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ASYMPTOTIC FREEDOM AND THE CALLAN - GROSS RELATION

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A B S T R A C T

We discuss the validity of free quark model sum rules in an asymptotically free gauge theory of the strong interactions. The predictions for the Callan-Gross relation are studied in detail and comparison is made with experiment. We conclude that data on σ_L/σ_T at higher energies should provide a strong test of asymptotic freedom.

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If the strong interactions are described by an asymptotically free gauge theory (AFGT) then, by use of the renormalization group, we can uniquely determine the effects of the strong interactions in the deep Euclidean region, where all momenta are large and spacelike ¹⁾.

"Conventionally" the strong interactions are taken to be mediated by an $SU(n)$ gauge group coupling to colour. The colour group commutes with the usual Gell-Mann - Zweig $SU(3)$ and any other internal quantum numbers. Then in the deep Euclidean region the strong interactions may be parametrized by an effective coupling constant $\bar{g}$ which is dependent on the large space-like momentum transfer $Q^2 = -q^\mu q_\mu$

$$\bar{g}^2(Q^2) = \frac{g^2}{1 + B g^2 \ln \left[\frac{Q^2}{\mu^2} \right]} \quad (1)$$

Here g^2 is the value of the coupling at $\mu^2 = -q^\mu q_\mu$. The appearance of two parameters is illusory for we can re-express $\bar{g}$ in terms of another subtraction point Λ^2

$$\bar{g}^2(Q^2) = \frac{1}{B \ln \left[\frac{Q^2}{\Lambda^2} \right]} \quad \text{where} \quad \bar{g}^2(\mu^2) = g^2 = \frac{1}{B \ln \left[\frac{\mu^2}{\Lambda^2} \right]} \quad (2)$$

The constant B depends on the gauge group and the fields in the theory. With $(n=3)$ three coloured quark quartets $(P, N, \lambda \text{ and } P')$ $16\pi^2 B = 25/3$. In what follows we will use the three quartet model but our results are insensitive to the number of quarks. For instance, for $SU(3)$ quarks (P, N, λ) $16\pi^2 B = 9$. The behaviour of the theory at short distances may now be determined in terms of $\bar{g}(Q^2)$. For instance by applying the renormalization group equations to the Wilson expansion of two currents at short distances it is found that the Fourier transforms of the coefficients of the operators appearing in the expansion have the form ²⁾

$$\begin{aligned} \tilde{F}_k^n \left(\frac{Q^2}{\mu^2}, g \right) &= \tilde{F}_k^n(1, \bar{g}) \left[\frac{\bar{g}^2}{g^2} \right]^{a_k^n/B} \\ &= \tilde{F}_k^n(1, \bar{g}) \left[\frac{\ln \left[\frac{\mu^2}{\Lambda^2} \right]}{\ln \left[\frac{Q^2}{\Lambda^2} \right]} \right]^{a_k^n/B} \end{aligned} \quad (3)$$

where the anomalous dimension of the operator associated with F_k^n is, to leading order in $\bar{g}$

$$\gamma_k^n(\bar{g}) = a_k^n \bar{g}^2 \quad (4)$$

Then, since for $O_k^n = J_k^\mu$, a conserved current, $\gamma_k^n = 0$ equal time commutators will obey the current algebra commutation rules.

Near the light cone the exponential factors in Eq. (3) give definite predictions of how $\tilde{F}_k^n((Q^2/\mu^2), g)$ deviates from its free field value. This has observable consequences in the deep inelastic scattering region where the $\tilde{F}_k^n$ may be related to moments of the structure functions $F(x, Q^2)$ by

$$\int_0^1 dx x^n F(x, Q^2) = \tilde{F}_k^n\left(\frac{Q^2}{\mu^2}, g\right) M_k^n \quad (5)$$

where M_k^n is the matrix element of the relevant operator.

The a_k^n have been computed in the $SU(n)$ colour model ^{2),3)}. The $(\log Q^2)^\alpha$ deviations from Bjorken scaling may be looked for either by directly forming the moments from the data or by using knowledge of $F(x, q^2 = q_0^2)$ to determine the M_k^n and hence predict $F(x, q^2)$ at other values of q^2 ⁴⁾. These predictions depend on the parameter Λ^2 . In practice for reasonable values of Λ^2 the variations over experimentally accessible energies are rather small and experiments of high statistical accuracy will be needed to confirm or reject the presence of logarithmic terms.

For e^+e^- annihilation in our AFGT the leading scaling term is unchanged since the leading operator contributing here is the unit operator and has no anomalous dimensions. Non-leading terms ($\propto \bar{g}^2$) are suppressed by inverse powers of $\log Q^2/\Lambda^2$ ⁵⁾. In this case again high accuracy is needed to distinguish the $\log Q^2$ terms from a constant term.

However, there are circumstances in which the AFGT predicts the presence of terms $\propto (\log Q^2/\Lambda^2)^{-1}$ whereas other models predict terms varying as inverse power of Q^2 . This is a much more dramatic difference than the cases discussed above and may give the best hope for a test of the ideas of asymptotic freedom. These terms arise as follows.

Since the F_k^n carry all the Lorentz and internal symmetry information the sum rules following from the quark light-cone algebra will be true when $\bar{g}=0$. However, for $\bar{g} \neq 0$ these sum rules will not, in general, be true and since $\bar{g} \rightarrow 0$ only logarithmically in Q^2 the sum rules will be approached only logarithmically. This may give a dramatic difference from other models in which the sum rule should be approached like a power of Q^2 .

In this note we consider the approach to the Callan-Gross relation. If we apply the renormalization group equations to the longitudinal structure functions then to zeroth order in $\bar{g}$,

$$W_L\left(\frac{Q^2}{\mu^2}, \bar{g}\right) \xrightarrow{Q^2, \nu \rightarrow \infty} 0$$

where

$$W_L = \left(1 - \frac{\nu^2}{Q^2}\right) W_2 - W_1$$

This follows from the fact that, to zeroth order in $\bar{g}$, the leading singularity is

$$\begin{aligned} & [J_\mu(x), J_\nu(0)]_{\text{Symmetric in } \mu, \nu} \\ &= \frac{1}{8\pi} \delta'(x^2) \text{Tr}\{\gamma_\mu \gamma_\lambda \gamma_\nu \gamma_0\} \epsilon(x_0) x_\lambda \left[\tilde{f}_6(x,0) - \tilde{f}_6(0,x) \right] \quad (6) \end{aligned}$$

where $\tilde{f}_6(x,0)$ is a bilocal operator.

As pointed out by Mandula ⁶⁾, this leading singularity predicts that νW_L should scale. Due to the presence of logarithmic terms in Eq. (3) this will not be true in asymptotically free gauge theories but the logarithmic deviations will be determined by the anomalous dimensions of the operators contributing. Then to zeroth order in $\bar{g}$

$$\nu W_L\left(\frac{Q^2}{\mu^2}, \bar{g}\right) = 2\pi \left[f(x, Q^2) - \frac{\partial}{\partial x} h(x, Q^2) \right] ; \quad x = \frac{Q^2}{2M_N^2} \quad (7)$$

$f(x, Q^2)$ comes from the twist two operators in $\tilde{f}_6(x,0)$, which are the same as those contributing to deep inelastic electron scattering. It may be related to the structure functions by

$$2x f(x, Q^2) = \nu W_2(x, Q^2) \quad (8)$$

[The logarithmic deviations from Bjorken scaling in $f(x, Q^2)$ are the same as those in $\nu W_2(x, Q^2)$.] $h(x, Q^2)$ comes from operators whose spin is two units below that of the leading spin operator in each coefficient of the Taylor expansion of $\mathcal{F}_G(x, 0)$ in powers of x . The (twist 3) operators contributing are of the form

$$\bar{\psi} \partial_{\mu_1} \partial_{\mu_2} \dots \partial_{\mu_n} \psi \quad ; \quad n \text{ even}, n \geq 0$$

The anomalous dimensions of these operators are straightforward to evaluate and are, for $SU(n)$ singlet and non-singlet operators, given by

$$\gamma_L^n = \frac{g^2}{8\pi^2} C_2(R) \left[1 - \frac{4}{(n+1)} + 2 \sum_{j=2}^{n+1} \frac{1}{j} \right] \quad (9)$$

This result is for an $SU(n)$ colour group. $C_2(R)$, and $C_2(G)$, $T(R)$ used later, are defined in Ref. 2).

$xh'(x, q^2) (= x(\partial/\partial x)h(x, q^2))$ will also be governed by these anomalous dimensions as may be seen by integrating by parts.

For comparison we note the anomalous dimensions of $xf(x, q^2)$. Since we will be interested both in $SU(4)$ singlet and non-singlet terms we have to consider them both. For non-singlet terms

$$\gamma_{xf}^{n, NS} = \frac{g^2}{8\pi^2} C_2(R) \left[1 - \frac{2}{(n+2)(n+3)} + 4 \sum_{j=2}^{n+2} \frac{1}{j} \right] \quad (10)$$

For singlet terms the vector gluon operators can contribute also and the operators, for each n , must be diagonalized to find the lowest eigenvalue for γ^n . It turns out that there is only significant mixing for $n=0$ when the lowest eigenvalue is zero. For the other terms $\gamma_{xf}^{n, S} \approx \gamma_{xf}^{n, NS}$.

From (9) and (10) we see that for all values of n

$$\gamma_h^n < \gamma_{xf}^{n,ns}, \gamma_{xf}^{n,s}$$

Thus for large q^2 at least we expect xh' to dominate the zeroth order contribution to W_L .

In order to compute $W_L(1, \bar{g})$ to the next order in $\bar{g}$ one-gluon corrections to the free field terms must be computed. This has been done ⁷⁾. For $SU(4)$ non-singlet structure functions only quark operators contribute and give

$$\frac{F_L^n(\text{quark})}{F_2^n(\text{quark})} = \frac{\bar{g}^2}{16\pi^2} C_2(R) \frac{4}{(n+3)} \quad (11)$$

where we write $F_L \equiv F_2 - 2xF_1 = 2x W_L$ for notational simplicity. For $SU(4)$ singlet structure functions, gluon operators too contribute at $O(\bar{g}^2)$ and give

$$\frac{F_L^n(\text{gluon})}{F_2^n(\text{quark})} = \frac{\bar{g}^2}{16\pi^2} C_2(G) \frac{16}{(n+3)(n+4)} \quad (12)$$

Then at order $\bar{g}^2$ we have, from Eq. (5)

$$\begin{aligned} \int_0^1 dx x^n F_L(x, Q^2) &= F_L^n(\text{quark}) \left[\frac{Q^2}{\mu^2}, g \right] M_{\text{quark}}^n \\ &+ F_L^n(\text{gluon}) \left[\frac{Q^2}{\mu^2}, g \right] M_{\text{gluon}}^n \end{aligned} \quad (13)$$

For the structure functions F_2 to zeroth order in $\bar{g}^2$ even for the singlet structure functions only the quark operators contribute. These operators (twist 2) are the same as those contributing to F_L . Thus we can determine M_{quark}^n from a knowledge of F_2 . Although for $n=0$ quark and gluon operators mix, the mixing is the same for F_L and F_2 and again the matrix elements of the mixture may be found. What stops us using this to write a relation between F_L and F_2 for singlet structure functions is the presence in Eq. (13) of the gluon terms. They have anomalous dimensions given for $n \neq 0$ by

$$\gamma_{\text{gluon}}^n = \frac{\bar{g}^2}{8\pi^2} \left\{ C_2(G) \left[\frac{1}{3} - \frac{4}{(n+1)(n+2)} - \frac{4}{(n+3)(n+4)} + 4 \sum_{j=2}^{n+2} \frac{1}{j} \right] + \frac{4}{3} T(R) \right\} \quad (14)$$

for $n=0$ the non-leading eigenvalue is

$$\gamma_{\text{gluon}}^0 = \frac{\bar{g}^2}{8\pi^2} \left\{ \frac{8}{3} C_2(R) + \frac{4}{3} T(R) \right\} \quad (15)$$

Since these dimensions are smaller than the corresponding ones in Eq. (10), asymptotically the terms corresponding to $F_L^n(\text{gluon})$ will be negligible compared to the terms from $F_L^n(\text{quark})$. We may use this to put a bound on the gluon contribution. If q_0^2 denotes the onset of scaling where Eq. (13) is applicable we have

$$-F_L^n(\text{gluon}) \left[\frac{q_0^2}{\mu^2}, g \right] M_{\text{gluon}}^n \leq F_L^n(\text{quark}) \left[\frac{q_0^2}{\mu^2}, g \right] M_{\text{quark}}^n$$

where we have neglected the $O(1)$ term. Since the $O(1)$ term is $\propto 1/\nu$ and ν may be varied independently of q^2 this term may be minimized and in the applications considered here it does not affect this bound.

Then for $Q^2 > q_0^2$

$$-F_L^n(\text{gluon})\left[\frac{Q^2}{\mu^2}, g\right] M_{\text{gluon}}^n \leq \left(\frac{\bar{g}^2(Q^2)}{\bar{g}^2(q_0^2)}\right)^{\gamma_{\text{Non leading}}^n - \gamma_{\text{leading}}^n} F_L^n(\text{quark})\left[\frac{Q^2}{\mu^2}, g\right] M_{\text{quark}}^n \quad (16)$$

where

$$\gamma_{\text{Non leading}}^n = \begin{cases} \gamma_{\text{Gluon}}^n & n \neq 0 \\ \gamma_{\text{N.L.}}^n & n = 0 \end{cases}$$

and

$$\gamma_{\text{leading}}^n = \gamma_{zf}^{n,S}$$

This means we can use Eqs. (13) and (16) to drop the gluon term and give the bound

$$\int_0^1 dx x^n F_L(x, Q^2) \geq \left[1 - \left(\frac{\bar{g}^2(Q^2)}{\bar{g}^2(q_0^2)}\right)^{\gamma_{\text{Non leading}}^n - \gamma_{\text{leading}}^n}\right] F_L^n(\text{quark})\left[\frac{Q^2}{\mu^2}, g\right] M_{\text{quark}}^n \quad (17)$$

Using Eq. (16) together with Eq. (11) we find an integral equation for F_L which, together with the zeroth order terms of Eq. (7) gives

$$\begin{aligned} \nu F_L(\omega, Q^2) = & 4C_2(R) \frac{\bar{G}^2}{16\pi^2} \frac{\nu}{\omega^2} \int_0^{\omega} d\omega' \omega' F_2(\omega', Q^2) \\ & + \frac{2}{\omega} F_2(\omega, Q^2) - \left(\frac{2}{\omega}\right)^2 h'(\omega, Q^2) \end{aligned} \quad (18)$$

Here for SU(4) singlet structure functions

$$\bar{G}^2 \geq \left[1 - \left(\frac{\bar{g}^2(Q^2)}{\bar{g}^2(q_0^2)} \right)^{\text{Min} \left\{ \gamma_{\text{non leading}}^n - \gamma_{\text{leading}}^n \right\}} \right] \bar{g}^2$$

and for SU(4) non-singlet structure functions

$$\bar{G}^2 = \bar{g}^2$$

Then

$$\nu \frac{\sigma_L}{\sigma_T} = \nu \frac{F_L}{F_2}$$

is simply obtained from Eq. (18).

Before confronting Eq. (18) with the data we should look at its expected range of validity. As discussed by Gross⁸⁾, near threshold ($\omega \rightarrow 1$) the effective F_2^n that contributes to F_2 is of order $n = 4G(\ln \alpha / \ln \omega)$ where α is the effective perturbation expansion parameter, $\alpha = \bar{g}^2 / 4\pi^2$, and

$$G = \frac{3 C_2(R)}{11 C_2(G) - 4 T(R)}$$

(= 4/25 in the three quartet model). Then since i^{th} order terms $\sim (\ln n)^{3i}$ we must have

$$\alpha \left[\ln \left(\frac{4G \ln \alpha}{\ln \omega} \right) \right]^3 \ll 1 \quad (19)$$

For large ω , in the singlet channel, the effective value of n is of order $n+1 = \sqrt{(\ln \alpha / \ln \omega)}$. Since the i^{th} order terms $\sim (1/n+1)^{3i}$ then we must have

$$\alpha \left[\frac{\ln \omega}{9G \ln \frac{1}{\alpha}} \right]^{3/2} \ll 1 \quad (20)$$

Of course it is possible to speculate that the leading terms still have some meaning outside their strict range of validity⁹⁾. If we adopt this (doubtful) attitude then we may say something about the continuation of Eq. (18)

to the Regge region. We may see from Eq. (14) that the $O(\bar{g}^2)$ term relative to F_2^n is regular at $n = -1$, the leading singularity in the Regge region. So we expect for large ω the two terms to have the same ω behaviour. Keeping only the leading terms gives, for $SU(4)$ singlet structure functions ⁹⁾

$$\frac{F_L(\omega, Q^2)}{F_2(\omega, Q^2)} \xrightarrow{\omega \rightarrow \infty} 2 C_2(R) \frac{\bar{G}^2}{16\pi^2} + O\left(\frac{1}{\nu}\right) \quad (21)$$

where we have assumed the Pomeron dominates $F_2(\omega, Q^2)$. For $SU(4)$ non-singlet terms, if $F_2 \sim \omega^{-\frac{1}{2}}$ as $\omega \rightarrow \infty$

$$\frac{F_L(\omega, Q^2)}{F_2(\omega, Q^2)} \xrightarrow{\omega \rightarrow \infty} \frac{8}{3} C_2(R) \frac{\bar{g}^2}{16\pi^2} + O\left(\frac{1}{\nu}\right) \quad (22)$$

The $O(1/\nu)$ terms we expect to have large ω behaviour governed by the leading Regge singularities. For a trajectory of intercept β , $F_L(\omega, Q^2) \sim \omega^{\beta-1}$ and $F_2 \sim \omega^{\beta-1}$. This means $h'(\omega, q^2)/\omega^2$ should $\sim \omega^\beta$ and again is expected to dominate the F_2/ω term, this time for large ω . A ω^β singularity corresponds to a $n = \beta - 1$ singularity in the moments of $h'(\omega, q^2)/\omega^2$. Such a singularity does not arise naturally in the anomalous dimensions [cf. Eq. (9)] and we would have to attribute it to a singularity in M_K^n . If this pole is present there is a definite Q^2 dependence implied for the leading term

$$\left(\frac{2}{\omega}\right)^2 h'(\omega, Q^2) \xrightarrow{\omega \rightarrow \infty} \text{const. } \omega^c [\bar{g}^2(Q^2)]^c \quad (23)$$

For $\beta = 1$, $c = 0$ and for $\beta = \frac{1}{2}$, $c = -0.35$. c negative would imply a growth of νF_L with q^2 for this term.

We now consider a comparison of Eq. (18) with the data. The data available ¹⁰⁾ are principally for ep scattering. For en the data are not sufficiently accurate to allow a meaningful separation of the isoscalar and isovector components. However, we may use Eq. (18) to obtain a bound for $\bar{g}^2$ from ep scattering alone. Since ep is a mixture of $I = 0$ and $I = 1$ components the $\bar{G}^2$ term in Eq. (18) will consist of two terms, one

with $\bar{G}^2 = \bar{g}^2$ from the $I=1$ component and one with $\bar{G}^2 \geq \bar{g}^2$ from the $I=0$ part. Moreover the $I=1$ component $\propto \bar{g}^2$ is uniquely predicted given a knowledge of $\nu W_2^{I=1}$.

A significant feature of the ep data is that it is consistent with

$$\nu R \equiv \nu \frac{\sigma_L}{\sigma_T}(Q^2, \omega) = \nu \frac{\sigma_L}{\sigma_T}(\omega)$$

in the region $1.5 \leq \omega \leq 5$. For $\omega \geq 5$ there is an indication of a rise with Q^2 .

From the limits on a Q^2 rise in $\nu(\sigma_L/\sigma_T)$ we may obtain, using the experimentally measured forms for $\nu W_2^{I=0}$ and $\nu W_2^{I=1}$ (11), limits on $\bar{G}^2$. If we neglect for the moment the possible gluon contribution, then $\bar{G}^2 = \bar{g}^2$. From Eq. (2) we may compute Λ^2 and the value of Λ^2 at one standard deviation $\Lambda^2 + \delta\Lambda^2$. In the Table, we give Λ^2 and $\delta\Lambda^2$ for various values of ω . [Possible systematic errors in the data are ignored in this analysis.] Where no limit is given the data cannot put a meaningful bound on Λ^2 since in these cases $\bar{g}^2$ is so large that the perturbation expansion is invalid.

It may be seen that, without the possible gluon term, there is a strong constraint on $\bar{g}^2$

$$\frac{\bar{g}^2}{4\pi^2}(Q^2 = 1 \text{ GeV}^2) < 0.27 \quad (1 \text{ standard deviation})$$

If we now include the possible effects of the gluon term using the inequalities of Eq. (18) and optimistically taking $q_0^2 = 1 \text{ GeV}^2$, we find

$$\frac{\bar{g}^2}{4\pi^2}(Q^2 = 1 \text{ GeV}^2) < 0.4 \quad (1 \text{ standard deviation})$$

Such values are just compatible with the assumption that a perturbation expansion in the effective parameter $\alpha = \bar{g}^2/4\pi^2$ makes sense at the onset of scaling $q_0^2 \approx 1 \text{ GeV}^2$. A value much larger than this and the perturbation expansion would be meaningless. A value much smaller

would be equally difficult to understand for how could $\bar{g}$ get so small just at the onset of scaling ? Moreover for extremely small $\bar{g}^2$ the predictions of asymptotic freedom differ from free field theory by a negligible amount until astronomical energies, and one might ask whether the difference is then more than an abstract theoretical question. It becomes crucial then to transform the bound obtained above to a definite number. For asymptotic freedom to be true linear terms in Q^2 must be present in νR and, as argued above, with $\bar{g}^2/4\pi^2$ of the order of 0.1. Better errors on νR will help. But more likely going to high energies will improve the bound. At NAL Q^2 of the order of 200 GeV^2 are available. Moreover for $\Lambda^2 \leq 0.2$ the difference in structure functions between $Q^2 = 1 \text{ GeV}^2$ and $Q^2 = 200 \text{ GeV}^2$ are relatively small ⁵⁾. (< 50% at all values of ω with small ω relatively suppressed and large ω enhanced.) Thus the coefficient of the Q^2 dependent term in Eq. (18) should not change significantly. If νR can be measured to an accuracy comparable to the existing measurements then the absence of a linear term would give a bound of the order of

$$\frac{\bar{g}^2(Q^2 = 1 \text{ GeV}^2)}{4\pi^2} < 0.01$$

Alternatively a term with $\bar{g}^2/4\pi^2 (Q^2 = 1 \text{ GeV}^2) = 0.1$ would give a contribution five times the expected νR scaling term. Such measurements should provide a dramatic test of the relevance of asymptotic freedom to the observed phenomena.

Of course, the presence of such large terms will not, in themselves, confirm asymptotic freedom for other models do not predict νR should scale. Models with more general light cone singularities can have any rate for the approach to the Callan-Gross relation ¹²⁾. However, when combined with Regge behaviour, they usually favour a $\nu^{-\frac{1}{2}}$ type approach and this again should be distinguishable from the AFGT prediction. The presence of scalar partons at a low level could also allow νR initially to scale but add a linear Q^2 term at high Q^2 . To distinguish the AFGT prediction from this one would have to identify the detailed ω dependence predicted in Eq. (18).

Finally what can be said at present energies for the large ω data. Unless a large positive gluon contribution suddenly appears the $O(\bar{g}^2)$ term cannot give the observed rise for $\omega > 5$ for it requires $\bar{g}^2$ to be so big that it itself rapidly decreases [Eq. (2)] and cancels the

Q^2 rise. It may be that the rise could be due to a term like Eq. (23) with $\beta = \frac{1}{2}$. In this case we would expect the rise rapidly to flatten as $\bar{g}(Q^2)$ varies more slowly with Q^2 . Alternatively it may be a (non-scaling) threshold effect since R must vanish at threshold.

In conclusion, the approach to the Callan-Gross relation already puts strong constraints on the parameters of an AFGT. More accurate data and an extension of the range of Q^2 covered should be crucial in testing for AFGT. The approach to other (scaling) sum rules should also be analyzed with a view to finding if a consistent choice of parameter values in an AFGT can explain all the data.

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ω	Λ^2	$\delta\Lambda^2$
1.5	-	-
1.75	0.1	2.6
2.0	0.6	2.0
2.5	0.04	0.8
3.0	0.8	0.3
4.0	0.0	0.2
5.0	0.0	0.3
6.0	-	-
7.0	-	-
8.0	-	-

TABLE : Λ^2 and $\delta\Lambda^2$ for various values of ω

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