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HIGH ENERGY SCATTERING AND THE NATURE OF THE
HADRONIC PHASE TRANSITION

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ABSTRACT

General arguments and the Reggeon calculus suggest that high energy scattering may be describable as a phase transition. If the renormalized Pomeron intercept is precisely 1, this transition is expected to be of second order. In this case, several inequalities and relations can be derived between the critical exponents describing the high energy behaviour of diffractive scattering processes. These relations ensure that many requirements of s channel unitarity are satisfied. Some experimental tests are suggested which may enable data at ISR energies to be useful in a phenomenological analysis of the nature of the hadronic phase transition.

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1. INTRODUCTION

The asymptotic (large s , small t) behaviour of high energy scattering is a process in which the large distance structure of the theory is of primary importance. One way to see this is to Fourier transform the configuration space matrix elements into momentum space. This introduces factors like $\exp i p_\mu x^\mu = \exp i(p_+ x_- + p_- x_+ - \vec{p}_\perp \cdot \vec{x}_\perp)$ multiplying the matrix elements over which we integrate, where $p_\pm = 1/\sqrt{2}(p_0 \pm p_\parallel)$ and $x_\pm = 1/\sqrt{2}(x_0 \pm x_\parallel)$. If the matrix elements are not too pathological, the major contribution to the integral will come when $p_\mu x^\mu$ does not get too large. For large s small t scattering (p_0 large, $p_\perp$ small) it is easy to see that all four components of x_μ can get large ($|x_+|, |x_\perp| \rightarrow \infty$, $|x_-| \rightarrow 0$), and still $p_\mu x^\mu$ will be finite. Hence, in asymptotic scattering we must concern ourselves with the large distance structure of the theory. [Note, on the other hand, that in high energy, large angle scattering ($s, p_\perp$ large) the large distance structure is suppressed by the oscillating exponential, and we explore only the detailed, short distance behaviour of the theory.]

This viewpoint is made more explicit in recent work on the Reggeon calculus ¹⁾ in which the problem of the superposition of Regge poles and cuts near $\alpha_p(0) = 1$ is studied by examining the behaviour of an analogue field theory in the infra-red limit: $Y \equiv \ln s \rightarrow \infty$, $x_\perp \rightarrow \infty$. It should be stressed, however, that although we shall primarily work in the framework of the Reggeon calculus, the focus on infra-red or long range behaviour in this kinematical limit is more general than the model.

Large distance behaviour also plays an important role in the study of statistical systems undergoing a phase transition. In particular, the behaviour of statistical systems near a second-order phase transition ^{*)} can be related to the infra-red behaviour of field theories with massless particles ²⁾. However, in the Reggeon calculus a Pomeron with intercept exactly at one behaves formally like a particle with zero mass or energy gap, and so we expect that the infra-red behaviour of the Reggeon calculus will have the characteristics of a second order phase transition ³⁾. This is certainly true in four transverse space dimensions, and is supported in the physical case of two transverse space dimensions by various estimates of the critical exponents ⁴⁾⁻⁷⁾.

*) In statistical systems (field theories) there is a partition function (generating functional), the logarithm of which is the free energy. Crudely speaking, a phase transition is of n^{th} order if the n^{th} derivative of the free energy with respect to a source term diverges as the phase transition is approached, but the $(n-1)^{\text{th}}$ derivative does not.

A very important property of second order phase transitions is universality -- the idea that the critical exponents are independent of the details of the interactions ⁸⁾. The possibility that we can determine the asymptotic behaviour of hadronic scattering, independent of the dynamical details of the underlying physics, is certainly attractive. But this is likely to be true only if we are dealing with a second, or higher order phase transition -- that is with a critical phenomenon.

In this paper we explicitly abstract the phase transition language, and specifically study the consequences for high energy scattering of the second order phase transition expected to be associated with some Pomeron singularity having $\alpha_p(0) = 1$. We derive various inequalities between the critical exponents describing asymptotic diffraction processes. For example, (in the standard notation) we find

$$\eta < \frac{D\nu}{2}$$

where D is the number of transverse space dimensions. These relations had been previously discussed by Gribov et al. as requirements of s channel unitarity -- for instance, $\eta < (D\nu)/2$ ensures $\sigma_{el}(s) < \sigma_{tot}(s)$ asymptotically. Thus we find that many necessary conditions for s channel unitarity are satisfied by models (including the Reggeon calculus) which exhibit a second order phase transition.

We then give plausibility arguments independent of the phase transition hypothesis, that lead us to expect the Froissart bound $\eta < D$ to be obeyed by a wide class of theories. Additional speculative arguments improve this bound to $\eta < 2/3$ in the physical case, $D = 2$.

Whether a second or first order phase transition is relevant to high-energy scattering can only finally be resolved by phenomenological studies of high-energy diffractive processes. To this end, we collect combinations of experimental quantities which should exhibit critical behaviours similar to those of traditional statistical mechanical quantities such as the magnetization, entropy, specific heat, and so on. The experimental behaviours of these quantities will help to tell us whether the critical phenomenon picture is relevant, and if so, which order of phase transition is occurring. The distinction is important, since geometrical scaling and most eikonal models correspond to a first order phase transition.

Some of our relations between critical indices have also been derived by Cardy and Sugar ⁹⁾ and by Cardy ¹⁰⁾, using a lattice analogue to the Reggeon field theory. We find, however, that most of these relations

follow directly from the Kadanoff scaling laws for a second order phase transition, without having to introduce a specific model for the Pomeron.

The arrangement of this paper is as follows: in Section 2 we review the Kadanoff scaling laws expected to apply to the multi-Pomeron Green's functions near the critical region. Similar laws would be obeyed by both continuum and lattice analogue models in the same universality class. In the first part of Section 3, we derive the relations and inequalities valid if high-energy scattering is described by a second order phase transition. These conditions ensure that many requirements of s channel unitarity are satisfied, even though the Reggeon calculus was constructed to be explicitly only t channel unitary. In the latter part of Section 3, we supplement the phase transition conditions with other arguments indicating that a class of Reggeon theories may obey improved versions of the Froissart bound. In Section 4 we collect combinations of experimental high energy quantities expected to have similar critical behaviour as fundamental thermodynamic quantities, enabling one to examine phenomenologically the nature of the phase transition. We use the Feynman fluid analogue to help motivate the possibility that ISR energies may suffice for making such a study. Finally, in Section 5 we summarize our conclusions.

2. SCALING LAWS FOR ASYMPTOTIC SCATTERING

In this section we review and extend the results of Ref. 11) (hereafter referred to as BESZ) on the scaling laws for high energy scattering. We will start within the framework of the Reggeon calculus, but as explained in the Introduction, we expect the scaling forms to be more generally valid in any model with a second order phase transition.

We recall that in the Reggeon calculus of Gribov ¹⁾, the Pomeron can be described by a complex field, ψ , in D space dimensions corresponding to impact parameter ($D=2$ in the physical case), and one time dimension corresponding to rapidity. The propagator may, in general, be relativistic or non-relativistic, and the theory may have arbitrarily complicated interactions. Contributions of multiple Pomeron exchanges to high energy scattering are then described by matrix elements obtained from the Green's functions of this field theory, by treating ordinary particles as external sources. Examples are the single-Pomeron exchange of Fig. 1a, the two-Pomeron cut of Fig. 1b, the triple-Pomeron contribution of Fig. 1c, and so on.

In what follows, it will be useful to have this theory in mind as an example. However, the scaling hypotheses should be valid generally in models with second order phase transitions, as expected if $\alpha_p(0) = 1$.

Calculation of the critical exponents requires a knowledge of the scaling properties of the theory near the critical point of the phase transition. Let us denote by ϵ the difference between the temperature of the (say, lattice) theory and the critical temperature, and introduce external sources H and J coupling to the two spins ϕ and χ which represent the real and imaginary parts of the complex field, ψ ^{*}) (see, for instance, BESZ). Then the Green's functions of the theory can be obtained from a Gibbs potential

$$\mathcal{G}(\epsilon, H, J)$$

In BESZ, it was assumed, in analogy with statistical systems, that $\mathcal{G}$ is a generalized homogeneous function

$$\mathcal{G}(\lambda^{a_\epsilon} \epsilon, \lambda^{a_H} H, \lambda^{a_J} J) = \lambda \mathcal{G}(\epsilon, H, J) \quad (2.1)$$

in the neighbourhood of the critical point. This immediately implies

$$\mathcal{G}(\epsilon, 0, 0) \approx \epsilon^{1/a_\epsilon} \quad (2.2)$$

as $\epsilon \rightarrow 0$. Differentiating $\mathcal{G}$ with respect to $H(J)$ gives the "magnetizations" $\mathcal{M}$ ($\mathcal{N}$) obeying

$$\mathcal{M}(\epsilon, 0, 0) \approx \epsilon^{(1-a_H)/a_\epsilon}, \quad \mathcal{N}(\epsilon, 0, 0) \approx \epsilon^{(1-a_J)/a_\epsilon} \quad (2.3)$$

and the "isothermal susceptibilities" $\chi_{HH}, \chi_{HJ}, \chi_{JJ}$:

$$\chi_{HH} \approx \epsilon^{(1-2a_H)/a_\epsilon}, \quad \chi_{HJ} \approx \epsilon^{(1-a_H-a_J)/a_\epsilon}, \quad \chi_{JJ} \approx \epsilon^{(1-2a_J)/a_\epsilon} \quad (2.4)$$

Consideration of the two field Green's functions then indicated ¹¹⁾ that

$$a_H = a_J \quad (2.5)$$

^{*}) The lattice system proposed by Cardy and Sugar ⁹⁾ has only one spin which represents χ , the imaginary part of the field, ψ . Presumably, correlation functions of this spin have behaviour similar to that described below for the ϕ spins.

In the subsequent discussion we will just consider correlation functions between $\emptyset$ "spins": the extension to include the χ "spins" is straightforward.

In order to discuss the space-time scaling properties of the theory, it was proposed in BESZ to generalize the familiar Kadanoff construction for statistical systems, and group the lattice sites into blocks of size L in the space directions and of size L^p in the time direction, for some unknown power, p . This then led to a homogeneity law for the pair correlation function:

$$\Gamma(|\underline{r}|, \tau, \epsilon) \approx \epsilon^A g(|\underline{r}|/\epsilon^B, \tau/\epsilon^C) \quad (2.6)$$

where $(\underline{r}, \tau) \equiv (\underline{x}_\perp, iY)$ is the (space-time) separation of the two $\emptyset$ "spins", $C/B = p$ and

$$\Gamma(0, 0, \epsilon) \approx \epsilon^A ; \quad A = 2(1 - a_H)/a_\epsilon \quad (2.7)$$

The time integral of $\Gamma(\underline{r}, \tau, \epsilon)$ was denoted by $\Gamma_t(\underline{r}, \epsilon)$, and the space integral by $\Gamma_s(\tau, \epsilon)$: they have scaling laws

$$\left. \begin{aligned} \Gamma_t(|\underline{r}|, \epsilon) &\approx \epsilon^{A+C} g_t(|\underline{r}|/\epsilon^B) \\ \Gamma_s(\tau, \epsilon) &\approx \epsilon^{A+DB} g_s(\tau/\epsilon^C) \end{aligned} \right\} \quad (2.8)$$

The space-time integral of $\Gamma(\underline{r}, \tau, \epsilon)$ is just the "isothermal susceptibility" χ_{HH} :

$$\chi_{HH} \approx \epsilon^{A+DB+C} \quad (2.9)$$

Comparing with (2.4) we see that

$$A+DB+C = (1-2a_H)/a_\epsilon \quad (2.10)$$

The connection between the parameters A, B, C etc., and the critical exponents of the Reggeon calculus, was also given in BESZ. The scaling law (2.6) implies that the single exchange contribution of Fig. 1a

contributes to the elastic amplitude an amount proportional to

$$(\ln s)^{\frac{A+DB}{c}} f(k^2 (\ln s)^{\frac{2B}{c}}) \quad (2.11)$$

so that the usual Reggeon exponents η and ν are given by

$$\eta = \frac{A+DB}{c} \quad \nu = \frac{2B}{c} \quad (2.12)$$

Also of interest are some of the other critical indices defined by Gribov et al. ^{1),12)}. In BESZ it was indicated how the Kadanoff construction could be used to derive the familiar relation between η , ν and the triple Pomeron vertex index γ :

$$\gamma = 1 + \frac{3A}{2c} + \frac{DB}{c} \quad (2.13)$$

implying

$$2\gamma = 3\eta - \frac{D\nu}{2} + 2 \quad (2.14)$$

Multiple diffractive processes are asymptotically described by another index, β : the cross-section for the production of $(n+2)$ low-mass clusters as shown in Fig. 2 is asymptotically ^{*)},13)

$$\sigma_{n+2}(s) \approx (\ln s)^{2\eta - \frac{D\nu}{2} - n\beta} \quad (2.15)$$

The connection between β and A , B and C was not derived in BESZ, and just quoted in Ref. 6). A derivation is given in the Appendix of this paper. The result is

$$\beta = 1 + \frac{DB}{c} + \frac{2}{c} \quad (2.16)$$

*) Note that this includes the two-cluster production ($n=0$) corresponding to one of the discontinuities of Fig. 1b.

For reference, we also quote the relation of Gribov et al. ^{1),12)} between β and the index γ' , describing the scaling behaviour of the two-Pomeron-one-particle vertex:

$$\beta = 2\gamma' + \frac{D\nu}{2} - 2\eta - 1 \quad (2.17)$$

corresponding to [see Eq. (A.6) of the Appendix]

$$\gamma' = 1 + \frac{A + DB + 1}{C} \quad (2.18)$$

This completes our discussion of the basic scaling laws for high energy scattering. Equations (2.7) and (2.10) summarize the relations between the conventional statistic model "thermal" and "magnetic" exponents a_e and a_H and the Reggeon correlation function exponents A , B and C . Equations (2.12), (2.13), (2.16) and (2.18) give the relations between A , B and C and the Gribov exponents η , ν , γ , β and γ' . Equations (2.14) and (2.17) give two well-known equalities between combinations of the Gribov exponents.

3. THE CONSEQUENCES OF A SECOND ORDER PHASE TRANSITION

3.1 General consequences

Scaling laws such as those discussed in the previous section have often been introduced as assumptions (Kadanoff scaling hypothesis), but they also follow as solutions of the renormalization group equations ⁴⁾. The asymptotic behaviour of the Green's functions and scattering amplitudes are then characterized by giving values to these exponents. Such scaling laws are expected to hold in the vicinity of a second order phase transition, as found in the Reggeon calculus when the number of transverse space dimensions is near 4. We would expect such a transition in theories with $\alpha_p(0) = 1$ (corresponding to zero energy gap in the Reggeon field theory), a massless ϕ^4 field theory, or a statistical mechanical system at the critical temperature. We will now derive the conditions which the critical exponents must satisfy, in order that we have a second order phase transition. It will be seen that several of these constraints impose restrictions already known to be required by s channel unitarity. As we remarked earlier, various low order approximation schemes yield estimates of the critical exponents which are consistent with the existence of a second order phase transition ⁴⁾⁻⁷⁾.

The general requirements that we have on n^{th} order phase transition are simple to state. From the scaling laws of the previous section, we have

$$\frac{\partial^n \mathcal{G}(\epsilon, 0, 0)}{\partial H^n} \approx \epsilon^{(1-na_H)/a_\epsilon} \quad (3.1)$$

$$\frac{\partial^n \mathcal{G}(0, H, 0)}{\partial H^n} \approx H^{(1-na_H)/a_H} \quad (3.2)$$

For an n^{th} order phase transition ($n > 0$), we require

$$\frac{1}{n-1} > a_H > \frac{1}{n} \quad (3.3)$$

so that $\partial^n \mathcal{G} / \partial H^n$ diverges at the critical point, but $\partial^{[n-1]} \mathcal{G} / \partial H^{[n-1]}$ does not. It is also usual that if the order of a phase transition > 1 , $\partial \mathcal{G} / \partial \epsilon$ (entropy) be continuous at $T = T_c$.

Before embarking on a general discussion of the consequences of these conditions for high energy scattering, it may be useful to consider a trivial example of a scaling theory which has the characteristics of a second order phase transition.

Consider a simple Regge pole model for the pair correlation function

$$\Gamma = \frac{i}{E - \alpha' k^2 - \Delta} \quad (3.4)$$

where $E \equiv 1 - j$. This is the propagator generated by the free Lagrangian density

$$\mathcal{L} = i \psi^\dagger \overleftrightarrow{\partial}_t \psi - \alpha' \nabla \psi^\dagger \cdot \nabla \psi + \Delta \psi^\dagger \psi \quad (3.5)$$

The usual arguments for equivalence between field theories and statistical systems ²⁾ indicate that for this system near the critical point $\Delta \simeq 0$ and $T \simeq T_c$, we have $\Delta \simeq \epsilon = (T - T_c)$. Therefore Eq. (3.4) may be written in the form

$$F(k^2, E, \epsilon) = \epsilon^{-1} \frac{i}{(E/\epsilon - \alpha' k^2/\epsilon - 1)} = \epsilon^{-1} f(|k|/\epsilon^{1/2}, E/\epsilon) \quad (3.6)$$

whose Fourier transform may be written

$$\Gamma(|\mathbf{r}|, \tau, \epsilon) \simeq \epsilon^{D/2} g(|\mathbf{r}|/\epsilon^{-1/2}, \tau/\epsilon^{-1}) \quad (3.7)$$

Comparison of (3.7) with the postulated scaling law (2.6) tells us that in this case

$$A = D/2, \quad B = -\frac{1}{2}, \quad C = -1 \quad (3.8)$$

Using (2.7) and (2.10), we find

$$a_\epsilon = \frac{2}{D+2}, \quad a_H = \frac{D+4}{2(D+2)} \quad (3.9)$$

For $D > 0$,

$$0 \leq a_\epsilon < 1, \quad \frac{1}{2} \leq a_H < 1 \quad (3.10)$$

so that the first derivatives of $\mathcal{G}$ are continuous, while the susceptibility diverges. The theory thus has the characteristics of a second order phase transition. Notice also that for $D > 2$, $a_\epsilon < \frac{1}{2}$, so that C_H does not diverge at the critical point. Using (2.16) and (3.8), we can also see that this means that $\rho \geq 0$, a necessary condition in order that s channel unitarity be satisfied. For later reference, we remind the reader that the pole scaling form (3.4), in addition to describing a non-interacting Reggeon field theory, also describes (to within logarithmic factors) the infra-red behaviour of the propagator in the standard interacting theory when $D = 4$.

We now want to ask what are the general, model independent consequences for asymptotic scattering of the requirement that the hadronic phase transition be of second order.

First, what is implied by the requirement that the transition is not of 0th order? From (3.1)-(3.3), this means that $1/a_H, 1/a_\epsilon > 0$. Using (2.7) and (2.10), we find

$$\left. \begin{aligned} \frac{1}{a_\epsilon} &= -DB - C > 0 \\ \frac{1}{a_H} &= \frac{DB + C}{\frac{1}{2}A + DB + C} > 0 \end{aligned} \right\} \quad (3.11)$$

so

$$-DB - C > \frac{1}{2}A \quad (3.12)$$

From the first inequality in (3.11) and the scaling expression (2.6), we see that some combination of $|\underline{r}|$ and $|\tau|$ is becoming infinite when $\epsilon \rightarrow 0$. In fact, for the phase transition picture to be applicable, we really need both B and $C < 0$ individually. The reason is that when $\epsilon \rightarrow 0$ we want to be able to explore the large space (time) structure of the theory, and so $B(C) < 0$ in order that $|\underline{r}|/\epsilon^B$ (τ/ϵ^C) stay fixed as $\epsilon \rightarrow 0$ with $|\underline{r}|$ ($|\tau|$) $\rightarrow \infty$.

Now, using (2.12), (3.11), (3.12), and $C < 0$, we find

$$\frac{Dv}{4} + \frac{1}{2}\eta > -1 \quad (3.13)$$

$$\frac{Dv}{2} > 0 \quad (3.14)$$

Moreover, from (2.16) and (2.17), we have

$$\beta < 1 + \frac{Dv}{2} \quad (3.15)$$

$$\gamma' < 1 + \eta \quad (3.16)$$

(3.13)-(3.16) follow simply from the absence of a 0th order phase transition.

Suppose now we require, in addition, that the phase transition not be first order. We then have

$$1 - a_H > 0$$

which, using (3.11), implies

$$A > 0 \quad (3.17)$$

Since $C < 0$, this leads immediately to

$$\eta < \frac{D\nu}{2} \quad (3.18)$$

which can also be expressed in the form

$$\gamma < 1 + \frac{D\nu}{2}$$

If in addition we want the entropy $[\alpha(\partial G/\partial \epsilon)|_H]$ to be continuous at the phase transition (that is, that there be no latent heat), then we are led to the requirement $a_\epsilon < 1$, which means

$$DB + C < -1 \quad (3.19)$$

Using (2.16) and (3.19) we find

$$\beta > -1 - \frac{D\nu}{2} \quad (3.20)$$

which can be combined with (3.15) to read

$$|\beta| < 1 + \frac{D\nu}{2} \quad (3.21)$$

Expression (3.18) is a particularly important inequality, and has an interesting connection with s channel unitarity. Reference back to Eq. (2.15) shows that the cross-section for diffractively producing two low-mass clusters of particles is

$$\sigma_2(s) \approx (\ln s)^{2\eta - \frac{D\nu}{2}} = (\ln s)^\eta \times (\ln s)^{\eta - \frac{D\nu}{2}} \quad (3.22)$$

Thus the condition (3.18) ensures that the simple exchange diagram of Fig. 1a dominates asymptotically over two-cluster production: in particular $\sigma_{el}(s) < \sigma_T(s)$ asymptotically. Consider also the asymptotic contribution to the total cross-section of the connected N point Pomeron Green's functions shown in Fig. 3. This is

$$\sigma_T^{[N]} \propto (\ln s)^{\eta + \frac{1}{2}(N-2)(\eta - \frac{D\nu}{2})} \quad (3.23)$$

Hence, the asymptotic dominance of σ_T by the two-point function is a consequence of condition (3.18), that is, of the absence of a first order phase transition.

The inequality (3.18) also ensures that the asymptotic two-point function (and thus the scattering amplitude) respects another s channel unitarity condition, namely that the opacity is bounded (and in fact decreases) as $\ln s \rightarrow \infty$. To sum up, the inequality (3.18) implies that the simple diagram of Fig. 1a dominates the asymptotic amplitude with no subtle resummation over all Pomeron Green's functions being necessary. Furthermore, this form obeys several of the requirements of s channel unitarity.

Having established the consequences of the phase transition being of higher order (so that we are dealing with a critical phenomenon), we discuss the consequences of assuming that the phase transition is precisely of second order. From (3.3), we see that this means that

$$a_H = \frac{\frac{1}{2}A + DB + c}{DB + c} \geq \frac{1}{2} \quad (3.24)$$

so

$$A \leq -(DB + c) \quad (3.25)$$

from which it is easy to derive

$$\eta \geq -1 \quad (3.26)$$

and

$$\gamma + \frac{D\nu}{4} \geq -\frac{1}{2}$$

and, using (2.17)

$$\gamma' + 1 + \frac{D\gamma}{2} \geq \frac{\beta}{2} \quad (3.27)$$

In discussing the order of the phase transition, we have primarily been concerned with derivatives of $\mathcal{G}$ with respect to H . We may wonder what is the behaviour and meaning of derivatives of $\mathcal{G}$ with respect to ϵ ? In particular, how may we expect the specific heat

$$C_H \approx \frac{\partial^2 \mathcal{G}}{\partial \epsilon^2} \approx \epsilon^{(1-2a_\epsilon)/a_\epsilon}$$

to behave near the critical point? In the two-dimensional Ising model, C_H has a logarithmic singularity at the critical point, so that $a_\epsilon = \frac{1}{2}$. In three dimensions C_H diverges like a power so that $a_\epsilon > \frac{1}{2}$. On the other hand, there are many examples of systems in which C_H does not diverge when $\epsilon \rightarrow 0$ ¹⁴⁾, so we can place no rigorous upper limits on a_ϵ . In the example of a simple pole described in the beginning of this section, we saw that for $2 < D$, $C_H \rightarrow 0$ as $\epsilon \rightarrow 0$. Now when $D=4$, the simple pole behaviour is the correct solution for the interacting Reggeon calculus, so we may expect -- although this is far from rigorous [indeed, there are counter-examples ¹⁴⁾] -- that $C_H \rightarrow 0$ for the interacting theory also when $D=2$. If we assume this, then

$$a_\epsilon = \frac{-1}{DB+C} \leq \frac{1}{2} \implies -DB-C \geq 2 \quad (3.28)$$

which implies immediately that

$$\beta \geq 0 \quad (3.29)$$

As seen from the formula (2.15), this is just the condition that is necessary to ensure that multiple low mass diffraction processes be asymptotically smaller than the total cross-section calculated from the graph of Fig. 1a. A violation of condition (3.29) would clearly mean an asymptotic violation of s channel unitarity.

There is another inequality which is imposed by s channel unitarity, namely

$$2\gamma' \leq \gamma$$

This was discussed by Gribov et al. ¹⁾, by Migdal et al. ¹²⁾, and by Cardy ¹⁰⁾: this inequality does not follow from simple arguments about the nature of the phase transition.

In this subsection we have explored the exponent inequalities which result from imposing the condition that we are dealing with a second order phase transition. These results are summarized in Table 1. Perhaps the most important result is inequality (3.18) which follows from the condition that the magnetization does not diverge at the phase transition. This inequality is important for the consistency of the Reggeon calculus -- at least without having to perform a resummation of diagrams -- and distinguishes it from eikonal models and geometrical scaling models which violate (3.18) and generally have the characteristics of a first order phase transition, as we shall see in Section 4.

Among the other inequalities is (3.29) which follows from the finiteness of the specific heat at the critical point and is necessary for our system to obey s channel unitarity. While this is not a strict requirement for a second order phase transition, it is certainly plausible and is consistent with it. Conspicuous by its absence from this discussion, is the inequality $\nu \leq 2$ which leads, via (3.18), to the Froissart bound. This is discussed in subsection 3.2.

Condition	Results	
	Scaling exponents	Physical exponents
Order of phase transition > 0	$B, C < 0$ $-DB - C > A/2$	$\beta < 1 + D\nu/2$ $\nu > 0$ $\gamma' < 1 + \eta$ $\frac{1}{2}\eta + D\nu/4 > -1$
Order of phase transition > 1	$A > 0$	$\eta < D\nu/2$ $\gamma < 1 + D\nu/2$
Entropy continuous	$DB + C < -1$	$\beta > -1 - D\nu/2$
Order of phase transition $= 2$	$A \leq -(DB + C)$	$\eta \geq -1$ $\gamma + D\nu/4 \geq -\frac{1}{2}$ $\gamma + 1 + D\nu/2 \geq \beta/2$
$C_H \neq \infty$	$-DB - C \geq 2$	$\beta \geq 0$

3.2 The Froissart bound

We saw in the previous subsection that if the hadronic phase transition is second order, then the simple exchange of Fig. 1a dominates the cross-section, with a scaling law (2.11) for the elastic amplitude and the inequality (3.18)

$$\eta < \frac{D\nu}{2}$$

which is required by s channel unitarity. But s channel unitarity also requires constraints $\nu \leq 2$ and thus $\eta \leq D$: are these conditions likely to be satisfied in the Reggeon calculus and associated models?

One heuristic argument can be formulated by considering a Regge model with an input Schwarz trajectory:

$$j \simeq 1 + \bar{\alpha} \sqrt{-t} \quad ; \quad t \rightarrow 0 \quad (3.30)$$

This trajectory is self-reproducing under t channel unitarity iterations, as a consequence of the formula

$$\alpha_c(t) = 2\alpha_p(t/4) - 1 \quad (3.31)$$

for the position of the two-Reggeon cut. This property is reflected in the Reggeon field theory where to get a trajectory of the form (3.30), it is necessary to take a relativistic Pomeron propagator which has

$$E^2 = (j-1)^2 = \bar{\alpha}^2 k^2 = -\bar{\alpha}^2 t \quad (3.32)$$

in which $\bar{\alpha}$ takes the role of the velocity of light and is unrenormalized. Reference to the scaling law (2.11) shows that the condition (3.30) or (3.32) corresponds to $\nu = 2$. Thus the non-renormalization result says that if $\nu_{\text{bare}} = 2$, then $\nu_{\text{ren}} = 2$ also.

Let us now consider any theory in which $\nu_{\text{bare}} < 2$: an example is the simple linear trajectory

$$j = 1 + \alpha' t \quad ; \quad t \rightarrow 0 \quad (3.33)$$

which has $\nu_{\text{bare}} = 1$. This trajectory is less steep than (3.30) at $t \approx 0$, and correspondingly has less shrinkage for $t \approx 0$. It is intuitively plausible (and seems to be true in models) that this property of less shrinkage is retained when the theory is subjected to t channel iteration. Hence, we might be led to expect that if we take any Reggeon theory with $\nu_{\text{bare}} < 2$, then the non-renormalization theorem for $\nu = 2$ will also imply $\nu_{\text{ren}} < 2$. If this argument were correct, then it would suggest via (3.18) that any theory which had an input trajectory less steep than (3.30) at $t \approx 0$, and which had a second order phase transition, would satisfy the Froissart bound as a strict inequality^{*)}.

Reference back to subsection 2 shows that satisfying the Froissart bound corresponds to $B \leq C$. Now we should remember that when D is near 4, $B < C$, and so, a violation of the Froissart bound in the physical case $D = 2$ requires a crossing of the eigenvalues B and C at some $2 < D < 4$. Such a crossing would result in the "non-relativistic" linear and "relativistic" Schwarz trajectory models, having identical values of $\nu = 2$ for some $D > 2$ which might seem unlikely.

The requirement that $B \leq C$ makes it clear why a second order phase transition which only gives constraints on A and on the combination $DB + C$, cannot by itself imply satisfaction of the Froissart bound. Thus, in this approach it may be difficult to do any better than the emphatically non-rigorous arguments presented here.

If the reader is still prepared to suspend her or his disbelief, we can improve significantly on the Froissart bound by making even more speculative assumptions. Consider a theory in 1 time and 1 transverse space dimension. Then a theory with the free propagator $\propto i/(E - \alpha'k)$ has "relativistic" symmetry between E and k and hence $\nu_{\text{ren}} = 2$. If a 1+1 dimensional theory is described by a free propagator such as $i/(E - \alpha'(k)^2)$, then as above it probably has $\nu_{\text{ren}} < 2$. Therefore if it has a second order phase transition, it will have $\eta < (D\nu)/2 < 1$. In going now to two-transverse dimensions, several speculations suggest themselves.

- 1) In many phase transitions studied experimentally, anomalous dimensions and other critical indices seem to be concave functions of $\epsilon = 4 - D$. This is also generally true of ϵ expansions, and specifically of the ϵ expansions for η and ν in the Reggeon field theory. If ν is a concave function of $\epsilon = 4 - D$, then $\nu < 1 + [(4-D)/3]$ (since $\nu = 1$ is the canonical value, and $\nu < 2$ in $D = 1$). Therefore in two-transverse dimensions $\nu < 5/3$ and $\eta < \nu$ implies $\eta < 5/3$.

*) Notice that if the Reggeon calculus has a phase transition whose order is > 1 , and if it satisfies s channel unitarity, then the Froissart bound is satisfied as a strict inequality.

- 2) It certainly seems plausible that the anomalous dimension will be a monotonic function of $\epsilon = 4 - D$. Therefore, since in one transverse dimension $\eta < D/2 \times \nu < 1$, one might expect in two dimensions also that $\eta < 1$.
- 3) If, further, η is a concave function of $\epsilon = 4 - D$, then in two dimensions $\eta < 2/3$.

Needless to say, these bounds have very low credibility, but they are stringent enough to be interesting.

4. SOME PHENOMENOLOGICAL IMPLICATIONS

A detailed comparison of the asymptotic solutions of the Reggeon calculus with experiment is not possible until the scaling terms dominate. The energies necessary to achieve this asymptopia depend, of course, on the size of the next-to-leading terms in the expressions of interest. While there is no reason to believe that present (ISR) energies are sufficient to clearly see the final scaling terms ¹⁵⁾, we can argue that ISR energies are sufficient to qualitatively see the onset of critical behaviour. Roughly, the argument is as follows: the beginning of the transition region between low energies, where simple perturbation theory is valid, and asymptopia, should occur when renormalization effects begin to be significant. The simplest graph contributing to renormalization is shown in Fig. 4. In order to meaningfully draw this graph we need at least six units of rapidity, since at least two units of rapidity are necessary for each sub-amplitude to be represented by a Pomeron. Hence, for $\ln s \lesssim 6$ we are certainly in the low energy region, while for $\ln s \gtrsim 6$ we may hope to begin to see the effects of renormalization. The length of the transition region will determine what energies are necessary to enter asymptopia and to see the true scaling behaviour, but in any event, we expect to enter the transition region when $\ln s \sim O(6)$. Since the ISR ranges up to $\ln s \sim 10$, we have roughly four units of rapidity in which to search for the effects of the approach to asymptopia. Of course, this is only an order of magnitude argument, and our estimate for the beginning of the transition region could be off by some factor of order 1. Nevertheless, it is worth while to ask what general, qualitative features may be expected in the transition region, and to try to compare these expectations with available data.

To help us understand what to expect in this region, it is useful to consider for a moment the s channel states which build up the total cross-section. In particular, we want to examine qualitatively how the intermediate states change when we approach the critical region. We can do

this by letting $T \rightarrow T_c$, or alternatively, by considering what happens when $\alpha_p(0) \rightarrow 1$ ³⁾.

Suppose $\alpha_p(0) < 1$ ($T > T_c$). Then the leading j plane singularity comes from exchange of a simple pole, and we generally expect that the important intermediate states which build this singularity are those with a uniform distribution of particles in rapidity, as shown in Fig. 5. If we now let $\alpha_p(0) \rightarrow 1$ ($T \rightarrow T_c$), graphs such as those shown in Figs. 4 and 6 become equally important. Some of the intermediate states which build these graphs consist of high mass clusters of particles widely separated in rapidity, as indicated in the rapidity "snapshot" of Fig. 7. In addition, of course, graphs like Figs. 4 and 6 have some contributions with no rapidity gaps, like those in Fig. 4. In the language of the Feynman fluid, we see that as $T \rightarrow T_c^+$ [$\alpha(0) \rightarrow 1^-$] we begin to observe droplets -- the high mass clusters of particles -- condense out of the gas phase. The existence of these droplets asymptotically results in large fluctuations in the particle density distribution, and causes certain thermodynamic quantities to diverge -- that is, we see a phase transition.

Thus, in both the language of the Reggeon calculus and in the language of the Feynman fluid analogy we can understand the appearance of a singularity which gives rise to a phase transition. There is, however, an important conceptual difference in these languages which should be stressed. In the Reggeon calculus (field theory, or lattice theories in the thermodynamic limit), we deal with a system which a priori has an infinite number of degrees of freedom. The critical singularity is approached in these systems by varying some parameter, say the temperature, or by looking at the large distance behaviour of Pomeron correlation functions embedded in a space of infinite extent. In the Feynman fluid, on the other hand, the critical singularity is approached by first setting $T = T_c$ [$\alpha_p(0) = 1$], and then increasing the number of degrees of freedom -- i.e., the size of the cylinder and the number of produced particles.

Armed with this understanding, we can begin to formulate some tests for the existence of a second order phase transition which might be useful at present energies. In what follows, we shall describe only the simplest of these droplets asymptotically results in large fluctuations in the particle density distribution, and causes certain thermodynamic quantities to

In statistical physics, it is often true that thermodynamic quantities behave in a way qualitatively similar to their critical behaviour, even when relatively few degrees of freedom are involved. For example, the tendency of the susceptibility to diverge as $T \rightarrow T_c$ may show up at reasonably low

orders in the high temperature expansion, although only very few lattice spacings are involved. Experimentally, the tendency of the susceptibility to diverge as $T \rightarrow T_c$ may be apparent even when T is relatively far from T_c . Using the scaling laws, we can find experimentally measurable quantities which have the same divergence when $\ln s \rightarrow \infty$ as do standard thermodynamic quantities when the critical point is approached, and we may hope they will also have qualitatively similar behaviour in the presently accessible transition region.

Consider, for instance, the susceptibility χ_T which behaves when $T \rightarrow T_c$ as

$$\chi_T(\epsilon) \approx \epsilon^{A+DB+C} \approx \delta^{-\left(\frac{A+DB}{C}\right)-1} \quad (4.1)$$

where $\delta = \epsilon^{-C}$, and since $C < 0$, $\delta \rightarrow 0$ as $\epsilon \rightarrow 0$. Now, defining

$$\chi_1(s) \equiv \sigma_T(s) \ln s \quad (4.2)$$

we see that

$$\chi_1 \approx \left[(\ln s)^{-1} \right]^{-\left(\frac{A+DB}{C}\right)-1} \equiv r^{-\left(\frac{A+DB}{C}\right)-1}$$

where $r = (\ln s)^{-1}$, and has the same qualitative behaviour with r as the susceptibility has with δ , and so for a second order phase transition χ_1 should diverge.

We turn now to the specific heat which is expected not to be divergent in the critical region. The scaling term of this object has the form

$$C_H \approx \epsilon^{-(DB+C+2)} \approx \delta^{\frac{DB+C+2}{C}} \quad (4.3)$$

This behaviour is mocked as a function of r by the function $C_1^{(n)}$ defined by:

$$C_1^{(n)} = \left[\frac{\sigma_{n+2}}{\sigma_{22}} \right]^{1/n} \quad (4.4)$$

which behave as

$$C_1^{(n)} \approx r^{\frac{DB+2}{c}+1} = r^\beta$$

All such functions $C_1^{(n)}$ should $\rightarrow 0$ asymptotically, and eventually at the same rate. As an example, the double Pomeron exchange diagram of Fig. 8, in which a dipion system is produced in the central region, behaves like σ_3 when $\ln s \rightarrow \infty$, and so gives a test of the nature of the specific heat.

The magnetization also has a scaling term which should $\rightarrow 0$ as $\epsilon \rightarrow 0$. It is given by

$$m \approx \epsilon^{A/2} \approx \int \epsilon^{\frac{-A}{2c}} \quad (4.4)$$

Translating this exponent into observables, we can write

$$m_1^2 = \sigma_{el}/\sigma_T \quad (4.5)$$

which should behave as $r^{-A/c}$, so that m_1^2 should approach zero, asymptotically. As we have discussed before, the physical implication of a non-divergent magnetization in the Reggeon calculus is just that the elastic cross-section not grow as rapidly, as the total cross-section asymptotically.

Turning now to the entropy, we expect it to be continuous at the critical point, and so its singular term,

$$S \approx \epsilon^{-(DB+c+1)} \approx \int \epsilon^{\frac{DB+c+1}{c}} \quad (4.6)$$

should approach zero as $\delta \rightarrow 0$. Hence, if we introduce

$$S_1^2 = \left[\frac{\sigma_{m2}}{\sigma_{el}} \right]^{1/n} \frac{\sigma_{el}}{\sigma_T^2 \ln s} \quad (4.7)$$

then it should behave as

$$S_1^2 \approx r^{\frac{2(DB+c+1)}{c}}$$

and S_1^2 ought to approach zero asymptotically.

Finally, the singular part of the Gibbs free energy,

$$\mathcal{G} \approx \varepsilon^{-DB-C} \approx \int \frac{DB}{\varepsilon} + 1 \quad (4.8)$$

should approach zero in the critical region, which means that

$$\mathcal{G}_1 = \frac{\sigma_{el}}{\sigma_T^2 \ln s} \quad (4.9)$$

which should behave as $r^{(DB/C)+1}$, should approach zero asymptotically.

From the scaling laws of Section 2 it is apparent that

$$\left. \frac{\sigma_{el}}{\sigma_T^4} \frac{d}{dt} \left(\frac{d\sigma}{dt} \right)_{el} \right|_{t=0} \sim \text{const.} \quad (4.10)$$

as $\ln s \rightarrow \infty$. A check of (4.10) would therefore be a check on the scaling laws we have been assuming. Equally well, Eq. (4.10) could be combined with the previous equations to give alternative measurable experimental quantities analogous to thermodynamic quantities. For example, an alternative to χ_1 defined in (4.2) would be

$$\chi_2 = \frac{1}{\sigma_T \ln s} \left[\sigma_{el} \frac{d}{dt} \left(\frac{d\sigma}{dt} \right)_{el} \right]_{t=0}^{1/2} \quad (4.11)$$

for the susceptibility. Checking that χ_1 and χ_2 (and similar combinations for other thermodynamic quantities) have similar behaviour for large $\ln s$ would indicate the size of non-leading terms in these expressions and serve as a check on the Kadanoff scaling laws assumed in this paper.

It is clear that some of these expressions will be more useful than others in testing the theory. First, the behaviour of some of the quantities, for example (4.9), is trivial and will follow from any sensible theory. Furthermore, the scaling property of the elastic amplitude, which is responsible for expression (4.10) is also possessed by eikonal and geometrical scaling models. Thus, these theories also predict identical asymptotic behaviour of the quantities χ_1 and χ_2 , etc. Moreover, these

theories also predict asymptotic divergence of χ_1 and χ_2 , although the exponent will generally be larger than the one predicted by the Reggeon calculus.

A quantity which can clearly distinguish between non-critical theories and the second order phase transition solution is $\mathcal{M}$, the magnetization. Most s channel models predict $\mathcal{M}_1 \rightarrow \text{const.}$ asymptotically. It is only in a critical theory, such as the one discussed here that the phase transition becomes second order, so that $\mathcal{M} \rightarrow 0$. Unfortunately, this asymptotic behaviour may be difficult to see at present energies, since the exponent is probably rather small, so non-leading terms may have a significant effect. Phenomenologically, geometrical scaling which has $\mathcal{M}_1^2 = \sigma_{\text{el}}/\sigma_{\text{T}} \rightarrow \text{const.}$ ($\eta = \nu$) and hence corresponds to a first order phase transition, is a reasonable approximation to present data. It remains to be seen whether this is an asymptotic truth or an approximation to behaviour in a limited energy range.

We want to mention one other experimental test, whose significance can be most easily understood in the Feynman fluid language. For large rapidity separations, we expect to be able to parametrize the correlation functions as

$$\frac{1}{\sigma_{\text{T}}} \frac{d^2\sigma}{dy_1 dy_2} - \frac{1}{\sigma_{\text{T}}^2} \frac{d\sigma}{dy_1} \frac{d\sigma}{dy_2} \sim e^{-y/3(Y)}$$

where $y = y_1 - y_2$, and if we sit at the critical temperature, the correlation length, $\xi \sim Y^\alpha$ ($Y = \ln s$). This asymptotic divergence of ξ is caused by the onset of large fluctuations in the fluid, and is the sine qua non of a second order phase transition. Hence we should look for a breakdown of the picture of simple short range correlations as the energy increases.

It is obviously going to be difficult to see the most important of the behaviours we have discussed in the present ISR data, and one may have to squint a lot in order to draw even tentative conclusions. The effects which we seek, however, may show up more clearly by making judicious cuts on the data which will enhance the critical region. Choosing primarily high multiplicity and very small t events are the sorts of restrictions which are likely to be helpful. In any case, it is clear that there are a host of experimental quantities, only the simplest of which we have discussed, which can be examined to search for signals of a second order phase transition. The qualitative tendency of these quantities to behave as expected asymptotically, may lend credence to the notion that high energy scattering is a critical phenomenon.

5. SUMMARY

The purpose of this paper has been to explore the consequences of the existence of a second order phase transition in the description of high energy scattering. One important feature of this condition should be emphasized: it is only when the phase transition is of second order that we can expect universality to be obeyed -- that is, only if the phase transition is of second order, is it likely that the asymptotic behaviour of hadronic scattering will be independent of the details of the physics.

Having assumed the Kadanoff scaling laws which should be valid near the critical region (and whose validity is supported by the renormalization group solutions of the Reggeon calculus), we proceeded to derive the relations among the critical point exponents which follow if the phase transition is of second order. Among these relations are some that, from another point of view, follow from the imposition of s channel unitarity. It is rather remarkable that these inequalities follow from simple assumptions concerning the order of the phase transition, without explicit reference to s channel constraints. On the other hand, not all s channel unitarity requirements are implied by our assumptions (notably the Froissart bound). To draw any inferences about these requirements, one must rely on more dynamical information, or, as we have done, have recourse to plausibility arguments.

We also discussed briefly some experimental tests which may shed some light on the applicability of these ideas. Relying partly on intuitions gained from the Feynman fluid analogy, we argued that present ISR energies may be sufficient to see the approach to the critical region, even though the asymptotically dominant terms cannot be measured. There are undoubtedly other tests of the theory which, in the same spirit, may be useful at present energies. A careful analysis of the data to look for trends of the type described here would be most welcome.

The view of high energy scattering as a critical phenomenon is most appealing. It is supported by the beautiful strong coupling solution of the Reggeon calculus, and would probably emerge from a proper treatment of hadronic scattering in the Feynman fluid language. Moreover, universality is expected to be applicable so that the asymptotic behaviour of scattering amplitudes is independent of the details of the underlying physics. These properties make it an intriguing subject for further theoretical and experimental investigation.

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APPENDIX

We recall that in the standard Reggeon field theory models, the difference $\Delta = 1 - \alpha_p(0)$ of the Pomeron intercept from 1 is represented by a $\psi^+\psi$ type term in the Lagrangian. Thus this "energy gap" term formally resembles a mass term in a relativistic field theory. It is well known that in making the transition from an Ising type model to a relativistic ϕ^4 field theory analogue, the temperature factor $1/kT$ in the lattice free energy introduces a temperature dependence into the parameters of the Lagrangian, including the mass term. Varying the temperature in the neighbourhood of T_c corresponds to varying the bare unrenormalized field theoretic mass, in such a way that the renormalized mass is near zero. Taking derivatives of the free energy with respect to $\epsilon = T - T_c$ in the neighbourhood of $\epsilon = 0$, corresponds to considering field theoretic Green's functions with mass insertions. A similar chain of arguments applies to Reggeon theories, implying that letting the operator $\partial/\partial\epsilon$ act on the Gibbs potential of the lattice analogue model is like inserting the $\psi^+\psi$ operator into the field theoretic Green's functions. This connection is of interest because the simplest operator for emitting a real particle from a multi-Pomeron Green's function is (see, for example, Fig. 2):

$$M\psi^+\psi \quad (A.1)$$

where M is some c number source.

If we now work with a lattice analogue model and introduce a parameter $\bar{A}$:

$$\frac{\partial G}{\partial \epsilon} \approx \epsilon^{1/4\epsilon-1} \approx \epsilon^{-DB-C-1} \approx \epsilon^{\bar{A}} \quad \text{as } \epsilon \rightarrow 0 \quad (A.2)$$

then we have

$$\bar{A} = -DB-C-1 \quad (A.3)$$

Applying the generalized Kadanoff construction to a Green's function

Γ_{n,m_1,m_2} which has n $\psi^+\psi$ insertions, m_1 external ψ fields, and m_2 external ψ^+ fields, then for $\epsilon \sim 0$:

$$\Gamma_{n,m_1,m_2} \approx \epsilon^{n\bar{A} + \frac{1}{2}(m_1+m_2)A} \Gamma(|x_i|/\epsilon^B, \gamma/\epsilon^C) \quad (A.4)$$

In particular, if we consider the case $n=m_1=m_2=1$ corresponding to the simplest vertex for the production of a real particle in the central region, then in $(E, \underline{k})$ space the vertex with external legs amputated obeys

$$\bar{\Gamma}(E, \underline{k}^2) \simeq E^{-\bar{A}/c} \simeq E^{\frac{A+DB+C+1}{c}} \quad \text{for } E \rightarrow 0 \quad (\text{A.5})$$

where we have used Eq. (A.3). This exponent is called γ' by Gribov et al. ^{1),12)} and so,

$$\gamma' = \frac{A+DB+C+1}{c} \quad (\text{A.6})$$

a result quoted in the text as Eq. (2.18). The analysis can easily be extended to the case $n>1$, $m_1=m_2=1$ which is illustrated by the diagram in Fig. 2. Using the scaling law (A.4), it can be seen that the cross-section for diffractively producing $(n+2)$ low mass clusters of particles, behaves as

$$\sigma_{nn} \simeq (hs)^F ; \quad F = \frac{2h\bar{A}+2A+(n+1)DB+hc}{c} \quad (\text{A.7})$$

Comparing Eq. (A.7) with the definition of β in Eq. (2.15) and using the expression (A.3), we see that

$$\beta = 1 + \frac{DB}{c} + \frac{2}{c} \quad (\text{A.8})$$

a result quoted in the text as Eq. (2.16), and used without proof in the calculations of Ref. 6). Combining (A.8) with the relations (2.12) and (A.6), we finish up with

$$\beta = -2\eta + 2\gamma' - 1 + \frac{D\nu}{2} \quad (\text{A.9})$$

quoted in the text as Eq. (2.17).

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FIGURE CAPTIONS

- Fig. 1a : Exchange of a single renormalized Pomeron.
- 1b : Exchange of two disconnected renormalized Pomerons.
- 1c : The renormalized three-point Pomeron Green's function.
- Fig. 2 : Production of $(n+2)$ low-mass clusters separated by $(n+1)$ large rapidity gaps.
- Fig. 3 : Connected, renormalized N point Pomeron Green's function.
- Fig. 4 : Lowest order graph contributing to the renormalization of the Pomeron.
- Fig. 5 : Intermediate states which are expected to be important in building up a simple pole, and their expected distribution in rapidity.
- Fig. 6 : Complicated diagram contributing to Pomeron renormalization.
- Fig. 7 : Intermediate states with high-mass clusters coming from discontinuities of graphs like Figs. 4 and 6, and a rapidity snapshot of the state.
- Fig. 8 : A process which behaves asymptotically like σ_3 , defined in the text.

FIG.1

FIG.2

FIG.3

FIG.4

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FIG. 7

FIG. 8