commutator from the variation of e^{gV} given by (2.1)

$$\delta e^{gV} = ig (e^{gV} \Lambda - \Lambda^+ e^{gV}) \quad (3.7)$$

One gets

$$\delta V = i \sum_{n=0}^{\infty} (a_{2n} (gV)^{2n} \{\Lambda - \Lambda^+\} + a_{2n+1} (gV)^{2n+1} \{\Lambda + \Lambda^+\}) \quad (3.8)$$

where

$$V^j \{\Lambda\} = [V, V^{j-1} \{\Lambda\}] \quad , \quad V^0 \{\Lambda\} = \Lambda$$

$$a_j = \sum_{k=0}^{j-1} \frac{(-1)^{j-k+1}}{(j-k+1)!} a_k \quad , \quad a_0 = 1 \quad (3.9)$$

In terms of components one has

$$C_{ba} = i \sum_{j=0}^{\infty} (2ig)^j a_j f_{bb,c} f_{b,b,c} \dots f_{b_{j-1},b_j,c_j} V_{c_1} \dots V_{c_j} \quad (3.10)$$

Therefore the variation of the action I yields an expression of the following structure

$$\int dS \wedge \Sigma + \int d\bar{S} \bar{\Lambda} \bar{\Sigma} = 0 \quad (3.11)$$

Hence the following $2(N^2 - 1)$ Ward identities are true

$$\Sigma = 0, \quad \bar{\Sigma} = 0 \quad (3.12)$$

the latter being the complex conjugate of the former. To the variation of I, defined by eq. (3.1), only the sources and the gauge-breaking term will contribute.

Consider first the source term: we have

$$\begin{aligned} \delta I_{\text{Sources}} = & \int dV (\Lambda^a C_{ab} J^b + \bar{\Lambda}_a \bar{C}_{ab} J^b) + \\ & + i \frac{g}{2} \int dS \Lambda^a (-\tau_{ij}^a S_{+j} J_{-i} + \tau_{ji}^a S_{-j} J_{+i}) + \\ & - i \frac{g}{2} \int d\bar{S} \bar{\Lambda}^a (-\bar{\tau}_{ij}^a \bar{S}_{+j} \bar{J}_{-i} + \bar{\tau}_{ji}^a \bar{S}_{-j} \bar{J}_{+i}) \end{aligned} \quad (3.13)$$

To compute the variation of I_{break} , we consider B^a as a functional of V, according to appendix B. Under an infinitesimal gauge transformation one has

$$\begin{aligned} \delta B^a(z_1) = & \int dS(z_2) \Lambda^b(z_2) \frac{\delta B^a(z_1)}{\delta \Lambda^b(z_2)} + \\ & + \int d\bar{S}(z_2) \bar{\Lambda}^b(z_2) \frac{\delta B^a(z_1)}{\delta \bar{\Lambda}^b(z_2)} = \\ = & \sum_i \int dS_i(z_2) \Lambda_i^b(z_2) \frac{\delta B^a(z_1)}{\delta \Lambda_i^b(z_2)} \end{aligned} \quad (3.14)$$

where $\Lambda_1^b = \Lambda^b$, $\Lambda_2^b = \bar{\Lambda}^b$. We also use the following normalization convention

$$\text{Tr } \lambda_a \lambda_{a'} = \delta_{aa'} \quad (3.15)$$

Therefore we get

$$\begin{aligned} \frac{\alpha}{2} \oint I_{\text{BREAK}} &= \int dV(z_1) \oint (B^a(z_1) \bar{B}^a(z_1)) = \\ &= \int dV(z_1) \oint dS_i(z_2) \left[\Lambda_i^b(z_2) \frac{\oint B^a(z_1)}{\oint \Lambda_i^b(z_2)} \bar{B}^a(z_1) + \Lambda_i^b(z_2) \frac{\oint \bar{B}^a(z_1)}{\oint \Lambda_i^b(z_2)} B^a(z_1) \right] \end{aligned} \quad (3.16)$$

From (3.13) and (3.16) one gets the Ward identity

$$\begin{aligned} \frac{g}{\alpha} \int dV(z_1) \left(\frac{\oint B^a(z_1)}{\oint \Lambda^b(z_2)} \bar{B}^a(z_1) + \frac{\oint \bar{B}^a(z_1)}{\oint \Lambda^b(z_2)} B^a(z_1) \right) + \\ + \bar{D} \bar{D} C_{ba} J^a(z_2) + \\ + i \frac{g}{2} (-\tau_{ij}^b S_{+j} J_{-i} + \tau_{ji}^b S_{-j} J_{+i})(z_1) = 0 \end{aligned} \quad (3.17)$$

together with its complex conjugate.

Now, in order to linearize the Ward identity, we use the Faddeev-Popov ghost method, which will lead us to the Slavnov equations. We need only to generalize to superfields the method used in ref. [16]. Let us introduce two Faddeev-Popov (FP) ghost chiral fields $c_{\pm}^a$ together with their complex conjugate $\bar{c}_{\pm}^a$, belonging to the adjoint representation of the Yang-Mills group. The FP contribution to the action is defined as follows

$$\begin{aligned} I_{\text{FP}} = -i \frac{2}{\alpha} \int dV(z_1) \oint dS_i(z_2) C_{+i}^b(z_2) \left(\frac{\oint B^a(z_1)}{\oint \Lambda_i^b(z_2)} \bar{C}_{-}(z_1) + \right. \\ \left. + \frac{\oint \bar{B}^a(z_1)}{\oint \Lambda_i^b(z_2)} C_{-}(z_1) \right) \end{aligned} \quad (3.18)$$

and moreover we have

$$C_{+i} C_{+j} = -C_{+j} C_{+i}, C_{+i} C_{-j} = -C_{-j} C_{+i}, C_{-i} C_{-j} = -C_{-j} C_{-i} \quad (3.19)$$

One has still to add source terms, the external sources being chiral anticommuting superfields $\xi_{\pm}$

$$I_{FP, sources} = \sum_{j=1}^2 (-1)^j \int dS_j(z_1) (C_{+i}^a \xi_{-i}^a(z_1) + C_{-i}^a(z_1) \xi_{+i}^a(z_1)) \quad (3.20)$$

The form of the equations of motion for the Faddeev-Popov fields reads

$$-i \frac{2}{\alpha} \int dV(z_1) \left(\frac{\delta B^a(z_1)}{\delta \Lambda_j^b(z_2)} \bar{C}_{-}^a(z_1) + \frac{\delta \bar{B}^a(z_1)}{\delta \Lambda_j^b(z_2)} C_{-}^a(z_1) \right) - (-1)^j \xi_{-j}^b(z_2) = 0 \quad j=1,2 \quad (3.21)$$

and

$$i \frac{2}{\alpha} \sum_{i=1}^2 \int dS_i(z_2) C_{+i}^b(z_2) \frac{\delta D D B^a(z_1)}{\delta \Lambda_i^b(z_2)} + \bar{\xi}_{+}^a(z_1) = 0$$

$$i \frac{2}{\alpha} \sum_{i=1}^2 \int dS_i(z_2) C_{+i}^b(z_2) \frac{\delta \bar{D} \bar{D} \bar{B}^a(z_1)}{\delta \Lambda_i^b(z_2)} - \xi_{+}^a(z_1) = 0 \quad (3.22)$$

The FP action part, defined by eq. (3.18), will contribute to the Ward identities.

We assume that the FP fields are gauge-invariant so, performing an infinitesimal gauge transformation of parameter Λ_j^c ($j = 1, 2$), we have

$$\delta I_{FP, sources} = 0 \quad (3.23)$$

$$\delta I_{FP} = -i \frac{2}{\alpha} \sum_{i,j=1}^2 \int dV(z_1) dS_i(z_2) dS_j(z_3) C_{+i}^b(z_2) \Lambda_j^c(z_3)$$

$$\cdot \left[\frac{\delta^2 B^a(z_1)}{\delta \Lambda_j^c(z_3) \delta \Lambda_i^b(z_2)} \bar{C}_{-}^a(z_1) + \frac{\delta^2 \bar{B}^a(z_1)}{\delta \Lambda_j^c(z_3) \delta \Lambda_i^b(z_2)} C_{-}^a(z_1) \right] \quad (3.24)$$

The complete Ward identities are obtained adding the contribution from the expression above to the left-hand side of (3.17) and of its complex conjugate.

$$\begin{aligned}
 & \frac{2}{\alpha} \int dV(z_1) \left(\frac{\delta B^a(z_1)}{\delta \Lambda_i^b(z_2)} \bar{B}^a(z_1) + \frac{\delta \bar{B}^a(z_1)}{\delta \Lambda_i^b(z_2)} B^a(z_1) \right) - \\
 & - i \frac{2}{\alpha} \sum_{j=1}^2 \int dV(z_1) dS_j(z_3) C_{+j}^c(z_3) \left[\frac{\delta^2 B^a(z_1)}{\delta \Lambda_i^b(z_2) \delta \Lambda_j^c(z_3)} \bar{C}_{-}^a(z_3) \right. \\
 & \left. + \frac{\delta^2 \bar{B}^a(z_1)}{\delta \Lambda_i^b(z_2) \delta \Lambda_j^c(z_3)} C_{-}^a(z_3) \right] + \text{source terms} = 0
 \end{aligned} \tag{3.25}$$

We now multiply the Ward identity (3.25) by $c_i^b(z_2)$ integrate over z_2 , and sum over i, b . Because of the anticommuting properties of the $c_{\pm i}^a(z)$ superfields we find

$$\begin{aligned}
 & \frac{2}{\alpha} \sum_i \int dV(z_1) dS_i(z_2) C_{+i}^b(z_2) \left[\frac{\delta B^a(z_1)}{\delta \Lambda_i^b(z_2)} \bar{B}^a(z_1) + \right. \\
 & \left. + \frac{\delta \bar{B}^a(z_1)}{\delta \Lambda_i^b(z_2)} B^a(z_1) \right] - i \frac{2}{\alpha} \sum_{i,j} \int dV(z_1) dS_i(z_2) dS_j(z_3) \cdot \\
 & \cdot C_{+i}^b(z_2) C_{+j}^c(z_3) \frac{1}{2} \left[\frac{\delta^2 B^a(z_1)}{\delta \Lambda_i^b(z_2) \delta \Lambda_j^c(z_3)} - \frac{\delta^2 \bar{B}^a(z_1)}{\delta \Lambda_i^b(z_2) \delta \Lambda_j^c(z_3)} \right] \bar{C}_{-}^a(z_3) \\
 & - i \frac{2}{\alpha} \sum_{i,j} \int dV(z_1) dS_i(z_2) dS_j(z_3) C_{+i}^b(z_2) C_{+j}^c(z_3) \cdot
 \end{aligned} \tag{3.26}$$

$$\frac{1}{2} \left[\frac{\delta^2 \bar{B}^a(z_1)}{\delta \Lambda_i^b(z_2) \delta \Lambda_j^c(z_3)} - \frac{\delta^2 \bar{B}^a(z_1)}{\delta \Lambda_i^c(z_2) \delta \Lambda_j^b(z_3)} \right] C_-^a(z_1) + \text{source terms} = 0$$

The previous identity can be drastically simplified. Indeed for the first line of eq. (3.26), using the property that $dV(z_1) = dS(z_1) D_1 \bar{D}_1 = d\bar{S}(z_1) \bar{D}_1 D_1$ and the fact that $D_\alpha \bar{B} = \bar{D}_\alpha B = 0$, making use of the equations of motion (3.22) one gets

$$i \int d\bar{S}(z_1) \bar{\xi}_+^a(z_1) \bar{B}^a(z_1) - i \int dS(z_1) \xi_+^a(z_1) B^a(z_1) \quad (3.27)$$

In order to simplify the second and third lines of eq. (3.26) we use the group property given by (B.10)

$$\begin{aligned} & i \frac{1}{2} \int dV(z_1) \sum_i dS_i(z_2) f_{bcd} C_{+i}^b(z_2) C_{+i}^c(z_2) \cdot \\ & \cdot \left[\frac{\delta B^a(z_1)}{\delta \Lambda_i^d(z_2)} \bar{C}_-^a(z_1) + \frac{\delta \bar{B}^a(z_1)}{\delta \Lambda_i^d(z_2)} C_-^a(z_1) \right] = \quad (3.28) \\ & = \frac{1}{2} \sum_j \int dS_j(z_2) (-1)^j f_{abc} C_{+i}^a(z_2) C_{+i}^b(z_2) \xi_{-i}^c(z_2) \end{aligned}$$

where we used the equations of motion given by (3.21). Collecting together (3.13), (3.27) and (3.28), we can write the complete Slavnov identity for the supersymmetric Yang-Mills theory.

$$\begin{aligned} & \int dS (-\xi_+^a B^a + \frac{i}{2} f_{abc} C_+^a C_+^b \xi_-^c) + \\ & + \int d\bar{S} (\bar{\xi}_+^a \bar{B}^a - \frac{i}{2} f_{abc} \bar{C}_+^a \bar{C}_+^b \bar{\xi}_-^c) + \\ & - i \int dV (C_+^a C_{ab} J^b + \bar{C}_+^a \bar{C}_{ab} J^b) + \quad (3.29) \\ & \frac{g}{2} \int dS (-J_{-i} C_{+ij} S_{+j} + S_{-i} C_{+ij} J_{+j}) \\ & - \frac{g}{2} \int d\bar{S} (-\bar{S}_{+i} C_{+ij}^\dagger \bar{J}_{-j} + \bar{J}_{+i} C_{+ij}^\dagger \bar{S}_{-j}) = 0 \end{aligned}$$

where

$$C_{+ij} = C_+^a \tau_{ij}^a, \quad C_{+ij}^\dagger = \bar{C}_+^a \bar{\tau}_{ji}^a$$

One can interpret this Slavnov identity as the integrated Ward identity associated with an exact global symmetry of the total Lagrangian, the so-called Slavnov symmetry. The infinitesimal Slavnov transformations are read out from the Slavnov identity; they are

$$\begin{aligned} \delta V &= \varepsilon \sum_{n=0}^{\infty} (a_{2n}(gV)^{2n} \{C_+ - C_+^\dagger\} + a_{2n+1}(gV)^{2n+1} \{C_+ + C_+^\dagger\}) \\ \delta S_{+i} &= -\varepsilon g C_{+ij} S_{+j}, \quad \delta \bar{S}_{+i} = \varepsilon g \bar{S}_{+j} C_{+ji}^\dagger \\ \delta S_{-i} &= \varepsilon g S_{-j} C_{+ji}, \quad \delta \bar{S}_{-i} = -\varepsilon g C_{+ij}^\dagger \bar{S}_{-j} \\ \delta C_+^a &= -\frac{i}{2} \varepsilon f^{abc} C_+^b C_+^c, \quad \delta \bar{C}_+^a = -\frac{i}{2} \varepsilon f^{abc} \bar{C}_+^b \bar{C}_+^c \\ \delta C_-^a &= \varepsilon B^a, \quad \delta \bar{C}_-^a = \varepsilon \bar{B}^a \end{aligned} \quad (3.30)$$

where the infinitesimal real parameter ε anticommutes with the FP fields and sources and commutes with the other fields.

From (3.30) one can realize that the Slavnov transforms of the fields V , $S_\pm$, $\bar{S}_\pm$ are gauge transforms of the type (3.6) with the replacement

$$i\Lambda^a \rightarrow \varepsilon C_+^a, \quad -i\bar{\Lambda}^a \rightarrow -\varepsilon \bar{C}_+^a \quad (3.31)$$

Up to now the gauge-breaking term $B^a \bar{B}^a$ was completely unspecified. We now consider, for a concrete application, the case where

$$B_a = \bar{D} \bar{D} V_a \quad (3.32)$$

Such a term gives rise to the conventional gauge-breaking term $(1/2\alpha)(\partial_\mu v^\mu)^2$ for the vector field contained in the supermultiplet V . In this case the z_2 integration in eq. (3.18) can be explicitly performed because

$$\begin{aligned} I_{FP} &= -i \frac{1}{2} \int dV(z_1) \sum_i dS_i(z_2) C_{+i}^b(z_2) \{ \bar{D}_1 \bar{D}_1 (d_{S_i}(z_2, z_1) C_{b a}^i(z_1)) \cdot \\ &\cdot \bar{C}_-^a(z_1) + D_1 D_1 (d_{S_i}(z_2, z_1) C_{b a}^i(z_1) C_-^a(z_1)) \} = \\ &= -i \frac{1}{2} \int dV(z_1) \sum_i C_{+i}^b(z_1) C_{b a}^i(z_1) (\bar{D} \bar{D} \bar{C}_-^a(z_1) + D D C_-^a(z_1)) \end{aligned} \quad (3.33)$$

after integration by part and use of the property

$$\frac{\delta V^a(z_1)}{\delta \Lambda_i^j(z_2)} = \delta_{S_i}(z_2, z_1) C_{ba}^i(z_1) \quad (3.34)$$

and C_{ba}^i has been defined through eq. (3.10). Using (3.8) we get

$$\begin{aligned} I_{FP} = & \frac{2}{\alpha} \int dV T_2 \sum_{n=0}^{\infty} \{a_{2n} (gV)^{2n}\} \{c_+ - c_+^\dagger\} + \\ & + a_{2n+1} (gV)^{2n+1} \{c_+ + c_+^\dagger\} \{(\bar{D}\bar{D}c_-^\dagger + DDc_-)\} \end{aligned} \quad (3.35)$$

and, using a trace property, we can rewrite the previous formula as

$$\begin{aligned} I_{FP} = & \frac{2}{\alpha} \int dS \bar{h} 16 c_+ \square c_- - \frac{2}{\alpha} \int d\bar{S} \bar{h} 16 c_+^\dagger \square c_-^\dagger + \\ & + \frac{2}{\alpha} \int dV T_2 \left[\sum_{n=1}^{\infty} (c_+ - c_+^\dagger) a_{2n} g^{2n} V^{2n} \{ \bar{D}\bar{D}c_-^\dagger + DDc_- \} \right. \\ & \left. - (c_+ + c_+^\dagger) a_{2n+1} g^{2n+1} V^{2n+1} \{ \bar{D}\bar{D}c_-^\dagger + DDc_- \} \right] \end{aligned} \quad (3.36)$$

Of course the free part of the ghost term is always the same, as long as we choose a gauge-breaking term of the form given by (2.24).

Note that we assign to the superfields c_+ and c_- dimensions 1 and 0, respectively; the usual scalar Faddeev-Popov ghosts have to be identified with the A component of c_+ and with the F component of c_- .

The Slavnov identities (3.29) can be written also for 1PI Green's functions, following the method of ref. [13]. We introduce an external source for each composite field appearing in the Slavnov identity. Such composite fields are

$$\begin{aligned} \tilde{B}^a = & B^a - \bar{D}\bar{D}V^a, \quad \frac{i}{2} f_{abc} c_+^b c_+^c, \\ & -i c_+^b C_{ba} - c_{+a}, \quad -c_{+ij} S_{+j}, \quad S_{-j} c_{+ji} \end{aligned} \quad (3.37)$$

and their complex conjugates. Their corresponding sources will be denoted by

$$T^a, X^a, Y^a, Z_{-i}, Z_{+j} \quad (3.38)$$

and their complex conjugates.

Let us add to the action the source terms

$$\begin{aligned} I_{\text{COMP, SOURCES}} = & - \int dV (i c_+^b c_{ba} + c_{+a}) \psi^a + \\ & + \int dS (T^a \tilde{B}^a + i f_{abc} c_+^b c_+^c X^a + \\ & + Z_{-i} c_{+ij} S_{+j} + S_{-i} c_{+ij} Z_{+j}) + h.c. \end{aligned} \quad (3.39)$$

and introduce the generating functional for connected Green's functions with insertions of composite fields given in (3.37)

$$G^c \{ J, J_{\pm i}, \bar{J}_{\pm i}, \xi_{\pm}, \bar{\xi}_{\pm}; T, X, \psi, Z_{\pm i}, \bar{T}, \bar{X}, \bar{\psi}, \bar{Z}_{\pm i} \} \quad (3.40)$$

For instance

$$\left. \frac{\delta G^c}{\delta Z_{-i}} \right|_{Z_{-i}=0}$$

generates the connected Green's functions with one insertion of the composite field $-c_{+ij} S_{+j}$. In the tree approximation, G^c is nothing else than the action including the source terms for all fields and composite fields (multiplied by i).

Thus, the Slavnov identity (3.29) may be written as a functional differential equation for G^c

$$\begin{aligned} & \left\{ \int dS \left(-\xi_+^a \bar{D} \bar{D}_i \frac{\delta}{\delta J^a} - \xi_+^a \frac{\delta}{\delta T^a} + \xi_-^a \frac{\delta}{\delta X^a} + \right. \right. \\ & + \frac{g}{2} J_{-i} \frac{\delta}{\delta Z_{-i}} + \frac{g}{2} J_{+i} \frac{\delta}{\delta Z_{+i}} \left. \right) + \int dV \left(J_{-i}^a \frac{\delta}{\delta \xi_{-a}} + \right. \\ & \left. \left. + J_{+i}^a \frac{\delta}{\delta \psi^a} \right) - h.c. \right\} G^c = 0 \end{aligned} \quad (3.41)$$

The 1PI Green's functions are obtained from G^c by a Legendre transform [17]. Let us define the classical fields

$$V = \frac{\delta G^c}{i \delta J}, \quad S_{\pm i} = \frac{1}{2} \frac{\delta G^c}{i \delta J_{\mp i}}, \quad \bar{S}_{\pm i} = \frac{1}{2} \frac{\delta G^c}{i \delta \bar{J}_{\mp i}} \quad (3.42)$$

$$c_{\pm}^a = \frac{\delta G^c}{i \delta \xi_{\mp}^a}, \quad \bar{c}_{\pm}^a = -i \frac{\delta G^c}{\delta \bar{\xi}_{\mp}^a}$$

The generating functional for 1PI Green's functions with insertions of (3.37)

$$\Gamma\{V, S_{\pm}, \bar{S}_{\pm}, c_{\pm}, \bar{c}_{\pm}; T, X, Y, Z_{\pm}, \bar{T}, \bar{X}, \bar{Y}, \bar{Z}_{\pm}\} \quad (3.43)$$

is given by

$$\begin{aligned} \Gamma = G^c - i \int dV J^a V^a - i \int dS \left(\frac{1}{2} J_{+i} S_{-i} + \right. \\ \left. + \frac{1}{2} J_{-i} S_{+i} + \xi_{+}^a c_{-}^a + \xi_{-}^a c_{+}^a \right) + h.c. \end{aligned} \quad (3.44)$$

and we have the converse formulae of (3.42)

$$i J = - \frac{\delta \Gamma}{\delta V}, \quad \frac{i}{2} J_{\mp i} = - \frac{\delta \Gamma}{\delta S_{\pm i}}, \quad i \xi_{\pm}^a = \frac{\delta \Gamma}{\delta c_{\mp}^a} \quad (3.45)$$

Differentiation with respect to sources associated with composite fields is not altered, for instance

$$\frac{\delta G^c}{\delta T^a} = \frac{\delta \Gamma}{\delta T^a} \quad (3.46)$$

Finally, it is easy to write down the Slavnov identity for the 1PI Green's function generating functional

$$\begin{aligned} \int dS \left(i \bar{D} \bar{D} V^a \frac{\delta \Gamma}{\delta c_{-}^a} + i \frac{\delta \Gamma}{\delta T^a} \frac{\delta \Gamma}{\delta c_{-}^a} - i \frac{\delta \Gamma}{\delta X^a} \frac{\delta \Gamma}{\delta c_{+}^a} + \right. \\ \left. + i g \frac{\delta \Gamma}{\delta Z_{-i}} \frac{\delta \Gamma}{\delta S_{+i}} + i g \frac{\delta \Gamma}{\delta Z_{+i}} \frac{\delta \Gamma}{\delta S_{-i}} \right) + h.c. + \\ + \int dV \left(i c_{+}^a \frac{\delta \Gamma}{\delta V^a} + i \frac{\delta \Gamma}{\delta \psi^a} \frac{\delta \Gamma}{\delta V^a} \right) = 0 \end{aligned} \quad (3.47)$$

Such generalized Ward identities, the Slavnov identities for 1PI Green's functions, together with the equations of motion for the ghost fields (3.21) are the basic ingredients in order to prove the renormalization of the supersymmetric Yang-Mills theory, as we shall see in section 6.

4. ABELIAN CASE: SUPERSYMMETRIC Q.E.D.
AND MASSIVE VECTOR THEORY

In the present section we consider the Abelian theory; namely we identify $G = U(1)$. Of course, in this case, we have not the complication of the Faddeev-Popov term and the Ward identities are simpler.

The gauge-breaking term is exactly the previous one, namely

$$\frac{g}{\alpha} DDV \bar{D} \bar{D} V \quad (4.1)$$

where V is a real vector multiplet. The charged matter field can be identified with

$$S_{\pm} = S_1 \pm i S_2 \quad (4.2)$$

where S_1, S_2 are two ordinary chiral fields. Under a local (infinitesimal) gauge transformation, we have

$$\begin{aligned} \delta V &= i(\lambda - \bar{\lambda}) \\ \delta S_{\pm} &= \mp i g S_{\pm} \quad \text{or} \\ \delta S_1 &= g \wedge S_2 \quad \delta S_2 = -g \wedge S_1 \end{aligned} \quad (4.3)$$

in terms of usual chiral fields.

The complete Lagrangian, including the source terms, reads

$$\begin{aligned} & W^{\alpha} W_{\alpha} + \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} + \frac{g^2}{\alpha} DDV \bar{D} \bar{D} V + \\ & + \frac{1}{2} (S_+ \bar{S}_+ e^{gV} + S_- \bar{S}_- e^{-gV} - 4m S_+ S_- - 4m \bar{S}_+ \bar{S}_-) + \\ & + VJ + \frac{1}{2} (J_+ S_- + J_- S_+) + \frac{1}{2} (\bar{J}_+ \bar{S}_- + \bar{J}_- \bar{S}_+) \end{aligned} \quad (4.4)$$

which can be rewritten in the simpler form

$$2V \left[\frac{1}{2\alpha} (DD + \bar{D}\bar{D})^2 - D^{\alpha} \bar{D} \bar{D} D_{\alpha} \right] V +$$

$$\begin{aligned}
 & + S_1 \bar{S}_1 + S_2 \bar{S}_2 - 2m(S_1^2 + S_2^2) - 2m(\bar{S}_1^2 + \bar{S}_2^2) + \\
 & + (S_1 \bar{S}_1 + S_2 \bar{S}_2)(\cosh gV - 1) + i(S_2 \bar{S}_1 - S_1 \bar{S}_2) \sinh gV + \\
 & + VJ + J_1 S_1 + J_2 S_2 + \bar{J}_1 \bar{S}_1 + \bar{J}_2 \bar{S}_2
 \end{aligned} \tag{4.5}$$

Note that, in terms of the chiral fields S_1, S_2 the free part of the matter field is now diagonalized, i.e.

$$\langle S_1 S_2 \rangle = \langle S_1 \bar{S}_2 \rangle = 0 \tag{4.6}$$

The equations of motion for the Abelian theory are simply obtained by performing the variation with respect to $V, S_{\pm}$ and $\bar{S}_{\pm}$. One obtains

$$\begin{aligned}
 & [D^{\alpha} \bar{D} \bar{D} D_{\alpha} - \frac{1}{2\alpha} (D\bar{D} + \bar{D}D)^2] V = \\
 & = \frac{1}{4} J + \frac{g}{8} (S_+ \bar{S}_+ e^{gV} - S_- \bar{S}_- e^{-gV})
 \end{aligned} \tag{4.7}$$

$$\begin{aligned}
 & \bar{D} \bar{D} (\bar{S}_+ e^{gV}) - 4m S_- + \bar{J}_- = 0 \\
 & \bar{D} \bar{D} (\bar{S}_- e^{-gV}) - 4m S_+ + \bar{J}_+ = 0
 \end{aligned} \tag{4.8}$$

$$\begin{aligned}
 & D D (S_+ e^{gV}) - 4m \bar{S}_- + \bar{J}_- = 0 \\
 & D D (S_- e^{-gV}) - 4m \bar{S}_+ + \bar{J}_+ = 0
 \end{aligned} \tag{4.9}$$

Note that the vector superfield

$$I = \frac{g}{8} (S_+ \bar{S}_+ e^{gV} - S_- \bar{S}_- e^{-gV}) \tag{4.10}$$

satisfies the equation

$$\bar{D}\bar{D}I = \frac{9}{8} (S_- J_+ - S_+ J_-) \quad (4.11)$$

and its complex conjugate. This implies

$$-\frac{8}{\alpha} \square \bar{D}\bar{D}V = \frac{1}{4} \bar{D}\bar{D}J + \frac{9}{8} (S_- J_+ - J_- S_+) \quad (4.12)$$

which are nothing but the local Ward identities of the U(1) gauge group. They imply that $\bar{D}\bar{D}V$ is a free field on the mass shell and that the S-matrix is gauge-invariant [16].

Note that the real vector multiplet I is nothing but the supersymmetric U(1) gauge-invariant current. In fact, its vector component field I_μ is conserved and gauge-invariant. In the Wess-Zumino non-covariant gauge it reduces to the normal gauge-invariant e.m. current

$$I_\mu = i (A \overleftrightarrow{\nabla}_\mu A^* + B \overleftrightarrow{\nabla}_\mu B^* + \bar{\Psi} \gamma_\mu \Psi) \quad (4.13)$$

where $\nabla_\mu = \partial_\mu - igA_\mu$ is the covariant derivative and A, B, ψ are the charged fields of the matter multiplet.

There is no difficulty to pass to the massive vector theory. We simply add to the previous Lagrangian a mass term for the vector field, namely

$$\mathcal{L}_H(V) = -16 M^2 V^2 \quad (4.14)$$

(where the numerical factor has been fixed in order to identify M with the mass of the field V).

Equations (4.8) and (4.9) are not changed and (4.7) simply gives

$$\begin{aligned} & [D^\alpha \bar{D}\bar{D}D_\alpha + 8M^2 - \frac{1}{2\alpha} (\bar{D}D + D\bar{D})^2] V = \\ & = \frac{1}{4} J + \frac{9}{8} (S_+ \bar{J}_+ e^{gV} - S_- \bar{J}_- e^{-gV}) \end{aligned} \quad (4.15)$$

The (broken) Ward identity of the gauge group simply gives

$$8 (M^2 - \frac{1}{\alpha} \square) \bar{D}\bar{D}V = \frac{1}{4} \bar{D}\bar{D}J + \frac{9}{8} (S_- J_+ - J_- S_+) \quad (4.16)$$

which implies that the ghost field $\bar{D}D V$ of mass αM^2 behaves on shell as a free-field, i.e. can be decoupled from the physical sector of Fock space.

Equation (4.16) looks like a functional differential equation for the generating functional of connected Green's functions [see eq. (3.2)], which is written as

$$8(M^2 - \frac{1}{2}\square) \bar{D}D \frac{\delta G^c}{\delta iJ} - \frac{1}{4} \bar{D}D J + \frac{g}{4} \left(J_+ \frac{\delta G^c}{\delta iJ_+} - J_- \frac{\delta G^c}{\delta iJ_-} \right) = 0 \quad (4.17)$$

and its complex conjugate.

It is well known that the generating functional for 1-particle irreducible (1PI) Green's functions

$$\Gamma\{V, S_{\pm}, \bar{S}_{\pm}\} = \sum_{k \in m} \frac{1}{k! e! m!} \int dV(z_1) \dots dV(z_k) dS(x_1) \dots dS(x_e) \\ d\bar{S}(y_1) \dots d\bar{S}(y_m) V(z_1) \dots V(z_k) S_{\pm}(x_1) \dots S_{\pm}(x_e) \bar{S}_{\pm}(y_1) \dots \bar{S}_{\pm}(y_m) \quad (4.18) \\ \cdot \langle T(V(z_1) \dots V(z_k) S_{\mp}(x_1) \dots S_{\mp}(x_e) \bar{S}_{\mp}(y_1) \dots \bar{S}_{\mp}(y_m)) \rangle^{1PI}$$

is given by the Legendre transformation

$$\Gamma\{V, S_{\pm}, \bar{S}_{\pm}\} = G^c\{J, J_{\pm}, \bar{J}_{\pm}\} - i \int dV V J - \\ - \frac{i}{2} \int dS S_+ J_- - \frac{i}{2} \int dS S_- J_+ - \frac{i}{2} \int d\bar{S} \bar{S}_+ \bar{J}_- - \frac{i}{2} \int d\bar{S} \bar{S}_- \bar{J}_+ \quad (4.19)$$

with

$$V = \frac{\delta G^c}{\delta iJ}, \quad S_{\pm} = 2 \frac{\delta G^c}{\delta iJ_{\mp}}, \quad \bar{S}_{\pm} = 2 \frac{\delta G^c}{\delta i\bar{J}_{\mp}} \quad (4.20)$$

So the Ward identities for 1PI Green's functions derived from (4.17) by means of the Legendre transform (4.19) become

$$8(M^2 - \frac{1}{2}\square) \bar{D}D V + \left\{ -i \frac{1}{4} \bar{D}D \frac{\delta}{\delta V} + \right. \\ \left. + i \frac{g}{4} \left(S_- \frac{\delta}{\delta S_-} - S_+ \frac{\delta}{\delta S_+} \right) \right\} \Gamma = 0 \quad (4.21)$$

And we note that the term $\bar{D}D V$ will contribute only to the two-point function for the vector multiplet, namely

$$8 \left(M^2 - \frac{1}{\alpha} D \right) (\bar{D} D)_{z_1} \delta_V(z_1, z_2) - i (\bar{D} D)_{z_1} \langle T V(z_1) V(z_2) \rangle = 0 \quad (4.22)$$

The Ward identities (4.17), (4.21) are the essential tools in order to prove the renormalizability of the Abelian gauge theory. In fact from the ultraviolet analysis of Feynman graphs carried out in section 6, we shall see that, in spite of the fact that such a theory contains an infinity of primitively divergent graphs, the previous Ward identities, transferred to the one-particle irreducible connected functional, relate such a class of divergences to only two basic primitively divergent graphs. These graphs correspond to a wave-function renormalization (and possibly mass renormalization) for the vector multiplet and a wave-function renormalization for the matter field. No independent mass renormalization for the matter multiplet is needed, because of supersymmetry. This is in exact analogy with the pure chiral model [6] (super S^3 theory).

We shall see in section 5 that the gauge-breaking term provides for the propagator of the gauge supermultiplet the same ultraviolet property as the massless theory. This is just because of the presence of the ghost field of mass given by (4.16).

This is the exact analogue of ordinary massive vector theories. So we are led to claim that, if the massless theory is renormalizable, the same happens for the massive vector theory. In fact, such a theory, because of the same ultraviolet behaviour as the vector multiplet propagator, has the same ultraviolet divergences as the zero mass theory.

We would like to point out that, in terms of component fields, the massive vector theory is a completely non-trivial generalization of the massless theory. In fact the physical fields of the massive vector multiplet are a real pseudo-scalar field C , two Majorana spinors χ, λ and a vector field v_μ with the same mass M (see for instance the paper by Salam and Strathdee [18]). So, beyond the longitudinal component of the vector field, there are two new particles, namely

a pseudoscalar C and a Majorana spinor χ . They interact with a charged scalar A , a charged pseudoscalar B and a charged fermion ψ with the same mass m . This interaction gives the supersymmetric generalization of the massive vector theory.

As a final point, we would like to remark that, as shown by Fayet and Iliopoulos [19], another possible term can be added to the Lagrangian (4.4), which is consistent with supersymmetry and gauge invariance, but which violates parity, namely

$$\mathcal{L}_\xi(V) = \xi V \quad (4.23)$$

The authors of ref. [19] have shown that such a term triggers a spontaneous breakdown of supersymmetry as well as of local $U(1)$ symmetry just in the tree approximation. We will see the reason for this by using the superfield formalism in appendix D and comment on the possibility of extending the present approach to the renormalization of spontaneously broken theories.

5. FEYNMAN RULES

In order to construct the Feynman rules for the supersymmetric gauge theories, we have first to work out the propagators of the several multiplets involved; namely the propagator of the vector field, of the matter fields, as well as of the Faddeev-Popov fields.

The propagators of the chiral matter fields are already known from previous works [5]. We just rederive them according to our conventions. The free part of the equations of motion for the matter field is [eq. (4.9) with $g = 0$]

$$DD\bar{S} - 4m\bar{S} + \bar{J} = 0 \quad (5.1)$$

$$\bar{D}_\alpha S = 0$$

$$\bar{D}\bar{D}\bar{S} - 4mS + J = 0 \quad (5.2)$$

Applying $\bar{D}\bar{D}$ to (5.1) and substituting into (5.2) one obtains

$$S(1) = \frac{1}{16(m^2 - \square)} (\bar{D}\bar{D}J + 4mJ)(1) \quad (5.3)$$

This leads to the following expression for the propagator:

$$\begin{aligned}\langle S(1) \bar{S}(2) \rangle &= \frac{-i}{16(m^2+k^2)} \bar{D}_1 \bar{D}_2 (e^{\bar{\theta}_1 \gamma \theta_2 k} \delta^2(\bar{\theta}_{12})) = \\ &= \frac{-i}{16(m^2+k^2)} e^{\bar{\theta}_1 \gamma \theta_2 k - \theta_{12} \sigma \bar{\theta}_{12} k}\end{aligned}\quad (5.4)$$

$$\begin{aligned}\langle S(1) S(2) \rangle &= \frac{-im}{4(k^2+m^2)} e^{\bar{\theta}_1 \gamma \theta_2 k} \delta^2(\theta_{12}) = \\ &= \frac{im}{16(k^2+m^2)} e^{\bar{\theta}_1 \gamma \theta_2 k} \theta_{12} \theta_{12}\end{aligned}\quad (5.5)$$

In fact using (5.4) and (5.5), (5.3) can be rewritten as

$$S(1) = \int dS(2) \langle S(1) S(2) \rangle iJ(2) + \int d\bar{S}(2) \langle S(1) \bar{S}(2) \rangle i\bar{J}(2) \quad (5.6)$$

and one has

$$\frac{\delta S(1)}{\delta iJ(2)} = \langle S(1) S(2) \rangle, \quad \frac{\delta S(1)}{\delta i\bar{J}(2)} = \langle S(1) \bar{S}(2) \rangle \quad (5.7)$$

together with the analogous expression for $\bar{S}$.

In order to obtain the remaining propagators, it is sufficient to exchange $1 \leftrightarrow 2$ in (5.4) ($\theta_1 \rightarrow \theta_2$ $k \rightarrow -k$) and $\theta_1 \rightarrow \bar{\theta}_2$ in (5.3), so we get

$$\langle \bar{S}(1) S(2) \rangle = \frac{-i}{16(m^2+k^2)} e^{\bar{\theta}_1 \gamma \theta_2 k + \theta_{12} \sigma \bar{\theta}_{12} k} \quad (5.8)$$

$$\langle \bar{S}(1) \bar{S}(2) \rangle = \frac{im}{16(m^2+k^2)} e^{\bar{\theta}_1 \gamma \theta_2 k} \bar{\theta}_{12} \bar{\theta}_{12} \quad (5.9)$$

For the propagator of the massless vector field we start with the free equation of motion [eq. (4.7) with $g = 0$]

$$\left[4 D^\alpha \bar{D} \bar{D} D_\alpha - \frac{2}{\alpha} (D D + \bar{D} \bar{D})^2 \right] V = J \quad (5.10)$$

In order to invert this equation, we use the following properties, which are a consequence of the algebra of covariant derivatives (see appendix A)

$$D^\alpha \bar{D} \bar{D} D_\alpha D^\alpha \bar{D} \bar{D} D_\alpha = -8 \square D^\alpha \bar{D} \bar{D} D_\alpha \quad (5.11)$$

$$(D D + \bar{D} \bar{D})^2 (D D + \bar{D} \bar{D})^2 = 16 \square (D D + \bar{D} \bar{D})^2 \quad (5.12)$$

$$(D D + \bar{D} \bar{D})^2 D^\alpha \bar{D} \bar{D} D_\alpha = D^\alpha \bar{D} \bar{D} D_\alpha (D D + \bar{D} \bar{D})^2 = 0 \quad (5.13)$$

$$\frac{1}{2} (D D + \bar{D} \bar{D})^2 - D^\alpha \bar{D} \bar{D} D_\alpha = 8 \square \quad (5.14)$$

(remember that $D^\alpha \bar{D} \bar{D} D_\alpha = \bar{D}_\alpha D D \bar{D}^\alpha$).

Using these properties we get

$$-32 \square D^\alpha \bar{D} \bar{D} D_\alpha V = D^\alpha \bar{D} \bar{D} D_\alpha J \quad (5.15)$$

$$-\frac{32}{\alpha} \square (D D + \bar{D} \bar{D})^2 V = (D D + \bar{D} \bar{D})^2 J \quad (5.16)$$

which gives finally, using (5.14)

$$V = \frac{1}{4^4} \frac{1}{\square^2} \left(D^\alpha \bar{D} \bar{D} D_\alpha - \frac{\alpha}{2} (D D + \bar{D} \bar{D})^2 \right) J \quad (5.17)$$

Then it follows that

$$\begin{aligned} \langle T(V(k, \theta_1) V(-k, \theta_2)) \rangle &= -i \frac{1}{4^4} \frac{1}{k^4} (D_1^\alpha \bar{D}_1 \bar{D}_1 D_{1,\alpha} - \\ &\quad - \frac{\alpha}{2} (D_1 D_1 + \bar{D}_1 \bar{D}_1)^2) \delta^4(\theta_{12}) \end{aligned} \quad (5.18)$$

Using the property

$$(D_1 D_1 + \bar{D}_1 \bar{D}_1)^2 \theta_{12} \theta_{12} \bar{\theta}_{12} \bar{\theta}_{12} = e^{\bar{\theta}_1 \gamma \theta_2 k} 8 (4 - \theta_{12} \theta_{12} \bar{\theta}_{12} \bar{\theta}_{12} k^2) \quad (5.19)$$

$$D_1^\alpha \bar{D}_1 \bar{D}_1 D_{1\alpha} \theta_{12} \theta_{12} \bar{\theta}_{12} \bar{\theta}_{12} = e^{\bar{\theta}_1 \gamma \theta_2 k} 4 (4 + \theta_{12} \theta_{12} \bar{\theta}_{12} \bar{\theta}_{12} k^2)$$

we get

$$\begin{aligned} \langle T(V(k, \theta_1) V(-k, \theta_2)) \rangle &= -i \frac{1}{4^5} \frac{e^{\bar{\theta}_1 \gamma \theta_2 k}}{k^4} [4(1-\alpha) + \\ &+ (1+\alpha) \theta_{12} \theta_{12} \bar{\theta}_{12} \bar{\theta}_{12} k^2] \end{aligned} \quad (5.20)$$

so we have

$$V(1) = \int dV(2) \langle V(1) V(2) \rangle i J(2) \quad (5.21)$$

or equivalently

$$\frac{\delta V(1)}{\delta i J(2)} = \langle V(1) V(2) \rangle \quad (5.22)$$

Note that for $\alpha = 1, 0$ we get, respectively, the Feynman and the Landau gauges;

they correspond to

$$\langle V(1) V(2) \rangle_{\text{FEYNMAN}} = \frac{-2i}{4^5} \frac{1}{k^2} \theta_{12} \theta_{12} \bar{\theta}_{12} \bar{\theta}_{12} \quad (5.23)$$

$$\langle V(1) V(2) \rangle_{\text{LANDAU}} = \frac{-i}{4^5} \frac{e^{\bar{\theta}_1 \gamma \theta_2 k}}{k^4} (4 + \theta_{12} \theta_{12} \bar{\theta}_{12} \bar{\theta}_{12} k^2) \quad (5.24)$$

respectively.

Observe that, using (5.19), in the Landau gauge, the vector propagator satisfies

$$D_{1,2} D_{1,2} \langle V(1) V(2) \rangle = \bar{D}_{1,2} \bar{D}_{1,2} \langle V(1) V(2) \rangle = 0 \quad (5.25)$$

which is nothing but the supersymmetric generalization of the usual transversality equation

$$k^\mu \Delta_{\mu\nu}(k) = 0 \quad (5.26)$$

In the Feynman gauge one has the very interesting property

$$\langle V(1) V(2) \rangle_{\text{FEYNMAN}}^2 = 0 \quad (5.27)$$

which is very useful for computations of Feynman supergraphs.

We observe that, for computational use, another useful gauge is the one where $\alpha = -1$, i.e. in this gauge the vector component field has the form

$$\Delta_{\mu\nu} = \frac{g_{\mu\nu}}{k^2} - 2 \frac{k_\mu k_\nu}{k^4} \quad (5.28)$$

The gauge (5.28) is useful from the point of view of supergraphs, because it corresponds to an effective behaviour of the superfield propagator of the kind $1/k^4$, as we shall see in section 6.

We wish now to compute the vector propagator for the massive Abelian theory. We start with the free equations of motion given by (4.13) with $g = 0$. We get

$$(4D^\alpha \bar{D} \bar{D} D_\alpha + 32M^2 - \frac{2}{\alpha} (DD + \bar{D} \bar{D})^2) V = J \quad (5.29)$$

Applying $D^\alpha \bar{D} \bar{D} D_\alpha$ to (5.29) we obtain

$$D^\alpha \bar{D} \bar{D} D_\alpha V = \frac{1}{32} \frac{1}{M^2 - \frac{1}{\alpha} \square} D^\alpha \bar{D} \bar{D} D_\alpha J \quad (5.30)$$

Applying $(DD + \bar{D} \bar{D})^2$ to (5.29) we have

$$(DD + \bar{D} \bar{D})^2 V = \frac{1}{32} \frac{1}{M^2 - \frac{1}{\alpha} \square} (DD + \bar{D} \bar{D})^2 J \quad (5.31)$$

Using (5.29), (5.30) and (5.31), we obtain

$$V = \frac{1}{32M^2} \left[1 + \frac{1}{16\alpha} \frac{1}{M^2 - \frac{1}{\alpha} \square} (DD + \bar{D} \bar{D})^2 - \frac{1}{8} \frac{1}{M^2 - \frac{1}{\alpha} \square} D^\alpha \bar{D} \bar{D} D_\alpha \right] J \quad (5.32)$$

Equation (5.32) gives the following propagator for the V massive field

$$\begin{aligned} \langle T(V(k, \theta_1) V(-k, \theta_2)) \rangle &= \\ &= \frac{-i}{32M^2} \left[1 + \frac{1}{16\alpha} \frac{1}{M^2 + \frac{1}{\alpha} k^2} (D_1 D_1 + \bar{D}_1 \bar{D}_1)^2 - \frac{1}{8} \frac{1}{M^2 + k^2} D_1^\alpha \bar{D}_1 \bar{D}_1 D_{1,\alpha} \right] \delta^4(\theta_{12}) = \end{aligned}$$

$$\begin{aligned}
 &= \frac{-i}{2 \cdot 4^4 M^2} e^{\bar{\Theta}_1 \gamma \Theta_2 K} \left[\Theta_{1L} \Theta_{1L} \bar{\Theta}_{1L} \bar{\Theta}_{1L} + \frac{1}{2} \frac{1}{K^2 + \alpha M^2} (4 - \Theta_{1L} \Theta_{1L} \bar{\Theta}_{1L} \bar{\Theta}_{1L} K^2) \right. \\
 &\quad \left. - \frac{1}{2} \frac{1}{K^2 + M^2} (4 + \Theta_{1L} \Theta_{1L} \bar{\Theta}_{1L} \bar{\Theta}_{1L} K^2) \right] = \\
 &= \frac{-i}{4^4 M^2} e^{\bar{\Theta}_1 \gamma \Theta_2 K} \left(\frac{1}{K^2 + \alpha M^2} - \frac{1}{K^2 + M^2} \right) - \quad (5.33) \\
 &\quad - i \frac{\Theta_{1L} \Theta_{1L} \bar{\Theta}_{1L} \bar{\Theta}_{1L}}{4^5 M^2} \left(2 - \frac{K^2}{K^2 + \alpha M^2} - \frac{K^2}{K^2 + M^2} \right)
 \end{aligned}$$

Such a propagator describes a particle multiplet with mass $k^2 = M^2$ together with some ghost particles with mass $k^2 = \alpha M^2$. However, we find that, in spite of the fact that such ghosts decouple from S-matrix elements because of the Ward identities (4.14), they provide the same ultraviolet property for the supermultiplet propagator of the zero-mass theory as easily seen by inspection of (5.33).

Finally we have to write the propagator for the Faddeev-Popov fields $c_{\pm}$. We get, from the free Faddeev-Popov Lagrangian

$$\frac{32}{\alpha} \square c_{\pm} = \mp \zeta_{\pm} \quad (5.34)$$

$$\bar{D}_{\alpha} c_{\pm} = 0$$

$$\frac{32}{\alpha} \square \bar{c}_{\pm} = \mp \bar{\zeta}_{\pm} \quad (5.35)$$

so we simply get

$$c_{\pm} = \mp \frac{\alpha}{32 \square} \zeta_{\pm} \quad (5.36)$$

and the propagator

$$\langle T(c_+(k, \theta_1) c_-(-k, \theta_2)) \rangle = \frac{-i}{2 \times 4^3} \frac{e^{\bar{\theta}_1 \delta \theta_2 k}}{k^2} \theta_{12} \theta_{12} \quad (5.37)$$

$$\langle T(c_+(k, \theta_1) \bar{c}_-(-k, \theta_2)) \rangle = 0 \quad (5.38)$$

Note that c_+ is a chiral superfield of dimension 1, while c_- is a chiral superfield of dimension 0; the usual scalar ghost propagator has to be identified with

$$\langle A_{c_+}(k) F_{c_-}(-k) \rangle = \frac{\alpha i}{32} \frac{1}{k^2} \quad (5.39)$$

and the usual scalar ghost fields of conventional Yang-Mills theories on the fields $A_{c_+}(x)$, $F_{c_-}(x)$ which are, in fact, dimension 1 canonical fields.

Having the propagators and the functional differentiation rule for the vector and chiral multiplets given in appendix A, we can write down the generating functional for the amputated Green's functions of perturbation theory.

According to Hori [20], we have for the Yang-Mills Lagrangian

$$\begin{aligned} Z(\phi) = & \exp \frac{1}{2} \int dV(1) dV(2) \frac{\delta}{\delta V(1)} \Delta_V(1,2) \frac{\delta}{\delta V(2)} + \\ & + \int dS(1) dS(2) \frac{\delta}{\delta S(1)} \Delta_{SS}(1,2) \frac{\delta}{\delta S(2)} + \int d\bar{S}(1) d\bar{S}(2) \frac{\delta}{\delta \bar{S}(1)} \Delta_{\bar{S}\bar{S}}(1,2) \frac{\delta}{\delta \bar{S}(2)} + \\ & + \int dS(1) d\bar{S}(2) \frac{\delta}{\delta S(1)} \Delta_{S\bar{S}}(1,2) \frac{\delta}{\delta \bar{S}(2)} + \int d\bar{S}(1) dS(2) \frac{\delta}{\delta \bar{S}(1)} \Delta_{\bar{S}S}(1,2) \frac{\delta}{\delta S(2)} + \\ & + \int dC_+(1) dC_-(2) \frac{\delta}{\delta C_+(1)} \Delta_C(1,2) \frac{\delta}{\delta C_-(2)} + \int d\bar{C}_+(1) d\bar{C}_-(2) \frac{\delta}{\delta \bar{C}_+(1)} \Delta_{\bar{C}}(1,2) \frac{\delta}{\delta \bar{C}_-(2)} \\ & \exp i \int dV(3) [\mathcal{L}_G(3) + \mathcal{L}_{\text{BREAK}}(3) + \mathcal{L}_{\text{FP}}(3)]_{\text{INTERACTION}} \end{aligned} \quad (5.40)$$

where ϕ stands for V , $S_{\pm}$, $\bar{S}_{\pm}$, $C_{\pm}$, or $\bar{C}_{\pm}$, and obvious index summations have been omitted.

The three terms in the last exponential of (5.40) are already given by eqs. (3.1), (2.24) and (3.35) respectively (without sources). Expression (6.40) can be equally adapted to the Abelian theory dropping the Faddeev-Popov part and with the substitution of $\mathcal{L}_G(3)$ given by eq. (3.1) with eq. (4.4) (without sources).

Finally, for the massive vector theory we have simply to substitute for the massless vector propagator defined by (5.20) the massive one given by (5.33).

Therefore the amputated n-point Green's functions in perturbation theory are simply given by

$$G(\phi(A_1) \dots \phi(A_n)) = \frac{\delta}{\delta \phi(A_1)} \frac{\delta}{\delta \phi(A_2)} \dots \frac{\delta}{\delta \phi(A_n)} Z(\phi) \Big|_{\phi=0} \quad (5.41)$$

In the next section we will give a very simple power-counting formula for the degree of divergence of any supergraph of the perturbation expansion of supersymmetric gauge theories given by formulae (5.40) and (5.41).

6. ULTRAVIOLET ANALYSIS OF SUPERGRAPHS, POWER-COUNTING AND RENORMALIZATION

In the present section we will derive an upper bound for the superficial degree of divergence of any perturbation theory graph of the supersymmetric gauge theories.

We will consider first the Abelian theory and then the Yang-Mills theory. From our power-counting formulae we will be able to classify all primitively divergent diagrams of the gauge theory. Provided we assume that a regularization procedure exists, which preserves supersymmetry and gauge invariance, we will then show that such primitive divergences (in spite of the fact that they occur in infinite numbers at any n-order loop, just because of the Ward identities of gauge invariance) turn out to be all related and can be, in fact, expressed only through a very small number of primitively divergent graphs. Such independent divergences are a wave-function renormalization for the vector and for the scalar multiplet in the Abelian theory, and an additional charge renormalization for the vector

multiplet and wave-function renormalization for the FP ghosts in the non-Abelian case. Hence the previous discussion indicates that, once a gauge-invariant counter term has been introduced, which makes these few basic graphs finite, then, because of the Ward identities all other primitively divergent graphs are made automatically finite. Such gauge-invariant counter terms are completely fixed by supersymmetry, gauge invariance and power-counting.

We analyse first the Abelian theory. Consider a general graph. We use the following notations

$$\left. \begin{array}{ll} L & : \text{number of loops} \\ S, V & : \text{number of scalar and vector propagators, respectively} \\ n & : \text{number of vertices} \\ N_S, N_V & : \text{number of external scalar and vector lines.} \end{array} \right\} (6.1)$$

In order to count the degree of divergence of a general graph, we have to compute the k dependence coming from vertices, propagators, loops and θ factors; the last being due to the fact that, for any possible couple i, j , a factor like $\theta_{ij} \theta_{ij} \bar{\theta}_{ij} \bar{\theta}_{ij} k^2$ (k being a loop momentum) can arise, which increases the divergence of the graph. Then we have that

$$d = -2S - 4V + 4L + 2(n-1) \quad (6.2)$$

In fact, apart from the θ factor, each scalar propagator behaves as $1/k^2$, while each vector propagator behaves as $1/k^4$, and the maximum power of k^2 which can come from θ factors is just equal to the maximum number of independent differences of θ 's one can construct, which is just $n - 1$. Note that no additional dimensional factors are given by the vertices, because they are just point-like vertices with respect to superspace [see, for instance, eq. (4.4)].

Using now the two topological relations

$$L = S + V + 1 - n \quad (6.3)$$

$$2n = 2S + N_S \quad (6.4)$$

the last being due to charge conservation, substituting in (6.2), we get

$$d = 2 - N_S \quad (6.5)$$

A better estimate of d can be proved to be

$$d = 2 - N_S - N_{\bar{S}} - \frac{1}{2} |N_S - N_{\bar{S}}| \quad (6.6)$$

This is because the most divergent graph is when there are a maximum number of $S\bar{S}$ propagators. In fact, if one replaces an $S\bar{S}$ propagator by an SS propagator the graph is less divergent, because such a propagator carries a δ -function in θ space which decreases at least by 1 the degree of divergence of the graph. On the other hand, because all vertices are of the $S\bar{S}$ type, the fact that $N_S - N_{\bar{S}} \neq 0$ necessarily implies the existence of $\frac{1}{2}|N_S - N_{\bar{S}}|$ propagators of SS or $\bar{S}\bar{S}$ type in the graph which implies the lowering of d from (6.5) to (6.6).

Then we have that all graphs with $N_S > 2$ lines and an arbitrary number of vector lines and with $2S$ (or $2\bar{S}$) lines and an arbitrary number of vector lines are convergent.

Graphs with $1S$, $1\bar{S}$ lines and an indefinite number of vector lines are logarithmically divergent, while graphs with only vector lines are quadratically divergent.

Assuming a supersymmetric and gauge-invariant regularization procedure has been introduced, all logarithmically divergent graphs with $1S$, $1\bar{S}$ matter lines and an indefinite number of gauge lines are made finite by a unique gauge-invariant counter term; the only basic independent divergence being the wave-function renormalization of the matter field, which comes from the logarithmically infinite part of the 1PI two-point function $\langle S\bar{S} \rangle$. This is a consequence of the Ward identities (4.21) of gauge invariance. The counter term is

$$(Z_S - 1) \frac{1}{2} (\bar{S}_+ e^{gV} S_+ + \bar{S}_- e^{-gV} S_-) \quad (6.7)$$

Note that no independent mass counter term, which would be of the form

$$\int m S_+ S_- + h.c. \quad (6.8)$$

is generated in this theory. So we have that

$$\delta m = 0 \quad (6.9)$$

in exact analogy with the pure chiral model [1,6]. We observe that the term (6.8) is both gauge-invariant and supersymmetric so the result (6.9) is not a consequence of the symmetry but of the particular model we are considering.

Consider now the quadratic divergent graphs, where only vector lines are present. In the regularized theory such divergences, for graphs with $N_V > 2$, must cancel at any loop order. In fact, there is no gauge-invariant counter term of such a form; then gauge invariance, which is assumed to hold in the regularized theory, implies that they are absent. It is amusing to observe, also if we do not have any simple explanation for this, that in this supersymmetric gauge theory, at least at the one-loop level (see appendix C), such divergences cancel without introduction of any regularization procedure.

For $N_V = 2$ only the divergence of the longitudinal part of the two-point function must be absent, because of the Ward identities. The transverse part, defined as

$$D D \langle V V \rangle_T = \bar{D} \bar{D} \langle V V \rangle_T = 0 \quad (6.10)$$

which is logarithmically divergent (by power-counting) needs renormalization and can be made finite by the gauge-invariant counter term

$$(Z_V - 1) D^\alpha V \bar{D} \bar{D} D_\alpha V \quad (6.11)$$

i.e. a wave-function renormalization.

Summarizing the first part of this section, we have proved that, provided a suitable regularization procedure has been found, which maintains supersymmetry and gauge invariance, the supersymmetric Abelian gauge theory is renormalizable with only a wave-function renormalization, both for the vector and for the matter multiplet. Such a proof extends trivially to the massive vector theory with a possible addition of a mass renormalization of the vector multiplet. The crucial point of our previous derivation is that, because of the Ward identities (4.21),

the form of the counter terms is unique and all the different (infinite) primitively divergent graphs are related to only two independent infinities.

We consider now the non-Abelian theory, i.e. the Yang-Mills theory. First of all we have to generalize the power-counting argument to the non-Abelian case and to derive the analogue of (6.6). In such a theory we have three types of vertices. First pure Yang-Mills couplings given by eq. (2.4): the general structure of such a term being

$$V^m D_\alpha V^m \bar{D} \bar{D} (V^\ell D_\alpha V^j) g^{n+m+\ell+j-2} \quad (6.12)$$

Then coupling with matter fields and with Faddeev-Popov fields given by eqs. (2.18) and (3.36). By inspection of eq. (3.36) one realizes that, provided one defines the field c_+ , $\bar{c}'_- = DDc_-$, the Lagrangian has essentially the same structure as the matter part with the replacement of $S, \bar{S}$ by $c, \bar{c}'$. So, for what concerns the matter part and the Faddeev-Popov part, the power-counting is exactly the same as in the Abelian theory.

The power-counting is, however, modified from the new vertices (6.12), because, contrary to the matter and Faddeev-Popov vertices, each of them carries a dimension 2 as seen by (6.12).

Taking into account the previous discussion we get in the Yang-Mills theory

$$L = S + G + V - n_c - n_v + 1 \quad (6.13)$$

$$2n_c = 2(S + G) + N_s + N_g \quad (6.14)$$

$$d = -2(S + G) - 4V + 4L + 2(n_c + n_v - 1) + 2n_v \quad (6.15)$$

where G refers to ghost propagators and n_c, n_v refer, respectively, to chiral (matter or Faddeev-Popov) and to pure vector vertices. Substitution of (6.13) and (6.14) into (6.15) gives

$$d = 2 - N_s - N_g \quad (6.16)$$

and the only primitively divergent graphs are: those with two matter lines or two ghost lines and an arbitrary number of vector lines, which look logarithmically divergent; those with an arbitrary number of vector lines only, which are quadratically divergent. All the other graphs are convergent. Also in this case one could show that a better estimate of d is

$$d = 2 - N_S - N_G - \frac{1}{2} |N - \bar{N}| \quad (6.17)$$

being an N , an S , or a G line. So only graphs with $1S$, $1\bar{S}$ and $1c_+$, $1\bar{c}_+$ lines are divergent.

The proof of renormalizability runs now along the following lines. One assumes that a suitable Slavnov invariant and supersymmetric regularization procedure exists for such a theory, so that the Slavnov Ward identities derived in section 3, eq. (3.47), for the 1PI functional Γ hold in the regularized theory. The problem of regularization will be treated at the end of the section.

All divergences coming from graphs with only vector lines can be re-absorbed, because of Slavnov identities, into a wave-function renormalization and a charge renormalization for the vector field. In fact, once the two- and three-point functions for the V fields are made finite, by adding a suitable counter term, this makes automatically finite all higher Green's functions, because of the Slavnov identities.

All divergences coming from graphs with two matter or FP ghost lines and an undetermined number of vector lines, because of the Slavnov identities, can be shown to be finite, after wave-function renormalization for the matter and FP ghost fields. The structure of the counter terms for the matter and FP field, by means of the Slavnov identities, provides that all other graphs are made finite.

The renormalized Lagrangian would then have the following form

$$\mathcal{L}_R = Z_V e^{-g_V^V D^\alpha} e^{g_V^V \bar{D} \bar{D}} (e^{-g_V^V D_\alpha} e^{g_V^V V}) + \\ + h.c. + \frac{g}{\alpha} D D V \bar{D} \bar{D} V + \frac{1}{2} Z_S (\bar{S}_+ e^{g_V^V V} S_+ +$$

$$\begin{aligned}
 & \int d^4x e^{-g_V V} \bar{\psi} \psi - 2m \psi \psi - 2m \bar{\psi} \bar{\psi} + \\
 & + Z_G C + \frac{\delta V}{\delta \Lambda} (g_V) (\psi \psi + \bar{\psi} \bar{\psi}) + h.c.
 \end{aligned} \tag{6.18}$$

where Z_V , Z_S , Z_G and g_V are the wave-function and the charge-renormalization constants, respectively. We would like to point out that the details of the proof sketched in this last discussion are closely parallel to the argument given by Zinn Justin [14] for usual Yang-Mills theories: assuming the Slavnov identities given by eq. (3.47) are satisfied by the regularized LPI functional Γ , making a loopwise expansion, the structure of the counter terms is in fact uniquely fixed and it is such that the renormalized functional satisfies, order by order, the Slavnov identities. In fact the argument of ref. [14] applies equally well in our case because the Slavnov identities [eq. (3.47)] and the equations of motion for the ghosts (3.21) have exactly the same structure as in conventional Yang-Mills theories.

As a final point we have to discuss the problem of the regularization procedure one has to introduce in order to renormalize the theory. Generally, Yang-Mills theories are regularized using dimensional regularization, which automatically preserves the Ward identities [21]. However, it is not up to now completely understood how such a procedure can be extended to supersymmetric theories. In fact, dimensional regularization must preserve also the Ward identities associated with supersymmetry [22]. Other possible renormalization schemes, which look more complicated but can be probably generalized to supersymmetric theories in a straightforward way, are the Pauli-Villars method and the Normal Product approach. This last approach has been recently shown to work for the chiral model, the super S^3 theory [23].

Another possible scheme of regularization, which in fact is the one we have found simpler to examine in the context of supersymmetric gauge theories is the

higher covariant derivative method. Such a method, as known in conventional Yang-Mills theories [24], is only a partial regularization procedure, because it does not eliminate all the divergences, but restricts all divergent graphs to one-loop graphs^{*}). We shall see further that this is also the case in supersymmetric gauge theories, namely once a suitable supersymmetric covariant derivative has been introduced (which, in fact, reduces to the ordinary covariant derivative in the Wess-Zumino gauge) the higher covariant derivatives introduced in the Yang-Mills Lagrangian make all but the one-loop superdiagrams finite. The only complication one has in this case is that, also at the one-loop level there are still an infinite set of divergent graphs, because of the non-polynomial interaction. However, it is not difficult, in principle, to verify in this case the validity of the Ward identities. Let us first introduce the generalized covariant (under Yang-Mills transformations) derivatives; we define them as follows

$$\nabla_\alpha = e^{-gV} D_\alpha, \quad \bar{\nabla}_{\dot{\alpha}} = e^{gV} \bar{D}_{\dot{\alpha}} \quad (6.19)$$

They transform, respectively, as

$$\nabla_\alpha \rightarrow e^{-i\theta\Lambda} \nabla_\alpha e^{i\theta\Lambda^\dagger}, \quad \bar{\nabla}_{\dot{\alpha}} \rightarrow e^{-i\theta\Lambda^\dagger} \bar{\nabla}_{\dot{\alpha}} e^{i\theta\Lambda} \quad (6.20)$$

under Yang-Mills transformations. Therefore the quantities $\nabla_\alpha \bar{\nabla}_{\dot{\alpha}}$, $\bar{\nabla}_{\dot{\alpha}} \nabla_\alpha$ transform as

$$\nabla_\alpha \bar{\nabla}_{\dot{\alpha}} \rightarrow e^{-i\theta\Lambda} \nabla_\alpha \bar{\nabla}_{\dot{\alpha}} e^{i\theta\Lambda}, \quad \bar{\nabla}_{\dot{\alpha}} \nabla_\alpha \rightarrow e^{-i\theta\Lambda^\dagger} \bar{\nabla}_{\dot{\alpha}} \nabla_\alpha e^{i\theta\Lambda^\dagger} \quad (6.21)$$

under Yang-Mills transformations.

Let us consider now the gauge-invariant part of the Lagrangian given by eqs. (2.6), (2.15)

$$\frac{1}{g^2} \int dV \text{Tr} [\bar{D}^{\dot{\alpha}} (e^{-gV} D_\alpha e^{gV}) \bar{D}_{\dot{\alpha}} (e^{-gV} D_\alpha e^{gV})] + \text{h. c.} \quad (6.22)$$

^{*}) We acknowledge a useful discussion with J. Zinn Justin on this point.

$$\int dV \frac{1}{2} (\bar{S}_+ e^{gV} S_+ + S_- e^{-gV} \bar{S}_-) \quad (6.23)$$

respectively, for the pure Yang-Mills part and for the matter part.

It is not difficult to realize that the following expressions

$$\begin{aligned} & \frac{1}{g^2} \int dV T_1 [\dots [\nabla^\beta (\bar{\nabla}^{\dot{\beta}} [\bar{D}^{\dot{\alpha}} (\nabla^\alpha e^{gV})] e^{-gV}) e^{gV}] \dots]_2 \\ & \cdot [\dots [\nabla_\beta (\bar{\nabla}_{\dot{\beta}} [\bar{D}_{\dot{\alpha}} (\nabla_\alpha e^{gV})] e^{-gV}) e^{gV}] \dots]_2 \\ & + h.c. \end{aligned} \quad (6.24)$$

$$\begin{aligned} & \int dV \frac{1}{2} \bar{S}_+ (\bar{\nabla}^{\dot{\beta}} \nabla^\beta \bar{\nabla}_{\dot{\beta}} \nabla_\beta)^2 (e^{gV} S_+) + \\ & + \frac{1}{2} S_- (\nabla^\beta \bar{\nabla}^{\dot{\beta}} \nabla_\beta \bar{\nabla}_{\dot{\beta}})^2 (e^{-gV} \bar{S}_-) \end{aligned} \quad (6.25)$$

are supersymmetric and Yang-Mills invariant.

The verification of (6.25) is trivial because of (6.21) and of the transformation properties (2.15), (2.17). In order to verify (6.24) it is sufficient to observe the following chain of transformation rules under Yang-Mills transformations

$$\begin{aligned} & [\bar{D}^{\dot{\alpha}} (\nabla^\alpha e^{gV})] \rightarrow e^{-ig\Lambda} [\bar{D}^{\dot{\alpha}} (\nabla^\alpha e^{gV})] e^{ig\Lambda} - \\ & - 2ie^{-ig\Lambda} \sigma^{\alpha\dot{\alpha}} \partial_\alpha e^{ig\Lambda} ; \\ & \bar{\nabla}^{\dot{\beta}} [\bar{D}^{\dot{\alpha}} (\nabla^\alpha e^{gV})] \rightarrow \\ & \rightarrow e^{-ig\Lambda^\dagger} \bar{\nabla}^{\dot{\beta}} [\bar{D}^{\dot{\alpha}} (\nabla^\alpha e^{gV})] e^{ig\Lambda} , \\ & (\bar{\nabla}^{\dot{\beta}} [\bar{D}^{\dot{\alpha}} (\nabla^\alpha e^{gV})] e^{-gV}) \rightarrow \\ & \rightarrow e^{-ig\Lambda^\dagger} (\bar{\nabla}^{\dot{\beta}} [\bar{D}^{\dot{\alpha}} (\nabla^\alpha e^{gV})] e^{-gV}) e^{ig\Lambda^\dagger} \end{aligned} \quad (6.26)$$

therefore

$$\begin{aligned} & [\nabla^\beta (\bar{\nabla}^{\dot{\beta}} [\bar{D}^{\dot{\alpha}} (\nabla^\alpha e^{gV})] e^{-gV}) e^{gV}] \rightarrow \\ & e^{-ig\Lambda} [\nabla^\beta (\bar{\nabla}^{\dot{\beta}} [\bar{D}^{\dot{\alpha}} (\nabla^\alpha e^{gV})] e^{-gV}) e^{gV}] e^{ig\Lambda} \end{aligned}$$

and, finally,

$$\begin{aligned} & [\nabla^\beta (\bar{\nabla}^{\dot{\beta}} [\bar{D}^{\dot{\alpha}} (\nabla^\alpha e^{gV})] e^{-gV}) e^{gV}] \rightarrow \\ & e^{-ig\Lambda} [\nabla^\beta (\bar{\nabla}^{\dot{\beta}} [\bar{D}^{\dot{\alpha}} (\nabla^\alpha e^{gV})] e^{-gV}) e^{gV}] e^{ig\Lambda} \end{aligned}$$

Such a transformation implies, in fact, that the integrand of the expression (6.24) is invariant under Yang-Mills transformations.

Note that in the Abelian case the modification of (6.22) is trivial, because the quantity $\bar{D}\bar{D}D_\alpha V$ is invariant under local $U(1)$ transformations, so one can simply take the expression

$$\int dS \bar{D}\bar{D} D^\alpha V \bar{D}\bar{D} D_\alpha \square^2 V$$

The modification of (6.23) is exactly the same as in the non-Abelian case.

The free part of expressions (6.24), (6.25) reduce to

$$\bar{D}^{\dot{\alpha}} D^\alpha V \bar{D}_{\dot{\alpha}} D_\alpha \square^2 V, \quad \frac{1}{2} (\bar{S}_+ \square^2 S_+ + S_- \square^2 \bar{S}_-) \quad (6.27)$$

respectively. For the gauge-breaking term one simply considers the modified expression

$$D D V \bar{D} \bar{D} \square^2 V \quad (6.28)$$

which however would not affect the structure of the Ward identities and of the Faddeev-Popov part [see, for instance, the derivation of eqs. (3.33) to (3.36)].

The effect of introducing terms like (6.24), (6.25) and (6.26) is of increasing the power fall-off of the propagator by an amount $(k^2)^{-1}$, but correspondingly of introducing new vertices [see, for instance, eqs. (6.24), (6.25)] that increase the degree of divergence of the supergraphs. As a consequence, one must evaluate the net effect in the modification of the power-counting formula.

We consider the case of the matter part and of the pure Yang-Mills part separately. More general diagrams are easily seen not to behave worse. The first case includes also, as a particular case, the analysis of the Abelian theory.

One has for the matter part [see, for instance, eqs. (6.3), (6.4)]

$$L = S + V - n + 1$$

$$2n = N_S + 2S$$

$$\begin{aligned} d &= -2(1+r)S - 2(2+r)V + 4L + 2(n-1) + 2nr = (6.29) \\ &= (r-1)(2n-2S) - 2rV + 2 = 2 + r(2n-2S-2V) \\ &\quad - (2n-2S) = 2 + 2r(1-L) - N_S \end{aligned}$$

Let us now consider the pure Yang-Mills part. One has

$$L = V - n_V + 1$$

$$\begin{aligned} d &= -2(2+r)V + 4L + 2(n_V-1) + 2(1+r)n_V = (6.30) \\ &= -2rV + 2 + 2rn_V = 2 + 2r(n_V - V) = 2 + 2r(1-L) \end{aligned}$$

Formulae (6.29), (6.30) imply that, for any $r \geq 2$ every loop diagram with more than one loop ($L > 1$) is finite, while one-loop diagrams are exactly divergent, irrespective of the value of r , as in the unregularized case. Namely, the only divergent diagrams are the one-loop diagrams with an arbitrary number of external gauge lines and no more than two matter lines. We would like to recall that the degrees of divergence given in this section, namely eqs. (6.6), (6.10), (6.29), (6.30) must be considered as upper bounds, so they must be read as the maximum possible degree of divergence of a given superdiagram.

The previous analysis reduces the problem to the investigation of the one-loop superdiagram. However, explicit computations indicate that the Ward identities are satisfied at the one-loop level. In this connection see, for example, the diagrams considered in appendix C (two-leg diagrams).

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APPENDIX A

In the present appendix we give some useful formulae for calculating with superfields. These formulae essentially summarize the tensor calculus of the supersymmetry algebra needed in this work.

For convenience we take a basis where the momentum is diagonal. Then the covariant derivatives, as defined in ref. [8], take the following form

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} - (\sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha}}) \kappa_\mu, \quad \bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + (\theta^\alpha \sigma_{\alpha\dot{\alpha}}^\mu) \kappa_\mu \quad (A.1)$$

They satisfy the following properties

$$\{D_\alpha, \bar{D}_{\dot{\alpha}}\} = 2\kappa^\mu \sigma_{\mu\alpha\dot{\alpha}}, \quad \{D_\alpha, D_\beta\} = \{\bar{D}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}}\} = 0 \quad (A.2)$$

$$D_\alpha D_\beta D_\gamma = \bar{D}_{\dot{\alpha}} \bar{D}_{\dot{\beta}} \bar{D}_{\dot{\gamma}} = 0 \quad (A.3)$$

From (A.1), (A.2), (A.3) it follows that

$$\begin{aligned} [D^\alpha D_\alpha, \bar{D}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}}] &= -8(2\kappa^2 + \bar{D}^{\dot{\alpha}} \sigma_{\alpha\dot{\alpha}}^\mu D^\alpha \kappa_\mu) = \\ &= 8(2\kappa^2 + D^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \bar{D}^{\dot{\alpha}} \kappa_\mu) \end{aligned} \quad (A.4)$$

$$[D_\alpha, \bar{D}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}}] = 4(\sigma_{\alpha\dot{\alpha}}^\mu \bar{D}^{\dot{\alpha}}) \kappa_\mu, \quad [\bar{D}_{\dot{\alpha}}, D^\alpha D_\alpha] = -4(D^\alpha \sigma_{\alpha\dot{\alpha}}^\mu) \kappa_\mu \quad (A.5)$$

$$D^\alpha \bar{D} \bar{D} D_\alpha = \bar{D}_{\dot{\alpha}} D D \bar{D}^{\dot{\alpha}} \quad \bar{D} \bar{D} = \bar{D}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}}, \quad D D = D^\alpha D_\alpha \quad (A.6)$$

From eqs. (A.4), (A.5), (A.6) one can easily deduce eqs. (5.11) to (5.14) of the text. In particular, note that the two real operators

$$P_T = \frac{1}{8\kappa^2} D^\alpha \bar{D} \bar{D} D_\alpha, \quad P_L = -\frac{1}{16\kappa^2} (D D + \bar{D} \bar{D})^2 \quad (A.7)$$

are a complete set of orthogonal projectors, i.e. they satisfy the following properties:

$$P_T^2 = P_T, \quad P_L^2 = P_L, \quad P_T P_L = P_L P_T = 0, \quad P_T + P_L = I \quad (A.8)$$

The following differentiation properties hold

$$D^\mu D_\mu (\Theta^\rho \Theta_\rho) = -4 e^{\Theta \sigma_\mu \bar{\Theta} k^\mu} \quad (A.9)$$

$$\bar{D}_\alpha \bar{D}^\alpha (\bar{\Theta}_\beta \bar{\Theta}^\beta) = -4 e^{-\Theta \sigma_\mu \bar{\Theta} k^\mu}$$

Consider now two superpoints of coordinates (k, θ_1) and $(-k, \theta_2)$, respectively.

Then one can deduce

$$\begin{aligned} e^{-\bar{\Theta}_1 \gamma \theta_{1\mu} k} \bar{D}_{1\alpha} e^{\bar{\Theta}_1 \gamma \theta_{1\mu} k} &= \bar{D}_{1\alpha} \\ e^{\bar{\Theta}_1 \gamma \theta_{1\mu} k} D_{1\alpha} e^{-\bar{\Theta}_1 \gamma \theta_{1\mu} k} &= D_{1\alpha} \end{aligned} \quad \{D_{1\alpha}, \bar{D}_{2\dot{\alpha}}\} = 0 \quad (A.10)$$

From (A.9), (A.10) one obtains

$$\begin{aligned} \bar{D}_1 \bar{D}_1 (e^{\bar{\Theta}_1 \gamma \theta_{1\mu} k} \bar{\Theta}_{1\alpha} \bar{\Theta}_{1\dot{\alpha}}) &= e^{\bar{\Theta}_1 \gamma \theta_{1\mu} k} \bar{D}_1 \bar{D}_1 (\bar{\Theta}_{1\alpha} \bar{\Theta}_{1\dot{\alpha}}) \\ &= -4 e^{\bar{\Theta}_1 \gamma \theta_{1\mu} k} - \theta_{1\alpha} \bar{\Theta}_{1\dot{\alpha}} k \end{aligned} \quad (A.11)$$

Note that because of the identities

$$\begin{aligned} e^{\bar{\Theta}_1 \gamma \theta_{1\mu} k} \theta_{1\alpha} \bar{\Theta}_{1\dot{\alpha}} &= e^{\theta_{1\alpha} \bar{\Theta}_{1\dot{\alpha}} k - \theta_{1\alpha} \bar{\Theta}_{1\dot{\alpha}} k} \theta_{1\alpha} \bar{\Theta}_{1\dot{\alpha}} \\ e^{\bar{\Theta}_1 \gamma \theta_{1\mu} k} - \theta_{1\alpha} \bar{\Theta}_{1\dot{\alpha}} k &= e^{-(\theta_{1\alpha} \bar{\Theta}_{1\dot{\alpha}} + \theta_{1\alpha} \bar{\Theta}_{1\dot{\alpha}}) k + 2 \theta_{1\alpha} \bar{\Theta}_{1\dot{\alpha}} k} \end{aligned} \quad (A.12)$$

from (5.4), (5.5) we get

$$\begin{aligned} \langle S_1(1) \bar{S}_2(2) \rangle &= \frac{-i}{16(m^2 + k^2)} e^{2 \theta_{1\alpha} \bar{\Theta}_{1\dot{\alpha}} k} \\ \langle S_1(1) S_1(2) \rangle &= \frac{i m}{16(k^2 + m^2)} \theta_{1\alpha} \bar{\Theta}_{1\dot{\alpha}} \end{aligned} \quad (A.13)$$

where the subscripts 1, 2 refer to bases 1, 2 according to the definitions of ref. [8]. On such a basis the chirality properties of the propagators (A.13) are manifest. According to the definitions (3.5), which imply (3.6) we have the following δ functions in θ space

$$\begin{aligned} \delta^4(\theta_{1\alpha}) &= 1/2^4 \theta_{1\alpha} \theta_{1\alpha} \bar{\Theta}_{1\dot{\alpha}} \bar{\Theta}_{1\dot{\alpha}} \\ \delta^2(\theta_{1\alpha}) &= -1/2^2 \theta_{1\alpha} \theta_{1\alpha} \\ \delta^2(\bar{\Theta}_{1\dot{\alpha}}) &= -1/2^2 \bar{\Theta}_{1\dot{\alpha}} \bar{\Theta}_{1\dot{\alpha}} \end{aligned} \quad (A.14)$$

They satisfy the following identities

$$\int d^2\theta \delta^2(\theta) = \int d^2\bar{\theta} \delta^2(\bar{\theta}) = \int d^4\theta \delta^4(\theta) = 1$$

The corresponding δ functions in x, θ space are

$$\delta_V(1,2) = \delta^4(x_1 - x_2) \delta^4(\theta_{12}) \quad (\text{A.15})$$

$$\delta_S(1,2) = \delta^4(x_1 - x_2 + i\bar{\theta}_1 \gamma \theta_2) \delta^2(\theta_{12}) \quad (\text{A.16})$$

$$\delta_{\bar{S}}(1,2) = \delta^4(x_1 - x_2 + i\bar{\theta}_1 \gamma \theta_2) \delta^2(\bar{\theta}_{12})$$

They satisfy the normalization property

$$\int dV \delta_V = \int dS \delta_S = \int d\bar{S} \delta_{\bar{S}} = 1 \quad (\text{A.17})$$

We have also the useful property

$$\int dV = \int dS \bar{D} D = \int d\bar{S} D D \quad (\text{A.18})$$

which is, of course, a consequence of previous definitions.

From (A.15), (A.16) we have the following definition of functional differentiation in superspace

$$\frac{\delta V(2)}{\delta V(1)} = \delta_V(1,2) \quad (\text{A.19})$$

$$\frac{\delta S(2)}{\delta S(1)} = \delta_S(1,2), \quad \frac{\delta \bar{S}(2)}{\delta \bar{S}(1)} = \delta_{\bar{S}}(1,2) \quad (\text{A.20})$$

$$\frac{\delta S(2)}{\delta \bar{S}(1)} = \frac{\delta \bar{S}(2)}{\delta S(1)} = 0 \quad (\text{A.21})$$

where V, S are the vector and chiral fields, respectively. Note that (A.20) can be written as before in the manifestly chiral form

$$\frac{\delta S_1(x_1, \theta_1)}{\delta S_2(x_2, \theta_2)} = \delta^4(x_1 - x_2) \delta^2(\theta_{12}), \quad \frac{\delta \bar{S}_2(x_2, \bar{\theta}_2)}{\delta \bar{S}_1(x_1, \bar{\theta}_1)} = \delta^4(x_1 - x_2) \delta^2(\bar{\theta}_{12}) \quad (\text{A.22})$$

APPENDIX B

The group property (3.28) can be proved making use of a straightforward generalization of the derivation given in ref. [16].

Let $\mathcal{G}$ be a Lie algebra whose elements are linear superpositions of the generators of the semi-simple group considered in this paper. We call X_a the elements of a basis of $\mathcal{G}$, with commutation relations

$$[X_a, X_b] = i f_{abc} X_c \quad (a, b, c = 1, \dots, N^2-1 \text{ for } SU(N)) \quad (B.1)$$

An element $\Omega \in \mathcal{G}$ is represented by a couple

$$\Omega = (\Lambda, \bar{\Lambda}) = (\Lambda^a X_a, \bar{\Lambda}^b X_b) = (\Lambda_i^a X_a, i=1, 2) \quad (B.2)$$

where $\Lambda_i^a(x, \theta, \bar{\theta})$ are the infinitesimal parameters of the gauge transformation.

They satisfy the chiral properties

$$D_i \Lambda_i = 0 \quad (D = \bar{D}, D_2 = D) \quad (B.3)$$

We shall treat the gauge group like a global symmetry, with parameters $(a, i, x, \theta, \bar{\theta})$. A point in superspace will be denoted by z . Then a field at the point z transforms according to

$$\varphi_A^\Omega(z) = \varphi_A(z) + \sum_i \int d\zeta_i(z') \Lambda_i^a(z') t_{A a i}(z, z') \quad (B.4)$$

Let $F(z)$ be a covariant functional of the fields ψ_A , with $Z = (z'_1, \dots, z'_n)$ $n \geq 0$.

It will also transform according to

$$F^\Omega(Z) = F(Z) + \sum_i \int d\zeta_i(z') \Lambda_i^a(z') T_i^a(z, Z) \quad (B.5)$$

The group property means

$$\frac{\delta^2}{\delta \Lambda_i^a(z) \delta \Lambda_j^b(z)} (F^{\Omega \Omega'}(Z) - F^{\Omega' \Omega}(Z)) = \frac{\delta^2}{\delta \Lambda_i^a(z) \delta \Lambda_j^b(z)} F^{i[\Omega, \Omega']}(Z) \quad (B.6)$$

(here and in the formulae that follow, all functional derivatives are taken at the same point $\Omega = \Omega' = 0$). The left-hand side of (B.6) can be rewritten as

$$\left(\frac{\delta^2}{\delta \Lambda_i^a(z_1) \delta \Lambda_j^b(z_2)} - \frac{\delta^2}{\delta \Lambda_i^b(z_1) \delta \Lambda_j^a(z_2)} \right) F^{\Omega \Omega'}(z) \quad (B.7)$$

To evaluate the right-hand side of (B.6) we observe that $i[\Omega, \Omega']_F$ is the infinitesimal transformation of F given by (B.5) with parameters given by

$$\tilde{\Omega}_i = [\Omega_i, \Omega'_i] = i f_{abc} \Lambda_i^a \Lambda_i^b X_c \quad (B.8)$$

Hence

$$F^{\Omega, \Omega'}(z) = F(z) + \sum_i \int dS_i(z) i f_{abc} \Lambda_i^a(z) \Lambda_i^b(z) T_i^c(z; z) \quad (B.9)$$

and double differentiation taken at $\Lambda = \Lambda'$ gives the right-hand side of (B.6).

Therefore, we get the basic group property

$$\left(\frac{\delta^2}{\delta \Lambda_i^a(z_1) \delta \Lambda_j^b(z_2)} - \frac{\delta^2}{\delta \Lambda_i^b(z_1) \delta \Lambda_j^a(z_2)} \right) F^{\Omega \Omega'}(z) = \quad (B.10)$$

at $\Omega = \Omega'$

$$= -\delta_{ij} f_{abc} \delta S_i(z_1, z_2) \frac{\delta}{\delta \Lambda_i^c(z_1)} F^{\Omega}(z)$$

APPENDIX C

In the present section we show two typical examples of cancellation of divergences among different supergraphs, which occur in supersymmetric gauge theories, due to the combined presence of the gauge Ward identities and of supersymmetry invariance.

In the first example we show, at the one-loop level, the gauge invariance of the wave-function renormalization of the charged matter field. We consider, for simplicity, the Abelian case, but such a consideration can equally be applied in the non-Abelian situation.

Consider the two supergraphs given by figs. 1 which contribute to the (amputated) two-point function $\langle S(A)\bar{S}(B) \rangle$ at the one-loop level.

From our power-counting formula we know that they are both logarithmically divergent, but we shall see in a moment that the infinite part of such graphs, when added, is gauge-invariant.

These graphs can be easily computed using our functional expression (5.40) of the perturbation series, together with the propagators and Feynman rules defined in section 5. We get, up to an over-all multiplicative factor

$$\begin{aligned} \langle S_1(A)\bar{S}_1(B) \rangle = & \bar{D}_A \bar{D}_A D_B D_B (\Delta_V(A,B) \Delta_{S_2\bar{S}_2}(B,A)) - \\ & - \frac{i}{2} \Delta_V(0) \delta^4(x_A - x_B + i\bar{\theta}_A \gamma \theta_B - i\theta_{AB} \sigma \bar{\theta}_{AB}) \end{aligned} \quad (C.1)$$

The infinite part coming from the seagull (fig. 1b) is

$$Z_2(\alpha) = -\frac{i}{2} \int d^4k \frac{4(1-\alpha)}{k^4} \quad (C.2)$$

while the infinite part of the first term on the right-hand side of eq. (C.1) (fig. 1a) is

$$\begin{aligned} Z_1(\alpha) = & \bar{D}_A \bar{D}_A D_B D_B \left(\int d^4k e^{\theta_{AB} \sigma \bar{\theta}_{AB} k} \frac{(-i)}{16k^2} \left[4(1-\alpha) \frac{1}{k^4} + \right. \right. \\ & \left. \left. + \frac{(1+\alpha)}{k^2} \theta_{AB} \theta_{AB} \bar{\theta}_{AB} \bar{\theta}_{AB} \right] \right) = \end{aligned}$$

$$\begin{aligned}
 &= \bar{D}_A \bar{D}_A D_B D_B (\theta_{AB} \theta_{AB} \bar{\theta}_{AB} \bar{\theta}_{AB}) \frac{(-i)}{16} \int d^4 k \frac{2\alpha}{k^4} = \\
 &= -i \int d^4 k \frac{2\alpha}{k^4}
 \end{aligned} \tag{C.3}$$

where only the term coming from the maximum number of θ 's contributes.

Adding together we have

$$Z_2(\alpha) + Z_1(\alpha) = (-i) 2 \int \frac{d^4 k}{k^4} \tag{C.4}$$

i.e. a gauge-invariant result.

Note in particular that the seagull does not contribute in the Feynman gauge ($\alpha = 1$), while it gives the entire contribution in the Landau gauge ($\alpha = 0$). Of course, such a result remains true at any order of perturbation theory. In fact, from supersymmetry we have that the renormalization of the mass is only due to a wave-function renormalization; no additional independent mass renormalization δM occurs. On the other hand, gauge invariance provides the gauge independence of the renormalized physical mass.

We now consider the vacuum polarization graphs. From the discussion of section 6 we know they are at most quadratically divergent. We explicitly verify at the one-loop level that, as a consequence of gauge invariance and of supersymmetry, the quadratically divergent part is exactly zero.

The vacuum polarization is given at the one-loop level by the sum of two graphs, namely those given by figs. 2a, 2b.

We get, up to an over-all normalization factor

$$\begin{aligned}
 \langle V(A) V(B) \rangle &= \Delta_{S, \bar{S}_L}(A, B) \Delta_{S, \bar{S}_L}(B, A) \\
 &\quad - i \Delta_{S, \bar{S}}(0) \delta^4(x_A - x_B) \delta^4(\theta_{AB})
 \end{aligned} \tag{C.5}$$

The quadratically divergent part of the first term of the right-hand side of (C.5) gives

$$\begin{aligned} Z_1 &= \frac{-i}{16} \int d^4 k \, e^{2\theta_{AB} \bar{\theta}_{AB} k} \frac{1}{k^4} = \\ &= \frac{i}{16} \theta_{AB} \theta_{AB} \bar{\theta}_{AB} \bar{\theta}_{AB} \int d^4 k \, \frac{1}{k^4} \end{aligned} \quad (C.6)$$

and we have retained only the term with the maximum number of θ factors.

The seagull term gives directly

$$Z_2 = \frac{-i}{16} \theta_{AB} \theta_{AB} \bar{\theta}_{AB} \bar{\theta}_{AB} \int d^4 k \, \frac{1}{k^4} \quad (C.7)$$

So we get that the quadratic divergent part just cancels. Such a result, which is of course gauge-independent, has been in fact first observed in ref. [2].

APPENDIX D

In this final appendix we consider two different problems. First of all we show that, according to the general discussion given by Zumino [25], the vacuum supergraphs cancel in the supersymmetric Abelian gauge theory. This was explicitly shown, at the second-loop level, in ref. [24], for the super S^3 theory. We extend this proof, in the same second-loop approximation, to the supersymmetric vector theory. Note that the present proof also holds in the case where we give a mass to the vector multiplet.

The graphs which give contribution to the pure vacuum graph are the two listed in figs. 3. Note in fact that, contrary to the chiral model, no tadpole graph is allowed here because of the absence of $S_1 S_1 V$ or $\bar{S}_1 \bar{S}_1 V$ couplings.

The graph obtained, putting together the two seagulls, gives (fig. 3b)

$$\int dV(1) \Delta_V(q) \Delta_{S\bar{S}}(q) = \int d^4\theta \int d^4k \int d^4h \frac{1}{k^2} \frac{1}{(k+h)^2} = 0 \quad (D.1)$$

because

$$\int d^4\theta \text{ const} = 0$$

The other supergraph gives (fig. 3a)

$$\iint dV(1) dV(2) \Delta_V(1,2) \Delta_{S_1 \bar{S}_1}(1,2) \Delta_{S_2 \bar{S}_2}(2,1) \quad (D.2)$$

which can be written as

$$\iint d^4\theta_1 d^4\theta_2 f(\theta_{12}) \quad (D.3)$$

where $f(\theta_{12})$ is at most of degree four in the θ 's; $f(\theta_{12}) = f\theta_{12}\theta_{12}\bar{\theta}_{12}\bar{\theta}_{12} + \dots$ and therefore, after double integration, gives a vanishing result.

We have seen that the two supergraphs separately vanish, independently of gauge invariance. This is because, according to the discussion given in ref. [25], the vanishing of the vacuum graphs is a property of supersymmetric theories.

Preliminary computations at higher loops support the conclusion that vacuum graphs vanish at each order of perturbation theory in the supersymmetric vector theory.

As a final problem we consider what happens if we add to the original gauge-invariant Lagrangian given by eq. (4.4) the further invariant

$$\xi V \quad (D.4)$$

Note that $\int dV \xi V$ is gauge-invariant because

$$\int dN (\lambda - \bar{\lambda}) = 0 \quad (D.5)$$

Of course, such a term explicitly violates parity, because $V \rightarrow -V$ under a parity transformation. It was shown in ref. [19] that such a term, just in the tree approximation, triggers a spontaneous breakdown of supersymmetry [and possibly of local $U(1)$ symmetry], so we would like to understand how this manifests in the covariant formalism. We consider the original gauge-invariant part of the Lagrangian and we add to it the term (D.4)

$$\begin{aligned} \mathcal{L}_G = & -2V D^\alpha \bar{D} \bar{D} D_\alpha V + \frac{1}{2} (S_+ \bar{S}_+ e^{gV} + \\ & + S_- \bar{S}_- e^{-gV}) - 4m (S_+ S_- + \bar{S}_+ \bar{S}_-) + \xi V \end{aligned} \quad (D.6)$$

Equation (D.6) is still supersymmetric and gauge-invariant. The linear terms can be eliminated, making the substitution $V \rightarrow V' + (a/16) \theta \theta \bar{\theta} \bar{\theta}$; in fact we get, in terms of V' , the new Lagrangian

$$\begin{aligned} \mathcal{L}_G = & -2V' D^\alpha \bar{D} \bar{D} D_\alpha V' - 2a V' + \\ & + \frac{1}{2} (S_+ \bar{S}_+ e^{gV'} + g a/16 \theta \theta \bar{\theta} \bar{\theta} + S_- \bar{S}_- e^{-gV'} - g a/16 \theta \theta \bar{\theta} \bar{\theta}) \\ & - 4m S_+ S_- - 4m \bar{S}_+ \bar{S}_- + \xi V' \end{aligned} \quad (D.7)$$

So we get $a = \xi/2$ and

$$\begin{aligned} \mathcal{L}_G = & -2V' D^\alpha \bar{D} \bar{D} D_\alpha V' + \frac{1}{2} (S_+ \bar{S}_+ e^{gV'} + S_- \bar{S}_- e^{-gV'} \\ & - 4m S_+ S_- - 4m \bar{S}_+ \bar{S}_-) + \frac{g\xi}{4} (S_+ \bar{S}_+ - S_- \bar{S}_-) \times \\ & \times \theta \theta \bar{\theta} \bar{\theta} \end{aligned} \quad (D.8)$$

We see that the interaction has exactly the same form as before plus the last term, i.e. a mass term

$$\frac{g^2}{4^3} (A_+ \bar{A}_+ - A_- \bar{A}_-) \theta \theta \bar{\theta} \bar{\theta} \quad (D.9)$$

This implies a mass shift for the scalar components of the chiral multiplets of an amount proportional to $\pm \xi g$ already indicating a spontaneous breakdown of supersymmetry. Note that this can be already seen from the transformation law of the new vector multiplet V' under supersymmetry which becomes

$$\delta V'(x, \theta) = (i \bar{\alpha} \gamma_\mu \theta \partial^\mu + \bar{\alpha} \frac{\partial}{\partial \theta}) V'(x, \theta) + 2 \xi \bar{\alpha} \theta \bar{\theta} \theta \quad (D.10)$$

(α, θ being Dirac spinors), which implies that the λ component of V' transforms as

$$\delta \lambda = 2 \xi \alpha + \text{q-number terms} \quad (D.11)$$

therefore λ is a Goldstone spinor [19].

The apparent difficulty of extending the proof of renormalizability, discussed in the present work, to the spontaneously broken gauge theory, lies in the fact that we no longer can add to the previous supersymmetric Lagrangian the supersymmetric gauge-breaking term introduced in eq. (4.4). In fact, as easily seen, it would also produce a linear term in the field V' given by^{*)}

$$\frac{2 \alpha}{\alpha} V' \quad (D.12)$$

which then would give rise to a gauge-dependent mass shift proportional to

$$\frac{\alpha}{\alpha-1} \frac{3}{2} \quad (D.13)$$

*) We would like to thank Professor B. Zumino for a discussion on this point.

The previous argument shows that, in order to treat the quantized version of the spontaneously broken gauge theory, one must abandon the supersymmetric gauge-breaking term previously introduced. This can be done in a straightforward way introducing the regularized expression (6.26) which for any $r \neq 0$ gives the result (D.9), (D.11). Presumably the analysis carried out in the present paper can also be used in order to investigate the renormalization of the spontaneously broken theory [19].

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$$\text{---} = \langle S \bar{S} \rangle$$

a

b

Fig. 1

a

b

Fig. 2

a

b

Fig. 3