CM-P00061554

ARCHIVES.

Ref.TH.2038-CERN

A SPACE-TIME DESCRIPTION OF QUARKS AND HADRONS

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A B S T R A C T

A more concrete formulation of the previously developed massive quark model (MQM) is given by describing hadrons as finite space-time domains (bags) in which simple quark systems can be found. A well-defined and appealing picture of hadrons and their interactions emerges.

Ref.TH.2038-CERN

20 June 1975

1. - INTRODUCTION

In particle physics there seems today little to dispute the usefulness of the notion of hadronic constituents as an important key in the solution of the problem of hadronic interactions.

Turning this notion into a concrete theory of hadrons and their interactions has, however, met with formidable difficulties. The experimental elusiveness of these constituents has been the main stumbling block in our effort to build a kind of "nuclear physics" of hadrons in terms of quarks. Recently, a considerable effort has gone into the exploration of the possibility that hadronic constituents may be confined, i.e., disappear from the space of physical states. It appears that only time will tell whether the various valiant attempts to put the notion of confinement in a conventional (Lagrangian) field theoretical framework are in fact worth while.

In this paper we shall take a different attitude which, however, is not necessarily orthogonal to the field theoretical one.

Guided by the vast amount of experimental information we shall introduce suitable mathematical objects (quark Green's functions) to describe hadronic states as simple composite of quarks. As a means of visualizing the quark degree of freedom we shall associate it with a "field" whose properties, however, appear quite unconventional. The peculiar features of such a description have been discussed at length in a series of papers dealing with the massive quark model (MQM) ^{*)}.

The work presented here is an attempt to give the general ansatz of the MQM a more concrete formulation in order to considerably sharpen up its calculational tools.

Before going into the details, we would like to sketch briefly the physical picture we have in mind. The elusive hadronic constituents, quarks, partons or whatever we choose to call them, are imagined to be localized lumps of energy, momentum, charge, etc., within the extended

^{*)} The most recent and complete account of the MQM ideas and calculations is in Ref. 1) ; there one can also find the references to earlier work.

hadron states they comprise. In order to keep contact with reality we impose the condition that such states carry zero triality and that their constituents span finite space-time domains in their relative co-ordinates ^{*)}. We emphasize that we can offer a deeper explanation for neither of these two constraints ; zero triality and confinement are taken as two prerequisites which are necessary to introduce the notion of constituents in hadron physics.

A further constraint which we would like to impose to this picture is scaling in deep inelastic phenomena. Light cone and parton physics strongly suggests that such behaviour results when the hadronic constituents at short space-time distances propagate as free particles with no or little interaction. We are thus immediately led to a class of "primitive" hadronic states which are the simplest zero-triality combinations of quarks (mesons : $q\bar{q}$; baryons : qqq). They are confined in finite space-time domains, inside which they move freely ^{**)}. We shall call such states, whose structure will be studied in detail, "allowed quark orbits". We then envisage scattering and decay processes as transitions from a set of allowed orbits to other allowed orbits, their probability amplitude being essentially given by the "space-time overlap" between initial and final configurations.

The plan of the paper is as follows : in Section 2, we treat the basic $q\bar{q}$ system in the not very realistic approximation of scalar quarks carrying no internal quantum number ; in Section 3, the picture of the interactions among $q\bar{q}$ systems is described in detail. Section 4 is devoted to the description of the decay of an excited $q\bar{q}$ state, while Section 5 contains a discussion of the structure of the $q\bar{q}$ four-point function. The conclusions and an outlook are in Section 6.

^{*)} In this respect we are quite close to the views of the MIT bag [Ref. 2].

^{**)} Again we are paralleling the MIT bag.

2. - THE BASIC $q\bar{q}$ SYSTEM

We want to study the simplest zero triality system : the meson $q\bar{q}$ system. Throughout this paper only scalar quarks shall be considered in order to illustrate the main ideas of this approach in the simplest possible situation. The extension to the realistic case where quarks have spin $\frac{1}{2}$ and carry $SU(3)$ quantum numbers shall be carried out elsewhere.

A. - Equations of motion

Let us then consider the $q\bar{q}$ wave function (w.f.) $\psi(P; x_1, x_2)$ appearing in Fig. 1.

Figure 1 The $q\bar{q}$ wave function.

In a conventional field theoretical framework it would be given by

$$\psi(P; x_1, x_2) = \langle 0 | T(q(x_1) \bar{q}(x_2)) | P \rangle \quad (2.1)$$

This representation will turn out to be useful in sorting out some general properties of the w.f. For example, we can use it to work out the separation of the centre-of-mass motion from the relative motion. In fact by translating by $(x_1 + x_2)/2 = R$ we get

$$\begin{aligned} \psi(P; x_1, x_2) &= e^{iPR} \langle 0 | T(q(\frac{x_1 - x_2}{2}) \bar{q}(\frac{-x_1 + x_2}{2})) | P \rangle \\ &= e^{iPR} \psi(P; x) \end{aligned} \quad (2.2)$$

where we have set $x = (x_1 - x_2)$.

We have now to impose on this wave function the requirement that the quarks are permanently confined. The simplest and admittedly quite crude way of achieving this is to set

$$\psi(p; x) = 0 \quad \text{for } x \notin R(p) \quad (2.3)$$

where $R(p)$ is a certain finite space-time domain which in principle depends on the momentum p of the hadron we wish to describe ^{*)}.

The next question to answer is to determine how quarks will propagate in the "bag" region $x \in R(p)$. As mentioned in the introduction the obvious guess is that for small x both quarks obey free equations. We should expect some deviations from free motion when x is near the boundary of $R(p)$, we choose for the time being to neglect these surface effects and we will try to take them into account when their presence cannot be consistently ignored. However, this approximation of "sharp boundaries" is only expected to become unreliable for certain low energy phenomena where the wavelengths involved become comparable with the hadronic size.

We shall then impose on $\psi(p; x_1, x_2)$ the two conditions ^{**)}

$$\begin{aligned} -\square_{x_1}^2 \psi(p; x_1, x_2) &= m^2 \psi(p; x_1, x_2) \\ -\square_{x_2}^2 \psi(p; x_1, x_2) &= m^2 \psi(p; x_1, x_2) \end{aligned} \quad x \in R(p) \quad (2.4)$$

(m is the quark mass inside the bag).

By employing the translational invariant wave function (2.2), we have the two equations :

$$\begin{aligned} \left(\frac{p}{2} - \frac{1}{i} \frac{\partial}{\partial x}\right)^2 \psi(p; x) &= m^2 \psi(p; x) \\ \left(\frac{p}{2} + \frac{1}{i} \frac{\partial}{\partial x}\right)^2 \psi(p; x) &= m^2 \psi(p; x) \end{aligned} \quad (2.5)$$

^{*)} This is required, for instance, by Lorentz invariance.

^{**)} Similar ideas are contained in Ref. 3).

These equations can be cast in the following form :

$$p \cdot \frac{\partial}{\partial x} \psi(p; x) = 0 \quad (2.6a)$$

$$\left(\frac{M^2}{4} - \square_x^2 \right) \psi(p; x) = m^2 \psi(p; x) \quad (2.6b)$$

for $x \in R(p)$.

Let us look at the Eqs. (2.6) in the rest frame $p \equiv (M, \vec{0})$. In this frame we see that space and time get decoupled and in order to obtain compatibility with the boundary conditions $R(M, \vec{0}) = R(M)$ should also split into a "time-bag" and a "space-bag". Thus we have

$$\frac{\partial}{\partial t} \psi(M; t, \vec{x}) = 0 \quad (2.7a)$$

for $|t| \leq R_t(M)$, and

$$\left(\frac{M^2}{4} + \vec{\nabla}^2 - m^2 \right) \psi(M; t, \vec{x}) = 0 \quad (2.7b)$$

for $|\vec{x}| \leq R_s(M)$.

Equation (2.7a) is quite important, it stipulates that in the rest frame the wave function is time-independent, thus washing out the dynamics of the relative time co-ordinate which has always been a source of troubles in relativistic Bethe-Salpeter approaches ^{*)}. A simple definition of electromagnetic current, as it will be discussed later, requires continuity of the wave function at the boundary ; thus we must expect the existence of a "skin" region in time where (2.7a) cannot hold and should be modified by a "potential" term giving a $\psi(t)$ of the type shown in Fig. 2. This kind of surface effect is expected to be appreciable only at low energy.

*) A discussion of the problem of ghosts in B.S. equation can be found for example in Ref. 4).

Figure 2 A possible shape of the time-w.f.

Equation (2.7b) will provide us with a spectrum once we impose the continuity of $\psi(M; \vec{x}, t)$ for $|\vec{x}| = R_s(M)$. The set of basic $\bar{q}q$ states is then :

$$\psi(M; t, \vec{x}) = \psi_M(t) \varphi_{M, n\ell}(\vec{x}) \quad (2.8)$$

where

$$\varphi_{M, n\ell}(\vec{x}) = N_{n\ell} j_\ell(\sqrt{\frac{M^2}{4} - m^2} |\vec{x}|) Y_\ell^m(\theta, \varphi) \quad (2.9)$$

where $N_{n\ell}$ is a normalization factor to be determined, $j_\ell(z)$ is the spherical Bessel function of order ℓ and Y_ℓ^m is a normalized spherical harmonic. The continuity condition implies the spectrum equation

$$[M_{n\ell}^2 - 4m^2]^{\frac{1}{2}} = \frac{2}{R_s(M)} \beta_{n, \ell} \quad (2.10)$$

where $\beta_{\ell, n}$ is the n^{th} positive zero of the Bessel function $j_\ell(z)$; asymptotically it is well known that

$$\beta_{n, \ell} \xrightarrow{n \text{ large}} \pi (n + \frac{\ell}{2}) \quad (2.11).$$

The spectrum equation (2.10) will give us a definite prediction only if the dependence of $R_s(M)$ is given. We shall discuss possible criteria to restrict the form of $R_s(M)$ in a short while.

B. - Normalization

We now take up the question of normalization. Inside the bag the free-field equations (2.4) imply that the bilinear operator

$$j_{\mu}(x) = q^{\dagger}(x) \overleftrightarrow{\partial}_{\mu} q(x) \quad (2.12)$$

is conserved and can be identified with the electromagnetic current. This form is also strongly suggested by scaling at short distances. We should, however, expect modifications of (2.12) for quarks near the boundaries, but to the extent that we are neglecting "surface" effects we shall ignore such modifications.

In order to compute the charge, however, we need to know not only the form (2.12) but also the rules governing the coupling of the bag states. This will be taken up in the next section, for the time being we shall work out the normalization of the w.f.'s assuming that the electromagnetic current operator between bag states is given by (Q_i is the quark charge)

$$j_{\mu}^{op}(x) = \mu^2 Z \sum_i Q_i 2 \partial_{\mu} \delta^4(x-x_i) \quad (2.13)$$

The meaning of μ^2 , a mass parameter, and Z , a renormalization constant, shall become apparent later. Using (2.13) we get the normalization condition

$$\frac{2}{\mu^2 Z} = \int d^4x |\psi(p;x)|^2 \quad (2.14)$$

which determines $|N_{n\ell}|$ in Eq. (2.9) as

$$|N_{n\ell}| = \frac{\sqrt{2}}{\mu Z^{1/2}} \frac{2}{\pi^{1/2}} \frac{\beta_{n,\ell}^{1/2}}{[2R_t(M)]^{1/2}} \frac{1}{R_s(M)^{3/2}} \frac{1}{|J_{\ell+3/2}(\beta_{n,\ell})|} \quad (2.15)$$

C. - The w.f. in momentum space. The MQM criterion

We shall now compute the Fourier transform of (2.8), which will provide us with an expression of the w.f. in momentum space (Fig. 3).

Figure 3 The momentum-space wave function.

We have

$$\psi(M, k) = \int d^4x e^{ikx} \psi(M; t, \vec{x}) \quad (2.16),$$

and performing the four-dimensional integration in a straightforward way we get

$$\psi(M, k) = (2\pi)^2 \frac{2\sqrt{2}}{\pi \mu Z^{1/2}} \frac{R_s^2(M)}{R_t(M)} \frac{\sin k_0 R_t(M)}{k_0} \frac{R_{n,e}}{\beta_{ne}^2 - |\vec{k}|^2 R_s^2(M)} j_l(|\vec{k}| R_s(M)) Y_l^m(\theta_k, \varphi_k) \quad (2.17)$$

In order to appreciate the basic properties of $\psi(M, k)$, let us first express the variables k_0 , and $\vec{k}$ in the rest frame in terms of the quark "masses" $(\frac{P}{2} \pm k)^2 = m_{1,2}^2$. We have

$$k_0 = \frac{1}{2M} (m_1^2 - m_2^2) \quad (2.18)$$

$$|\vec{k}|^2 - \frac{M^2}{4} + m^2 = \frac{1}{4M^2} (m_1^2 - m_2^2)^2 - \frac{1}{2} (m_1^2 + m_2^2) \quad (2.19),$$

and we can immediately see that (2.17) is an entire function of the variables m_1^2 and m_2^2 . This is one of the basic ansätze for the MQM ¹⁾ and a general property of permanently confined quarks, so it is not surprising we get it in our picture. But there is more ; if we look at the function $(\sin k_0 R_t(M))/k_0$ ^{*} we see that in the variable k_0 it is peaked at $k_0 = 0$, with a width $\Delta k_0 \sim \pi/R_t(M)$, and from (2.18) the peak is at $m_1^2 = m_2^2$ with a width $\Delta((m_1^2 - m_2^2)/2) \sim \pi(M/R_t(M))$. A similar analysis applies to the variable $|\vec{k}|$, and using (2.19) we get a peaking of the w.f. at $((m_1^2 + m_2^2)/2) = m^2$ with a width $\Delta((m_1^2 + m_2^2)/2) \sim \pi M/R_s(M)$. Again, this is one of the fundamental assumptions of the MQM, which stipulates that the quark Green's functions are peaked at small values of the quark masses (we should then choose $m \sim 300$ MeV). But the MQM can recover all the appealing aspects of scaling and parton models provided such peaking is universal, i.e., the widths of the mass distributions do not depend on the particular hadronic state. This provides us with an important and well-defined criterion to determine the functions $R_s(M)$ and $R_t(M)$. We should in fact set

$$R_s(M) \sim R_t(M) \xrightarrow[M \text{ large}]{} R^2 M \quad (2.20)$$

Actually, as a smooth interpolating formula for $R_s(M)$ and $R_t(M)$ we shall choose :

$$R_s^2(M) = R_t^2(M) = R_0^2 \frac{\beta_{n,e}}{\pi} \quad (2.21)$$

The spectrum equation (2.10) then becomes

$$M_{ne}^2 = 4m^2 + \frac{4\pi}{R_0^2} \beta_{n,e} \quad (2.22)$$

and, comparing with (2.20),

$$R^2 = \frac{R_0^2}{2\pi} \quad (2.23)$$

^{*}) This leads to quark propagators of a similar structure to that recently considered by Polkinghorne ⁵⁾.

One remarkable consequence of the MQM universal quark mass behaviour is the linearity of Regge trajectories and the absence of odd daughters implied by (2.22) ; furthermore if we want to obtain a Regge slope $\alpha' \simeq 1 \text{ GeV}^{-2}$, we have

$$R_0^2 \simeq 2\pi^2 \text{ GeV}^{-2} \quad (2.24),$$

which, for the lowest lying state ($\ell=0, n=1$) gives a spatial extension

$$\bar{R} \simeq 1F \quad (2.25),$$

a result which is quite satisfactory.

A hadronic "radius" $R(M)$, which by virtue of (2.20) increases linearly with the mass of the hadron, can, however, lead us in serious trouble if we do not provide some way of coupling only those angular momenta for which

$$\ell \leq \ell_{\max} = \frac{M}{2} R_1 \quad (2.26)$$

The absence of such a limitation on the angular momentum states which couple to the $q\bar{q}$ system would lead us to a linear shrinkage of the diffractive peaks, and in fact will also destroy the desired scaling behaviour obtained in the MQM. The condition (2.26) is very crucial in order to make contact with reality and seems to be realized in the MIT bag model ²⁾. There in fact using the virial theorem one gets for large mass

$$\frac{M}{2} \sim BV \sim \frac{\pi}{2} B R_1^2 R_1 \quad (2.27)$$

where the classical volume spanned by the quarks has been taken as a cylinder with radius $R_1/2$ and height $2R_1$. Thus we see that the only way R can increase linearly with M is to have a constant "transverse dimension".

From all this we are led to the spatial shape of the highly excited $q\bar{q}$ orbits appearing in Fig. 4 ^{*)}.

Figure 4 The highly excited $q\bar{q}$ orbit.

D. - The $q\bar{q}$ w.f. in any frame

In momentum space the form (2.17) only holds in the rest frame of the meson, i.e., for $p \equiv (M, \vec{0})$. In order to have a w.f. in any moving frame, we need only consider a boost under a particular direction which we choose as the z axis. In this way we obtain $\psi(p, k)$ by substituting in (2.17) for k_0 (2.18); for $|\vec{k}|$ (2.19); for θ_k

$$\bar{\theta}_k = \arccos \left[\frac{Ek_3 - pk_0}{M|\vec{k}|} \right]$$

where $\vec{k}$ is given by (2.19), while φ_k remains the same.

In performing the calculations a convenient form for wave functions of hadrons of mass $\gtrsim 2$ GeV is

$$\begin{aligned} \psi_{nl}(p, k) \simeq & \frac{2\sqrt{2}}{\mu R^{1/2}} (2\pi)^2 \left(\frac{\pi}{R^2} \right) \delta_{R^2} \left[\frac{m_1^2 + m_2^2}{2} - m^2 \right] \\ & \times \delta_{R^2} \left[\frac{m_1^2 - m_2^2}{2} \right] Y_l^h(\bar{\theta}_k, \varphi_k) \end{aligned} \quad (2.28)$$

^{*)} Such shapes correspond to states constructed out of a well-defined coherent superposition of the $(\bar{l}/2)^2 \simeq (R_L/4)^2 M^2$ degenerate states of mass M .

where $M^2 \simeq (\pi/R^2)(2n+l)$ and $\delta_{R^2}(z)$ is a δ like function whose prototype is

$$\delta_{R^2}(z) = \frac{1}{\pi} \frac{\sin R^2 z}{z}.$$

$\delta_{R^2}(z)$ has the following properties :

$$\int_{-\infty}^{+\infty} dz \delta_{R^2}(z) = 1 \quad (2.29)$$

and

$$\delta_{R^2}(0) = \frac{R^2}{\pi} \quad (2.30).$$

In the limit of $R^2 \rightarrow \infty$ $\delta_{R^2}(z)$ obviously tends to a Dirac δ function. For low mass hadrons (2.28) becomes a much too imprecise representation and we should rather use (2.9) and (2.15).

3. - A GEOMETRICAL DESCRIPTION OF HADRONIC INTERACTIONS. THE GRAPHICAL RULES

In the previous section we have described a set of simple hadronic states in terms of their quark-like constituents. We must now construct a scheme of coupling among such hadronic states.

A possible scheme has been previously discussed in general in the context of the MQM ^{*)}, and has been devised by totally relying on the field-theoretical visualization of the quark degree of freedom.

If we forget for the time being about baryons, the coupling scheme amounts to a diagrammatic expansion of the S matrix which parallels completely that of a $\lambda\varphi^3$ theory provided we make the following identifications :
(Fig. 5)

^{*)} See Ref. 1), especially Sections I and III.

Figure 5 The dictionary leading from $\lambda\varphi^3$ to the MQM expansion.

- i) φ propagator $\rightleftharpoons$ $q\bar{q}$ Green's function G ;
- ii) φ^3 vertex $\rightleftharpoons$ irreducible six-point kernel k_6 .

To any given order in λ , which should then be a relevant expansion parameter, we can draw the diagrams of $\lambda\varphi^3$ and then proceed to substitute the φ propagators with the $q\bar{q}$ Green's function and the φ^3 vertex with the irreducible kernel. Crossing and unitarity will then be implemented order by order in λ , provided both G and k_6 are crossing symmetric.

The whole coupling scheme boils down to determining the structure of G and k_6 and the rules governing their juxtaposition.

A. - The $q\bar{q}$ Green's function

The Green's function G embodies the space-time propagation property of a $q\bar{q}$ (qq) system. The simple bag states constructed in Section 2 are the obvious candidates to describe such propagation properties. We shall therefore write the crossing symmetric Green's function as

$$G = G_s + G_t \quad (3.1)$$

where G_s is built up of bag states in the s channel and G_t out of bag states in the t channel (see Fig. 6).

Figure 6 Construction of the Green's function G .

We defer to Section 5 a discussion of the main properties of G .

B. - The irreducible kernel k_6

In determining the form of k_6 we shall be guided by a geometrical view of hadronic interactions. Let us look at the coupling of three bag states (Fig. 7)

Figure 7 The coupling of three bag states.

k_6 gives the probability amplitude for the quarks of one bag, say p_1 , disappearing and reappearing as the constituents of a two bag system, p_2, p_3 . According to our idea that quark exists only in small space-time domains which we associate with a zero-triality hadronic state, this amplitude should be zero whenever the space-time regions spanned by the three bags do not overlap. This can only be achieved if k_6 is given by :

$$k_6 [x_1, \dots, y_3] = \mu^6 \delta^4(x_1 - x_2) \delta^4(y_2 - y_3) \delta^4(x_3 - y_1) + \text{permutations} \quad (3.2)$$

where μ^2 is a mass parameter, which has been introduced here for dimensional reasons, playing the rôle of a coupling constant (in the $\lambda\varphi^3$ analogy we have $\lambda \rightleftharpoons \mu^6$). According to (3.2) the three-bag coupling in momentum space amounts to the calculation of the diagrams depicted in Fig. 8.

Figure 8 The diagrams corresponding to the three-bag coupling.

The meaning of these diagrams is specified by a set of graphical rules which we are going to discuss.

C. - Graphical rules

A general quark diagram can be reduced to a network of quark lines joining quark Green's function and w.f.'s in such a way that each loop consists of no more and no less than three quark lines. The rules to evaluate any such diagram are the following :

- i) each incoming "bag" state of momentum p_i and relative momentum k_i is introduced through the wave functions $\psi(p_i, k_i)$, whose explicit expression is given by (2.17) ;
- ii) each outgoing bag state of momentum p_f , and relative momentum k_f is given by $\psi^*(p_f, k_f)$;
- iii) for each quark line we must multiply by μ^2 ;
- iv) four-momentum at each two-quark hadron vertex must be conserved ;
- v) for each loop we have an integral $\int d^4\ell / (2\pi)^4$, where ℓ is the loop momentum and the integration is over the real axis ;

- vi) $q\bar{q}$ Green's functions are totally connected and have discontinuities corresponding to physical hadron states ;
- vii) any weak and electromagnetic current is represented by a pointlike coupling to a $q\bar{q}$ Green's function (see Fig. 9) ; the two-quark loop is calculated without multiplying by the factor μ^2 for each quark line.

Figure 9 The current $q\bar{q}$ coupling.

Equipped with these rules we can construct amplitudes having any degree of complexity. As remarked, the viability of this approach rests on the possibility of treating μ^2 as a perturbative parameter. We shall check later that this is indeed the case.

D. - The electromagnetic vertex

As a particularly important example of the graphical expansion we consider the electromagnetic vertex. We have seen that its structure determines the normalization of hadronic wave functions, and consequently that of the $q\bar{q}$ Green's function. To lowest order in (μ^2) we have the diagram in Fig. 10.

Figure 10 The lowest order electromagnetic vertex of the particle p .

A very interesting feature of this diagram is that it naturally contains vector meson dominance through the vector meson poles appearing in the Green's function G . Keeping only the ρ intermediate state in the Green's function ^{*}) and taking into account the normalization (2.15), we easily obtain

$$Z \simeq \sqrt{2} \left(\frac{\mu}{m_\rho} \right) \quad (3.3)$$

which represents a useful connection between the parameters Z and μ defined in (2.13) and (3.2) ^{**}).

Before ending this section we would like to make a comment on the meaning of the constant μ^2 in the realm of the geometrical description we have pursued here. A look at Eq. (3.2) shows that we can interpret μ^2 as the probability amplitude for a quark to disappear from a given bag state and reappear in another bag state but at the same space-time point.

^{*}) In the next section [see Eqs. (4.3) and (4.6)] it will be shown that the coupling to higher states drops fast enough.

^{**}) It is perhaps amusing to note that (2.15) implies that the bag w.f. diverges when the "coupling" constant μ goes to zero, a fact which has been very recently stressed in a quite different context by Christ and Lee ⁶⁾.

4. - THE DECAY OF AN EXCITED BAG

In this section we shall consider the very important question of determining how a highly excited (large M) bag de-excites itself.

A. - The three-bag vertex

Following the graphical rules given in the preceding section the diagrams in Fig. 8 are easily calculated as :

$$\begin{aligned} \mathcal{M}(1 \rightarrow 2+3) &= (\mu^2)^3 \int \frac{d^4 k}{(2\pi)^4} \psi_1(p_1, k) \psi_2^*(p_2; \frac{p_1-p_2}{2} + k) \psi_3^*(p_3; -\frac{p_2}{2} + k) \\ &+ (2 \leftrightarrow 3) \end{aligned} \quad (4.1)$$

On substituting (2.28) for the w.f.'s we have :

$$\begin{aligned} \mathcal{M}(1 \rightarrow 2+3) &\simeq \left(\frac{2\sqrt{2}\mu}{2^{1/2}} \right)^3 (2\pi)^2 \int d^4 k \delta_{R^2}(m_1, k_0) \delta_{R^2}(\vec{k}^2 - \frac{m_1^2}{4}) \cdot \\ &\cdot \delta_{R^2} \left[\frac{m_2^2 + m_3^2 - m_1^2}{4} - \frac{|\vec{p}_1| m_1}{2} \cos \theta_{kp} \right] Y_{l_1}^{m_1} Y_{l_2}^{m_2} Y_{l_3}^{m_3} \\ &+ (2 \leftrightarrow 3) \end{aligned} \quad (4.2)$$

where we have repeatedly made use of the properties (2.29) and (2.30) of the δ_{R^2} function. A little analysis of the kinematics shows that the peaking of the quark masses at small value requires that in the rest frame of p_1 the decay momentum $\vec{p}$ is in the direction of the initial quark relative momentum $\vec{k}$. Thus, calling $h_{2,3}$ the helicities of $p_{2,3}$, we get :

$$\begin{aligned} \mathcal{M}(1 \rightarrow 2+3) &\simeq \frac{2\sqrt{2}}{2^{3/2}} \mu^3 (2\pi)^2 (2\ell_2+1)^{1/2} (2\ell_3+1)^{1/2} \delta_{h_2 0} \delta_{h_3 0} \cdot \\ &\cdot Y_{l_1}^{m_1}(\theta, \varphi) \int_{-1}^{+1} dz \delta_{R^2}(a-bz) \end{aligned} \quad (4.3)$$

where θ, φ are the polar co-ordinates of the decay momentum $\vec{p}$, and

$$\begin{aligned} a &= \frac{1}{4} (m_1^2 - m_2^2 - m_3^2) \\ b &= \frac{1}{4} [m_1^4 + (m_2^2 - m_3^2)^2 - 2m_1^2 (m_2^2 + m_3^2)]^{1/2}. \end{aligned} \quad (4.4)$$

The z integral in (4.3) is crucial in determining the decay property of the excited bag p_1 . Approximating the δ_{R^2} by an appropriate exponential we get the explicit expression

$$f(m_1^2; m_2^2, m_3^2) = \int_{-1}^{+1} dz \delta_{R^2}(a-bz) \simeq \frac{1}{b} e^{-2R^2 a} \sinh 2R^2 b \quad (4.5).$$

The function $f(m_1^2; m_2^2, m_3^2)$ is strongly peaked for $a=b$, i.e., for either m_2^2 or $m_3^2 \simeq 0$. In this case we have

$$f(m_1^2; 0, m_3^2) \simeq \frac{4}{m_1^2 - m_3^2} [1 - e^{-R^2(m_1^2 - m_3^2)}] \quad (4.6).$$

Equation (4.6) expresses the quite intuitive fact that the decay amplitude is largest for the emission of a small mass bag (the π !?) and a very excited bag whose mass m_3 is very close to that of the decaying bag. In fact if we picture the decaying bag as Fig. 4, it is clear that the space-time overlap is going to be maximal for the transition to a bag state close to the original one and a small mass bag.

In order to get a feeling about the magnitude of μ^2 we compute the width for the decay of the state with $\ell=1$ (which we may call ρ , with mass m_ρ) into two $\ell=0$ states (π , with mass m_π). We get the simple result

$$\Gamma_{\rho \rightarrow 2\pi} \simeq 32\pi^2 m_\rho \left(\frac{\mu^2}{m_\rho^2 Z} \right)^3 \quad (4.7)$$

which setting $\Gamma_{\rho \rightarrow 2\pi} \simeq 150 \text{ MeV}$ gives $(\mu/Z^{1/2}) \simeq 220 \text{ MeV}$, a quite small value compared with a typical hadronic mass ^{*}.

^{*}) Inclusion of spin and internal quantum numbers leads us to the following approximate values: $\mu \simeq 300 \text{ MeV}$ and $Z \simeq 1$.

B. - The linear decay chain

The preceding results are very important to reach an understanding of the structure of final states in hadronic collisions. Due to the very substantial enhancement of the decay configuration $M \rightarrow M' + \pi$ ^{*)} with M' close in mass with M , to a good approximation the final states evolve through a linear decay chain as depicted in Fig. 11.

Figure 11 The linear chain decay of an excited $q\bar{q}$ system.

Such a picture implies that a $q\bar{q}$ system, moving on a highly excited orbit, gets de-excited through the emission of pions while cascading down through a series of states of decreasing mass. This resembles very much the cascade decay of an excited atom through photon emission. The final states produced by this cascade will then be composed of a large number of pions distributed in rapidity and transverse momentum in a characteristic way which shall be discussed below ^{**)} .

If we want to compare this picture with the usual quark-parton description, we notice at once the quite different meaning the notion of "q $\bar{q}$ sea" acquires here. Such sea is represented by a large number of pions and arises as a product of a violent collision which brings the initial hadronic system in a highly excited state.

Taking up the atomic physics analogy again, it does not appear useful anymore to speak about the "q $\bar{q}$ sea" as a property of the initial hadrons, than to try to include the "photon sea" in the wave function of the atomic ground state.

^{*)} Actually other low mass bags like K , ρ , etc., do not give totally negligible contributions, and should be also considered.

^{*)} Here we may be in close contact with the thermodynamical approach of R. Hagedorn and co-workers.

The linear decay chain problem can be solved by means of the integral equations appearing in Fig. 12.

Figure 12 The integral equations governing the decay of highly excited states.

The details of the solution of this problem shall be reported elsewhere ⁷⁾. Here we quote only the results :

- i) if we call $\Gamma(M)$ the width of an excited state of mass M , asymptotically we have ($Z=1$)

$$M\Gamma(M) \xrightarrow{M \text{ large}} 2 (4\mu^2 R^2)^3 \frac{2\pi}{R^2} \quad (4.8)$$

recalling that $\mu^2 R^2 \simeq 1/\pi$ and that the level spacing is asymptotically $\Delta M^2 = (2\pi/R^2)$ we see that (4.8) implies that the poles present in G when $\mu \rightarrow 0$ are completely washed out, at high energy, thus giving to G a very smooth behaviour ; in view of our retaining only the π state (4.8) should be considered a lower limit for $M\Gamma(M)$;

- ii) the one-particle yield exhibits Feynman scaling ;
- iii) for small x the inclusive distribution shows a plateau, thus giving rise to a multiplicity increasing logarithmically with s :

$$\langle n \rangle \sim c \log s \quad (4.9).$$

- iv) there is no correlation between impact parameter and rapidity ^{*)},
and there is a cut-off in transverse momentum

$$\langle k_{\perp}^2 \rangle \simeq \frac{\pi}{R^2} \simeq .1 \text{ GeV}^2.$$

- v) the π 's are emitted independently, thus following a Poisson distribution.

It is clear that these results look quite close to what is going on in hadronic final states. It is also very gratifying that we have derived those crucial properties of the MQM which were a direct consequence of the ansatz of general Regge behaviour of quark Green's functions ¹⁾.

5. - THE $q\bar{q}$ GREEN'S FUNCTION

We shall now derive the main properties of the $q\bar{q}$ Green's function G which plays a central rôle in all our discussion. As discussed in Section 3, we write G as a sum of two pieces, representing the contribution of bag states in the s and in the t channel, respectively [see Eq. (3.1)].

A. - The G_s contribution

Let us study the G_s contribution first. According to the first term in Fig. 6, we have :

$$G_s = \sum_{ne} \frac{1}{s - M_{ne}^2 + i M \Gamma_{ne}} \psi_{ne}(p, k) \psi_{ne}^*(p, k') \quad (5.1)$$

^{*)} This seems to be borne out by recent analyses.

where $\psi_{n\ell}(p,k)$ is the normalized wave function corresponding to the quantum numbers n, ℓ ; $p^2 = S$ is the centre-of-mass energy squared. We will evaluate (5.1) in the high energy limit. Using (2.28), the spectrum equation

$$M_{n\ell}^2 \simeq \frac{2}{R^2} \beta_{n\ell} \simeq \frac{2\pi}{R^2} (n + \frac{\ell}{2}) \quad (5.2)$$

and the result (4.8) $(M\Gamma)_{n\ell} \sim \lambda (2\pi/R^2)$, we have

$$G_S \simeq \delta_{R^2} \left(\frac{m_1^2 + m_2^2}{2} - m^2 \right) \delta_{R^2} \left(\frac{m_1^2 - m_2^2}{2} \right) \cdot \delta_{R^2} \left(\frac{m_1'^2 + m_2'^2}{2} - m^2 \right) \delta_{R^2} \left(\frac{m_1'^2 - m_2'^2}{2} \right) G_S(s, t) \quad (5.3)$$

where

$$G_S(s, t) = (2\pi)^3 \frac{4}{\mu^2 Z} \left(\frac{\pi}{R^2} \right)^2 \sum_{n\ell} \frac{(2\ell+1) P_\ell(\cos\theta_s)}{s - \frac{2\pi}{R^2} (n + \frac{\ell}{2}) + i\lambda \frac{2\pi}{R^2}} \quad (5.4)$$

In (5.3) $m_{1,2}^2$ and $m_{1,2}'^2$ are the incoming and outgoing quark masses, respectively, in (5.4) $\cos\theta_s = (1+2t/s)$.

The double sum appearing (5.4) can be performed by fixing the integer $N = 2n + \ell$ first and then summing over ℓ up to $\ell_{\max}$ given by (2.26). Asymptotically in s we have

$$\begin{aligned} \sum_{\text{even } \ell}^{R_1 \sqrt{s}/2} (2\ell+1) P_\ell \left(1 + \frac{2t}{s} \right) &\simeq \sum_{\text{odd } \ell}^{R_1 \sqrt{s}/2} (2\ell+1) P_\ell \left(1 + \frac{2t}{s} \right) \simeq \\ &\simeq \frac{8R_1^2}{4} \frac{J_1(R_1 \sqrt{t})}{R_1 \sqrt{t}} \end{aligned} \quad (5.5)$$

and inserting into (5.4)

$$G_S(s, t) = \frac{4}{\mu^2 Z} (2\pi)^3 \left(\frac{\pi}{R^2} \right)^2 R_1^2 s \frac{J_1(R_1 \sqrt{t})}{R_1 \sqrt{t}} \sum_N \frac{1}{s - \frac{\pi}{R^2} (N + 2i\lambda)} \quad (5.6)$$

In order to evaluate (5.6) we must perform a subtraction in view of the divergence of the series. Thus we get

$$G_s(s,t) \cong \frac{(2\pi)^3}{\mu^2 Z} \frac{\pi}{R^2} R_1^2 \frac{J_1(R_1 \sqrt{-t})}{R_1 \sqrt{-t}} s \left\{ \psi \left(1 - \frac{R^2 s}{\pi} + 2i\lambda \right) + c \right\}$$

$$\xrightarrow{\text{larges}} \frac{(2\pi)^3}{\mu^2 Z} \frac{\pi R_1^2}{R^2} \frac{J_1(R_1 \sqrt{-t})}{R_1 \sqrt{-t}} s \left[\log \left(-\frac{R^2 s}{\pi} + 2i\lambda \right) + c \right] \quad (5.7)$$

where ψ denotes the digamma function, and c is a subtraction constant.

Thus we see that the very simple ideas which have led us to determine the structure of hadronic wave functions do provide for the existence of the "primeval Pomeron" ^{*}) which, in the MQM framework, plays a fundamental rôle in ensuring the validity of scaling and current algebra. The structure of $G_s(s,t)$ in the J plane is thus a fixed pole at $J=1$. The t dependence of (5.7) is typical of a grey-disc of diameter R_1 .

B. - The G_t contribution and the "Regge states"

As we have previously discussed, crossing symmetry requires that G also receives a contribution from "bag" states in the t channel. Proceeding exactly as in the previous paragraph, we have :

$$G_t \simeq \delta_{R^2} \left[\frac{m_1^2 + m_1'^2}{2} - m^2 \right] \delta_{R^2} \left(\frac{m_1^2 - m_1'^2}{2} \right)$$

$$\delta_{R^2} \left[\frac{m_2^2 + m_2'^2}{2} - m^2 \right] \delta_{R^2} \left(\frac{m_2^2 - m_2'^2}{2} \right) G_t(s,t) \quad (5.8)$$

where

$$G_t(s,t) = \frac{4}{\mu^2 Z} (2\pi)^3 \left(\frac{\pi}{R^2} \right)^2 \sum_{n\ell} \frac{(2\ell+1) P_\ell(\cos\theta_t)}{t - m_{n\ell}^2 + i(m\Gamma)_{n\ell}} \quad (5.9)$$

and $\cos\theta_t = (1 + 2s/(t - 4m^2))$. To determine the behaviour of (5.9) for s large, we use the Sommerfeld-Watson transformation :

^{*}) See Ref. 1), especially Section II. An application of the "primeval Pomeron" notion to high energy hadron scattering has been made in Ref. 9).

$$\sum_{n\ell} (2\ell+1) P_{\ell}(\cos\theta_t) \frac{1}{t-m_n^2} = \frac{1}{2i} \sum_n \int_C \frac{d\ell (2\ell+1)}{\sin\pi\ell} P_{\ell}(-\cos\theta_t) \frac{1}{t-m_n^2(\ell)}$$

where C is the usual contour encircling the positive real ℓ axis.
Defining the trajectory function $\alpha_n(t)$ as the solution of

$$t = m_n^2(\alpha_n(t))$$

we have for large s and small t

$$G_t(s,t) \rightarrow \frac{4}{\mu^2 Z} (2\pi)^3 \left(\frac{\pi}{R^2}\right)^2 \sum_n \frac{\pi \alpha_n'(t) (2\alpha_n(t)+1)}{\sin\pi\alpha_n(t)} \frac{\Gamma(1+2\alpha_n(t))}{\Gamma(1+\alpha_n(t))^2} \left(-\frac{s}{4m^2}\right)^{\alpha_n(t)} + \text{background} \quad (5.10)$$

Thus $G_t(s,t)$ exhibits at high energy Regge behaviour, which is controlled by a set of Regge trajectories $\alpha_n(t)$ precisely corresponding to the "bag states". These trajectories are exactly exchange degenerate, a property which is crucial in order to ensure the absence of physical states, discontinuities for $s < 0$, which would correspond to an unobserved non-zero triality state. This constitutes a very important check of self-consistency of our description. Thus (5.10) has only a right-hand cut corresponding to zero-triality states; the question now is how to interpret physically such "Regge states". As we have seen these states arise in our picture by virtue of crossing and should be regarded as genuine physical states of a character very different from simple bag states: their spectrum is continuous and the typical logarithmic Regge shrinkage shows that their "transverse dimension" increases logarithmically with energy. This suggests that the hadronic discontinuity of $G_t(s,t)$ corresponds to hadrons in a configuration of a "multiperipheral" kind ⁸⁾.

To conclude, we have found that crossing forces us to extend the set of $q\bar{q}$ states to include besides the simple bag states also the "Regge states". These states exhibit a continuous spectrum and give rise to final state configurations of multiperipheral type quite different from those arising from the decay of excited bags. Another very important distinction between "bag" and "Regge" states is that the former build up the diffractive scattering while the latter generate normal Regge pole behaviour.

6. - CONCLUSION AND OUTLOOK

In this paper we have presented a possible way to go about making the general ideas of the MQM more concrete. We have tried to use a quite rudimentary form of the bag idea to describe hadrons as confined space-time regions where "lumps" of energy momentum, charge, etc. (quarks) can be found. This has led us to envisage a world in which quarks can propagate only in certain "allowed orbits" (hadrons), and the probability for the transition between "allowed orbits" is essentially determined by their geometry. Going further we have seen the emergence of well-defined hadronic final states, which resemble closely what has been observed experimentally both in diffractive and non-diffractive scattering. The notions of diffraction as a shadow process and of normal Regge behaviour have also naturally emerged, showing remarkably simple aspects of high energy scattering.

Applications of these ideas to high energy and deep inelastic phenomena will be presented in forthcoming publications.

We would like to conclude by listing a number of problems whose discussion has not been presented here, which will form the object of future study :

- i) inclusion of spin and internal symmetry $SU(3)$ $[SU(4) ?]$ in the $q\bar{q}$ system ;
- ii) discussion of meson form factors, and low energy meson parameters ;
- iii) baryons as systems of three quarks ; their spectrum, form factors, etc.

We can anticipate that a preliminary study yields results which compare very satisfactorily with the available experimental information.

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