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ELECTRON-POSITRON ANNIHILATION ABOVE CHARM THRESHOLD

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A B S T R A C T

A first exploratory attempt is made to describe quantitatively, in the charm scheme, the behaviour of $\sigma(e^+e^- \rightarrow \text{hadrons})$ in the region above charm threshold. Indications are given for the important rôle of ψ'' and possibly of ψ''' , the second and third radial excitations of ψ .

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Shortly after the spectacular discovery of the two narrow width resonances $\psi(3.1)$ ¹⁾ and $\psi'(3.7)$ ²⁾ in e^+e^- annihilation a broad enhancement around 4.15 GeV has also been discovered ³⁾. Though it is not at all clear whether this broad bump has anything to do with the two narrow width peaks, there is a common belief that these two phenomena are somehow connected.

This connection is naturally established in the charm scheme. Within this model $\psi(3.1)$ is believed to be the 1^3S_1 $c\bar{c}$ bound state, $\psi'(3.7)$ its first radial excitation 2^3S_1 ⁴⁾ and the enhancement at 4.15 GeV is interpreted as the second radial excitation $\psi''(4.15)$. The narrow widths of ψ and ψ' are attributed to the validity of the phenomenological Zweig rule : the decays of the $c\bar{c}$ bound states into zero charm hadrons are strongly suppressed and the a priori allowed decay into charmed meson pairs are energetically forbidden. On the other hand, the large width of ψ'' can then only be understood if one assumes it to lie already above the first charmed meson pair threshold. A priori, of course, the bump at 4.15 GeV could inside this picture as well be interpreted as a pure threshold effect stemming from the opening of the lightest charmed hadron decay channels.

Adopting the point of view of charmonium it is our main purpose in this paper to compute the height and the shape of the ratio $R = \sigma(e^+e^- \rightarrow \text{hadrons}) / \sigma(e^+e^- \rightarrow \mu^+\mu^-)$ in the neighbourhood of the charm threshold and especially to investigate the possible rôle of ψ'' (and even of the next radial excitation ψ''') in this energy region ⁵⁾.

It seems to be natural to assume that for such energies the three vector meson poles (ψ, ψ', ψ'') dominate the charmed hadron production in e^+e^- annihilation. It is clear, however, that this vector meson dominance (VDM) assumption does not tell us the relative coupling strengths of ψ, ψ', ψ'' to the photon nor to the charmed mesons.

To get this information we use the $SU(8)$ version of the quark pair creation model of Carlitz and Kislinger ⁶⁾, especially its more dynamical form proposed by Le Yaouanc et al. ⁷⁾. This seems to us an appropriate extension of the non-relativistic $c\bar{c}$ bound state hypothesis for the ψ particles, and, furthermore, it can be generalized to include radial excitations ⁸⁾. The three meson vertices are defined by the quark

diagram in Fig. 1, where the bubbles denote the products of spin, SU(4), and spatial wave functions. The light quark pair is created from the vacuum in a spin triplet p wave state of zero total angular momentum with a universal coupling strength γ ; the quarks of the initial meson propagate freely. The T matrix element corresponding to the process in Fig. 1 is

$$\langle M_B M_C | T | M_A \rangle = \delta \cdot \sum_m C(1, 1, m, -m | 0, 0) \langle \chi_B \chi_C | \chi_A \chi_{-m} \rangle I_{A, BC}^m \quad (1)$$

where χ_A, χ_B, χ_C are the SU(8) wave functions of the mesons, χ_{-m} is the spin triplet SU(4) singlet quark-antiquark spurion, C is the appropriate Clebsch-Gordan coefficient for L-S coupling, and $I_{A, BC}^m$ are overlap functions explicitly calculable from the meson spatial wave functions with the help of

$$I_{A, BC}^m(\vec{k}_B, \vec{k}_C; \vec{k}_A) = \frac{1}{8} \delta(\vec{k}_B + \vec{k}_C) \int d^3k \varphi_C^*(\vec{k}) \varphi_B^*(-\vec{k}) \varphi_A(\vec{k}_B + \vec{k}) \chi_A^m(\vec{k}_B - \vec{k})$$

From Eq. (1), without specifying the value of γ and the binding potential, we can calculate the decay branching ratios of ψ'' into mass degenerate⁹⁾ charmed pseudoscalar (D, F, globally denoted by P_c) and vector (D^* , F^* , denoted by V_c) mesons:

$$\frac{\Gamma(\psi'' \rightarrow P_c \bar{P}_c)}{\Gamma(\psi'' \rightarrow V_c P_c)} = \frac{1}{2} ; \quad \frac{\Gamma(\psi'' \rightarrow V_c \bar{V}_c)}{\Gamma(\psi'' \rightarrow P_c \bar{P}_c)} = 3 \left(1 + \frac{1}{3} \frac{m_{\psi''}^2}{m_{V_c}^2} \right) \quad (2)$$

$SU(8)_W$ would give the same prediction only for the first ratio, but would lead to a different one in the second case. This illustrates the fact that the model implies a kind of broken static SU(8) which is similar but not identical to the $SU(8)_W$ symmetry.

To determine also the relative magnitudes of the ψ, ψ', ψ'' couplings to the charmed mesons as well as to the photon, we have to choose an explicit form of the wave functions, i.e., of the quark binding potential. As a first approach, we adopt an oscillator potential.

Having made this choice, the meson wave functions $\varphi_n(\vec{x})$ are given in terms of the respective average square radii ($\varphi_n(r) \sim e^{-r^2/2R^2}$). For the ψ 's this radius is, within the model, connected with the leptonic width which is given by

$$\Gamma_{\psi}^{\text{lept}} = \frac{4\pi\alpha^2}{3} \frac{m_{\psi}}{f_{\psi}^2} \approx \frac{16\pi\alpha^2}{3m_{\psi}^2} \frac{4}{9} |\varphi_{\psi}(0)|^2 \quad (3)$$

From the experimental value of $\Gamma(\psi \rightarrow e^+e^-) \cong 5 \text{ keV}$ we get $R_{\psi}^2 = 1.3 \text{ GeV}^{-2}$ and furthermore :

$$f_{\psi} = 11.76 \quad f_{\psi_1} = 12.50 \quad f_{\psi_{11}} = 13.30$$

In addition we have to know the square radius R^2 of the charmed mesons (D, D^*). Since $R_{\psi}^2 \approx 1.3 \text{ GeV}^{-2}$ and $R_{\rho}^2 \approx 8 \text{ GeV}^{-2}$ we may expect a value $R_D^2 \approx 4 \text{ GeV}^{-2}$, if we take into account the fact that in this non-relativistic picture R^2 is inversely proportional to the reduced constituent mass. The final results are insensitive within 10% to a change of R_D^2 by two units around its chosen value.

Having determined the radii we end up with wave functions containing no free parameters. The corresponding overlap integrals $I_{\psi_i} (\equiv I_{\psi_i, BC})$ are given in the Table. Of course, these overlap integrals depend on k , the three momentum of the final state particle in the centre-of-mass system. In using this formula for off-mass shell ψ 's the question arises how to interpret then the k^2 dependence of the overlap. We shall consider here two choices :

$$\text{Choice A : } k^2 = 0$$

$$\text{Choice B : } k^2 \equiv k_s^2 = \frac{1}{4} (s - 4m^2)$$

(m is the final meson mass).

Near the resonance region the results for R will not be sensitive to a change of the above choices.

In order to give definite results for $\sigma(e^+e^- \rightarrow \text{hadrons})$ we need to know the mass spectrum of the charmed mesons.

According to the simplest SU(4) mass formulae, the mass splitting of the various P_c and V_c should be very small. Thus for the sake of simplicity we assume that all pseudoscalars have the same mass and so do all charmed vector mesons.

As to the difference between P_c and V_c masses we distinguish between the following alternative : either all channels ($P_c \bar{P}_c$, $P_c \bar{V}_c$, $\bar{P}_c V_c$, $V_c \bar{V}_c$) are open to ψ'' decay or only the pseudoscalar threshold ($P_c \bar{P}_c$) lies below $\sqrt{s} = 4.15$ GeV.

Treating the first possibility, we proceed as follows.

From the shape of the measured cross-section near $\sqrt{s} = 4.15$ GeV, we estimate $\Gamma_{\text{exp}}(\psi'') \simeq 280$ MeV ($\cong 20\%$ accuracy). Using Eq. (3), we obtain the following partial widths (same 20% accuracy as for Γ_{exp} : $\Gamma(\psi'' \rightarrow P_c \bar{P}_c) \cong 7.5$ MeV for each of the three pseudoscalar-pseudoscalar channels ; $\Gamma(\psi'' \rightarrow V_c \bar{P}_c) \cong 15$ MeV for each of the six pseudoscalar-vector channels and $\Gamma(\psi'' \rightarrow D^* \bar{D}^*) = 56$ MeV for each of the vector-vector channels ¹⁰⁾. Comparing the theoretical prediction of the model with $\Gamma(\psi'')_{\text{exp}} \gamma^2$ is obtained as a function of the common threshold energy $t_c (= 2m_{P_c} = 2m_{V_c})$. For instance,

$$\Gamma(\psi'' \rightarrow P_c \bar{P}_c) = \quad (4)$$

$$\frac{1}{6\pi m_{\psi''}^2} \left(\frac{m_{\psi''}^2 - t_c^2}{4} \right)^{3/2} g_{\psi'' P_c \bar{P}_c}^2$$

$$g_{\psi'' P_c \bar{P}_c} = \gamma (4\pi)^{1/4} m_{\psi''}^{3/2} I_{\psi''} \quad (5)$$

For γ we thus get values between 1 and 1.4 (for $3.7 \text{ GeV} < t_0 < 4 \text{ GeV}$) similar to those calculated in the same model from the ρ_{NN} coupling ⁷⁾.

Once γ is known we can easily compute the total charmed hadron production cross-section in e^+e^- annihilation assuming the dominance of the vector meson poles ψ, ψ', ψ'' . In terms of the conventional R ratio, one has :

$$R(e^+e^- \rightarrow \text{charmed hadrons}) = \quad (6)$$

$$= 24 \left(\frac{k}{\sqrt{s}} \right)^3 \left[\left(1 + \frac{s}{t_0^2} \right) |a|^2 + s |b|^2 \right] \theta(s - t_0^2)$$

$$a = \frac{g_\psi}{f_\psi} \frac{m_\psi^2}{s - m_\psi^2} + \frac{g_{\psi'}}{f_{\psi'}} \frac{m_{\psi'}^2}{s - m_{\psi'}^2} + \frac{g_{\psi''}}{f_{\psi''}} \frac{m_{\psi''}^2}{s - m_{\psi''}^2 + i \Gamma_{\text{tot}} m_{\psi''}} \quad (7)$$

$$b = \frac{g_\psi}{f_\psi} \frac{m_\psi}{s - m_\psi^2} + \frac{g_{\psi'}}{f_{\psi'}} \frac{m_{\psi'}}{s - m_{\psi'}^2} + \frac{g_{\psi''}}{f_{\psi''}} \frac{m_{\psi''}}{s - m_{\psi''}^2 + i \Gamma_{\text{tot}} m_{\psi''}}$$

with the f 's and g 's defined as in Eqs. (3) and (5). We add to this value a constant background B_{lq} due to uncharmed hadron production. For this we choose $B_{lq} \sim 2.5$ which is suggested by experiments. The resulting curves for different values of t_0 are plotted in Fig. 2. They correspond to choice A (i.e., $k^2 = 0$ in the overlap integrals I), a choice appropriate for vector meson dominance. Very small differences appear in the analogous results using choice B. At the bottom of Fig. 2, we have also plotted the value of R (without B_{lq}) excluding from our fit ψ'' but keeping the same value of γ . It is clear that these contributions in themselves are not able to account for the experimental situation (at least in this vector dominance calculations), unless unreasonably large values are chosen for γ ¹¹⁾.

The values of the couplings (for $t_0 = 3.8 \text{ GeV}$ ¹²⁾) as given by the model are : $g_\psi = 1.33$, $g_{\psi'} = -1.83$, $g_{\psi''} = 2.03$. Since $\Gamma_{\text{tot}}(\psi'')$ is taken from the experimental shape, the prediction of the theory concerns mainly the value of R at the maximum of the peak (R_{max}), which can be decomposed as

$$R_{\text{max}} = B_{lq} + B_{cq} + R_{\psi''}$$

where $R_{\psi''} = (9/\alpha^2) \Gamma(\psi'' \rightarrow e^+e^-) / \Gamma_{\text{tot}}(\psi'')$ and B_{cq} is the charmed quark background at $\sqrt{s} = 4.15$ GeV. In our VDM model B_{cq} comes from the ψ, ψ' and from their interference with ψ'' . We obtain $R_{\psi''} \cong 3$ and $B_{cq} \sim 0.2 - 0.5$ (depending on the threshold). The smallness of the background is important in obtaining $R_{\text{max}} \cong 6$ (for reasonable values of t_0) in good agreement with the data. In that respect the destructive interference coming from the minus sign of $g_{\psi'}$, plays an important rôle. With positive $g_{\psi'}$, B_{cq} would be $B_{cq}^{(+)} \sim 1.2 + 3.5$ (depending on the value of t_0). Above $\sqrt{s} = 4.2$, R falls down very rapidly to B (see Fig. 2).

Let us now consider the second alternative, where only the pseudoscalar-pseudoscalar channels are open for the decay of ψ'' (threshold $t_0 < 4.1$ GeV), whereas the channels involving vector particles open at a higher threshold $t_1 > 4.2$ GeV¹³⁾. Again, γ is fixed from the experimental $\Gamma(\psi'')$ in terms of t_0 . Since a smaller number of channels is open γ is larger. Typically γ lies between 3.5 and 4. The corresponding predictions for $R(s)$ for different values of t_0 and t_1 are plotted in Fig. 3 (again with choice A). The predictions for R_{max} are essentially the same as in the previous case, because $R_{\psi''}$ is fixed only in terms of $\Gamma(\psi'' \rightarrow e^+e^-)$ and B_{cq} remains small. We note that in this case the opening of the charmed channels above the ψ'' resonance gives a considerable increase of R , unlike in the former case. The exact amount of this increase depends crucially on the mass spectrum.

The preliminary data³⁾ for R seem to indicate a further rise for $\sqrt{s} > 4.6$ GeV. It is not difficult to find arguments for such an increase within the charm framework. Either there are further radial excitations of ψ , or/and additional charmed channels are open ($\rightarrow 2\eta_c$, radial excitations of D and D^* , charmed baryons, etc). We shall study here the possible rôle of the third radial excitation ψ''' . In the numerical computation we take two possible mass values: $m_{\psi'''} = 4.5$ GeV and 4.8 GeV. The coupling to the mesons depends strongly on the choice one makes for k^2 in the overlap integral. For choice A), we obtain $g_{\psi'''} = -2.0$ for $m_{\psi'''} = 4.5$; $g_{\psi'''} = -2.2$ for $m_{\psi'''} = 4.8$. Note the alternation in sign: g is positive for ψ and ψ'' , negative for ψ' and ψ''' . This is important in deciding the type of interference (constructive or destructive). The value of γ is known from the fit to $\Gamma_{\text{tot}}(\psi'')$. We can therefore predict $\Gamma_{\text{tot}}(\psi''')$ depending on the choices for the thresholds. For $m_{\psi'''} = 4.5$ GeV we find $\Gamma_{\text{tot}}(t_0 = 3.8 \text{ GeV}, t_1 = 3.8 \text{ GeV}) = 0.78 \text{ GeV}$, and $\Gamma_{\text{tot}}(t_0 = 3.8 \text{ GeV}, t_1 = 4.4 \text{ GeV}) = 0.9 \text{ GeV}$. For $m_{\psi'''} = 4.8$ GeV, we find

$\Gamma_{\text{tot}}(t_0 = 3.8 \text{ GeV}, t_1 = 3.8 \text{ GeV}) = 1.6 \text{ GeV}$, $\Gamma_{\text{tot}}(t_0 = 3.8 \text{ GeV}, t_1 = 4.4 \text{ GeV}) = 3.0 \text{ GeV}$. These predictions may be wrong by factors of two because of the uncertainties (the k^2 dependence) and the unphysical features of the potential ¹¹⁾, nevertheless the qualitative result of a very large width for ψ''' is presumably general. The results for the ratio R after inclusion of ψ''' , but on the same assumptions as before, are shown in Fig. 4.

We conclude from all the preceding calculations that the non-relativistic quark model described above gives a natural explanation of R around the 4.15 enhancement. It indicates that the contribution from the ψ and ψ' tail in this region is small not only due to the expected fall-down of the pole terms but also because it is additionally damped by destructive interference effects. Furthermore, the experimental data cannot be explained by a pure threshold effect (unless unreasonable high values of γ are invoked). Finally, we find it interesting to observe that the gradual opening of each charmed hadron channel above but in the neighbourhood of $\sqrt{s} = 4.15 \text{ GeV}$ gives a rise of R , $\Delta R \cong 0.1 + 1.0$ depending on the charm spectrum. Inclusion of the contribution from the third radial excitation ψ''' requires in the model only the mass value for ψ''' and improves the fit to the experimental data.

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$$I_{\psi} = R_{\psi}^{-\frac{3}{4}} F(R_{\psi}^2, R_D^2, k^2)$$

$$I_{\psi'} = \sqrt{\frac{3}{2}} R_{\psi}^{-\frac{3}{4}} \left(1 - \frac{4}{3} x \frac{\partial}{\partial x}\right) F(x, R_D^2, k^2) \Big|_{x=R_{\psi}^2}$$

$$I_{\psi''} = \sqrt{\frac{15}{8}} R_{\psi}^{-\frac{3}{4}} \left(1 - \frac{8}{15} x \frac{\partial}{\partial x} + \frac{16}{15} x^2 \frac{\partial}{\partial x^2}\right) F(x, R_D^2, k^2) \Big|_{x=R_{\psi}^2}$$

$$F(x, r^2, k^2) = \left(\frac{xr}{x+2r}\right)^{\frac{3}{2}} \frac{x+r}{x+2r} \exp\left(-\frac{k^2}{4} \frac{xr}{x+2r}\right)$$

TABLE : Overlap integrals

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- 5) Due to the numerous uncertainties we employ a non-relativistic description (as suggested by the relatively small mass-splittings). We neglect the possible mixings with higher ℓ states. An effect of such mixings would be the appearance of presumably much smaller couplings of 3^3D_1 , 4^3D_1 , 5^3D_1 , etc., to the virtual photon.
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- 8) To keep with the smallest number of parameters we rely on a SU(8) description for the spin SU(4) factors with the low-lying mesons belonging to a 63 plet : one pseudoscalar SU(4) 15 plet and one vector SU(4) (15+1) plet with ideal mixing such that ψ is pure $c\bar{c}$. The model implies a broken SU(8) but different from SU(8)_W.
- 9) In the actual situation the mass splitting between, say, D^{+*} and D^+ may be comparable to that between F and D^+ . A detailed calculation will require a better knowledge of the masses.
- 10) We obtain equal cross-sections for the resonant reactions $e^+e^- \rightarrow \psi \rightarrow D^+D^-$, or $\rightarrow D^0\bar{D}^0$, or $\rightarrow F\bar{F}$ (and also equal cross-sections for the six $P_c\bar{V}_c$ and $\bar{V}_cP_c$ channels, and the three $V_c\bar{V}_c$ channels). By contrast, exact SU(4) gives vanishing amplitudes for $e^+e^- \rightarrow D^0\bar{D}^0$ and $e^+e^- \rightarrow D^{*0}\bar{D}^{*0}$. The cancellation arises, however, from compensation between the components $\frac{2}{3}\bar{u}u$ and $\frac{2}{3}\bar{c}c$ in the electromagnetic current and it would clearly be unreasonable to expect it in our resonant model.

- 11) We are now performing the calculations of the overlap integrals with a linear plus Coulomb potential and comparing with the fits by Eichten et al. and Appelquist et al. [E. Eichten, K. Gottfried, T. Kinoshita, J. Kogut, K.D. Lane and T.M. Yan - Phys.Rev.Letters 34, 369 (1975) ; T. Appelquist, A. De Rujula, H.D. Politzer and S.L. Glashow - Phys.Rev.Letters 34, 365 (1975)].
- 12) If the total width of $\psi'(3.7)$ turns out to be much larger than the sum of $\Gamma(\psi' \rightarrow \psi + 2\pi) + \Gamma(\psi \rightarrow \text{hadrons})$ one possibility (already suggested by Bjorken and by Glashow) is that a real decay into $D\bar{D}$ occurs ($t_0 < 3.7$). In our model we obtain, with choice A, $\Gamma(\psi' \rightarrow D\bar{D}) = 420 \text{ keV}, 210 \text{ keV}, 60 \text{ keV}$ for $m_D = 1.82 \text{ GeV}, 1.83 \text{ GeV},$ and 1.84 GeV , respectively. The computation is done for one final channel only, for instance $\psi' \rightarrow D^+ D^-$ (electromagnetic mass splitting could allow only for one channel).
- 13) For D^*, F^* one expects strong decay modes $D^* \rightarrow D + \pi$, and electromagnetic ones $D^* \rightarrow D + \gamma, F^* \rightarrow F + \gamma$. If pion emission is kinematically forbidden or depressed (depending on the mass differences) the main decay mode will be through γ emission. The onset of D^*, F^* production ($e^+ e^- \rightarrow D^* \bar{D}, D^* \bar{D}^*,$ etc.) will in this case be accompanied by increase in γ emission ($E_\gamma \sim 70 + 150 \text{ MeV}$).

FIGURE CAPTIONS

Figure_1 The three-meson vertex.

Figure_2 R versus $2E=\sqrt{s}$ when all the charmed meson channels are simultaneously open and for choice A (i.e., $k^2=0$ within the overlap integrals). The threshold values t_0 are 3.8 GeV for ———, 3.7 GeV for -----, and 4 GeV for -.-.-. The lower curves give R without background and ψ'' (i.e., R comes only from ψ and ψ' with $\gamma \sim 1$). The vertical lines are the data points ³⁾.

Figure_3 R versus $2E=\sqrt{s}$ with $P_c P_c$ channels open at t_0 and the channels involving vector mesons ($P_c V_c$ and $V_c V_c$) open at t_1 . Choice A for k^2 in the overlap integrals. The threshold values are : $t_0=3.8$, $t_1=4.4$ for ———, $t_0=3.7$, $t_1=4.4$ for ----- ; $t_0=4$, $t_1=4.5$ for -.-.-

Figure_4 R versus $2E=\sqrt{s}$ with inclusion of the fourth resonant state ψ''' at 4.5 GeV or at 4.8 GeV. The values of the thresholds are $t_0=3.9$ GeV and $t_1=4.3$ GeV.

FIG. 1

FIG. 2

FIG 3

FIG 4