MESON MASSES AND WIDTHS IN A GAUGE THEORY
WITH LINEAR BINDING POTENTIAL

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A B S T R A C T

A linear binding potential with one-gluon exchange corrections, within a colour gauge field theory including four types of quarks, is applied to the description of the meson masses and leptonic widths (both for light and heavy mesons). First order relativistic corrections are calculated by means of the Breit-Fermi Hamiltonian. Mixing effects and the validity of the non-relativistic approximation are also discussed.

The results show a considerable agreement with the experimental data for mesons of masses $\gtrsim 1$ GeV, whereas the breakdown of the non-relativistic expansion puts limits on the analysis for the lighter particles. A link is also made with the e^+e^- annihilation cross-section, showing how an explanation of the ratio $R(s)$ in the region $\sqrt{s} \simeq 3.5 + 6$ can be accounted for.

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INTRODUCTION

The $SU(4) \otimes SU(3)$ gauge model of strong interactions involving four types of fractionally charged quarks, each in three colours and coupled to an $SU(3)$ octet of massless gauge gluons, seems to incorporate in a rather natural way the phenomenology ¹⁾ of the newly discovered ²⁾ narrow boson resonances $\psi(3,1), \psi'(3,7)$. Such a model suggests a possible approximate description of hadrons: the quarks are confined within colourless hadrons by a long range force (infra-red slavery), and at short distances they interact via the one-gluon exchange (asymptotic freedom).

In a recent attempt to apply quantitatively these ideas to the (charmed and uncharmed) hadron spectroscopy, De Rujula, Georgi and Glashow ³⁾ have made the following two additional assumptions: i) the hadrons can be described mainly by a non-relativistic dynamics; ii) the effective long range binding potential is independent of quark spins and quark masses, depending just on the spatial separations of the constituent quarks. The only breaking of $SU(4)$ is introduced by allowing the various quark types (p,n, λ ,c) to have different masses $m_p = m_n (\equiv m_d)$, m_λ and m_c , respectively.

In this paper we adopt these two assumptions as well. As for assumption (i), we are going to test its self-consistency. Concerning assumption (ii), we think that its consequences are worth while investigating, even though we are aware of the fact that its validity is an open question within the underlying field theory.

Restricting ourselves, for example, to a colour neutral quark-antiquark mesonic state, the strong effective Hamiltonian for the relative motion of the constituent quarks has then the following form

$$H = V(r) + m_1 + m_2 + \frac{m_1 + m_2}{2m_1m_2} \vec{p}^2 - \left(\frac{4}{m_1} + \frac{4}{m_2} \right) \left(\frac{\vec{p}^2}{2} \right) - \frac{4}{3} \alpha_s \left(\frac{1}{r} + S_{BF} \right) \quad (1)$$

where $V(r)$ describes the universal binding interaction, $\vec{r}$ and $\vec{p}$ are the relative position and momentum operators, and S_{BF} is the two-body Breit-Fermi interaction between Dirac particles arising together with the Coulomb-like term from the one-gluon exchange.

The explicit form of S_{BF} is given by ^{*)}

*) We refer to any standard textbook of relativistic quantum mechanics for the derivation of the Breit-Fermi potential.

$$S_{BF} = V_B + V_{SS} + V_{SO} + V_T \quad , \quad (2)$$

where

$$V_B = -\frac{1}{2m_1m_2r} \left(\vec{p}^2 + \frac{\vec{r}(\vec{r}\vec{p})\vec{p}}{r^2} \right) \quad (2a)$$

$$V_{SS} = -\frac{\pi}{2} \delta^3(\vec{r}) \left(\frac{1}{m_1^2} + \frac{1}{m_2^2} + \frac{4}{3} \frac{\vec{\sigma}_1 \cdot \vec{\sigma}_2}{m_1 m_2} \right) \quad (2b)$$

$$V_{SO} = \frac{1}{m_1 m_2 r^3} \vec{S} \cdot \vec{L} + \frac{1}{4r^3} \left(\frac{1}{m_1^2} \vec{\sigma}_1 + \frac{1}{m_2^2} \vec{\sigma}_2 \right) \cdot \vec{L} \quad (2c)$$

$$V_T = \frac{3}{2m_1m_2r^3} \left[\frac{(\vec{S} \cdot \vec{r})(\vec{S} \cdot \vec{r})}{r^2} - \frac{1}{3} \vec{S}^2 \right] \quad (2d)$$

Here $\vec{S}$ is the total spin $\vec{S} = \frac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2)$ and $\vec{L}$ the total orbital angular momentum of the bound state. In Eq. (1) the relativistic correction proportional to $(\vec{p}^2)$, present in a consistent expansion of the kinetic energy, is kept mainly in order to have a control of the importance of the relativistic corrections themselves, which are neglected in higher orders (see later).

Having formulated the starting point and the basic assumptions, two different lines can be followed to explore their consequences. One can parametrize in a general way the expectation value of the Breit-Fermi interaction between the wave functions of degenerate multiplets and deduce results independent of the explicit form of the binding potential. Alternatively, a specific potential can be assumed and the complete diagonalization of the Hamiltonian (1) can be performed up to first order in the relativistic corrections.

The first line of approach, pursued by De Rujulà, Georgi and Glashow³⁾ with good results, is in principle more interesting, but it has the shortcoming that many parameters must be introduced in the analysis : certainly the multiplets of different orbital angular momentum have to be treated separately ; furthermore the bound states containing charmed quarks are

essentially uncorrelated with the usual hadrons because of the large charmed-uncharmed quark mass differences. In fact this mass splitting affects very much the pure non-relativistic calculations in a way strictly dependent on the explicit form of the potential.

The second line of approach, relying on a specific potential, is less general in character but also more ambitious, involving very few parameters only. In particular, it allows to test whether a non-relativistic description is indeed tenable. Furthermore, having explicitly the different wave functions, various decay modes can also be estimated. In this paper we shall pursue this second line of investigation, restricting ourselves to the spectroscopy of mesons which is technically much simpler to tackle.

THE POTENTIAL

Out of the various potentials showing quark confinement, we choose to work with that one increasing linearly with the separation between the quarks. Field theoretical analysis ^{4),5)}, as well as bag models including gauge fields ⁶⁾ indeed seem to suggest an r behaviour for the long range binding potential. In addition, potentials with such a long range part have already been applied ^{7),8)} quite successfully to the phenomenological analysis of the newly discovered resonances, interpreted as charmonia.

In the effective Schrödinger equation that we are going to solve numerically, we will then work with the following potential

$$V_1(r) = \lambda r - V_0 - \frac{4}{3} \frac{\alpha_s}{r} \quad (3)$$

The Coulomb-like term dominating at short distances is taken from the one-gluon exchange, and the constant V_0 determines how to interpolate between the r and $1/r$ asymptotic terms [in the notation of Eq. (1), we have taken $V(r) = \lambda r - V_0$]. The perturbation term in the Hamiltonian (1)

$$V_2 = -\frac{1}{8} \left(\frac{1}{m_1^3} + \frac{1}{m_2^3} \right) (\vec{p}^2)^2 - \frac{4}{3} \alpha_s S_{BF} \quad (4)$$

will be treated up to first order.

PARAMETERS

We have three parameters in the potential (α_s, λ, V_0) , and three parameters for the values of the quark masses (m_d, m_λ, m_c) . However, in the actual calculation of the meson masses we will have only three free parameters, namely m_c, λ, V_0 . The values of m_d and m_λ are taken from the general analysis of baryons by De Rujulà, Georgi and Glashow³⁾, ($m_d = 0.34$ and $m_\lambda = 0.54$), giving the correct "hyperfine splitting", as well as the magnetic moments of baryons in absence of quark anomalous moments. Furthermore, in the charmonium picture, the branching ratio $\Gamma(\psi \rightarrow \text{hadrons})/\Gamma(\psi \rightarrow e^+e^-)$ fixes the value of the effective gluon coupling constant $\alpha_s(M_\psi^2)$ in the sense of renormalization group improved perturbation theory¹⁾. We then make the simplifying assumption that the strong fine structure constant α_s appearing in the Hamiltonian describing a bound state of squared invariant mass M^2 is $\alpha_s(M^2)$. Typically, for a $c\bar{c}$ bound state we take $4/3 \alpha_s^c = 0.27$, for a $\lambda\bar{\lambda}$ bound state $4/3 \alpha_s^\lambda = 0.36$ and for a $d\bar{d}$ bound state $4/3 \alpha_s^d = 0.42$. That such increasing values for bound states of decreasing mass must indeed be taken is suggested by the fact that the mean momenta carried by the constituent quarks (and so the mean momenta flowing through this gluon line) are decreasing with their masses.

In all the final results we will use the following values for the parameters : $\lambda = 0.25 \text{ GeV}^2$, $V_0 = -0.76 \text{ GeV}$, $m_c = 1.64 \text{ GeV}$. These values will account correctly for the measured masses $m(\psi) = 3.095 \text{ GeV}$, $m(\psi') = 3.684 \text{ GeV}$ and the leptonic width of the ψ particle $\Gamma(\psi \rightarrow e^+e^-) = 5.2 \text{ keV}$, which are then our experimental inputs.

RELATIVISTIC CORRECTIONS

Once the non-relativistic wave functions are computed by solving the Schrödinger equation with the potential V_1 , the first order relativistic corrections are given by the matrix elements of V_2 between these wave functions.

Working out the spin-orbit and the spin-spin interactions and reducing, mainly for purpose of numerical work, the derivatives of the wave functions by means of the Schrödinger equation and by partial integrations, the diagonal matrix elements of V_2 can be written in the following way :

i) for S states, described by wave functions $\psi_S(\vec{r}) = Y_{00}(\hat{r})\phi_S(r)$,

$$\begin{aligned} \langle JSL=0 | V_2 | JSL=0 \rangle = & -\frac{\mu^2}{2} \left(\frac{1}{m_1^3} + \frac{1}{m_2^3} \right) \int_0^\infty \phi_S^* r^2 (E_{NR} - V_1)^2 \phi_S dr + \\ & -\frac{8\mu\alpha_s}{3m_1m_2} \int_0^\infty \phi_S^* r (E_{NR} - V_1) \phi_S dr + \frac{\alpha_s}{6} \left(\frac{1}{m_1^2} + \frac{1}{m_2^2} \right) |\phi_S(0)|^2 \left(\langle S | S^2 | S \rangle + \frac{8}{3} \right) \end{aligned} \quad (5)$$

ii) for P and D waves, described by wave functions $\psi_{P(D)}^m(\vec{r}) = Y_{1(2)m}(\hat{r})\phi_{P(D)}(r)$

$$\begin{aligned} \langle JSL=1(2) | V_2 | JSL=1(2) \rangle = & -\frac{\mu^2}{2} \left(\frac{1}{m_1^3} + \frac{1}{m_2^3} \right) \int_0^\infty \phi_{P(D)}^* r^2 (E_{NR} - V_1)^2 \phi_{P(D)} dr + \\ & -\frac{8\mu\alpha_s}{3m_1m_2} \int_0^\infty \phi_{P(D)}^* r (E_{NR} - V_1 - \frac{L(L+1)}{4\mu r^2}) \phi_{P(D)} dr + \frac{4\alpha_s}{3m_1m_2} C(J,S,L) \int_0^\infty \phi_{P(D)}^* \frac{1}{r} \phi_{P(D)} dr \end{aligned} \quad (6)$$

where

$$C(J,S,L) = \langle JSL | \left[\left(1 + \frac{m_2^2 + m_1^2}{4m_1m_2} \right) S \cdot L - \left(\frac{3}{5} \delta_{L1} + \frac{2}{21} \delta_{L2} \right) \left((S \cdot L)^2 - \frac{4}{3} S^2 L^2 \right) \right] | JSL \rangle$$

Here E_{NR} stands for the non-relativistic unperturbed energy levels, corresponding to the potential V_1 , $\mu = (m_1 m_2) / (m_1 + m_2)$ is the reduced mass, and $\vec{J}$ the total angular momentum. The first terms in the right-hand sides of Eqs. (5), (6), not proportional to α_s , correspond to the relativistic correction to the kinetic energy, i.e., the $(\vec{p}^2)^2$ term in Eq. (1); the remaining terms come from the Breit-Fermi potential (both the spin independent and the spin-orbit and spin-spin interactions).

Furthermore, there exist off-diagonal matrix elements between the 1^{--} (s wave) and the 1^{--} (d wave) states of the tensor term V_T present in S_{BF} , Eq. (2d), as well as off-diagonal matrix elements between the non-neutral elements of the 1^{++} and 1^{+-} 16-plets of the spin-orbit potential, V_{SO} , Eq. (2c), also present in S_{BF} . They are given by

$$\langle 1^{--} L=0 | \frac{4}{3} \alpha_s V_T | 1^{--} L=2 \rangle = \frac{4\alpha_s}{3\sqrt{2} m_1 m_2} \int_0^\infty \phi_S \frac{1}{r} \phi_D dr \quad (7)$$

$$\langle 1^{++} | \frac{4}{3} \alpha_s V_{so} | 1^{+-} \rangle = \frac{4\alpha_s}{3} \frac{m_2^2 - m_1^2}{2\sqrt{2} m_1^2 m_2^2} \int_0^\infty \phi_P \frac{1}{r} \phi_P dr \quad (8)$$

RESULTS FOR THE MESON MASS SPECTRUM

The Schrödinger equation with the potential of Eq. (3) has been solved numerically for different reduced masses and different α_s , corresponding to $c\bar{c}$, $c\lambda$, $\lambda\lambda$, $c\bar{d}$, $\lambda\bar{d}$ and $d\bar{d}$ type bound states. Having got in this way the non-relativistic energy levels and the non-relativistic wave functions, the first order relativistic corrections have been computed according to the matrix elements given in Eqs. (5), (6).

The obtained energy levels for the S wave spin triplet vector mesons states (M_V) and for their pseudoscalar partners (M_P) are given in Table I. The mass differences ($M_V - M_P$) come from the spin dependent part of the Breit-Fermi interaction. The ground states as well as the first radial excitations are calculated. For the $c\bar{c}$ bound states also the next two higher radial excitations are considered. The final values of the masses are obtained as sums of three terms : the sum of the masses of the constituent quarks ($m_1 + m_2$), using the values quoted before, the non-relativistic energies E_{NR} and the full relativistic corrections E_{RC} , quoted in the Table for the spin triplet states. For completeness also the contribution E_{BF} coming only from the Breit-Fermi interaction is given (again for the triplet states). Finally in the last column the values of the parameter

$$\xi \equiv \frac{\langle \frac{1}{8} (\frac{1}{m_1^3} + \frac{1}{m_2^3}) (p^2)^2 \rangle}{\langle \frac{1}{2\mu} p^2 \rangle} \quad (9)$$

are quoted, which gives a way of estimating how good the non-relativistic approximation is in the various cases since it represents a sort of non-relativistic expansion parameter. In Table I, the experimental values of

the various meson masses, when known, are reported ^{*)}. By looking at the values of the parameter ξ we can conclude that indeed, although the relativistic corrections are not negligible (especially for the radial excitations), the cc , $\lambda\lambda$ and $c\lambda$ type bound states may be regarded as non-relativistic systems. For the bound states made up only of charmed quarks the results essentially agree with previous purely non-relativistic calculations ⁷⁾. The strange quark bound states (ϕ and χ^0) fit also fairly well with calculation. The predictions for the masses of the charmed mesons should then be considered as quite firm. In particular, we argue that : i) the decays of the $c\bar{c}$ vector mesons into the corresponding pseudoscalars plus pions are energetically forbidden ; ii) the $\psi'(3.7)$ lies at threshold for its decay into two charmed pseudoscalar mesons, which is perhaps of help in understanding its total width value ⁹⁾.

The agreement with the experimental masses of the remaining low-lying vector mesons (ρ, ω, K^*) is also acceptable. These results, however, should be taken with great care. In fact, the relativistic corrections to all the bound states containing at least one d type quark turn out to be quite large. The non-relativistic expansion parameter ξ is in this case near 1, and it usually exceeds 1 for the first radial excitations. At least for the essentially linear potential that we have used, this shows that the non-relativistic description is presumably not adequate for these bound states ^{**)}. Specifically we interpret the failure in getting the correct (K^*-K) and $(\rho-\pi)$ splittings as a failure of the non-relativistic approximation. Given this situation, we also do not consider relevant here a possible attempt to apply the mixing scheme proposed by De Rujulà, Georgi and Glashow ³⁾ to the neutral pseudoscalar mesons.

^{*)} In making the comparison, one should not forget that the theoretical values refer to states of unmixed quark content. However, except perhaps for the η , which in any case, as it will be made clear later, is outside the scope of the present calculation, we believe that the displacements in masses produced by the mixings of quark content are within the precision of the results that we are presenting here.

^{**)} A serious difficulty in attempting a fully relativistic description is the equal mass problem. A Dirac equation description could be adequate for λc and $d c$ type bound states where recoil corrections are presumably smaller than relativistic corrections.

Analogous results for the P wave multiplets ($J^{PC} = 2^{++}, 1^{++}, 1^{+-}, 0^{00}$) ^{*)} and for the D wave charmonium-like bound states are given in Table II. We emphasize that at this stage of the calculation no new free parameter is introduced. Again the relativistic corrections to the states containing at least a d type quark are large. The agreement with the experimental data (when available on a sufficiently sound basis) is acceptable. We remark in particular a good agreement for the $f' - \varphi$ "hyperfine" splitting

$$(f' - \varphi)_{th} \simeq (f' - \varphi)_{exp} \simeq 490 \text{ MeV}$$

as well as for the $K^{**} - K^*$ splitting

$$(K^{**} - K^*)_{th} \simeq (K^{**} - K^*)_{exp} \simeq 530 \text{ MeV}$$

MIXINGS

Slight corrections to the above mass values are introduced by the existence of the mixings as has been already pointed out. More importantly relevant mixings of quantum numbers could be induced. Let us first comment on the mixing between S and D waves 1^{--} states induced by the tensor term of the Breit-Fermi Hamiltonian restricting to the almost degenerate charmonia $\psi_S'(3.685)$ and $\psi_D(3.785)$. The off-diagonal element of the mass matrix

$$\begin{pmatrix} V_{11} = 3.685 & V_{12} \\ V_{21} & V_{22} = 3.785 \end{pmatrix}$$

between unperturbed wave functions corresponding to these states, computed according to Eq. (7), is $V_{12} = 2 \text{ MeV}$. The mixing angle, defined by $\text{tg} 2\theta = 2V_{12}/(V_{11} - V_{22})$, is then $\theta \simeq 2 \times 10^{-2} \text{ rad}$, and therefore the branching ratio (see below)

^{*)} For $c\lambda$, cd and λd states the quantum number C stands for $(-)^{L+S}$.

$$\frac{\Gamma(\psi_D \rightarrow e^+e^-)}{\Gamma(\psi_S' \rightarrow e^+e^-)} = \sin^2\theta \simeq 4 \times 10^{-4} \quad (10)$$

is very small, implying an essential decoupling of ψ_D in e^+e^- annihilation ^{*)}.

The non-neutral and unequal-mass elements of the P wave 1^{++} and 1^{+-} multiplets are mixed by the spin-orbit potential. The values of the off-diagonal matrix elements, given in Eq. (8) for the $c\lambda$, cd and λd type bound states are 19 MeV, 32 MeV, and 19 MeV, respectively. From the diagonal matrix elements that can be read off from Table II, one then learns that the mixing angles are in this case quite important. Typically, $\tan 2\theta \simeq 1$. But unfortunately the experimental information here is not accurate enough to have any test of this prediction, especially because the mixing of quantum numbers ($S=0$ and $S=1$) does not lead to any dramatic effect in this case.

We also want to mention that other mixings would exist between all the states of given quark content and fixed L and S (the ground states and their corresponding radial excitations). We do not pursue the calculation of these mixings since they do not introduce mixings at relevant quantum numbers and they produce only very small changes in the mass values.

LEPTONIC WIDTHS

In a non-relativistic approximation the various decay matrix elements are given by proper overlap integrals of the initial and final state non-relativistic wave functions. We can then calculate the leptonic, semi-leptonic, radiative and two-body decay matrix elements. Part of this work has already been done ⁷⁾.

*) The fact that this mixing angle turns out to be small seems to be a potential independent conclusion. The analogous calculation in the case of the harmonic oscillator potential, when the unperturbed energy levels are exactly degenerate, gives also a relatively small angle, because the diagonal matrix elements of the perturbation potential V_2 , and so their difference, is still relatively large with respect to the off-diagonal matrix element.

Let us discuss first the e^+e^- decays of the neutral vector mesons. Our basic formula ¹⁰⁾ is

$$\Gamma(V \rightarrow e^+e^-) = 3 e_v^2 \alpha^2 \frac{16\pi}{3} \frac{|\psi(0)|^2}{m_V^2} \left(1 - \frac{16}{3\pi} \alpha_s(m_V^2)\right) \quad (11)$$

where e_v is the charge of the constituent quark. The factor in parentheses properly accounts for a first order radiative correction having exactly the same origin as the Breit-Fermi interaction S_{BF} in Eq. (1). It arises from the exchange of a transverse gluon, with a propagator not containing the Coulomb part. Again the effective gluon coupling is taken at the squared mass of the corresponding bound state.

The obtained values for the leptonic widths of ψ 's, ϕ , ρ , ω are quoted in Table III, together with the values of the non-relativistic wave functions at the origin.

As to the weak leptonic decays, the corresponding widths can also be computed, starting from the usual form of the weak Hamiltonian for the quark and lepton fields

$$H_W = \frac{G_F}{\sqrt{2}} (\bar{J}_\mu J_\mu^\dagger + c.c.)$$

$$J_\mu = \bar{p} \gamma_\mu (1 + \gamma_5) (n \cos \theta + \lambda \sin \theta) + \bar{c} \gamma_\mu (1 + \gamma_5) (\lambda \cos \theta - n \sin \theta) + \bar{\nu}_e \gamma_\mu (1 + \gamma_5) e + \bar{\nu}_\mu \gamma_\mu (1 + \gamma_5) \mu \quad (12)$$

For a meson bound state of mass M , wave function $\psi(\vec{r})$, and total spin $S=0,1$ one gets

$$\Gamma(M \rightarrow e\nu) = 3 \frac{G_F^2}{2\pi} |\psi(0)|^2 \left(1 - \frac{m_e^2}{M^2}\right)^2 \cdot \left[\frac{2M^2}{3} \left(1 + \frac{m_e^2}{2M^2}\right) \frac{\langle S^2 \rangle}{2} + m_e^2 \frac{\langle 2 - S^2 \rangle}{2} \right] \quad (13)$$

where G should be read as $G_F \cos\theta$ for pn and $c\lambda$ type states and $G_F \sin\theta$ for $p\lambda$ and cn type bound states. The first term in the right-hand side of Eq. (13), proportional to $\langle \vec{S}^2 \rangle$, is contributing to the vector decays, whereas the second term, proportional to $\langle 2-\vec{S}^2 \rangle$, corresponds to the pseudoscalar decays. The calculation of the radiative corrections is not attempted in this case, because Eq. (13) is presumably affected by other more important ambiguities. For the cases where the weak decays are not overwhelmed by strong or electromagnetic decays, the obtained values are reported in Table III.

As previously discussed, in this Table only the width of the $\psi(3.1)$ is fitted. Again it should be emphasized that the numbers for ρ and ω are only tentative ones. For example, in these cases, the radiative correction in Eq. (11) is exceedingly large. The very large values obtained for the leptonic width of the K and π clearly indicate that the non-relativistic approximation is not applicable to the low-lying pseudoscalar mesons. Nevertheless, the results given in Table III as a whole perhaps may be considered as indicative for a possible over-all validity of the model and show the quite important rôle of the introduction of the non-infra-red radiative corrections in the e^+e^- decay amplitude. Other non-negligible corrections are, of course, expected from different sources, like relativity and unitarity.

As to the semi-leptonic and non-leptonic weak decays, here we only make the remark that, as far as the non-relativistic description is valid, there is no natural argument for the enhancement of the non-leptonic decay mode. Since the non-relativistic approximation seems to be more adequate for the heavy charmed mesons, we would expect that their non-leptonic decays are not enhanced as much as the non-leptonic decay modes of the low-lying mesons¹¹⁾. Recent experimental data¹²⁾, if we interpret them within the charm model, indicate the importance of the semi-leptonic decay modes of the charmed mesons.

HADRONIC COUPLINGS AND e^+e^- ANNIHILATION

A crucial question, demanding at least a tentative answer, concerns the interpretation of the e^+e^- annihilation total cross-section data ¹³⁾. A first explanatory attempt to describe quantitatively the behaviour of $\sigma(e^+e^- \rightarrow \text{hadrons}; s)$ in the region above charm threshold has already been made by the present authors ⁹⁾. There we assume that : i) the total charmed hadron production cross-section in e^+e^- annihilation for $\sqrt{s} \simeq 3.5 + 6$ GeV is mainly given by the two-body charmed meson final states ; ii) the form factors of the charmed mesons are dominated by the vector meson poles ($\psi, \psi', \dots$).

To have then a quantitative prediction for $\sigma(e^+e^- \rightarrow \text{hadrons})$, the knowledge of the three meson vertices ($\psi \rightarrow DD, \psi' \rightarrow DD, \psi \rightarrow FF$, etc.) is needed. For this we use an extension of the quark pair creation model ¹⁴⁾, where the three meson vertices are defined by the quark diagram in Fig. 1. The bubbles denote the products of spin SU(4) wave functions and of the spatial wave functions that we have just computed. The three meson couplings will involve an over-all universal coupling strength γ for the creation of the light quark pair in a spin-triplet p wave state of zero total angular momentum. According to the model, these effective three meson couplings are given as follows ⁹⁾ :

i) vector $\rightarrow$ 2 pseudoscalars

$$\langle P_1(k_1) P_2(k_2) | V(k, \epsilon_\mu) \rangle = 2 g_{VP_1P_2} k_1 \cdot \epsilon$$

ii) vector (of mass m_1) $\rightarrow$ vector + pseudoscalar

$$\langle P(k) V_2(k_2, \epsilon_\nu^{(2)}) | V_1(k_1, \epsilon_\mu^{(1)}) \rangle = \frac{2 g_{V_1V_2P}}{m_1} \epsilon^{\mu\nu\lambda\sigma} \epsilon_\mu^{(1)} \epsilon_\nu^{(2)} k_{1\lambda} k_{2\sigma}$$

iii) vector $\rightarrow$ 2 vectors

$$\begin{aligned} & \langle V_2(k_2, \epsilon_\nu^{(2)}) V_3(k_3, \epsilon_\lambda^{(3)}) | V_1(k_1, \epsilon_\mu^{(1)}) \rangle = \\ & = g_{V_1V_2V_3} \left[2 (\epsilon^{(1)} \cdot \epsilon^{(3)}) (\epsilon^{(2)} \cdot k_3) - (\epsilon^{(1)} \cdot \epsilon^{(3)}) (\epsilon^{(2)} \cdot k_1) + (\epsilon^{(1)} \cdot \epsilon^{(2)}) (\epsilon^{(3)} \cdot k_2) \right] \end{aligned}$$

where the coupling constant g_{ABC} is connected with the spatial overlap integrals in the following way :

$$g_{ABC} = \frac{\pi^{3/2}}{\sqrt{2}} \gamma m_A^{3/2} \int_0^\infty r_1^2 dr_1 \left(2\psi_A(r_1) + \frac{r_1}{3} \psi_A'(r_1) \right) \cdot \int_0^\infty r_2^2 dr_2 \psi_B^*(r_2) \int_{-1}^{+1} dz \psi_C^*\left(\sqrt{r_1^2 + r_2^2 - 2r_1 r_2 z}\right) \quad (14)$$

here ψ_A, ψ_B, ψ_C are the spatial wave functions for the meson states A, B, C, not depending on angles because we are restricting ourselves to S wave mesons ; the integrations in Eq. (14) are performed numerically, using the wave functions previously computed.

The obtained values for g_{ABC}/γ are listed in Table IV (since the spatial wave functions of S wave vector mesons are the same as for the corresponding pseudoscalars, the couplings have to be computed only for one configuration, e.g., $V \rightarrow PP$). With these values for g_{ABC} and with the mass spectrum given in Table I both the various partial widths of the ψ resonances and the shape of the conventional ratio

$$R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons}; s)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-; s)}$$

can be given in terms of the only one unknown parameter γ . In computing R a constant background $B_{\ell q}$ due to uncharged hadron production is added, with a value $B_{\ell q} \sim 2.5$ which is suggested not only by the experimental value of R at $\sqrt{s} < 3.7$ GeV¹³⁾, but also by the light quark parton model (including asymptotic freedom corrections).

Figure 2 shows the resulting curve (A) in the region $3.5 < \sqrt{s}/\text{GeV} < 6$ with $\gamma = 0.45$, with the interferences between the various vector meson resonances $[\psi(3.1), \psi'(3.7), \psi''(4.1), \psi'''(4.5)]$ not taken into account. The shape of the experimental curve around the ψ'' mass can then be roughly accounted for even with the relatively small leptonic width predicted by the model for this resonance $[\Gamma(\psi'' \rightarrow e^+e^-) \simeq 2.7 \text{ keV}]$.

How to deal with the interferences between the various very nearby resonances in this specific VMD model is not a problem free from ambiguities. We then take here a purely phenomenological attitude. The dotted line (B) in Fig. 2 shows a best fit of the experimental data allowing for the presence of

relative phases between the various resonances and with the absolute values of the couplings as given in Table IV. To be more specific, at the amplitude level the Breit-Wigner poles corresponding to the three radial excitations $\psi'(3.7)$, $\psi''(4.1)$, $\psi'''(4.5)$ of the $\psi(3.1)$ introduced in the model get multiplied by the phase factors $\exp(i\varphi')$, $\exp(i\varphi'')$, $\exp(i\varphi''')$, respectively. The fit reproduced in Fig. 2 corresponds to $\gamma = 0.57$, $\varphi' = 2.7$ rad, $\varphi'' = 1.5$ rad, $\varphi''' = 3.76$. A fit with all these phases put equal to zero, which amounts in taking strictly the prediction of the model even for the interference terms, produces the curve (A) in Fig. 3, with $\gamma = 0.57$, $\varphi' = 2.7$ rad, corresponds to a sort of maximum destructive interference. Taking maximum constructive interference, i.e., all positive signs for the various coupling constants and no other phase factor, results in the dotted curve (B) shown in Fig. 3 ($\gamma = 0.40$). Regarding the large constructive interference contribution as a phenomenological simulation of the non-resonating background, this curve can be considered as a more realistic description of the situation *).

Corresponding to the different curves displayed in Figs. 2, 3, each of them obtained with a fixed value of γ , the following charmed hadronic widths are got for the resonances ψ'' , ψ''' above the charm threshold :

- i) to curve (A), Fig. 2, $\Gamma(\psi'') = 190$ MeV, $\Gamma(\psi''') = 1040$ MeV ;
- ii) to curve (B), Fig. 2, $\Gamma(\psi'') = 310$ MeV, $\Gamma(\psi''') = 1700$ MeV ;
- iii) to curve (A), Fig. 3, $\Gamma(\psi'') = 150$ MeV, $\Gamma(\psi''') = 830$ MeV ;
- iv) to curve (B), Fig. 3, $\Gamma(\psi'') = 150$ MeV, $\Gamma(\psi''') = 830$ MeV.

It is interesting to see how the opening of the various charmed meson channels, as predicted by the mass spectrum, can give a rise of R from the uncharmed hadron background quite consistent with the experimental data so far.

*) Remark that the obtained values of the γ are not very different from the values of the effective quark-gluon coupling constant.

CONCLUSIONS

Here we have presented a way to describe the masses and widths of the mesons, including the newly discovered narrow resonances. The explicit model we have used rests on the following main assumptions : i) the dynamics of the constituents inside the mesons is essentially non-relativistic ; ii) their binding force is described by a spin-independent linear potential ; iii) at short distances they weakly interact via a one-gluon exchange. The over-all resulting picture is perhaps suggested by an underlying gauge field theory.

The considerable agreement with the experimental data for mesons of masses $\gtrsim 1\text{GeV}$ and the subsequent semi-phenomenological description of the ratio $R(s)$ in the region $\sqrt{s} \simeq 3.5 - 6 \text{ GeV}$ seem to support these assumptions.

The amount of the computed relativistic corrections shows that the non-relativistic description is indeed consistent for the $cc, c\lambda, \lambda\lambda$ bound states. This does not seem to be the case for the $cd, \lambda d$ and dd type bound states ^{*}), where the mean velocity of the constituents reaches values near to 1, though an acceptable agreement with the data is still obtained in these cases, at least for the uncharmed vector mesons.

In order to obtain with minimal number (three) of free parameters the remarkable agreement with the data, the inclusion of the one-gluon exchange has been important. The fact that the effective quark-gluon coupling constant increases with decreasing mass values of the constituents (consistently with an underlying asymptotically free theory), leads to important corrections which shift the mass and width values nearer to the experimental results.

^{*}) Rumours circulate that events of the type $e^+e^- \rightarrow (e_{\mu}^{\pm} \mp + \text{no charged particles})$ may exist in the e^+e^- annihilation SLAC-LBL experiment at $\sqrt{s} = 4.8 \text{ GeV}$. If one accepts as preferential interpretation of these events the production of a charmed vector meson pair $e^+e^- \rightarrow V^+V^-$, with relatively large leptonic decay modes $V \rightarrow \mu + \nu_{\mu}$ and $V \rightarrow e + \nu_e$, the charmed vector mesons should lie below their pseudoscalar partners or should be almost degenerate with them. With the scheme proposed, this could only be interpreted as a breakdown of the non-relativistic expansion for the $cd, c\lambda$ type bound states, since the one-gluon exchange correction gives $M_V(cd) - M_P(cd) \simeq M_V(c\lambda) - M_P(c\lambda) \simeq 70 \text{ MeV}$.

With the computed wave functions, we could calculate the three-meson couplings within the non-relativistic pair creation model. This enabled us, using vector meson dominance, to get the e^+e^- annihilation cross-section by introducing only one new free parameter (the dimensionless coupling strength for the creation of a light quark pair). Important interferences have been obtained between the ψ' , ψ'' , ψ''' pole contributions. With constructive interference we could explain the R ratio in the $\sqrt{s} \simeq 3.5 - 6$ GeV region, which may indicate the importance of the multi-quark thresholds and consequently the non-asymptotic nature of this regime. This would imply that the asymptotic value for $R(s)$ ($10/3$) will be reached only at much higher energies.

It is clear that this work is only a part of a more comprehensive analysis that has to be performed in this framework. As to the mesons, we have to wait now for the results of the experimental searches for heavier meson states and see if they fit in the predicted scheme. On the other hand, it seems to be natural to extend the analysis to baryons. Very interesting first results have already been obtained in a similar direction by De Rujulà, Georgi and Glashow ³⁾.

Needless to say any insight into the connection between the stated assumptions and an underlying field theory would be of greatest importance.

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	$M_V (1^{--})$	$M_V^{exp} (1^{--})$	$M_P (0^{-+})$	$M_P^{exp} (0^{-+})$	$E_{NR} + V_0$	E_{RC}	E_{BF}	ξ
cc	3095	$\psi (3095)$	3041	-	607	- 29	6	0.17
(cc)'	3680	$\psi' (3684)$	3635	-	1255	- 94	- 16	0.19
(cc)''	4111	$\psi'' (4100)?$	4061	-	1766	- 175	- 28	0.25
(cc)'''	4471	-	4435	-	2239	- 287	- 58	0.30
c λ	2105	-	2034	-	832	- 144	23	0.60
(c λ)'	2607	-	2543	-	1616	- 427	- 12	0.75
c $\bar{d}$	1912	-	1842	-	995	- 301	33	0.90
(c $\bar{d}$)'	2200	-	2137	-	1879	- 898	10	1.44
$\lambda\lambda$	1131	$\varphi (1020)$	959	$X^0 (958)$	978	- 163	11	0.53
($\lambda\lambda$)'	1685	-	1541	$E (1416)?$	1883	- 517	- 79	0.70
$\lambda\bar{d}$	918	$K^* (892)$	678	$K (494)$	1083	- 281	18	0.90
($\lambda\bar{d}$)'	1315	-	1118	-	2060	- 864	- 85	1.13
d $\bar{d}$	716	$\omega (783)$ $\vec{\rho} (770)$	448	$\eta (549)$ $\vec{\pi} (140)$	1168	- 370	- 19	0.90
(d $\bar{d}$)'	1033	$\vec{p}^* (1250)?$	755	-	2215	-1100	-143	1.30

TABLE I : Masses (MeV) of the S wave bound states.

	$M(0^{++})$	$M^{\text{exp}}(0^{++})$	$M(1^{++})$	$M^{\text{exp}}(1^{++})$	$M(1^{+-})$	$M^{\text{exp}}(1^{+-})$	$M(2^{++})$	$M^{\text{exp}}(2^{++})$	E_{NR}	ξ
$(cc)_P$	3425	-	3466	-	3482	-	3503	-	1025	0.11
$(c\lambda)_P$	2385	-	2456	-	2488	-	2528	-	1324	0.49
$(cd)_P$	2125	-	2205	-	2246	-	2294	-	1539	0.93
$(\lambda\lambda)_P$	1326	-	1480	D (1285)	1540	-	1619	f' (1516)	1540	0.45
$(\lambda d)_P$	1030	K (~1300)?	1220	$Q_A(1242)?$	1297	$Q_B(\sim 1300)?$	1400	$K^{**}(1420)$	1680	0.70
$(dd)_P$	781	$S^*(993)?$ $\delta^*(976)$	1039	$\vec{A}_1(1100)$	1141	$\vec{B}(1237)$	1274	$f(1270)$ $\vec{A}_2(1310)$	1791	0.72
	$M(1^{--})$	$M(2^{--})$	$M(2^{+-})$	$M(3^{--})$						
$(cc)_D$	3778	3794	3798	3811					1351	

TABLE II : Masses (MeV) of the P and D wave bound states.

	ψ	ψ'	ψ''	ψ'''	ϕ	ρ	ω	F	D	K	π
Γ_e	5.2 keV	3.1 keV	2.7 keV	1.7 keV	2.1 keV	10.6 keV	1.2 keV	2.7×10^{10}	9.1×10^8	6.7×10^8	8.0×10^9
Γ_e^{exp}	5.2 keV	2.2 keV	-	-	1.44 keV	6.0 keV	0.8 keV	-	-	5.13×10^7	3.84×10^7
$4\pi \psi(0) ^2$ (GeV) 3	0.808	0.679	0.751	0.538	0.209	0.110	0.110	0.335	0.201	0.110	0.110

TABLE III : Leptonic widths of the S wave bound states (weak decays in sec^{-1}).

	ψ	ψ'	ψ''	ψ'''
DD	3.62	-5.86	4.58	-3.33
DD'	-0.53	2.82	-4.99	6.00
D'D'	3.24	-4.23	2.77	-3.13
FF	3.56	-5.12	3.25	-1.93
FF'	-0.69	3.34	-5.21	5.25
F'F'	3.14	-3.74	0.14	-4.03
<p>For example, the first two figures in the first line should be read as $g_{\psi DD}/\gamma = 3.62$, $g_{\psi' DD}/\gamma = -5.86$.</p>				

TABLE IV : Effective three-meson couplings.

REFERENCES

- 1) T. Appelquist and H.D. Politzer - Phys.Rev.Letters 34, 43 (1975) ;
A. De Rujulà and S.L. Glashow - Phys.Rev.Letters 34, 46 (1975).
- 2) J.J. Aubert et al. - Phys.Rev.Letters 33, 1404 (1974) ;
J.E. Augustin et al. - Phys.Rev.Letters 33, 1453 (1974) ;
G.S. Abrams et al. - Phys.Rev.Letters 33, 1453 (1974).
- 3) A. De Rujulà, H. Georgi and S.L. Glashow - Harvard Preprint, to be published.
- 4) H. Nielsen and P. Olesen - Nuclear Phys. B61, 45 (1973) ;
B. Zumino - CERN Preprint TH. 1779 (1973) ;
G. 't Hooft - Nuclear Phys. B35, 167 (1974).
- 5) J. Schwinger - Phys.Rev. 128, 2425 (1962) ;
A. Casher, J. Kogut and L. Susskind - Phys.Rev.Letters 31, 494 (1973) ;
K. Wilson - Phys.Rev. D10, 2445 (1974).
- 6) P. Hasenfratz, J. Kuti and A. Szalay - KFKI Preprint, Budapest, KFKI-75-30 (1975).
- 7) E. Eichten, K. Gottfried, T. Kinoshita, J. Kogut, K.D. Lane and T.M. Yan - Phys.Rev.Letters 34, 369 (1975) ;
B.J. Harrington, S.Y. Park and A. Yildiz - Phys.Rev.Letters 34, 168 (1975).
- 8) Extended works on all the hadron spectroscopy, in a context different from ours but still using a linear potential have been made by :
J.F. Gunion and R.S. Willey - Pittsburgh University Preprint PITT-144 (1975) ;
J.S. Kang and H.J. Schnitzer - Brandeis University Preprint (1975).
- 9) For estimates, see :
R. Barbieri, R. Gatto, R. Kögerler and Z. Kunszt - to appear in Phys. Letters B.
- 10) R. Barbieri, R. Gatto, R. Kögerler and Z. Kunszt - submitted to Phys. Letters B.
- 11) J. Ellis, M.K. Gaillard and D.V. Nanopoulos - CERN Preprint TH. 2030 (May 1975).
- 12) A.M. Boyarski et al. - Preprint SLAC-PUB-1583 (May 1975).
- 13) Talk presented by J.A. Kadyk at the International Colloquium on High Energy Neutrino Physics, Paris (18-20 March 1975).
- 14) R. Carlitz and M. Kislinger - Phys.Rev. D2, 336 (1970) ;
A. Le Yaouanc, L. Olivier, O. Pène and J.C. Raynal - Phys.Rev. D7 2223 (1973).

FIGURE CAPTIONS

Figure 1 The three-meson vertex.

Figure 2 The ratio $R(s) = \sigma(e^+e^- \rightarrow \text{hadrons}) / \sigma(e^+e^- \rightarrow \mu^+\mu^-)$. The curve (A) (full line) has been calculated without the interference contributions (with $\gamma = 0.45$). The dotted curve (B) has been obtained by allowing for three arbitrary relative phases between the resonances $\psi, \psi', \psi'', \psi'''$ and performing a least square fit ($\gamma = 0.57$).

Figure 3 $R(s)$ in the VDM model. The curve (A) (full line) is the strict prediction of the non-relativistic model (real coupling constants, destructive interference, $\gamma = 0.40$). The dotted curve (B) has been computed with maximum constructive interference (with $\gamma = 0.40$).

FIG. 1

FIG. 2

FIG. 3