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NEW AMPLITUDES FOR DATA ANALYSIS

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ABSTRACT

We propose sets of quasi-two-body amplitudes which express the results of an amplitude analysis concisely, and are very simply related to helicity amplitudes. They are particularly suited to meson - baryon scattering with a polarized target or polarization - analyzing baryon decay. In this case they can be used to give optimal bounds on moduli and relative phases of helicity amplitudes.

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In meson-baryon scattering processes with target or recoil baryon polarization information (but not both), an almost complete amplitude analysis can be made ^{1),2)}. At each energy and momentum transfer, the amplitudes are completely determined except for two parameters. If the analysis is done in terms of transversity amplitudes ³⁾, these two parameters are the absolute phases of two groups of the amplitudes ^{1),2)}. Traditionally, amplitude analyses have been made with transversity amplitudes (or the closely related Byers-Yang amplitudes). This is because their parity properties make the formalism very simple. Transversity amplitudes are of some interest in their own right [for instance, for testing quark model predictions ⁴⁾], but the main interest of an amplitude analysis is probably to extract features of helicity amplitudes ⁵⁾; for this it is necessary to fix the undetermined phases, using some theoretical prejudices -- which are probably prejudices about how helicity amplitudes behave.

The relation between transversity and helicity amplitudes is usually rather complicated. Therefore one cannot say much about helicity amplitude structure from looking at graphs of transversity amplitudes; also, theoretical ideas cannot easily be used to fix transversity amplitude phases. In this note we propose sets of amplitudes which, like transversity amplitudes, can be used to express the results of an amplitude analysis (the two undetermined parameters are again two over-all phases) but, unlike transversity amplitudes, are very simply related to s or t channel helicity amplitudes. Graphs of these amplitudes give immediate information about helicity amplitude structure.

Transversity amplitudes $T_{P_3 P_4 P_2 P_1}$ for the process $12 \rightarrow 34$ are related to s channel helicity amplitudes $H_{\lambda_3 \lambda_4 \lambda_2 \lambda_1}$ by:

$$T_{P_3 P_4 P_2 P_1} = \sum_{\lambda_3 \lambda_4 \lambda_2 \lambda_1} X_{P_3 \lambda_3}^{J_3} X_{P_4 \lambda_4}^{J_4} X_{P_2 \lambda_2}^{*J_2} X_{P_1 \lambda_1}^{*J_1} H_{\lambda_3 \lambda_4 \lambda_2 \lambda_1} \quad (1)$$

$$H_{\lambda_3 \lambda_4 \lambda_2 \lambda_1} = \sum_{P_3 P_4 P_2 P_1} X_{P_3 \lambda_3}^{*J_3} X_{P_4 \lambda_4}^{*J_4} X_{P_2 \lambda_2}^{J_2} X_{P_1 \lambda_1}^{J_1} T_{P_3 P_4 P_2 P_1}$$

The X matrices are D matrices for a rotation of $\pi/2$ about the x axis, which takes the helicity quantization axis into the transversity quantization axis. For a few low-spin cases, they are given by:

$$X^{\frac{1}{2}} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} \\ \frac{-i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \quad (2a)$$

$$X^1 = \begin{bmatrix} \frac{1}{2} & \frac{-i}{\sqrt{2}} & \frac{-1}{2} \\ \frac{-i}{\sqrt{2}} & 0 & \frac{-i}{\sqrt{2}} \\ \frac{-1}{2} & \frac{-i}{\sqrt{2}} & \frac{1}{2} \end{bmatrix} \quad (2b)$$

$$X^{\frac{3}{2}} = \begin{bmatrix} \frac{1}{\sqrt{8}} & -\frac{i\sqrt{3}}{8} & -\frac{\sqrt{3}}{8} & \frac{i}{\sqrt{8}} \\ -\frac{i\sqrt{3}}{8} & \frac{1}{\sqrt{8}} & \frac{-i}{\sqrt{8}} & -\frac{\sqrt{3}}{8} \\ -\frac{\sqrt{3}}{8} & \frac{-i}{\sqrt{8}} & \frac{1}{\sqrt{8}} & -\frac{i\sqrt{3}}{8} \\ \frac{i}{\sqrt{8}} & -\frac{\sqrt{3}}{8} & -\frac{i\sqrt{3}}{8} & \frac{1}{\sqrt{8}} \end{bmatrix} \quad (2c)$$

These equations show the main drawback of transversity amplitudes; nearly all elements of the X matrices are non-zero, so each helicity amplitude is a sum of nearly all transversity amplitudes (and vice versa). For a process like $K^-p \rightarrow \omega\Lambda$, summing six complex transversity amplitudes, with their experimental errors, is not convenient or easy.

From now on we shall specialize to meson-baryon scattering with a spin-zero incoming meson.

In processes with an unpolarized target, but with recoil polarizations measured [e.g., $K^-p \rightarrow \omega\Lambda$, $K^-p \rightarrow \pi^- Y^{*+}(1385)$], the transversity amplitudes can be divided into two groups with proton transversity $p_2 = +\frac{1}{2}$ and $-\frac{1}{2}$, respectively. No measured quantities depend on interference between amplitudes in these two groups, so that although all moduli and relative phases within each group can be measured, the relative phase of the two groups cannot. Thus the two undetermined parameters of an amplitude analysis are the over-all phases of the two groups of transversity amplitudes. For these processes, we shall define mixed amplitudes $M_{\lambda_3\lambda_4 p_2 p_1}$, which use helicity quantization axes for the final particles 3 and 4, and transversity quantization for the initial particles 1 and 2. Their relation to the other sets of amplitudes is:

$$\begin{aligned} M_{\lambda_3\lambda_4 p_2 p_1} &= \sum_{p_3 p_4} X_{p_3\lambda_3}^{* J_3} X_{p_4-\lambda_4}^{* J_4} T_{p_3 p_4 p_2 p_1} \\ T_{p_3 p_4 p_2 p_1} &= \sum_{\lambda_3\lambda_4} X_{p_3\lambda_3}^{J_3} X_{p_4-\lambda_4}^{J_4} M_{\lambda_3\lambda_4 p_2 p_1} \end{aligned} \quad (3a)$$

$$M_{\lambda_3 \lambda_4 P_2 P_1} = \sum_{\lambda_2 \lambda_1} X_{P_2 - \lambda_2}^{*J_2} X_{P_1 \lambda_1}^{*J_1} H_{\lambda_3 \lambda_4 \lambda_2 \lambda_1} \quad (3b)$$

$$H_{\lambda_3 \lambda_4 \lambda_2 \lambda_1} = \sum_{P_2 P_1} X_{P_2 - \lambda_2}^{J_2} X_{P_1 \lambda_1}^{J_1} M_{\lambda_3 \lambda_4 P_2 P_1}$$

Each mixed amplitude $M_{\lambda_3 \lambda_4 P_2 P_1}$ is a sum of transversity amplitudes $T_{P_2 P_4 P_2 P_1}$ in the same phase group (with the same initial transversity P_2), whose moduli and relative phases are known from data. Therefore, the moduli of all mixed amplitudes can be found; so can the relative phases of mixed amplitudes with the same P_2 . However, the over-all phase of each such group of mixed amplitudes is not fixed by data.

In the usual case when particle 1 is spinless and particle 2 has spin $\frac{1}{2}$, the relation between mixed and helicity amplitudes (3b) is extremely simple. X^{J_1} is just 1, and X^{J_2} is a 2×2 matrix, given in Eq. (2a). Therefore each helicity amplitude is a sum of only two mixed amplitudes, and vice versa.

If, for convenience, we define

$$A_{\lambda_3 \lambda_4} = \frac{1}{\sqrt{2}} M_{\lambda_3 \lambda_4 \frac{1}{2} 0} \quad (4)$$

$$B_{\lambda_3 \lambda_4} = \frac{i}{\sqrt{2}} M_{\lambda_3 \lambda_4 -\frac{1}{2} 0}$$

then the helicity amplitudes are given by

$$H_{\lambda_3 \lambda_4 \frac{1}{2} 0} = A_{\lambda_3 \lambda_4} + B_{\lambda_3 \lambda_4} \quad (5)$$

$$H_{\lambda_3 \lambda_4 -\frac{1}{2} 0} = i[A_{\lambda_3 \lambda_4} - B_{\lambda_3 \lambda_4}]$$

Recall that the moduli of all A amplitudes, and their relative phases, can be extracted directly from the data; similarly for the B amplitudes. The absolute phase of each group cannot be got from data, so some theory input is still necessary for a determination of helicity amplitudes. However, because Eq. (5) is so simple, some properties of helicity amplitudes can be tested simply by looking at experimental mixed amplitudes.

1. HELICITY FLIP PROPERTIES

Each mixed amplitude is a sum of two s channel helicity amplitudes whose helicity flip $|\lambda_3 - \lambda_4 + \lambda_2 - \lambda_1|$ differs by one. For instance, in $\pi p \rightarrow KY^*(1385)$, $A_{0\frac{3}{2}}$ and $B_{0\frac{3}{2}}$ are both combinations of a single-flip and a double-flip amplitude, and should both vanish in the forward direction like $\sqrt{-t}$. If helicity-flip amplitudes are little affected by absorption, then $A_{0\frac{3}{2}}$ and $B_{0\frac{3}{2}}$ should have Regge pole properties such as factorization and shrinkage. They need not have Regge pole phases, as they are related to helicity amplitudes by complex coefficients.

2. BOUNDS ON MODULI OF HELICITY AMPLITUDES

From Eq. (5) one can bound the magnitude of any helicity amplitude:

$$||A_{\lambda_3\lambda_4}| - |B_{\lambda_3\lambda_4}|| \leq |H_{\lambda_3\lambda_4\lambda_2\lambda_1}| \leq |A_{\lambda_3\lambda_4}| + |B_{\lambda_3\lambda_4}| \quad (6)$$

This is the strongest bound that can be made from the data without further theoretical assumptions.

As a special case of (6), a helicity amplitude $H_{\lambda_3\lambda_4\lambda_2\lambda_1}$ can only be zero if the two corresponding mixed amplitudes $A_{\lambda_3\lambda_4}$ and $B_{\lambda_3\lambda_4}$ have equal moduli. This gives an immediate test for helicity amplitude zeros (e.g., absorptive zeros at $t \simeq -0.2$, or Regge pole zeros at $d \simeq 0$) predicted by models. Then, forcing one helicity amplitude to zero fixes the relative phase of the two groups of mixed amplitudes -- so the moduli and relative phases of all other helicity amplitudes can be found. For instance, at small t one could reasonably set a double-flip amplitude zero, to extract all the other helicity amplitudes.

3. BOUNDS ON RELATIVE PHASES OF HELICITY AMPLITUDES

If θ is the relative phase of $H_{\lambda_3\lambda_4\frac{1}{2}0}$ and $H_{\lambda_3\lambda_4-\frac{1}{2}0}$, then $\sin \theta$ has the same sign as $|A_{\lambda_3\lambda_4}| - |B_{\lambda_3\lambda_4}|$. Furthermore,

$$|\sin \theta| \geq \frac{||A_{\lambda_3\lambda_4}|^2 - |B_{\lambda_3\lambda_4}|^2|}{|A_{\lambda_3\lambda_4}|^2 + |B_{\lambda_3\lambda_4}|^2} \quad (7)$$

so θ is restricted to some region around $+\pi/2$ or $-\pi/2$. If two helicity amplitudes are not related to the same pair of mixed amplitudes, then the

bound on their relative phase is more complicated, and is best found numerically in individual cases.

As a particular case of the bound (7), two helicity amplitudes $H_{\lambda_3 \lambda_4 \frac{1}{2} 0}$ and $H_{\lambda_3 \lambda_4 -\frac{1}{2} 0}$ can only have equal phases if the corresponding mixed amplitudes $A_{\lambda_3 \lambda_4}$ and $B_{\lambda_3 \lambda_4}$ have equal moduli ^{*)}. This might occur if $H_{\lambda_3 \lambda_4 \frac{1}{2} 0}$ and $H_{\lambda_3 \lambda_4 -\frac{1}{2} 0}$ are spin-flip and double-flip amplitudes dominated by the same Regge pole, or are real because of exchange degeneracy. Mixed amplitudes give a convenient way of testing these hypotheses against data.

For completeness we record the parity properties of helicity, transversity, and mixed amplitudes:

$$H_{-\lambda_3 -\lambda_4 -\lambda_2 -\lambda_1} = \eta_3 \eta_4 \eta_2 \eta_1 (-)^{J_3 - \lambda_3} (-)^{J_4 - \lambda_4} (-)^{J_2 - \lambda_2} (-)^{J_1 - \lambda_1} H_{\lambda_3 \lambda_4 \lambda_2 \lambda_1} \quad (8a)$$

$$T_{P_3 P_4 P_2 P_1} = \eta_3 \eta_4 \eta_2 \eta_1 (-)^{P_2 - P_1 + P_4 - P_3} T_{P_3 P_4 P_2 P_1} \quad (8b)$$

$$M_{-\lambda_3 -\lambda_4 P_2 P_1} = \eta_3 \eta_4 \eta_2 \eta_1 (-)^{J_3 - \lambda_3} (-)^{J_4 - \lambda_4} e^{-i\pi(P_2 + P_1)} M_{\lambda_3 \lambda_4 P_2 P_1} \quad (8c)$$

The mixed amplitudes in Eqs. (3) and (4) are useful when one has recoil baryon polarization data, but no target polarization. With a polarized target, but no recoil polarization measurement (e.g., $\pi^- p \rightarrow \rho^0 n$ with polarized target), a different set of amplitudes must be used. In this case the transversity amplitudes divide into two groups with different final baryon transversity P_4 , such that relative phases within a group are determined, but the absolute phase of each group is unknown. Therefore, we define mixed amplitudes $M_{\lambda_3 P_4 \lambda_2 P_1}$ using transversity quantization for particles 1 and 4, helicity quantization for 2 and 3. From formulae analogous to Eqs. (5) and (6) it follows that:

- a) moduli of all mixed amplitudes, and relative phases of amplitudes with the same P_4 , can be extracted from data;

*) One can work the other way: if $A_{\lambda_3 \lambda_4}$ and $B_{\lambda_3 \lambda_4}$ have equal moduli, then either $H_{\lambda_3 \lambda_4 \frac{1}{2} 0}$ and $H_{\lambda_3 \lambda_4 -\frac{1}{2} 0}$ have equal phases, or one of them vanishes

- b) each helicity amplitude is a sum of two mixed amplitudes with different P_4 (and vice versa).

These mixed amplitudes can be used in the same ways (Sections 1-3 above) as the other mixed amplitudes.

In cases with a polarized baryon target, and a high-spin baryon resonance in the final state (e.g., $\pi N \rightarrow \pi \Delta$ or $\pi N \rightarrow \rho \Lambda$), one can make a complete amplitude analysis, with only the over-all phase undetermined ⁶⁾. Then the results can be presented as helicity amplitudes, and there is little point in defining mixed amplitudes.

In baryon-baryon scattering there are so many basically different cases that one cannot define one set of amplitudes which is useful for all of them. Useful sets can be defined for individual cases. For instance, in $pp \rightarrow \Delta^{++} \Delta^0$ with a polarized beam and unpolarized target, only two phases are undetermined ⁶⁾; here one can define mixed amplitudes (using transversity quantization for the target, helicity quantization for everything else) which have all the useful properties found in meson-baryon cases.

So far we have used s channel helicity quantization axes to define s channel mixed amplitudes which are simply related to s channel helicity amplitudes. However, we could equally well have used t channel or u channel helicity quantization axes to define t channel mixed amplitudes, related to t channel helicity amplitudes as in Eq. (8). These could then be used to give bounds on moduli of t channel helicity amplitudes, bounds on relative phases, etc., exactly as before.

In the simplest spin case, $0^{-\frac{1}{2}+} \rightarrow 0^{-\frac{1}{2}+}$, mixed amplitudes are proportional to transversity amplitudes. In higher spin cases, they have the advantages over transversity amplitudes which we have already described; but they have a disadvantage. Unlike transversity amplitudes (but like helicity amplitudes), they have no simple connection ^{1),2)} with the exchanged natural parity $\mp P$. One could define combinations of mixed amplitudes with definite natural parity [in the case $0^{-\frac{1}{2}+} \rightarrow 1^{-\frac{1}{2}+}$, these are Byers-Yang amplitudes ⁷⁾] but they would no longer be so simply connected to helicity amplitudes.

If the results of an amplitude analysis are presented with a complete error matrix (not just error bars for individual amplitudes), then, in principle, it does not matter what set of amplitudes is used; no information is lost in converting from one set to another. However, for practical convenience, and for easy visual tests of helicity amplitude structure, we believe that mixed amplitudes are the most useful.

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