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NON-ABELIAN GAUGE THEORY IN TWO DIMENSIONS

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ABSTRACT

Non-Abelian gauge theory of quarks and gluons with $SU(n)$ symmetry, in one-space and one-time dimensions, is investigated in the large n limit. Special attention is given to the infra-red regularization. It is shown that in the $n \rightarrow \infty$ approximation, tachyons appear in the two-point function of the scalar density for mass $m = 0$ of the quarks. It is argued that this remains also for small $m \neq 0$, which entails that $1/n$ corrections must be important. It is also shown that the singlet vector current has a free two-point function for $m = 0$, which entails a consistency condition on bound state wave functions.

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1. INTRODUCTION

In this note we investigate a two-dimensional non-Abelian gauge theory with $SU(n)$ symmetry and with fermions in the quark representation. Our main result is that the local scalar operator $\sigma = :\bar{\psi}\psi:$ (ψ is the quark field) has tachyons in the two-point function, in the limit of large n and mass $m=0$ for the fermion. We also show that the singlet current $J_\mu = :\bar{\psi}\gamma_\mu\psi:$ has a free two-point function for $m=0$ and any n .

In the large n approximation we follow 't Hooft ¹⁾. [For a treatment of the Abelian $n=1$ case, see Ref. 2).] We first show that in the limit of $m=0$, the solution of Ref. 1) yields tachyons in the scalar channel. We then employ different infra-red regularizations, which we motivate, and for which we still get tachyons for the σ two-point function. We argue that the tachyons remain also for $m \neq 0$ and sufficiently small, in the $n \rightarrow \infty$ approximation. Since σ is a gauge invariant operator, we must eliminate the tachyons from their two-point function. Thus $1/n$ corrections are important.

The fact that J_μ has a free two-point function entails a consistency condition on bound states, which is satisfied by the wave functions of Ref. 1).

The programme of the paper is as follows. In Section 2 we discuss the infra-red structure of the gluon propagator and show that a natural choice leads to no cut-off dependence in the fermion propagator ³⁾, unlike the result of Ref. 1). In Section 3 we show that this solution still leads to tachyons in the propagator of the scalar density. In Section 4 we motivate another infra-red cut-off. We show, in Section 5, that this still leads to tachyons in the two-point function of the scalar density for $m=0$. We argue that this does not change when $m \neq 0$, but sufficiently small. The same result remains for yet another infra-red cut-off. In Section 6 we show that the two-point function for the singlet current is free. The latter result holds for any n . This leads to a condition on the wave functions of Ref. 1), which is satisfied.

Two appendices contain calculations necessary for our discussion.

2. PROPAGATORS AND INTEGRAL EQUATIONS

We start with the Lagrangian

$$\mathcal{L}(x) = -\frac{1}{4} F_{\mu\nu}^a(x) F^{\mu\nu a}(x) + \bar{\psi}(x) \left(\frac{i}{2} \overleftrightarrow{\not{D}} - m \right) \psi(x) - g \bar{\psi}(x) \gamma_\mu^a \psi(x) A^{\mu a}(x) \quad (2.1)$$

where ρ_μ^a is the Yang-Mills field, ψ is the spinor field, and

$$\begin{aligned} F_{\mu\nu}^a &= \partial_\mu \rho_\nu^a - \partial_\nu \rho_\mu^a - g f^{abc} \rho_\mu^b \rho_\nu^c \\ J_\mu^a &= \bar{\psi} \gamma_\mu \frac{1}{2} \lambda^a \psi \end{aligned} \quad (2.2)$$

λ^a are the usual traceless $n \times n$ matrices of $SU(n)$ with

$$\left[\frac{1}{2} \lambda^a, \frac{1}{2} \lambda^b \right] = i f^{abc} \left(\frac{1}{2} \lambda^c \right) \quad (2.3)$$

We work in one-space one-time dimensions and use the metric $g_{00} = -g_{11} = 1$ with

$$\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2g_{\mu\nu} \quad (2.4)$$

Defining

$$\tilde{A} = A^a \left(\frac{1}{2} \lambda^a \right) \quad (2.5)$$

we have

$$\tilde{F}_{\mu\nu} = \partial_\mu \tilde{\rho}_\nu - \partial_\nu \tilde{\rho}_\mu + ig [\tilde{\rho}_\mu, \tilde{\rho}_\nu] \quad (2.6)$$

The Lagrangian (2.1) is invariant under the local gauge transformation

$$\begin{aligned} \tilde{\rho}_\mu &\rightarrow \Lambda^{-1} \tilde{\rho}_\mu \Lambda - \frac{i}{g} \Lambda^{-1} \partial_\mu \Lambda \\ \psi &\rightarrow \Lambda^{-1} \psi \end{aligned} \quad (2.7)$$

with

$$\Lambda = e^{i\beta} \quad (2.8)$$

Note that $\text{Tr}(\Lambda^{-1} \partial_\mu \Lambda) = 0$, which is needed for consistency of the transformation in Eq. (2.7). Also

$$F_{\mu\nu} \rightarrow \Lambda^{-1} F_{\mu\nu} \Lambda \quad (2.9)$$

Following 't Hooft ¹⁾, we choose to quantize in the light-cone gauge

$$\rho_- = 0 \quad (2.10)$$

where

$$A_{\pm} = \frac{1}{\sqrt{2}} (A_0 \pm A_1) \quad (2.11)$$

In this gauge, the quantization does not require any ghost loops, unlike the case of a covariant gauge ⁴⁾. The equations of motion imply

$$\partial_-^2 \rho_+^a = -g J_-^a \quad (2.12)$$

which means one can eliminate the ρ field and deal with a non-local four Fermi interaction. Formally,

$$\rho_+^a(x) = -\frac{1}{2} g \int dx_+ - dy_+ / [J_-^a(y)]_{y_- = x_-} dy_+ \quad (2.13)$$

and

$$\mathcal{L}_I(x) d\dot{x} = -\frac{1}{2} g \int J_-^a(x) \rho_+^a(x) d\dot{x} = \frac{1}{4} g^2 \int \int dx_+ - dy_+ / \delta(x_- - y_-) J_-^a(x) J_-^a(y) d\dot{x} d\dot{y}$$

This, in turn, implies that the ρ propagator $D_{++}^{ab} \equiv D \delta^{ab}$ is (2.14)

$$D(x) = -\frac{i}{2} / x^- / \delta(x^+) = -\frac{i}{2} / x_+ / \delta(x_-) \quad (2.15)$$

and

$$\begin{aligned} D(k) &= \int d^2x e^{-ikx} D(x) = \\ &= -\frac{i}{2} \int dx^- / x^- / e^{-ikx^-} = \\ &= \frac{i}{2} \left[\frac{1}{(k_- + i\epsilon)^2} + \frac{1}{(k_- - i\epsilon)^2} \right] \end{aligned} \quad (2.16)$$

Expression (2.16) has a built-in infra-red cut-off ³⁾, unlike the case of a $1/k_-^2$ propagator ¹⁾ which requires an infra-red cut-off from the outside. We shall see later that Eq. (2.16) leads to a cut-off independent fermion propagator.

The integral equation for the fermion propagator $S(p)$ in the limit $n \rightarrow \infty$ with $\frac{1}{2}g^2 n = G^2$ fixed is ¹⁾

$$S^{-1}(p) = S_0^{-1}(p) + \frac{G^2}{4\pi^2} \int d^2k \, \sigma_- S(k) \sigma_- D(p-k) \quad (2.17)$$

From

$$i S_0^{-1}(p) = \sigma_+ p_- + \sigma_- p_+ - m \quad (2.18)$$

it immediately follows that

$$i S^{-1}(p) = \sigma_+ p_- - m + \sigma_- p_+ s(p) \quad (2.19)$$

with $s(p)$ to be determined. Also

$$\frac{1}{i} S(p) = \frac{1}{p_-^2 s - m^2 + i\epsilon} [\sigma_- p_+ s + \sigma_+ p_- + m] \quad (2.20)$$

Thus

$$p_+ s(p) = p_+ - \frac{iG^2}{4\pi^2} \int d^2k \frac{k_-}{k_-^2 s(k) - m^2 + i\epsilon} \left[\frac{1}{(k_- - p_- + i\epsilon)^2} + \frac{1}{(k_- - p_- - i\epsilon)^2} \right] \quad (2.21)$$

Following Ref. 1), we note that $p_+[s(p) - 1] = f(p)$ depends on p_- only, and that

$$\begin{aligned}
 \not{f}(p_-) &= -\frac{iG^2}{4\pi^2} \int d^2k \frac{1}{2\not{f}(k_-) + 2k_+ - \frac{m^2}{k_-} + i\epsilon k_-} \\
 &\quad \left[\frac{1}{(k_- - p_- + i\epsilon)^2} + \frac{1}{(k_- - p_- - i\epsilon)^2} \right] = \\
 &= -\frac{G^2}{8\pi} \int d^2k \epsilon(k_-) \left[\frac{1}{(k_- - p_- + i\epsilon)^2} + \frac{1}{(k_- - p_- - i\epsilon)^2} \right] = \\
 &= \frac{G^2}{4\pi} \left[\frac{1}{p_- - i\epsilon} + \frac{1}{p_- + i\epsilon} \right]
 \end{aligned} \tag{2.22}$$

Hence

$$\begin{aligned}
 S(p) &= i \left\{ \not{x}_+ p_- + \not{x}_- p_+ + \frac{G^2}{4\pi} \not{x}_- \left[\frac{1}{p_- - i\epsilon} + \frac{1}{p_- + i\epsilon} \right] + m \right\} \\
 &\quad \cdot \frac{1}{p^2 - m^2 + \frac{G^2}{\pi} + i\epsilon}
 \end{aligned} \tag{2.23}$$

This is the same as in Ref. 1), except for the absence of the cut-off dependent term $-G^2 \epsilon(p_-)/\pi\lambda$ in f (λ the small cut-off).

3. PROPAGATOR FOR SCALAR DENSITY

We would like to consider here the two-point function of the operator

$$\sigma = : \bar{\psi} \psi : \tag{3.1}$$

which is a gauge invariant quantity and may serve as a field for generating physical scalar states from the vacuum. We furthermore consider the case $m=0$. We claim that the set of all Feynman diagrams for

$$\Sigma(x) = \langle 0 | T \sigma(x) \sigma(0) | 0 \rangle \tag{3.2}$$

is given by the ones in Fig. 1. A contribution as in Fig. 2 is zero for the following reason. At y_1 and y_2 , the matrices γ_- appear. This means that from y_1 to x and from y_2 to x , only the γ_+ propagators can appear. But then we get zero, since at x we have the unit matrix.

We use

$$\begin{aligned}\gamma_+^2 &= \gamma_-^2 = 0 \\ \gamma_+ \gamma_- &= 1 - \gamma_5 \\ \gamma_- \gamma_+ &= 1 + \gamma_5\end{aligned}\quad (3.3)$$

Thus, for $m=0$

$$\Sigma(p; m=0) \propto \int d^2k \, \text{Tr} [S(k; m=0) S(k-p; m=0)] \quad (3.4)$$

Note that since the self-energy of the fermion is proportional to γ_- , it follows that for $m=0$ there are no diagrams contributing to the γ_- part of S . Thus, the γ_- part of S has to be like the free one. However, from Eq. (2.20) we note that the γ_- part for $m=0$ is

$$\frac{p_+ s}{p^2 s + i\epsilon} = \frac{p_+}{p^2 + i\epsilon s}$$

which is like the free case only when $s > 0$ near $p^2 = 0$. However, the solution for s , as obtained from Eq. (2.22), is $s = 1 + G^2/\pi p^2$. To avoid this problem we write, instead of Eq. (2.20) for s , the expression

$$\frac{1}{i} S(p) = \frac{\gamma_+ p_- + m}{p^2 s - m^2 + i\epsilon} + \frac{\gamma_- p_+ s}{p^2 s - m^2 + i\epsilon s} \quad (3.5)$$

This will not change the solution for s , since only the γ_+ part appears in the integral Eq. (2.21). Now

$$\frac{1}{i} S(p) = \frac{\gamma_+ p_- + m}{p^2 - m^2 + \frac{G^2}{\pi} + i\epsilon} + \frac{\gamma_- p_+ (p^2 + \frac{G^2}{\pi})}{p^2 (p^2 - m^2 + \frac{G^2}{\pi}) + i\epsilon (p^2 + \frac{G^2}{\pi})} \quad (3.6)$$

Note also that when $p^2 s - m^2 = p^2 - m^2 + G^2/\pi = 0$, we have $p^2 = m^2 - G^2/\pi$ and $s = m^2/(m^2 - G^2/\pi)$. Thus for $m^2 > G^2/\pi$ the two denominators in Eq. (3.5) are equal. However, for $m^2 < G^2/\pi$, we have a $-i\epsilon$ for the second term instead of the usual $i\epsilon$. Now, for $m=0$,

$$\frac{1}{i} S(p; m=0) = \frac{\gamma_+ p_-}{p^2 + \frac{G^2}{\pi} + i\epsilon} + \frac{\sigma_- p_+}{p^2 + i\epsilon} \quad (3.7)$$

We have a tachyon in the first term. One may argue that this is in the fermion propagator, which is a gauge-dependent quantity. However, going over to $\Sigma(p)$ we get from Eq. (3.4)

$$\Sigma(p; m=0) \propto \int d^4k \frac{k_- (k-p)_+}{(k^2 + \frac{G^2}{\pi} + i\epsilon) [(k-p)^2 + i\epsilon]} \quad (3.8)$$

The discontinuity of Σ , with respect to p^2 , is

$$\text{Im } \Sigma(p; m=0) \propto \mathcal{O}(p^2 + \frac{G^2}{\pi})$$

Thus there is an imaginary part also for $-G^2/\pi < p^2 < 0$, which is unacceptable. [Actually, this result remains unchanged also if we use Eq. (2.20) for S and not Eq. (3.5).]

Note that this problem is there also in the solution of Ref. 1). Denoting that solution by S_t , its $m=0$ limit would be

$$\frac{1}{i} S_t(p; m=0) = \frac{\gamma_+ p_-}{p^2 + \frac{G^2}{\pi} - \frac{2G^2/p-1}{\pi\lambda} + i\epsilon} + \frac{\sigma_- p_+}{p^2 + i\epsilon} \quad (3.9)$$

Also here it is only the diagrams of Fig. 1 that contribute. Thus

$$\Sigma_t(x; m=0) \propto \text{Tr} [S_t'(x; m=0) S_t'(-x; m=0)] \quad (3.10)$$

Denoting the first term on the right-hand side of Eq. (3.9) by $S_t^{(1)}$ and the second by $S_t^{(2)}$, we have

$$\begin{aligned} \Sigma_t(x; m=0) &\propto \text{Tr} [S_t^{(1)'}(x) S_t^{(2)'}(-x) + S_t^{(2)'}(x) S_t^{(1)'}(-x)] \\ &= 2 \text{Tr} [S_t^{(1)'}(x) S_t^{(2)'}(x)] \end{aligned} \quad (3.11)$$

Now

$$\begin{aligned}
 S_t^{(n)}(x) &= \frac{1}{(2\pi)^2} \delta_+ \int d^4k e^{ikx} \frac{k_-}{k^2 + \frac{G^2}{\pi} - \frac{2G^2|k_-|}{\pi\lambda} + i\epsilon} = \\
 &= \frac{1}{(2\pi)^2} \int dk_+ dk_- e^{ik_+x_-} e^{ik_-x_+} \frac{k_-}{2k_+k_- + \frac{G^2}{\pi} - \frac{2G^2|k_-|}{\pi\lambda} + i\epsilon} = \\
 &= \frac{1}{(2\pi)^2} \int dk_- e^{ik_-x_+} e^{-\frac{i2G^2|x_-|}{\pi\lambda}} e^{-\frac{iG^2x_-}{\pi k_-}} (\pi i) \cdot \\
 &\cdot [\theta(x_-)\theta(-k_-) - \theta(-x_-)\theta(k_-)] = e^{-\frac{i2G^2|x_-|}{\pi\lambda}} S_t^{(n)}(x) \quad (3.11)
 \end{aligned}$$

Thus

$$\Sigma_t(x; m=0) = e^{-\frac{i2G^2|x_-|}{\pi\lambda}} \Sigma(x; m=0) \quad (3.12)$$

Here two features are unacceptable: the cut-off dependence in Σ_t (our Σ is cut-off independent) and the part of the spectrum which has tachyons.

Note that for $m \neq 0$ also, diagrams as in Fig. 2 will contribute, and to calculate $\Sigma(x)$ will be more complicated. However, whenever there is continuity in $m \rightarrow 0$, the problems encountered above will remain also for small $m \neq 0$. This continuity holds for each Feynman diagram separately.

4. ANOTHER INFRA-RED CUT-OFF

We are going to investigate now the effects of changing the ρ propagator in the integral equation (2.17). In particular, we shall introduce a mass parameter, which we shall let go to zero after we solve the integral equation (still in the $\rho_- = 0$ gauge). We do not introduce a mass term in the Lagrangian, since this ruins renormalizability⁵⁾. Our aim is only to investigate the effects of various infra-red cut-offs. Also, by summing the geometric series of fermion bubbles in Fig. 3, one gets

$$\widehat{D}(k) = \frac{i}{2} \left[\frac{1}{(k_- + i\epsilon)^2} + \frac{1}{(k_- - i\epsilon)^2} \right] \frac{k^2}{k^2 - \mu^2 + i\epsilon} \quad (4.1)$$

$$\mu^2 = \frac{g^2}{4} = \frac{2G^2}{n\pi}$$

with $\mu^2 \rightarrow 0$ for $n \rightarrow \infty$. [Each bubble in Fig. 3 equals

$$i \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \frac{g^2}{2\pi} \delta^{ab}$$

the factor $\frac{1}{2}$ relative to the Abelian case ⁶⁾ coming from

$$\text{Tr} \left[\left(\frac{1}{2} \lambda^a \right) \left(\frac{1}{2} \lambda^b \right) \right] = \frac{1}{2} \delta^{ab} .]$$

Thus it is conceivable that infra-red cut-offs may come from the next-to-leading $1/n$ corrections.

We shall actually not take the expression Eq. (4.1), since its Fourier transform does not tend to Eq. (2.15) in the limit $\mu^2 \rightarrow 0$. In fact, the Fourier transform of the function Eq. (4.1) is

$$\begin{aligned} & \frac{1}{(2\pi)^4} \int d^4 k \frac{k^2}{k^2 - \mu^2 + i\epsilon} \frac{i}{2} \left[\frac{1}{(k_- - i\epsilon)^2} + \frac{1}{(k_- + i\epsilon)^2} \right] e^{ikx} = \\ & = -\frac{i}{2} \left\{ \chi_+ / \delta(x_-) - \frac{1}{4\pi} \left\{ \chi_+ \left[\frac{1}{x_- - i\epsilon} + \frac{1}{x_- + i\epsilon} \right] + \frac{1}{\mu^2} \left[\frac{1}{(x_- - i\epsilon)^2} + \frac{1}{(x_- + i\epsilon)^2} \right] \right\} \right\} + (\text{function which tends to zero as } \mu \rightarrow 0) . \end{aligned} \quad (4.2)$$

The calculation is given in Appendix A.

We thus already see the subtleties of the infra-red problem in this model, since the limit of $\mu \rightarrow 0$ and the Fourier transform cannot be interchanged. Since $\mu = G/\sqrt{n\pi}$, it shows that taking the $n \rightarrow \infty$ limit under the integrals involved is a questionable procedure.

We note that the extra terms in Eq. (4.2), which do not vanish for $\mu \rightarrow 0$, yields zero when ∂_-^2 is applied; this means that they represent some freedom in choosing a solution of Eq. (2.12).

Demanding a propagator that tends to $D(x)$ as $\mu \rightarrow 0$ we chose (Appendix A),

$$D_1(k) = 4i \frac{k_+^2}{(k^2 + i\epsilon)(k^2 - \mu^2 + i\epsilon)} + \pi \epsilon(k_+) \delta'(k_-) \quad (4.3)$$

Note that taking the limit of $\mu \rightarrow 0$ in the first term and then doing the Fourier transform, we get ambiguous results (Appendix A). However, we are going to use Eq. (4.3) for the propagator and let $\mu \rightarrow 0$ only at the end. Note also, that with our procedure, the first term has only $+i\epsilon$ in the denominators [see Appendix A for cancellation of $1/(k^2 - i\epsilon)$ term].

Note that the large x_+ behaviour of D_1 for given μ is different from $|x_+|$. However, any infra-red cut-off changes that behaviour (see Appendix A).

5. SOLUTIONS FOR FERMION AND SCALAR DENSITY PROPAGATORS

We now want to solve the fermion propagator from Eq. (2.17), using D_1 instead of D . Thus, instead of Eq. (2.21) we get

$$p_+ S_1(p) = p_+ - \frac{iG^2}{2\pi^2} \int d^2k \frac{k_-}{k^2 S_1(k) - m^2 + i\epsilon} \cdot \left\{ \frac{4(k_+ - p_+)^2}{[(k-p)^2 + i\epsilon][(k-p)^2 - \mu^2 + i\epsilon]} - i\pi \epsilon(k_+ - p_+) \delta'(k_- - p_-) \right\} \quad (5.1)$$

Using the calculation performed in Appendix B, we get the same result as for $s(p)$;

$$p^+ S_1(p) = p^2 + \frac{G^2}{11} \quad (5.2)$$

The limit $\mu \rightarrow 0$ was taken essentially at the end of the calculation. Thus with our procedure of carefully taking care of the infra-red problem, the fermion propagator still has the tachyon for $m^2 < G^2/\pi$.

Thus, in the $m=0$ limit, the two-point function of the scalar density, Eq. (3.2), has a spectrum starting from $p^2 > -G^2/\pi$ (see discussion in Section 3), which is unacceptable. Note that the discussion in Ref. 1) was about the spectrum in the vector channel. [Explicitly, in the channel $\psi_-^+(1)\psi_-(2)$.]

Taking as the gluon propagator

$$\bar{D}(x) = -\frac{i}{2\mu} (1 - e^{-\mu|x_+|}) \delta(x_-) \quad (5.3)$$

we again get the same solution as the one obtained from D of Eq. (2.16), thus leading again to tachyons in the two-point function for the scalar density (Appendix B).

Note that in the case of D_1 of Eq. (4.3), if only the first term is maintained, again a tachyon spectrum arises (Appendix B).

It is actually clear why we get the same results for the fermion propagator from the different cut-off procedures. Starting from the integral on the right-hand side of Eq. (2.17), and going over to x space, we get

$$\begin{aligned} \frac{G^2}{4\pi^2} \frac{1}{(2\pi)^2} \int d^2p \, e^{ipx} \int d^2k \, \delta_- S(k) \delta_- D(p-k) = \\ = G^2 \delta_- S(x) \delta_- D(x) \equiv \tilde{I}(x) \end{aligned} \quad (5.4)$$

For all the propagators, we chose

$$D(x) \xrightarrow{\mu \rightarrow 0} -\frac{i}{2} |x_+| \delta(x_-) \quad (5.5)$$

and thus the expression in Eq. (5.4) tends to

$$\tilde{I}(x) \xrightarrow{\mu \rightarrow 0} G^2 |x_+| \delta(x_-) \delta_- \frac{1}{(2\pi)^2} \left[(-i\partial_-) \int d^2p \, \frac{e^{ipx}}{p^2 S(p) - m^2 + i\epsilon} \right] \quad (5.6)$$

In all our cases $s(p)$ is a function of p^2 only. Writing

$$\frac{1}{p^2 S(p^2) - m^2 + i\epsilon} = \int_{-\infty}^{\infty} \frac{h(M^2) dM^2}{p^2 - M^2 + i\epsilon} \quad (5.7)$$

and using

$$(-i\partial_-) \int d^2p \, \frac{e^{ipx}}{p^2 - M^2 + i\epsilon} \xrightarrow{x_- \rightarrow 0} \frac{\pi}{x_+} \quad (5.8)$$

we get

$$\tilde{I}(x) \xrightarrow{\mu \rightarrow 0} \frac{G^2}{4\pi} C_1 \epsilon(x_+) \delta(x_-) \delta_- \quad (5.9)$$

$$C_1 = \int_{-\infty}^{\infty} h(M^2) dM^2$$

Returning to momentum space, we get for the integral equation (2.17),

$$\begin{aligned} S^{-1}(p) &= S_0^{-1}(p) + \frac{G^2}{4\pi} \gamma_- C_1 \int d^4x e^{-ipx} \epsilon(x_+) \delta(x_-) = \\ &= S_0^{-1}(p) - i \frac{G^2}{2\pi} \gamma_- C_1 \frac{1}{p_-} \end{aligned} \quad (5.10)$$

which leads to

$$p^2 S(p) = p^2 + \frac{G^2}{\pi} \quad (5.11)$$

and $C_1 = 1$.

6. THE SINGLET VECTOR CURRENT

An exact result in our two-dimensional model is that the singlet current

$$J_\mu = \bar{\psi} \gamma_\mu \psi \quad (6.1)$$

has, for $m=0$, a two-point function which is equal to that of the free case. The proof follows from an examination of diagrams. We show that all contributions, except for the bubble diagram of Fig. 4, vanish. [The latter has to be regulated to ensure current conservation. See, for example, Ref. 6).] A general diagram would have a part which is a fermion loop with at least one singlet current entering. When both singlet currents enter the same fermion loop, as in Fig. 5, that part is a tensor which is conserved and traceless in the two indices of the singlet currents. But such a tensor must vanish in two dimensions⁶⁾. When only one singlet current enters the fermion loop, we prove the vanishing of that part by induction. Suppose there are N vector $SU(n)$ currents entering: the divergence with respect to one of the latter is given by diagrams with $(N-1)$ vector $SU(n)$ currents. If the latter vanish, then the diagram with the N lines

vanishes too [by tracelessness and conservation ⁶⁾]. But it is obvious that for $N=1$ the diagram is zero, which completes the proof.

This property of the singlet current means that the wave function $\psi(p,r)$ of Ref. 1) has to obey $\int d^2p \psi(p,r) = 0$, since there the fermion-antifermion pair is in a singlet vector state. This, in turn, means that $\int dx \varphi(x) = 0$ for the φ there. But from Eq. (25) in Ref. 1), this condition is obeyed for $\alpha_1 = \alpha_2 = -1$ when $m_1 = 0$. The relation in Eq. (24) of Ref. 1) is consistent with that.

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APPENDIX A

In this Appendix we show:

- (a) the validity of Eq. (4.2);
- (b) that the Fourier transform of Eq. (4.3) tends to $-i|x_+|\delta(x_-)$ as $\mu \rightarrow 0$;
- (c) that the Fourier transform of $k_+^2/(k^2 + i\epsilon)^2$ is ambiguous;
- (d) the cancellation of all terms with $(-i\epsilon)$ as denominator in arriving to Eq. (4.3);
- (e) the large x_+ behaviour of D_1 and the propagator of Ref. 1).

We start with the calculation of the Fourier transform of $\tilde{D}(k)$ of Eq. (4.1).

$$\begin{aligned}
 & \frac{1}{(2\pi)^2} \int d^2k \frac{k^2}{k^2 - \mu^2 + i\epsilon} \frac{i}{2} \left[\frac{1}{(k_- + i\epsilon)^2} + \frac{1}{(k_- - i\epsilon)^2} \right] e^{ikx} = \\
 & = \frac{1}{(2\pi)^2} \int d^2k \frac{i}{2} \left[\frac{1}{(k_- + i\epsilon)^2} + \frac{1}{(k_- - i\epsilon)^2} \right] e^{ikx} \\
 & + \frac{1}{(2\pi)^2} \int d^2k \frac{\mu^2}{k^2 - \mu^2 + i\epsilon} \frac{i}{2} \left[\frac{1}{(k_- + i\epsilon)^2} + \frac{1}{(k_- - i\epsilon)^2} \right] e^{ikx} = \\
 & = -\frac{i}{2} |x_+| \delta(x_-) + \frac{1}{2} I(x)
 \end{aligned}$$

(A.1)

Now

$$\begin{aligned}
 I(x) &= \frac{1}{(2\pi)^2} \int d^2k \frac{\mu^2}{k^2 - \mu^2 + i\epsilon} i \left[\frac{1}{(k_- + i\epsilon)^2} + \frac{1}{(k_- - i\epsilon)^2} \right] e^{ikx} = \\
 &= \frac{1}{(2\pi)} \int dk_+ e^{ik_+ x} - \left\{ i|x_+| + \frac{2}{\mu^2} \epsilon(x_+) k_+ \right. \\
 &\quad \left. - \frac{4}{\mu^2} k_+ e^{i \frac{\mu^2 x_+}{2k_+}} [\theta(x_+) \theta(-k_+) - \theta(-x_+) \theta(k_+)] \right\} =
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2\pi} \int dk_+ e^{ik_+ x_-} \frac{4}{\mu^2} k_+ \left[e^{i \frac{\mu^2 x_+}{2k_+}} - 1 - i \frac{\mu^2 x_+}{2k_+} \right] \\
 &\quad \cdot [\theta(-x_+) \theta(k_+) - \theta(x_+) \theta(-k_+)] \\
 &+ \frac{1}{2\pi} \int dk_+ \left[\frac{2}{\mu^2} |k_+| + i x_+ \epsilon(k_+) \right] e^{ik_+ x_-} = \\
 &= -\frac{1}{2\pi} \left\{ x_+ \left[\frac{1}{x_- - i\epsilon} + \frac{1}{x_- + i\epsilon} \right] \right. \\
 &\quad \left. + \frac{2}{\mu^2} \left[\frac{1}{(x_- - i\epsilon)^2} + \frac{1}{(x_- + i\epsilon)^2} \right] \right\} \\
 &+ J(x)
 \end{aligned}$$

(A.2)

With

$$\begin{aligned}
 J(x) &= \frac{2}{\pi} \frac{1}{\mu^2} \int_0^\infty d\xi [\theta(x_+) e^{-i\xi x_-} + \theta(-x_+) e^{i\xi x_-}] \\
 &\quad \cdot \xi \left[e^{-i \frac{\mu^2 |x_+|}{2\xi}} - 1 + \frac{i \mu^2 |x_+|}{2\xi} \right] = \\
 &= \frac{1}{2\pi} \mu^2 x_+^2 \int_0^\infty d\eta \left[e^{-\frac{i+\epsilon}{2}\eta} - 1 + \frac{i}{2}\eta \right] \\
 &\quad \cdot \eta e^{-\frac{\epsilon}{4}\mu^2 x_+^2 \eta}
 \end{aligned}$$

(A.3)

Now $J(x) \rightarrow 0$ as $\mu \rightarrow 0$. We write the integral as $\int_0^N + \int_N^\infty$. In $\int_N^\infty$ we can expand $e^{-(i+\epsilon)\eta/2}$ in a power series, which converges rapidly for large N . We thus have a $\int_N^\infty (d\eta/\eta) e^{-(\mu^2 x_+^2 \eta)/4}$ + finite terms as $\mu \rightarrow 0$. The integral $\int_0^N$ is finite as $\mu \rightarrow 0$. Thus $J(x) \approx (\mu^2 x_+^2) \ln(\mu^2 |x^2|)$ for $\mu \rightarrow 0$.

We now choose the propagator $D_1(k)$ by adding the Fourier transform of second and third terms on the right-hand side of Eq. (4.2) to $\tilde{D}(k)$ of Eq. (4.1). We are thus assured that the Fourier transform of $D_1(k)$ tends to $-i|x_+|\delta(x_-)$ as $\mu \rightarrow 0$. Thus

$$D_1(k) = \frac{i}{2} \left[\frac{1}{(k_- + i\epsilon)^2} + \frac{1}{(k_- - i\epsilon)^2} \right] \frac{k^2}{k^2 - \mu^2 + i\epsilon} - \frac{2\pi}{\mu^2} |k_+| \delta(k_-) + \pi \epsilon(k_+) \delta'(k_-) \quad (\text{A.4})$$

Now, the first two terms can be rewritten as

$$\begin{aligned} & i \left[\frac{1}{k_- + i\epsilon} + \frac{1}{k_- - i\epsilon} \right] \frac{k_+}{k^2 - \mu^2 + i\epsilon} + \\ & + \frac{i}{\mu^2} |k_+| \left[\frac{1}{k_- - i\epsilon} - \frac{1}{k_- + i\epsilon} \right] = \\ & = i \left[\frac{1}{k^2 + i\epsilon} + \frac{1}{k^2 - i\epsilon} \right] \frac{2k_+^2}{k^2 - \mu^2 + i\epsilon} + \frac{2ik_+^2}{\mu^2} \cdot \\ & \cdot \left[\frac{1}{k^2 - i\epsilon} - \frac{1}{k^2 + i\epsilon} \right] = \\ & = 2ik_+^2 \frac{1}{k^2 + i\epsilon} \left[\frac{1}{k^2 - \mu^2 + i\epsilon} - \frac{1}{\mu^2} \right] \\ & + 2ik_+^2 \frac{1}{k^2 - i\epsilon} \left[\frac{1}{k^2 - \mu^2 + i\epsilon} + \frac{1}{\mu^2} \right] = \\ & = 4i \frac{k_+^2}{(k^2 + i\epsilon)(k^2 - \mu^2 + i\epsilon)} - 2ik_+^2 \frac{1}{\mu^2(k^2 - \mu^2 + i\epsilon)} \left[1 - \frac{k^2 + i\epsilon}{k^2 - i\epsilon} \right] = \\ & = 4i \frac{k_+^2}{(k^2 + i\epsilon)(k^2 - \mu^2 + i\epsilon)} \quad (\text{A.5}) \end{aligned}$$

We have thus proved (b) and (d). Actually, one can directly show that the Fourier transform of $D_1(k)$ of Eq. (4.3) tends to $-i|x_+|\delta(x_-)$ as $\mu \rightarrow 0$.

We are now going to discuss (c), namely calculate the Fourier transform of $k_+^2/(k^2 + i\epsilon)^2$. Doing it by a $\int dk_+ \int dk_-$ integration, we get a different result from that of $\int dk_+ \int dk_-$. The former is actually not consistent with Lorentz invariance.

$$\begin{aligned}
 f_1(x) &= \int dk_+ e^{-ik_+ x_-} \int dk_- e^{ik_- x_+} \frac{k_+^2}{(k^2 + i\epsilon)^2} = \\
 &= \frac{\pi x_+}{\sqrt{2}} \int dk e^{ikx_-} [\theta(k)\theta(-x_-) - \theta(-k)\theta(x_+)] = \\
 &= \frac{i\pi x_+}{\sqrt{2}} \left[\theta(x_-) \frac{1}{x_- - i\epsilon} + \theta(-x_-) \frac{1}{x_- + i\epsilon} \right] = \\
 &= \frac{1}{2} i\pi \left[i\pi |x_+| \delta(x_-) + 1 \right. \\
 &\quad \left. + \frac{1}{2} x_+ \left(\frac{1}{x_- + i\epsilon} + \frac{1}{x_- - i\epsilon} \right) \right]
 \end{aligned} \tag{A.6}$$

The constant term is not allowed by Lorentz invariance, since we expect the $++$ component of a tensor. Now,

$$\begin{aligned}
 f_2(x) &= \frac{1}{4} \int dk_+ e^{ik_+ x_-} \int dk_- e^{ik_- x_+} \frac{1}{(k_- + i\epsilon k_+)^2} = \\
 &= \frac{i\pi}{2} \int dk_+ e^{ik_+ x_-} (i x_+) [\theta(x_+) \theta(-k_+) - \theta(-x_+) \theta(k_+)] = \\
 &= \left(\frac{1}{2} i\pi \right) \left[i\pi |x_+| \delta(x_-) \right. \\
 &\quad \left. + \frac{1}{2} x_+ \left(\frac{1}{x_- + i\epsilon} + \frac{1}{x_- - i\epsilon} \right) \right]
 \end{aligned} \tag{A.7}$$

The reason for the ambiguity, namely the difference between (A.6) and (A.7), is that the covariant quantity

$$\gamma_{\mu\nu}(x) = \int d^2k e^{ikx} \frac{k_\mu k_\nu}{(k^2 + i\epsilon)^2} \quad (\text{A.8})$$

has an infinity trace, and thus a covariant calculation is not possible.

Note that our previous results, namely (a) and (b), are unambiguous and the same whether done as $\int dk_0 \int dk_1$ or $\int dk_+ \int dk_-$ (in this Appendix we explicitly calculated only in the latter form). Note also that Eq. (4.7) is the result of first Fourier transforming the term $k_+^2 / [(k^2 + i\epsilon)(k_-^2 + i\epsilon)]$ and then letting $\mu \rightarrow 0$.

We now go over to (e), namely the large x_+ behaviour. It follows from our discussion in this Appendix that

$$D_1(x) = \frac{1}{(2\pi)^2} \int d^2k e^{ikx} D_1(k) = -\frac{i}{4} |x_+| \delta(x_-) + \frac{1}{4} J(x) \quad (\text{A.9})$$

with $J(x)$ given by Eq. (A.3). For fixed x_+ and $x_- \rightarrow 0$, it is easy to show that

$$J(x) \xrightarrow{x_- \rightarrow 0} \frac{1}{4\pi} \mu^2 x_+^2 \ln \mu^2 |x^2| + \frac{i}{8} \mu^2 x_+^2 \epsilon(x^2) + \frac{1}{2\pi} C \mu^2 x_+^2 \quad (\text{A.10})$$

where C is a constant,

$$C = \frac{i\pi}{4} - \frac{3}{4} - \ln 2 + \gamma$$

Now, for fixed $x_- \neq 0$ and $x_+ \rightarrow \pm\infty$ ⁷⁾,

$$J(x) \sim \begin{cases} \frac{1}{\pi} \frac{x_+}{x_-} + \frac{2}{\pi} \frac{1}{\mu^2 x_-^2} + (\text{exponential decrease in } |x^2|); & x^2 < 0 \\ \frac{1}{\pi} \frac{x_+}{x_-} + \frac{2}{\pi} \frac{1}{\mu^2 x_-^2} + \sqrt{\frac{\pi}{2}} (1-i) \mu^2 x_+^2 e^{-i\sqrt{\mu^2 x^2}} (\mu^2 x^2)^{-\frac{5}{4}}; & x^2 > 0 \end{cases} \quad (\text{A.11})$$

Taking $D_1(x)$ and integrating symmetrically over a small neighbourhood of $x_- = 0$, the leading term would then be $-i|x_+|$.

The cut-off procedure of Ref. 1) is to take for the gluon propagator

$$D_t(k) = \frac{i}{k_-^2} \Theta(k_-^2 - \lambda^2) \quad (\text{A.12})$$

For this one,

$$\begin{aligned} D_t(x) &= \frac{1}{(2\pi)^2} \int d^2k \frac{i}{k_-^2} \Theta(k_-^2 - \lambda^2) e^{ikx} = \\ &= \frac{i}{2\pi} \delta(x_-) \int dk_- \Theta(k_-^2 - \lambda^2) \frac{e^{ik_- x_+}}{k_-^2} = \\ &= \frac{i}{\pi} \delta(x_-) \left[\frac{1}{\lambda} \cos \lambda x_+ - |x_+| \int_{\lambda}^{\infty} \frac{d\xi}{\xi^2} \sin \xi |x_+| \right] = \\ &= \frac{i}{\pi} \delta(x_-) \int_{\lambda}^{\infty} \frac{d\xi}{\xi^2} \cos \xi |x_+| \end{aligned} \quad (\text{A.13})$$

Thus

$$D_t(x) \xrightarrow{\lambda \rightarrow 0} \left[-\frac{i}{\lambda} |x_+| + \frac{i}{\pi \lambda} \right] \delta(x_-) \quad (\text{A.14})$$

and $D_t(x)$ decreases for $x_+ \rightarrow \pm\infty$, as $-(i/\pi\lambda)[1/(\lambda|x_+|)] \sin \lambda|x_+|$.

Note that $D_t(x)$, for $\lambda \rightarrow 0$, has an extra infinite constant.

APPENDIX B

In this Appendix we show:

- (a) that the solution of the integral Eq. (5.1) is $k^2 s_1(k) = k^2 + G^2/\pi$;
- (b) that the propagator Eq. (5.3) yields the same solution for the fermion propagator.

Let us start with (a). From Eq. (5.1) it follows, by Lorentz invariance, that $s_1(p)$ is a function of p^2 only. Now,

$$\begin{aligned}
 & \int d^4k \, k_- \, \ell(k^2) \, \epsilon(k_+ - p_+) \, \delta'(k_- - p_-) = \\
 & = - \int dk_+ \, \ell(2k_+ p_-) \, \epsilon(k_+ - p_+) \\
 & \quad - \int dk_+ \, (2k_+ p_-) \, \ell'(2k_+ p_-) \, \epsilon(k_+ - p_+) = \\
 & = - \int dk_+ \, \epsilon(k_+ - p_+) \, \frac{\partial}{\partial k_+} [k_+ \ell(2k_+ p_-)] = \\
 & = 2 p_+ \, \ell(p^2) \\
 & \quad - \left\{ [k_+ \ell(2k_+ p_-)]_{k_+=\infty} + [k_+ \ell(2k_+ p_-)]_{k_+=-\infty} \right\} = \\
 & = 2 p_+ \, \ell(p^2) \\
 & \quad - \frac{1}{2p_-} \left\{ [k^2 \ell(k^2)]_{k^2=\infty} + [k^2 \ell(k^2)]_{k^2=-\infty} \right\} \quad (B.1)
 \end{aligned}$$

Also,

$$\begin{aligned}
 & \int d^2k \frac{k_-}{k^2 - M^2 + i\epsilon} \frac{(k_+ - p_+)^2}{[(k-p)^2 + i\epsilon][(k-p)^2 - \mu^2 + i\epsilon]} = \\
 & = 2 \int d^2k \iint_{(\alpha_1 + \alpha_2) \leq 1} d\alpha_1 d\alpha_2 \frac{k_- (k_+ - p_+)^2}{[k^2 - 2(\alpha_1 + \alpha_2)k \cdot p + (\alpha_1 + \alpha_2)p^2 - \alpha_1 \mu^2 - (1 - \alpha_1 - \alpha_2)M^2 + i\epsilon]^3} = \\
 & = p_+ \int d^2k \iint_{(\alpha_1 + \alpha_2) \leq 1} d\alpha_1 d\alpha_2 (\alpha_3) \cdot \\
 & \quad \frac{p^2 \alpha_3 (1 - \alpha_3) - 2k^2}{[k^2 + \alpha_3 (1 - \alpha_3) p^2 - \alpha_1 \mu^2 - \alpha_3 M^2 + i\epsilon]^3} \\
 & \xrightarrow{\mu \rightarrow 0} p_+ \int d^2k \iint_{(\alpha_1 + \alpha_2) \leq 1} d\alpha_1 d\alpha_2 (\alpha_3) \frac{p^2 \alpha_3 (1 - \alpha_3) - 2k^2}{[k^2 + \alpha_3 (1 - \alpha_3) p^2 - \alpha_3 M^2 + i\epsilon]^3} = \\
 & \stackrel{(k \rightarrow \sqrt{s}k)}{=} p_+ \int d^2k \iint d\alpha_1 d\alpha_2 \frac{p^2 (1 - \alpha_3) - 2k^2}{[k^2 + (1 - \alpha_3) p^2 - M^2 + i\epsilon]^3} \\
 & = -i\pi p_+ \int_0^1 d\alpha_3 (1 - \alpha_3) \left\{ \frac{1}{M^2 - (1 - \alpha_3) p^2} + \frac{(1 - \alpha_3) p^2}{2[M^2 - (1 - \alpha_3) p^2]^2} \right\} = \\
 & = \frac{i\pi}{2} \frac{p_+}{p^2 - M^2 + i\epsilon} \tag{B.2}
 \end{aligned}$$

Since we can represent $i/[k^2 s_1(k) - m^2 + i\epsilon]$ as $\int_{-\infty}^{\infty} h(M^2) dM^2 / (p^2 - M^2 + i\epsilon)$, it follows that

$$\begin{aligned}
 & \int d^2k \frac{k_-}{k^2 s_1(k) - m^2 + i\epsilon} \frac{(k_+ - p_+)^2}{[(k-p)^2 + i\epsilon][(k-p)^2 - \mu^2 + i\epsilon]} \\
 & \xrightarrow{\mu \rightarrow 0} \frac{i\pi}{2} \frac{p_+}{p^2 s_1(p) - m^2 + i\epsilon} \tag{B.3}
 \end{aligned}$$

and

$$\int d^2k \frac{k_-}{k^2 s_1(k) - m^2 + i\epsilon} \left\{ \frac{4(k_+ - p_+)^2}{[(k-p)^2 + i\epsilon][(k-p)^2 - \mu^2 + i\epsilon]} - i\pi \epsilon(k_+ - p_+) \delta'(k_- - p_-) \right\}$$

$$\xrightarrow{\mu \rightarrow 0} \frac{i\pi}{p_-} C_2 \quad (B.4)$$

$$C_2 = \frac{1}{2} \lim_{k^2 \rightarrow \infty} \left[\frac{k^2}{k^2 s_1(k^2) - m^2 + i\epsilon} + \frac{-k^2}{-k^2 s_1(-k^2) - m^2 + i\epsilon} \right] \quad (B.5)$$

Thus, the integral Eq. (5.1) becomes

$$p^2 \tilde{s}_1(p^2) = p^2 + \frac{G^2}{\pi} C_2 \quad (B.6)$$

Combining (B.5) and (B.6), we get

$$C_2 = 1 \quad (B.7)$$

which completes the proof of (a).

Note that if we do not include the term $2\pi \epsilon(k_+) \delta'(k_-)$ in D_1 , the equation for s_1 in the limit $\mu \rightarrow 0$, would be

$$\tilde{s}_1 = 1 + \frac{G^2}{\pi} \frac{1}{k^2 \tilde{s}_1 - m^2 + i\epsilon} \quad (B.8)$$

$$\tilde{s}_1 = \frac{1}{2k^2} \left[(k^2 + m^2) + (k^2 - m^2) \sqrt{1 + \frac{4k^2}{(k^2 - m^2)^2} \frac{G^2}{\pi}} \right]$$

The sign of the square root was chosen so that $\tilde{s}_1 = 1$ for $G=0$. With this solution, the propagator $1/(k^2 \tilde{s}_1 - m^2 + i\epsilon)$ has no pole, but there is a cut for

$$-\left(\frac{G}{\sqrt{\pi}} - \sqrt{\frac{G^2}{\pi} - m^2}\right)^2 > p^2 > -\left(\frac{G}{\sqrt{\pi}} + \sqrt{\frac{G^2}{\pi} - m^2}\right)^2$$

where $G^2/\pi > m^2$. Thus for $m=0$ we have a tachyon cut $0 > p^2 > -4G^2/\pi$, which implies a cut in the scalar density propagator from $p^2 = -4G^2/\pi$ to $p^2 = +\infty$ (by considerations similar to those in Section 3). This is also unacceptable.

We now go over to show (b) of this Appendix. First we Fourier transform the propagator in Eq. (5.3)

$$\bar{D}(k) = i \left[\frac{1}{k_-^2 + \mu^2} - \frac{\pi}{\mu} \delta(k_-) \right] \quad (\text{B.9})$$

The integral equation for the fermion propagator is now

$$p_+ \bar{S}_1(p) = p_+ - \frac{i G^2}{2\pi^2} \int d^4 k \frac{k_-}{k^2 \bar{S}_1(k) - m^2 + i\epsilon} \left[\frac{1}{(k_- - p_-)^2 + \mu^2} - \frac{\pi}{\mu} \delta(k_- - p_-) \right] \quad (\text{B.10})$$

Thus $p_+ [\bar{S}_1(p) - 1] = \bar{F}_1(p)$ is a function of p_- only. Hence

$$\begin{aligned} & \int d^4 k \frac{k_-}{k^2 \bar{S}_1(k) - m^2 + i\epsilon} \frac{1}{(k_- - p_-)^2 + \mu^2} = \\ &= \frac{1}{2} \int dk_- \frac{1}{(k_- - p_-)^2 + \mu^2} \int dk_+ \frac{1}{k_+ + \bar{\gamma}_1(k_-) - \frac{m^2}{2k_-} + i\epsilon k_-} = \\ &= -\frac{1}{2} \pi i \int dk_- \frac{\epsilon(k_-)}{(k_- - p_-)^2 + \mu^2} = -\frac{\pi i}{\mu} \arctg \frac{p_-}{\mu}; \\ & \int d^4 k \frac{k_-}{k^2 \bar{S}_1(k) - m^2 + i\epsilon} \left[\frac{1}{(k_- - p_-)^2 + \mu^2} - \frac{\pi}{\mu} \delta(k_- - p_-) \right] = \\ &= -\frac{\pi i}{\mu} \epsilon(p_-) \left[\arctg \frac{|p_-|}{\mu} - \frac{1}{2} \pi \right] \xrightarrow{\mu \rightarrow 0} \frac{\pi i}{p_-} \end{aligned}$$

We get

$$\begin{aligned} p_+ \bar{s}_1(p) &= p_+ + \frac{G^2}{2\pi} \frac{1}{p_-} \\ p^2 \bar{s}_1(p) &= p^2 + \frac{G^2}{\pi} \end{aligned} \tag{B.11}$$

which is our assertion (b).

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(use 7.12 equations 23-24, and 7.13 equation 7).

FIGURE CAPTIONS

- Fig. 1 : All diagrams contributing to $\Sigma(x)$ of Eq. (3.2) for $m=0$.
A solid line represents the bare fermion propagator, and a line with a dashed bubble represents the full fermion propagator.
- Fig. 2 : Diagrams that vanish for $m=0$. The wavy line represents the gauge field propagator.
- Fig. 3 : The geometric series of fermion bubbles for the ρ propagator.
- Fig. 4 : Zeroth order contribution to two-point function of singlet current (the latter represented by dashed line).
- Fig. 5 : One fermion loop with two singlet currents and at least one vector $SU(n)$ current.
- Fig. 6 : One fermion loop with one singlet current and N vector $SU(n)$ currents.

FIG. 1

FIG. 2

FIG. 3

FIG. 4

FIG. 5

FIG. 6