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ON THE WEAK DECAYS OF HIGH MASS HADRONS

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A B S T R A C T

Several arguments for the dominance of certain types of non-leptonic weak decays of strange particles are examined for their applicability to high mass hadrons. Arguments based on asymptotic freedom, duality, current algebra and PCAC, and coloured fermion quarks are all found to be weaker than for strange decays. Specific calculations in the Glashow-Iliopoulos-Maiani charm scheme yield total semi-leptonic decay rates of 20% or more. Estimates of branching ratios for two-body decays of charmed mesons are given, and remarks are made about charmed baryon decays.

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1. - INTRODUCTION

The recent discovery ¹⁾ of high mass neutral vector mesons which seem to be hadrons ²⁾ suggests strongly the existence of other charged and neutral high mass hadrons, some of which should decay weakly. Indeed several experimental groups have reported indications that such particles may exist. One of the theoretical schemes including such particles is the SU(4) charm scheme with weak interactions of the form suggested by Glashow, Iliopoulos and Maiani ³⁾, though many other models are also consistent with the experimental data.

Phenomenological analyses of weak decays of strange particles indicate that semi-leptonic decay modes are weaker than non-leptonic modes, and that the non-leptonic weak Hamiltonian behaves predominantly as an octet under SU(3). The theoretical literature contains many proposed explanations of these facts, based for example on short distance behaviour and asymptotic freedom ^{4),5)}, duality ⁶⁾, current algebra, PCAC and coloured fermion quarks ⁷⁾. Several papers ⁸⁾⁻¹¹⁾ have recently tried to extrapolate these arguments to the weak decays of conjectured high mass hadrons, principally using the charm scheme.

In this paper we re-examine the applicability of these standard arguments to the decays of heavy hadrons. We find the short distance and coloured quark arguments somewhat weaker in this case, and the duality and current algebra arguments even more shaky. Many of these considerations could be true in models other than charm. We make approximate calculations in the charm scheme and find a total semi-leptonic branching ratio of the order of 20 or more, with perhaps 1/3 of the non-leptonic decays going via non-enhanced parts of the weak Hamiltonian.

We also make estimates of the two-body decay branching ratios of charmed mesons, finding numbers $\leq 4\%$. Naïve estimates of the two-body branching ratios of charmed baryons (made assuming they are stable against strong and electromagnetic decay) turn out considerably higher. More theoretical study is needed, but this may be an interesting indicator for experimental searches.

2. - SHORT-DISTANCE EFFECTS : OPERATOR ENHANCEMENT AND/OR SUPPRESSION

In the conventional charm scheme ^{3),5),11)} the effective charm-changing weak interactions are of the form

$$-f^2 \int d^4x D_F(x, m_w^2) T(J_{\mu c}^+(x) J^\mu(0)) + \text{H.C.} \quad (1)$$

where $D_F(x, m_w^2)$ is an intermediate boson propagator, $J_{\mu c}$ is the charm-changing charged current ^{*)}

$$J_{\mu c} = \bar{e} \gamma_\mu (1-\gamma_5) (\lambda \cos \Theta_c - n \sin \Theta_c) \quad (2)$$

and J^μ is either a leptonic current

$$J_L^\mu = \bar{\nu}_\mu \gamma^\mu (1-\gamma_5) \mu + \bar{\nu}_e \gamma^\mu (1-\gamma_5) e \quad (3)$$

or a charm-conserving charged current ^{*)}

$$J_N^\mu = \bar{p} \gamma^\mu (1-\gamma_5) (n \cos \Theta_c + \lambda \sin \Theta_c) \quad (4)$$

The time-ordered product of $J_{\mu c}^+$ and $J_{\mu N}$ may be expanded in the form

$$T(J_{\mu c}^+(x) J_N^\mu(0)) \approx \sum_{\alpha} \mathcal{F}^{(\alpha)}(x^2) O^{(\alpha)}(0) \quad (5)$$

where the $\mathcal{F}^{(\alpha)}(x^2)$ are c number functions and the $O^{(\alpha)}(0)$ are local operators related to products of quark fields. For our purposes, the relevant operators $O^{(\alpha)}$ are normal ordered ($\bar{q}q\bar{q}q$) quark field products, which can be decomposed into 20 and 84 representations of $SU(4)$. For the dominant $\cos^2 \Theta_c \Delta C = \Delta S = 1$ decays we will need the operators ^{**)}

$$O^{\mp} = \frac{1}{2} N \left\{ (\bar{e} \lambda)(\bar{n} p) \mp (\bar{e} p)(\bar{n} \lambda) \right\} \quad (6)$$

^{*)} Θ_c is the Cabibbo angle.

^{***)} The symbol N indicates normal ordering.

which belong to the $\underline{20}$ and $\underline{84}$ representations, respectively ^{*}). The coefficient functions $\overline{F}^{(\mp)}(x^2)$ have been calculated ⁵⁾ in the limit $x^2 \rightarrow 0$ in an asymptotically free gauge theory of coloured SU(4) quarks. Choosing a normalization mass μ_1

$$\overline{F}^{(\mp)}(x^2) \approx \left[1 - \frac{25 g^2(\mu_1)}{24 \pi^2} \ln(\mu_1 \sqrt{-x^2}) \right]^{-0.24} \quad \text{for } |\mu_1^2 x^2| \ll 1 \quad (7)$$

where g is the usual running coupling constant : it is assumed that $(g^2/4\pi) \simeq 1$ at a normalization mass μ_2 ; then

$$\frac{g^2(\mu_1)}{4\pi} \approx \left[1 + \frac{25}{6\pi} \ln(\mu_1/\mu_2) \right]^{-1}$$

Inserting (6) and (7) into (5) and integrating the expression (1) yields an expression for the dominant term in the effective non-leptonic $\Delta C = \Delta S = 1$ interaction

$$\frac{G_F}{\sqrt{2}} \left[c_- O^-(0) + c_+ O^+(0) \right] \cos^2 \theta_c \quad (8)$$

where the $c_{\mp}$ receive calculable contributions from the asymptotic parts of the x integral :

$$c_{\mp} = \left[\frac{1 + \frac{25}{6\pi} \ln(M_W/\mu_2)}{1 + \frac{25}{6\pi} \ln(\mu_1/\mu_2)} \right]^{-0.24} + \dots \quad (9)$$

the dots indicating contributions from $|x^2| \geq 0(1/\mu_1^2)$. In the case of $\Delta C = 0, \Delta S = 1$ decays the analogous coefficients were calculated using $M_W \simeq 100$ GeV, $\mu_1 \simeq \mu_2 \simeq 1$ GeV corresponding to the onset of scaling in deep inelastic scattering, and neglecting the $|x^2| \geq 0(1/\mu_1^2)$ part of the integral. Then one obtained ⁵⁾

$$\left[1 + \frac{25}{6\pi} \ln(M_W/1 \text{ GeV}) \right] \approx 7.1 \quad (10)$$

^{*}) We have suppressed reference to the spinor structure of the currents.

giving a non-leptonic enhancement factor and a $\Delta I = \frac{1}{2} / \Delta I = \frac{3}{2}$ ratio which were appreciably >1 , but insufficient to explain the phenomenologically observed enhancements, suggesting that extra factors were necessary in the hadronic matrix elements of the relevant operators $O^{(\alpha)*}$.

What happens to the above argument in the case of charmed particle decays? The choice of the subtraction point μ_2 is ideally: (a) the (average) effective mass of the weakly interacting quarks in the hadron (so that hard gluon radiative corrections may be neglected in evaluating matrix elements between physical states), and (b) below the onset of scaling [so that the perturbative calculation used in deriving Eq. (7) is reasonably valid]. We observe that since charmed particles are expected to have masses $O(2 \text{ GeV})$, corresponding perhaps to a charmed quark mass $O(1.5 \text{ GeV})$, scaling in processes involving charmed particles may be expected to set in at higher $|q^2|$ than for strange particles. Hence certainly μ_1 should be larger than in the $\Delta C=0$ case, say $\sim 2 \text{ GeV}$. Keeping $M_W = 100 \text{ GeV}$ and $[g^2(1)/4\pi] = 1$, we then find

$$\left[\frac{1 + \frac{25}{6\pi} \ln(M_W/1\text{GeV})}{1 + \frac{25}{6\pi} \ln(\mu_1/1\text{GeV})} \right] \approx 3.7 \quad \text{for } \mu_1 = 2\text{GeV} \quad (11a)$$

However, the lower onset of scaling may also be reflected in a slower fall off in the effective quark gluon coupling constant. If we instead assume for couplings of charmed quarks $[g^2(2)/4\pi] = 1$, we find

$$\left[1 + \frac{25}{6\pi} \ln(M_W/\mu_2) \right] \approx 6.2 \quad \text{for } \mu_1 = \mu_2 = 2\text{GeV} \quad (11b)$$

Although there is certainly an ambiguity as to the values of the subtraction constants, it is clear that the non-leptonic/leptonic and $20/84$ enhancement factors may be smaller than in the case (10) of strange particle decays. Since these enhancement factors are explicitly short-distance effects, we feel happier with these conclusions than with the recently proposed ¹²⁾ application of asymptotic freedom to the total hadronic widths of the new resonances ¹³⁾.

We should emphasize that similar qualitative asymptotic freedom considerations should apply to the weak decays of any new massive particles, even if they turn out not to be charmed. Later in this paper, however, we

*) We return to this point in Section 3.

make more detailed estimates in the conventional charm model. For this purpose we make naïve ball-park estimates of the factors $c_{\mp}$

$$c_{\mp} \approx \left[\frac{1 + \frac{25}{6\pi} \ln(\mu_1/\mu_2)}{1 + \frac{25}{6\pi} \ln(\mu_1/\mu_2)} \right]^{\begin{matrix} +0.48 \\ -0.24 \end{matrix}}$$

$$= \begin{cases} 1.9 \\ 0.7 \end{cases} \quad \text{for } \mu_1 = 2 \text{ GeV}, \mu_2 = 1 \text{ GeV} \quad (12a)$$

$$= \begin{cases} 2.4 \\ 0.6 \end{cases} \quad \text{for } \mu_1 \approx \mu_2 \approx 2 \text{ GeV} \quad (12b)$$

3. - MATRIX ELEMENT ENHANCEMENT AND/OR SUPPRESSION

We recalled earlier on that asymptotic freedom alone was not enough to explain the enhancement of $\Delta I = \frac{1}{2}$ strangeness changing non-leptonic decays relative to $\Delta I = \frac{3}{2}$ transitions. As discussed by Gaillard and Lee ⁵⁾, another mechanism seems to be needed to enhance the matrix elements of the $\Delta I = \frac{1}{2}$ operator. Several possible mechanisms have been proposed, including duality ⁶⁾, soft pion theorems and coloured fermion quark arguments ⁷⁾. How well do these arguments extend to charmed particle decays ?

3.1. - Duality arguments

In the duality arguments of Nussinov and Rosner ⁶⁾, the effective non-leptonic weak Hamiltonian is treated as a spurion S , and the decay $A \rightarrow B + \pi$ related to a process

$$S + A \rightarrow B + \pi \quad (13)$$

The amplitude for (13) is then assumed to have Regge behaviour :

$$|A(v, t)| \sim \left(\frac{v}{v_0}\right)^{\alpha(t)} \quad \text{as } |v| \rightarrow \infty \quad (14)$$

where

$$s = (p_B + p_\pi)^2, \quad u = (p_A - p_\pi)^2, \quad t = (p_A - p_B)^2, \quad v = \frac{s-u}{2}$$

and vector and tensor exchanges are supposed to contribute with the usual Regge phases. A finite contour dispersion relation is then written down for $A(v, t)$:

$$A(v, t) = \frac{1}{\pi} \int_{v_s}^L dv' \frac{\text{Im } A_s(v', t)}{v' - v} + \frac{1}{\pi} \int_{v_u}^L dv' \frac{\text{Im } A_u(v', t)}{v' + v} + \left(\begin{array}{c} \text{poles below} \\ \text{threshold} \end{array} \right) \\ + \frac{1}{\pi} \int_L^N dv' \frac{\text{Im } A_s(v', t)}{v' - v} + \frac{1}{\pi} \int_L^N dv' \frac{\text{Im } A_u(v', t)}{v' + v} + \frac{1}{2\pi i} \oint_C dv' \frac{A(v', t)}{v' - v} \quad (15)$$

where $\text{Im } A_s$ and $\text{Im } A_u$ are the absorptive parts above the thresholds v_s and v_u in the s and u channels, and C is a contour made of semicircles of radius N in the upper and lower halves of the v plane. It is then assumed that N is sufficiently large that the Regge representation (14) can be used for the C integral, and that L is chosen such that for $L \leq v' \leq N$ the absorptive parts can be approximated on the average by the Regge form. It is then apparent that (15) can be written in the form

$$A(v, t) = A_{(1)}(v, t) + A_{(2)}(v, t) \quad (16)$$

where $A_{(1)}$ corresponds to the top line of (15) and contains pole and low energy contributions, while $A_{(2)}$ reflects the structure of the Regge terms (14). It is then assumed that no exotic resonances or t channel trajectories contribute to $\text{Im } A(v, t)$. If S is a $(\bar{q}q)$ $SU(3)$ octet [or $SU(4)$ 16 plet] operator, these no-exotic assumptions can be realized with exchange degenerate vector and tensor trajectories, and the solution can be visualized

in terms of quark diagrams, as shown in Fig. 1. However, if S is an exotic operator (e.g., $\bar{q}q\bar{q}q$) then no such dual solution exists. For meson decays no connected quark diagram can be drawn, for baryon decays there are only diagrams like Fig. 2 with s and u channel baryon exchanges⁶⁾. The final step in the argument is to argue that in certain cases, e.g., S wave baryon decays, the low energy pole contributions to $A_{(1)}$ will be relatively small, and that the dual Regge $A_{(2)}$ contributions will dominate, thereby enhancing non-exotic $(\bar{q}q)$ parts of S over exotic $(\bar{q}q\bar{q}q)$ parts.

There are several places where this argument can be expected to break down for charmed particle decays.

- (i) The dominant Regge exchanges (14) now come from (F, F^*) or (D, D^*) trajectories, which probably^{11), 13)} have very low intercepts $\alpha(0) \sim -1$ to -2 . Hence the Regge contribution $A_{(2)}$ should be relatively small, and the dispersion integrals in (15) weighted towards the thresholds v_s, v_u .
- (ii) In addition to the physical contributions above thresholds, there will be substantial pole contributions below threshold coming from s and u channel uncharmed particles. This and the previous remark suggest that $|A_{(1)}|$ is not $\ll |A_{(2)}|$, and reminds us of similar problems with applying dual arguments to familiar $SU(3)$ processes such as pseudoscalar vector meson scattering.
- (iii) In the dominant $\Delta C = \Delta S = 1$ charmed particle decays, the spurions has $(\bar{c}\lambda)(\bar{n}p)$ quantum numbers and is a $(\bar{q}q\bar{q}q)$ operator which does not have 16plet $(\bar{q}q)$ quantum numbers. In contrast, for $\Delta C = 0, \Delta S = 1$ decay the spurion is a $(\bar{\lambda}n)(\bar{n}p)$ operator which has non-exotic $(\bar{\lambda}p)$ $SU(3)$ quantum numbers. Thus in the $\Delta C = 0, \Delta S = 1$ (or $\Delta C = 1, \Delta S = 0$) decays there could be contributions from spurions looking like Fig. 3a or Fig. 3b, whereas for $\Delta C = \Delta S = 1$ only Fig. 3b can contribute. Only a non-exotic spurion of the form of Fig. 3a could be enhanced by the Nussinov-Rosner duality argument on the structure of $A_{(2)}$.

[Superficially, one might think that contributions of the type of Fig. 3a should vanish due to the GIM³⁾ cancellation mechanism; however, this is not the case. While hard gluon exchange at the weak vertex is absorbed into the definition of the factors $c_{\pm}$, soft gluon exchange cannot be neglected in the evaluation of matrix elements of $O_{\pm}$; thus implicit in the diagram of Fig. 3a is soft

gluon exchange between the loop and the external quark lines.

The GIM suppression factor here is analogous to that for

$K_L \rightarrow \gamma\gamma$ ¹⁴⁾ : $\sim (M_c^2 - M_p^2)/M_G^2$ where M_G is the effective mass of the exchanged gluon system. Note that for $\Delta C=1, \Delta S=0$ amplitudes, where diagrams of this type may occur, the suppression factor is $\sim (M_\lambda^2 - M_n^2)/M_G^2$, so that these amplitudes may be effectively suppressed. $\square$

- (iv) The dominant decays of charmed particles are expected to be complicated multibody ones for which duality arguments may be less easily applied.

3.2. - PCAC and coloured quarks

In the case of strangeness changing decays, the $\Delta I = \frac{3}{2}$ amplitudes have been shown to vanish in the limit where all pions are soft, if - in the case of baryons - the soft pion theorem is complemented by the Pati-Woo colour quark argument ⁷⁾ to be discussed below. In the case of charm changing decays with appreciable energy release to a meson system, chiral $SU(3)$ can probably be considered a good symmetry; for the same reasons, however, soft pion theorems are presumably of little relevance. Moreover we will argue that even in the limit of all soft decay mesons enhancement of 20 relative to 84 is considerably weaker in the charm case.

First consider charmed meson decay. For exact chiral $SU(3)$ all amplitudes vanish in the total soft meson limit. This is because they reduce to amplitudes of the form $D, F \rightarrow \text{vacuum}$ proceeding via a diagram of the type in Fig. 3b. This represents an $SU(3)$ rotation of the non-leptonic Hamiltonian term $(\bar{c}\lambda)(\bar{n}p)$ which belongs to a combination of 6 and 15 representations, and hence cannot annihilate a D or F meson.

For strange baryon decay, in the soft pion limit, amplitudes reduce to the form $\langle B' | \mathcal{H}_{\text{non-leptonic}} | B \rangle$ for both S and P waves. Pati and Woo ⁷⁾ have shown that if the baryons are in the lowest symmetric representation of $SU(6)$ only the matrix elements of O^- in Eq. (6) (which has $\Delta I = \frac{1}{2}$) can contribute. This argument can be generalized ^{*)} in

^{*)} These arguments are true in the presence of coloured gluons if the total S wave wave function is dominant (M.K. Gaillard and B.W. Lee, unpublished).

the following way. Consider the quark diagrams of Fig. 4. The diagram of Fig. 4a has $\Delta I = \frac{1}{2}$; in 4b and 4c, only O^- contributes if the baryon wave function is totally symmetric. Thus, only Fig. 4d contributes to amplitudes with $\Delta I = \frac{3}{2}$. If the "mesons" are a single pseudoscalar, approximate conservation of the axial current suppresses this diagram (which vanishes in the soft meson limit). Consider now the decays of charmed mesons ; the analogous arguments become weaker on three counts :

- a) Fig. 4a does not contribute ;
- b) charmed baryons presumably have masses $\geq 0(2.5)$ GeV, and the initial state baryon system may well not be dominated by the lowest 120 representation of $SU(8)$; then O^+ can contribute to Figs. 4b and 4c ;
- c) the "mesons" may be in a total spin 1 state, so that the diagram of Fig. 4d is not suppressed.

It is apparent that many of the above arguments should apply to the weak decays of any new massive particles, and are substantially independent of details of the charm scheme. Exceptions include remarks about the graphs of Fig. 3a and Fig. 4a, which are dependent on details of the charm-changing current. In the Glashow-Iliopoulos-Maiani ³⁾ scheme, it is possible to make more detailed estimates of branching ratios for charmed particles, and this we now proceed to do.

4. - MESON DECAY

4.1. - Leptonic branching ratio

Because of the absence of the diagram of Fig. 3b, both leptonic and semi-leptonic decay amplitudes may be described by the diagram of Fig. 5a (where $P_c = D$ or F) ; Fig. 5b vanishes in the limit of conserved $\Delta C = 0$ currents. Neglecting soft gluon corrections at the weak vertex, the amplitude may be written in a factorized form

$$A \sim \langle \text{hadrons} | J_\mu' | P_c \rangle \langle \text{hadrons} | J^{\mu 2} | 0 \rangle$$

leptons

Thus the hadron/lepton branching ratio is determined by the number of ways a colour symmetric quark pair can be created at the weak vertex. Consider first the diagram of Fig. 6a, where the $(\bar{n}p)$ system is colour symmetric. The weak interaction of Eq. (8) may be written

$$\frac{G_F}{\sqrt{2}} \cos^2 \theta_c \frac{1}{2} \left[(C_+ + C_-) (\bar{\lambda} c) (\bar{p} n) + (C_+ - C_-) (\bar{p} c) (\bar{\lambda} n) \right] \quad (17)$$

where $(\bar{q}_1 q_2)$ is a colour symmetric V-A current. Under the Fierz-Michel transformation we have

$$(\bar{p} c) (\bar{\lambda} n) = \frac{1}{3} (\bar{\lambda} c) (\bar{p} n) + \frac{1}{2} (\bar{\lambda} \lambda^\alpha c) (\bar{p} \lambda^\alpha n) \quad (18)$$

where λ^α is an $SU(3)$ colour matrix : $(\bar{p} \lambda^\alpha n)$ is an octet and therefore the second term in (18) does not contribute to colour symmetric final states. Thus the total contribution to Fig. 6a is

$$\frac{G_F}{\sqrt{2}} \cos^2 \theta_c \frac{1}{3} (2C_+ + C_-) \langle \text{hadrons} | (\bar{\lambda} c) | P_c \rangle \quad (19)$$

Similarly the contribution to Fig. 6b is

$$\frac{G_F}{\sqrt{2}} \cos^2 \theta_c \frac{1}{3} (2C_+ - C_-) \langle \text{hadrons} | (\bar{p} c) | P_c \rangle \quad (20)$$

and the contribution to Fig. 6c is

$$\frac{G_F}{\sqrt{2}} \cos \theta_c \langle \text{hadrons} | (\bar{\chi}_c) | P_c \rangle \quad (21)$$

In the spirit of the present calculation, we assume

$$\langle \text{hadrons} | (\bar{p}_c) | P_c \rangle \approx \langle \text{hadrons} | (\bar{\chi}_c) | P_c \rangle$$

However, there is an ambiguity as to whether the amplitudes of Eqs. (19) and (20) should be added coherently or incoherently ; depending on interference effects we obtain

$$\frac{\Gamma(P_c \rightarrow \text{hadrons})}{\Gamma(P_c \rightarrow l + \nu + \text{hadrons})} \approx \begin{cases} \frac{16}{3} C_+^2 = 2.6 (1.9) : \text{destructive interference} \\ \frac{4}{3} C_-^2 = 4.8 (7.7) : \text{constructive interference} \\ \frac{8}{3} C_+^2 + \frac{2}{3} C_-^2 = 3.7 (4.8) : \text{incoherent} \end{cases} \quad (22)$$

where the first and second sets of numbers correspond to the estimates (12a) and (12b) for $C_{\mp}$. Since the diagrams of Figs. 6a and 6b in general lead to different final state configurations, interference effects may well be small ; thus we predict a leptonic branching ratio in the range

$$\frac{\Gamma(P_c \rightarrow l + \nu + \text{hadrons})}{\Gamma(P_c \rightarrow \text{all})} \approx \frac{\Gamma(P_c \rightarrow e + \nu + \text{hadrons})}{\Gamma(P_c \rightarrow \text{all})} \approx 15 \text{ to } 20 \% \quad (23)$$

Allowing for maximum interference effects the range becomes (10-25)%.

4.2. - Total lifetime

In Ref. 11), the total lifetime of a charmed particle was estimated assuming that it proceeds via the decay of an effectively free charmed quark through an effective local current-current interaction enhanced by an appropriate factor. There the phenomenological enhancement factor ($\sim \tan \theta_c^{-1}$ in amplitude) was inferred from the hadronic decay rates of strange particles. We shall make a similar estimate using the interaction of Eq. (8), with $c_{\pm}$ as determined above, and taking careful account of colour factors. Since $d\ell_{N.L.}$ is colour invariant, the final state must be a colour triplet :

$$c_{\alpha} \rightarrow \begin{cases} \lambda_{\alpha} & (\bar{u}\gamma) \\ \lambda_{\alpha} & (\bar{u}_{\beta} p_{\beta}) \\ p_{\alpha} & (\bar{u}_{\beta} \lambda_{\beta}) \end{cases}$$

If we sum incoherently over the two hadronic final states, we obtain (neglecting all masses except m_C) :

$$\Gamma_{tot} \approx 3 \left[\frac{8}{3} C_+^2 + \frac{2}{3} C_-^2 + 2 \right] \left(\frac{m_C}{m_{\mu}} \right)^5 \Gamma(\mu \rightarrow e \nu \bar{\nu}) \approx (6 \text{ to } 7) \times 10^{12} \text{ sec}^{-1} \quad (24)$$

or, allowing for maximum interference :

$$\Gamma_{tot} \approx (0.4 \text{ to } 1) \times 10^{13} \text{ sec}^{-1}$$

where we have taken $m_C = 1.5 \text{ GeV}$.

4.3. - Three-body semi-leptonic decays

We will see in Section 6 that the branching ratio for these decays may be relevant for the interpretation of high $p_{\perp}$ leptons in pp scattering. Their partial rates have been estimated in Ref. 11), using $SU(4)$ to determine the matrix elements of the vector current and assuming constant form factors. However, if the vector mesons are nearly degenerate with the pseudoscalars, the variation of the form factor f_+ may be appreciable (the contribution of f_- is negligible) ; for $D \rightarrow K \ell \nu$ we have $f_+(t) \sim (1 - t/M_{F^*}^2)^{-1}$

and in the physical region $0 \leq t \leq (M_D - M_K)^2$ so that the F^* pole is not far from the physical region. Accordingly we scale the estimates of Ref. 11) by a factor 2 and obtain

$$\left. \begin{aligned} \Gamma(D \rightarrow K \ell \nu) &\approx 3 \times 10^{11} \text{ sec}^{-1} \\ \Gamma(F \rightarrow \eta \ell \nu) &\approx 2 \times 10^{11} \text{ sec}^{-1} \end{aligned} \right\} \quad (25)$$

where we have assumed $M_D \approx M_F \approx 2 \text{ GeV}$. Using the total width estimate of Eq. (24), we obtain the branching ratios :

$$\left. \begin{aligned} \Gamma(D \rightarrow K \ell \nu) &\approx 0.3 \Gamma(D \rightarrow \ell \nu + \text{hadrons}) \approx 0.05 \Gamma(D \rightarrow \text{all}) \\ \Gamma(F \rightarrow \eta \ell \nu) &\approx 0.2 \Gamma(F \rightarrow \ell \nu + \text{hadrons}) \approx 0.03 \Gamma(F \rightarrow \text{all}) \end{aligned} \right\} \quad (26)$$

where $\ell = e$ or μ .

4.4. - Two-body hadronic decays

The branching ratio for these decays is relevant to the search for charmed particle states. The quark diagrams which can contribute are those of Figs. 6a and 6b. Again, neglecting soft gluon corrections at the weak vertex the amplitudes may be expressed in terms of factorized matrix elements of the form :

$$\langle K^- | (\bar{s}c) | D^0 \rangle \langle \pi^+ | (\bar{p}n) | 0 \rangle, \text{ etc.}$$

In order to lend a degree of confidence to this rather naïve estimate, we note that the same type of diagrams contribute to the $\Delta I = \frac{3}{2}$ amplitudes for $K \rightarrow 2\pi$. The analogous calculation ¹⁵⁾, using $c_1 = 0.62$ [cf. Eqs. (9) and (10)], gives

$$\alpha(K^+ \rightarrow \pi^+ \pi^0) \approx 2.8 \times 10^{-8} \text{ GeV} \quad (27)$$

while the experimental value is $1.8 \times 10^{-8} \text{ GeV}$.

To simplify estimates in the charm case, we set $f_K = f_\pi$ and find

$$\begin{aligned}
 \Gamma(D^+ \rightarrow \bar{K}^0 \pi^+) &= 2 C_+^2 \cos^2 \theta_c \left(\frac{m_D}{m_K} \right)^2 \frac{\Gamma(K^+ \rightarrow \pi^+ \pi^0)}{0.16} \\
 &= (1.2 \text{ to } 1.6) \times 10^{11} \text{ sec}^{-1} \\
 \Gamma(D^0 \rightarrow K^- \pi^+) &= 1.5 \Gamma(F^+ \rightarrow \pi^+ \pi^+) = \frac{1}{4} \left(\frac{C_-}{2C_+} + 1 \right)^2 \Gamma(D^+ \rightarrow \bar{K}^0 \pi^+) \\
 &= (2.2 \text{ to } 2.7) \times 10^{11} \text{ sec}^{-1} \\
 \Gamma(D^0 \rightarrow \bar{K}^0 \pi^0) &= \frac{1}{2} \Gamma(F^+ \rightarrow \bar{K}^0 K^+) = \frac{1}{8} \left(\frac{C_-}{2C_+} - 1 \right)^2 \Gamma(D^+ \rightarrow \bar{K}^0 \pi^+) \\
 &= (0.25 \text{ to } 3) \times 10^{10} \text{ sec}^{-1}
 \end{aligned} \tag{28}$$

These estimates would be reduced by a factor ~ 2 if we took the experimental $\alpha(K^+ \rightarrow \pi^+ \pi^0)$ instead of the theoretical estimate (27). An interesting result is that the decay $D^+ \rightarrow \bar{K}^0 \pi^+$, which is forbidden in the limit of 20 enhancement, $c_+/c_- \rightarrow 0$, is essentially unsuppressed, whereas the "allowed" decays $D^0 \rightarrow \bar{K}^0 \pi^0$ and $F^+ \rightarrow \bar{K}^0 K^+$ have become suppressed by a cancellation between 20 and 84 amplitudes.

Using the total width estimate of Eq. (24), we obtain the branching ratios ^{*} :

$$\begin{aligned}
 \Gamma(D^0 \rightarrow K\pi) / \Gamma(D^0 \rightarrow \text{all}) &\approx 4\% \\
 \Gamma(D^+ \rightarrow K\pi) / \Gamma(D^+ \rightarrow \text{all}) &\approx (2 \text{ to } 3)\% \\
 \Gamma(F^+ \rightarrow \pi^+ \pi^+ + \bar{K}K) / \Gamma(F^+ \rightarrow \text{all}) &\approx (2.5 \text{ to } 3)\%
 \end{aligned} \tag{29}$$

These estimates are, of course, very approximate, but they seem reasonable orders of magnitude, and may serve as indicators of the rates to be expected.

^{*}) Note that the branching ratio of 50% for $D^0 \rightarrow K\pi$ quoted in Ref. 11) should be regarded only as a weak upper limit ; what is estimated there is $\Gamma(D \rightarrow K\pi) / \sum_{n=0} \Gamma(D \rightarrow K\pi + n \text{ soft pions})$. Other final states including vector mesons, for example, were not included.

5. - BARYON DECAYS

An estimate of the total leptonic branching ratio is not straightforward in this case because of the uncontrollable contribution from the diagrams of Figs. 4b and 4c. However, the free quark decay estimate of the total width is in principle applicable to any decaying hadron, since it is basically a dimensional argument. Perhaps complexities due to the contributions of these diagrams can be absorbed into the allowance for interference effects. If we assume that Eq. (23) gives a reasonable estimate of the total baryon decay width, we may use it to estimate branching ratios for two-body hadronic decays.

Altarelli et al.¹⁰⁾ have estimated partial rates for these decays assuming :

- a) equal enhancement of $\Delta C=1$ and $\Delta C=0$ non-leptonic decays ;
- b) the selection rules associated with the dominance of 0^- ; while we expect these selection rules to be badly violated we do not expect the violation to change rate predictions by orders of magnitude ;
- c) exact $SU(4)$ symmetry ; a measure of the validity of this approximation is provided by a sum rule these authors have derived¹⁶⁾ for strange baryon decays and which is found to be satisfied to within 40%^{*)}.

In order to get a gross estimate of partial widths for the decays $B_c \rightarrow B + P$, we consider two possibilities.

- i) The estimates of Altarelli et al. (ACM)¹⁰⁾ should be scaled down by the ratio of the enhancement factors :

$$\Gamma(B_c \rightarrow B + P) \approx \left(\frac{C_-(\Delta C=1)}{C_-(\Delta C=0)} \right)^2 \Gamma_{ACM}$$

Then we obtain

$$\Gamma_2 = \sum_{\substack{2\text{-body} \\ \text{final states}}} \Gamma(B_c \rightarrow B + P) \sim (1 \text{ to } 2) \times M_{B_c} \times 10^{12} \text{ sec}^{-1} G_F V^{-1}$$

*) Note that if the diagram of Fig. 6a - which vanishes in the limit of exact $SU(4)$ - is important (and presumably it is dominant for K decay), this approximation should be very bad.

Since weak decays are relevant only for $M_{B_c} \lesssim 3$ GeV and since we expect $M_{B_c} \gtrsim 2.5$ GeV we obtain

$$\Gamma_2 \approx (2 \text{ to } 6) \times 10^{12} \text{ sec}^{-1} \quad (30)$$

ii) The ACM estimate should be scaled down by the ratio $(c_-/A)^2$, where $A \approx \cot \theta_c$ is the effective non-leptonic enhancement for strange baryon decay, the extra enhancement arising, presumably, from the contribution of Fig. 4a. Then we find

$$\Gamma_2 \approx (0.5 \text{ to } 1.5) \times 10^{12} \text{ sec}^{-1} \quad (31)$$

Naively, taking the estimates (30) and (31) and dividing by the estimated total decay rate (24), we find

$$\frac{\Gamma_2}{\Gamma(B_c \rightarrow \text{all})} \approx \begin{cases} (20 \text{ to } 50)\% & \text{in case (i)} \\ (10 \text{ to } 20)\% & \text{in case (ii)} \end{cases} \quad (32)$$

The former of these estimates seems absurdly high: it may very well fail precisely because of an anomalously high contribution from Fig. 4a in strange particle decays. The second estimate (32) is also surprisingly high, and we have no intuitive understanding why the two-body decays of charmed baryons should have a larger branching ratio than the two-body decays of charmed mesons. More study of this question is needed - experimental as well as theoretical!

6. - DISCUSSION

In Sections 2 and 3 of this paper we gave general arguments why high mass particles might have higher weak semi-leptonic branching ratios than the 5% or so expected from a straightforward generalization of the enhancement factors in strange particle decay. We also gave reasons why selection rules proposed for non-leptonic decays should perhaps be treated with caution. Then in Sections 4 and 5 we specialized to the case of charm, and tried to make quantitative the arguments of the previous sections. It may be worth while discussing the implications of the rates (23) for various experimental situations.

(A) - Dimuon events in neutrino collisions

For $x \geq 0.1$ in high energy neutrino experiments, the production of charmed particles should be a function $\simeq \sin^2 \theta_c \simeq 5\%$ of the charged current cross-section. The branching ratios (23) then suggest that a fraction ($\frac{1}{2}$ to 1)% of the charged current events with $x > 0.1$ could have a second muon from charmed particle decay, and that there should be a similar fraction of (μe) coincidences. In addition, there might be events with $x \leq 0.1$. The structure of the charged weak currents ensures that this mechanism yields lepton pairs of zero net charge. Benvenuti et al. ¹⁷⁾ quote a branching ratio for $(\mu^- \mu^+)$ events of $(0.9 \pm 0.3)\%$ for $E_\nu > 40$ GeV : the Caltech experiment ¹⁸⁾ apparently sees $0(1-2)\%$ when $E_\nu \simeq 150$ GeV.

(B) - Direct lepton production in hadronic collisions

Production of a charmed meson at large p_T followed by its subsequent decay to $\ell + \nu$ + a pseudoscalar meson P would give a lepton to pion ratio at large p_T ¹⁹⁾ :

$$\left(\frac{\ell}{\pi} \right) \approx \left(\frac{1}{200} \text{ to } \frac{1}{300} \right) \times \left(\frac{\text{charm}}{\pi} \right) \times \frac{\Gamma(\text{charm} \rightarrow \ell + \nu + P)}{\Gamma(\text{charm} \rightarrow \text{all})} \quad (33)$$

with $\Gamma(\text{charm} \rightarrow \ell + \nu + P)/\Gamma(\text{charm} \rightarrow \text{all}) \simeq 5\%$, a ratio $(\ell/\pi) \simeq 10^{-4}$ could be got from a large p_T (charm/ π) ratio $\simeq 1/4$ to $\frac{1}{2}$.

(C) - e^+e^- annihilation

According to the charm model, just above charm threshold $\approx 50\%$ of the events should contain charmed meson pairs. Hence with the branching ratios (23) over 20% of all events above threshold should contain leptons from charmed particle decay - a conclusion which may have trouble with the preliminary SLAC-LBL data ²⁾. Such semi-leptonic decays have on the average more neutral energy and possibly smaller charged multiplicity than non-leptonic decays. Further, a few per cent of these events may have two final state leptons with opposite charges. Assuming as above that $\Gamma(\text{charm} \rightarrow \ell + \nu + P) / \Gamma(\text{charm} \rightarrow \ell + \nu + \text{all}) \simeq \frac{1}{3}$, we estimate that about 10^{-3} of the final states in e^+e^- annihilation just above charm threshold should be of the form $e^+e^- \rightarrow e^\pm \mu^\mp + \text{no charged particles}$. Rumours circulate that such events may exist.

As far as purely hadronic decays are concerned, the two-body meson branching ratio estimates (29) are not unreasonable, and do not conflict with upper limits quoted by Feldman and Perl ²⁾ on the basis of SPEAR data. The two-body baryon branching ratios (32) are probably unreasonably high, but the qualitative feature of higher ratios than for meson decays may well be worth bearing in mind.

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FIGURE CAPTIONS

Figure__1 A $(\bar{q}q)$ spurion contribution to the decay (13).

Figure__2 A $(\bar{q}q\bar{q}q)$ spurion contribution to the decay (13).

Figure__3 a) $(\bar{q}q)$ and
 b) $(\bar{q}q\bar{q}q)$ spurions.

Figure__4 Diagrams for weak baryon decays.

Figure__5 Diagrams for weak meson decays.

Figure__6 Diagrams for charmed meson decays.

FIG. 1

FIG. 2

FIG. 3

FIG. 4

FIG. 5

(a)

(b)

(c)

FIG. 6