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CALCULATION OF THE ANNIHILATION RATE
OF P WAVE QUARK-ANTIQUARK BOUND STATES

R. Barbieri, R. Gatto ^{*)} and R. Kögerler
CERN -- Geneva

A B S T R A C T

In the framework of the gauge theory of strong interactions (in particular for the charm scheme) we calculate the annihilation rates of P wave quark-antiquark bound states of $J^{PC} = 0^{++}, 2^{++}$. Applications can be made to the decays of the $C=+1$ states lying between ψ and ψ' , to their gluonic production, and to f' decay. Annihilations into 2γ and the Primakoff productions are also discussed.

^{*)} and Istituto di Fisica dell'Università,
Rome, Italy.

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A most appealing scheme for strong interaction dynamics is based on colour triplets of fermions (quarks) interacting through colour octets of massless gauge vector-mesons (gluons). Within such a scheme and under some additional conjectures on the relevant dynamics, the ψ decay rate into hadrons, for instance, has been calculated in terms of the three-gluon annihilation of a heavy quark-antiquark S-wave bound system¹⁾.

We present here the rather more complex calculation of the rates for two-gluon annihilation of P-wave quark-antiquark bound states. A similar calculation does not exist in the positronium literature. Our calculation holds for any scheme where the relevant meson states are described in terms of heavy fermion pairs annihilating through massless gluons. The quantum numbers are, for quark-antiquark P-waves, $J^{PC} = 0^{++}, 1^{++}, 2^{++}, \text{ and } 1^{+-}$. Our two-gluon calculation concerns 0^{++} and 2^{++} . Annihilation into two gluons of 1^{++} is indeed forbidden²⁾. In particular we shall obtain for the ratio of the 2^{++} annihilation rate to the 0^{++} annihilation rate, in the approximation of zero binding, the result

$$\frac{\Gamma_{\text{ann}}(2^{++})}{\Gamma_{\text{ann}}(0^{++})} = \frac{4}{15} \quad , \quad (1)$$

independent of any parameter.

The main purpose of our work is to apply the results to the newly found states³⁾ in the mass region between ψ and ψ' , which have been described as P-wave bound states of a heavy quark (to be called Q) and its antiquark⁴⁾. The two-gluon annihilation rates $\Gamma_{\text{ann}}(2^{++})$ and $\Gamma_{\text{ann}}(0^{++})$ give then directly the measured annihilation rates into hadrons of such states (through violation of the Zweig rule⁵⁾). The same would apply to annihilation into non-strange hadrons (via Zweig-rule violation) of $f'(1514)$, if described as a 2^{++} P-wave ($s\bar{s}$)-state, except that our approximations may badly fail in that case.

Whereas the result in Eq. (1) for the zero binding approximation is independent of dynamical parameters, the formulae we obtain separately for the widths, are

$$\Gamma_{\text{ann}}(0^{++}) = \frac{96\alpha_s^2}{M^4} |\Phi'_P(0)|^2 \quad (2)$$

$$\Gamma_{\text{ann}}(2^{++}) = \frac{128}{5} \frac{\alpha_s^2}{M^4} |\Phi'_P(0)|^2 \quad , \quad (3)$$

where M is the bound state mass, $\alpha_s = g^2/4\pi$ the gluonic fine-structure constant, and $\Phi'_P(0)$ is the derivative at $r = 0$ of the radial wave function of the bound state.

If the lowest-order QED diagrams are employed to calculate annihilation into two photons [which might turn out to be a risky assumption⁵⁾], one gets for the ratios of two-photon to the above two-gluon annihilation

$$\frac{\Gamma(2^{++} \rightarrow \gamma\gamma)}{\Gamma_{\text{ann}}(2^{++})} = \frac{\Gamma(0^{++} \rightarrow \gamma\gamma)}{\Gamma_{\text{ann}}(0^{++})} = \frac{9}{2} e_Q^4 \left(\frac{\alpha}{\alpha_s} \right)^2, \quad (4)$$

where e_Q is the electric charge of the constituent quark. Similar considerations for the pseudoscalar lowest S-bound state 0^{-+} and its first radial excitation $(0^{-+})'$ give

$$\frac{\Gamma_{\text{ann}}((0^{-+})')}{\Gamma_{\text{ann}}(0^{-+})} = \frac{\Gamma(1^{--}' \rightarrow e^+e^-)}{\Gamma(1^{--} \rightarrow e^+e^-)} \quad (5)$$

$$\frac{\Gamma_{\text{ann}}(0^{-+})}{\Gamma(1^{--} \rightarrow e^+e^-)} = \frac{2}{3 e_Q^2} \left(\frac{\alpha_s}{\alpha} \right)^2 \quad (6)$$

$$\frac{\Gamma(0^{-+} \rightarrow \gamma\gamma)}{\Gamma_{\text{ann}}(0^{-+})} = \frac{9}{2} e_Q^4 \left(\frac{\alpha}{\alpha_s} \right)^2, \quad (7)$$

where 1^{--} and $(1^{--})'$ are the corresponding vector states (ψ and ψ' for the heavy quark system).

The transition amplitude of a J^P quark-antiquark bound state of mass M into two gluons of four-momenta $k_{1\mu}, k_{2\mu}$, polarization vectors $\epsilon_{1\mu}, \epsilon_{2\mu}$, and colour indices a, b is graphically represented in Fig. 1. The bubbles denote the Bethe-Salpeter bound-state wave function $\psi_{\alpha\beta}(J^P, M; p_\mu)$ dependent on the relative four-momentum p_μ . In the c.m.s. $k_{1\mu} = (M/2)(1, \hat{k})$, $k_{2\mu} = (M/2)(1, -\hat{k})$, $\epsilon_{1\mu} = (0, \hat{\epsilon}_1)$, $\epsilon_{2\mu} = (0, \hat{\epsilon}_2)$, $\hat{\epsilon}_1 \cdot \hat{k} = \hat{\epsilon}_2 \cdot \hat{k} = 0$.

In terms of this Bethe-Salpeter wave function the transition amplitude $\mathcal{M}_{ij}^{ab}(J^P, M; \hat{k}) \hat{\epsilon}_{1i}^* \hat{\epsilon}_{2j}^*$ can be written as

$$\mathcal{M}_{ij}^{ab}(J^P, M; \hat{k}) = -\frac{ig^2}{2\sqrt{3}} \delta^{ab} \left(d_\mu^\nu \text{Tr} \left[C N_{ij}(p, k_1) \Psi(J^P, M; p) \right] \right) \quad (8)$$

where

$$N_{ij} = \gamma_i \frac{\frac{1}{2} \not{K} + \not{p} - \not{k}_1 + m}{(\frac{1}{2} \not{K} + \not{p} - \not{k}_1)^2 - m^2} \gamma_j + \gamma_j \frac{-\frac{1}{2} \not{K} + \not{p} + \not{k}_1 + m}{(-\frac{1}{2} \not{K} + \not{p} + \not{k}_1)^2 - m^2} \gamma_i \quad (9)$$

$$\not{K} = \not{k}_1 + \not{k}_2 = (M, \vec{0}),$$

and the charge conjugation matrix C has been introduced in order to describe our quark-antiquark system by a fermion-fermion equal-mass bound state wave function $\psi(J^P, M; p)$. The elementary quark-quark-gluon vertex

$$-ig \frac{(\lambda^a)_{ij}}{2} \gamma_\mu$$

defines the coupling constant g .

In the extreme non-relativistic limit the dependence on p_μ of N_{ij} is neglected and the usual expression is obtained for the decay amplitude proportional to the non-relativistic wave function calculated at the origin in the relative coordinate space. This gives the well-known result for the 0^- state, which has zero relative angular momentum. However, to get a non-zero result for the P-waves, because of the vanishing of the non-relativistic wave function at the origin, the terms linear in $\vec{p}$ in the expansion of N_{ij} , as well as the terms linear in $\vec{p}$ coming from the small components of the relativistic wave function, must be retained. In this approximation the relativistic wave function can be expressed in terms of the non-relativistic wave function which has only large components:

$$\Phi(J^P, M; p) \equiv (1+\beta) \Psi(J^P, M; p) (1+\beta)/4$$

as

$$\begin{aligned} \Psi(J^P, M; p) &\simeq \left(1 + \frac{\vec{\alpha} \cdot \vec{p}}{2m}\right) \Phi(J^P, M; p) \left(1 - \frac{\vec{\alpha} \cdot \vec{p}}{2m}\right) \equiv \\ &\equiv \left(1 + \frac{\vec{\alpha} \cdot \vec{p}}{2m}\right) \chi(J^P, M; \vec{p}) \delta(p_0) \left(1 - \frac{\vec{\alpha} \cdot \vec{p}}{2m}\right) \end{aligned} \quad (10)$$

Here m is the constituent mass ($m \approx M/2$).

If this expression for the wave function is inserted in Eq. (9) and the expansion of N_{ij} is made up to linear terms in p , one gets

$$\mathcal{M}_{ij}^{ab}(J^P, M; \hat{k}) \simeq \frac{i}{\sqrt{3}} \frac{g^2}{M^2} \int d^3p \text{Tr} \left[C(\mathcal{N}_{ij}^{(0)}(\hat{k}) + p_\ell \mathcal{N}_{ij\ell}^{(1)}(\hat{k})) \chi(J^P, M; \vec{p}) \right] \quad (11)$$

where

$$\begin{aligned} \mathcal{N}_{ij}^{(0)}(\hat{k}) &= M(T_{ij}^A(\hat{k}) - \delta_{ij}) \\ p_\ell \mathcal{N}_{ij\ell}^{(1)}(\hat{k}) &= \vec{\alpha} \cdot \vec{p} T_{ij}^A(\hat{k}) + T_{ij}^A(\hat{k}) \vec{\alpha} \cdot \vec{p} - 2\vec{\alpha} \cdot \vec{p} \delta_{ij} - \\ &\quad - 2T_{ij}^S(\vec{p}) + 2\vec{p} \cdot \hat{k} (T_{ij}^S(\hat{k}) + i\varepsilon_{ijk} \Sigma_k) \end{aligned} \quad (12)$$

$$T_{ij}^S(\vec{q}) = \gamma_i \vec{q} \cdot \vec{\gamma} \gamma_j + \gamma_j \vec{q} \cdot \vec{\gamma} \gamma_i, \quad T_{ij}^A(\vec{q}) = \gamma_i \vec{q} \cdot \vec{\gamma} \gamma_j - \gamma_j \vec{q} \cdot \vec{\gamma} \gamma_i.$$

This is our explicit starting point for the calculation.

Because of Bose statistics for the gluons and parity conservation, the transition amplitude $\mathcal{M}_{ij}^{ab}(0^+; M; \hat{k})$ for the $J^P = 0^+$ state can be expressed in terms of two invariant amplitudes

$$\begin{aligned} \mathcal{M}_{ij}^{ab}(0^+, M; \hat{k}) &= \delta^{ab} [A \delta_{ij} + B \hat{k}_i \hat{k}_j] \\ A &= \frac{1}{16} (\mathcal{M}_{ii}^{aa} - \hat{k}_i \hat{k}_j \mathcal{M}_{ij}^{aa}) = -\frac{2i}{\sqrt{3}} \frac{g^2}{M^2} \int d^3p \text{Tr} [C \vec{\alpha} \cdot \vec{p} \chi(0^+, M; \vec{p})] \end{aligned} \quad (13)$$

The trace of the 4×4 matrix product can be immediately reduced to the trace of a 2×2 matrix, since $\chi(0^+, M; \vec{p})$ has only the high component 2×2 matrix $\chi^{NR}(0^+, M; \vec{p})$. One has

$$\text{Tr} [C \vec{\alpha} \chi] = -\text{Tr} [C \vec{\tau} \chi] = \text{Tr} [\sigma_2 \vec{\sigma} \chi^{NR}]^{NR}$$

and then

$$A = -\frac{2i}{\sqrt{3}} \frac{g^2}{M^2} \int d^3p \text{Tr} [\sigma_2 \vec{\sigma} \cdot \vec{p} \chi^{NR}(0^+, M; \vec{p})] \quad (14)$$

To evaluate the right-hand side of Eq. (14) the following formulae are useful:

$$\begin{aligned} \chi^{NR}(0^+, M; \vec{p}) &= \sum_{m, l}^{0, \pm} \langle 1100 | 1m 1l \rangle \tilde{\Phi}_1^m(\vec{p}) \chi_1^l \\ \int d^3p p^n \Phi_1^m &= \langle 1100 | 1n 1m \rangle i \frac{3}{\sqrt{4\pi}} \Phi_1'(0) \\ \text{Tr} (\sigma_2 \sigma^n \chi_1^l) &= \langle 1100 | 1n 1l \rangle (-i)\sqrt{6}, \quad \vec{\sigma} \cdot \vec{p} = \sum_n^{0, \pm} (-1)^n \sigma^n p^{-n}, \end{aligned} \quad (15)$$

where χ_1^l is the $S = 1$, $S_z = l$, spin part of the non-relativistic wave function and $\tilde{\Phi}_1^m(\vec{p})$ its orbital part in momentum space, whose Fourier transform is $\phi_1^m(\vec{x}) = \phi_p(r) Y_1^m(\hat{x})$, normalized to 1.

Inserting Eqs. (15) into Eq. (14) the final expression for the amplitude A is

$$A = \frac{ig^2}{M^2} \sqrt{\frac{6}{\pi}} \Phi_1'(0) \quad (16)$$

Correspondingly, for the width, one has

$$\begin{aligned} \Gamma(0^+, M) &= \int \frac{d\Omega_k}{26\pi^2} \mathcal{M}_{ij}^{ab}(0^+, M; \hat{k}) \mathcal{M}_{lm}^{ab*}(0^+, M; \hat{k}) (\delta_{il} - \hat{k}_i \hat{k}_l) (\delta_{jm} - \hat{k}_j \hat{k}_m) = \\ &= \frac{1}{\pi} |A|^2 = \frac{96\alpha_s^2}{M^4} |\Phi_1'(0)|^2 \end{aligned} \quad (17)$$

The decomposition into invariant amplitudes of the transition matrix element for a $J^P = 2^+$ state is slightly more complicated. Introducing a traceless symmetric tensor V_{ij} to describe the transformation properties under rotations of the $J = 2$ state (with definite J_z , e.g. $J_z = 0$), one has (sum over equal indices is understood):

$$\begin{aligned} \mathcal{M}_{ij}^{ab}(2^+, M; \hat{k}) &= \delta^{ab} [B V_{ij} + C V_{lm} \hat{k}_l \hat{k}_m \delta_{ij} + \\ &+ D V_{lm} \varepsilon_{lni} \varepsilon_{mrj} \hat{k}_n \hat{k}_r + E V_{lm} \hat{k}_l \hat{k}_m \hat{k}_i \hat{k}_j] \end{aligned} \quad (18)$$

With V_{ij} normalized to 1, using

$$V_{ij} V_{jl} V_{li} = \frac{1}{\sqrt{6}},$$

the coefficients B, ..., E can be projected out by a straightforward calculation. They are found to depend on the quantity

$$H = -V_{rs} \int d^3p \text{Tr} (p_r \sigma_2 \sigma_s \chi^{NR}(2^+, M; \vec{p})) = -\sqrt{\frac{3}{2\pi}} \Phi'_P(0). \quad (19)$$

The final expression for H is obtained with the help of Eqs. (15), after having made the usual decompositions

$$\chi^{NR}(2^+, M; \vec{p}) = \sum_{m, \ell}^{0, \pm} \langle 1120 | 1m 1\ell \rangle \tilde{\Phi}_1^m(\vec{p}) \chi_1^\ell \quad (20)$$

$$V_{rs} p_r p_s = \sum_{q, t}^{0, \pm} \langle 1120 | 1q 1t \rangle p^q \sigma^t \quad (21)$$

Finally, the width of the 2^+ state is given by

$$\Gamma(2^+, M) = \frac{128}{5} \frac{\alpha_s^2}{M^4} |\Phi'_P(0)|^2. \quad (22)$$

As a trivial by-product of Eqs. (11) and (12), from the term independent of $\vec{p}$, one gets for the two-gluon decay of the S-wave 0^- state

$$\Gamma(0^-, M) = \frac{8}{3} \frac{\alpha_s^2}{M^2} |\Phi_S(0)|^2, \quad (23)$$

where $\phi_S(r)$ is the non-relativistic wave function for the $L = 0$ state, normalized to $\int_0^\infty r^2 dr |\phi_S(r)|^2 = 1$. Equation (17) is the usually quoted formula (see Ref. 1).

The calculations that we have presented apply as well, with essentially no modification, to the two-photon decays when computed from the graphs analogous to those in Fig. 1. With the substitution rule $\alpha_s^2 \rightarrow (9/2)e_Q^4 \alpha^2$ one gets from the two-gluon decay widths the corresponding two-photon decay widths [see Eqs. (4) and (7) and footnote 6].

To apply our results to the phenomenology of the new states X observed in $\psi'(3.7) \rightarrow X + \gamma$ ³⁾, we assume that they are indeed P-wave bound states of two heavy components (possibly, but not necessarily, charmed quarks). We then get, independently of any parameter

$$\frac{\Gamma_{\text{ann}}(2^{++})}{\Gamma_{\text{ann}}(0^{++})} = 0.27$$

The absolute rates $\Gamma_{\text{ann}}(2^{++})$ and $\Gamma_{\text{ann}}(0^{++})$ depend on the not well-known parameter $\alpha_s^2 |\Phi'_P(0)|^2$. We take the value of this parameter from the masses and widths of ψ and ψ' following our previous systematic work⁷⁾. We find

$$\Gamma_{\text{ann}}(2^{++}) = 0.64 \text{ MeV}$$

$$\Gamma_{\text{ann}}(0^{++}) = 2.4 \text{ MeV}$$

From Eq. (5) we find

$$\frac{\Gamma_{ann}((0^{-+})')}{\Gamma_{ann}(0^{-+})} = 0.45 ,$$

whereas Eq. (6) gives

$$\Gamma_{ann}(0^{-+}) = 6.7 \text{ MeV}.$$

For the 2γ rates, from Eqs. (4) and (7), we get values

$$\begin{aligned} \Gamma(2^{++} \rightarrow 2\gamma) &= 0.93 \text{ keV} , & \Gamma(0^{++} \rightarrow 2\gamma) &= 3.5 \text{ keV} , \\ \Gamma(0^{-+} \rightarrow 2\gamma) &= 9.8 \text{ keV} , & \Gamma((0^{-+})' \rightarrow 2\gamma) &= 4.4 \text{ keV} . \end{aligned}$$

We stress that the presence of parameters (α_s and wave function) which are not well known (and which we have taken from a previous work) makes the numerical predictions not completely reliable at this time. We think that they could, however, provide for at least a frame for comparison.

The graphs of Fig. 1 also appear in a possible production mechanism of the states 0^{-+} , 0^{++} , 2^{++} , $(0^{-+})'$ through two-gluons, analogous to the Drell-Yan picture (see Fig. 2). Following Einhorn and Ellis⁸⁾, we take a gluon distribution within the proton of the form

$$\frac{3}{8} \frac{(1-x)^5}{x}$$

(normalized and averaged over colour and polarizations). Typical integrated production cross-sections obtained through such a mechanism are:

at $s = 500 \text{ GeV}^2$: $\sigma \sim 0.7 \mu\text{b}$ for 0^{-+} ; $\sigma \sim 0.1 \mu\text{b}$ for $0^{++}, 2^{++}$ and $(0^{-+})'$.

At $s = 2800 \text{ GeV}^2$ we find:

$$\sigma \sim 2.2 \mu\text{b} \text{ for } 0^{-+}; \quad \sigma \sim 0.4 \mu\text{b} \text{ for } 0^{++}, 2^{++} \text{ and } (0^{-+})'.$$

From the two-photon amplitudes of the type of Fig. 1 we have also obtained for the coherent Primakoff production in lead the following typical values for the cross-sections⁹⁾: at a laboratory photon momentum of 100 GeV,

$$\sigma(\gamma \rightarrow 0^{-+}) = 70 \text{ nb}, \quad \sigma(\gamma \rightarrow 0^{++}) = 4.1 \text{ nb}, \quad \sigma(\gamma \rightarrow 2^{++}) = 4 \text{ nb}, \quad \sigma(\gamma \rightarrow (0^{-+})') = 3.8 \text{ nb}$$

at a laboratory photon momentum of 150 GeV,

$$\sigma(\gamma \rightarrow 0^{-+}) = 160 \text{ nb}, \quad \sigma(\gamma \rightarrow 0^{++}) = 14 \text{ nb}, \quad \sigma(\gamma \rightarrow 2^{++}) = 15 \text{ nb}, \quad \sigma(\gamma \rightarrow (0^{-+})') = 14 \text{ nb}.$$

Other cross-sections determined from the 2γ amplitudes of the type shown in Fig. 1 are the two-photon cross-sections $e^+e^- \rightarrow e^+e^-X$ ¹⁰⁾, where in our case $X = 0^{-+}, 0^{++}, 2^{++}, (0^{-+})'$. We find at colliding beam energy $2E = 7 \text{ GeV}$:

$$\sigma_{2\gamma}(0^{-+}) = 2 \times 10^{-2} \text{ nb}, \quad \sigma_{2\gamma}(0^{++}) = 2 \times 10^{-3} \text{ nb}, \quad \sigma_{2\gamma}(2^{++}) = 3 \times 10^{-3} \text{ nb}, \quad \sigma_{2\gamma}((0^{-+})') = 3 \times 10^{-3} \text{ nb},$$

whereas for $2E = 30 \text{ GeV}$ $\sigma_{2\gamma}(0^{-+})$ is $1 \times 10^{-1} \text{ nb}$, whereas $\sigma_{2\gamma}(0^{++}), \sigma_{2\gamma}(2^{++})$

and $\sigma_{2\gamma}((0^{-+})')$ are all $2 \times 10^{-2} \text{ nb}$.

As we have already mentioned, application of our results to light mesons is *a priori* subject to doubt because of large relativistic effects, presumably large light-quark pairs contamination, etc. If we tentatively apply our formulae for $\Gamma_{\text{ann}}(2^{++})$ to $f'(1514)$ assumed to consist of $s\bar{s}$ only, with all parameters [α_s at the f' mass, $\phi'_p(0)$, and m_s] as suggested from a previous work^{7,11}), we obtain a value $\Gamma_{\text{ann}}(2^{++}) = 6 \text{ MeV}$ at the f' mass. Assuming that roughly two-thirds of this goes into states consisting of $u, d, \bar{u}, \bar{d}$ quarks, we find a branching ratio of 10% for decay into such states. This number, although large, is consistent with the present scanty experimental evidence¹²). Better experimental determinations would be of interest.

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- 2) Charge conjugation does *not* forbid a $J^{PC} = 1^{++}$ state to annihilate into three gluons: it only requires that the colour dependence of the final gluons is described by the antisymmetric f^{abc} tensor. In addition, a $J^{PC} = 1^{++}$ state can to leading order annihilate into one gluon plus one quark pair.
- 3) W. Braunschweig et al., Phys. Letters 57B, 407 (1975).
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- 5) The dynamical conjectures which underlie our calculation, and the identification of the gluonic widths with the widths for decay into ordinary hadrons of the new particles, are essentially the following. i) The low-lying poles of the $Q\bar{Q}$ four-point Green function are approximate eigenstates of the total Hamiltonian of the theory. The infrared structure of the theory, which gives the poles in the $Q\bar{Q}$ sector, is not producing any strong connection of them with other sectors of the theory. (The smallness of $\alpha_S(M)$, the sliding coupling constant at the typical masses of the $Q\bar{Q}$ poles, alone is not enough to guarantee that this is the case). ii) The binding mechanism and the actual parameters of the theory are such that the $Q\bar{Q}$ bound states are mainly non-relativistic systems. iii) The colour singlet gluon density correlation functions do not show resonant or threshold behaviours in the region of the $Q\bar{Q}$ poles, since this is likely to destroy their computability in perturbation theory even with a small coupling α_S . The above conjectures are implicit in the similar calculation for ψ and η_c (ref. 1).
- 6) We note that, as is well known, the applicability of the perturbation expansion in α_S rests on the absence of infrared divergences in each order. However, when a partial width is computed, logarithms of the mass ratio between light and heavy quarks are likely to appear. This kind of criticism applies also to the "naive" calculation of the γ -transitions between bound states (such as $\psi' \rightarrow P\text{-state} + \gamma$, $\psi \rightarrow \eta_c + \gamma$, etc.) where the infrared problem is even more severe, since additional infrared divergences appear in the limit of zero binding for the initial and final states involved in the transition. This circumstance may in fact be the main reason for the failure of such a calculation when making a comparison with experimental data.
- 7) R. Barbieri, R. Gatto, R. Kögerler and Z. Kunszt, Phys. Letters 57B, 455 (1975); and Ref. TH. 2036-CERN. The values we use are $\alpha_S = 0.18$, $|\phi_P'(0)|^2 = 0.09 \text{ (GeV)}^5$ for P-states, $|\phi_S(0)|^2 = 0.81 \text{ (GeV)}^3$ for S-states, $m_c = 1.64 \text{ GeV}$.
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- 11) For strange quarks we take $m_s = 0.54$ GeV; at the f' mass we take $\alpha_s = 0.21$,
and for $|\phi'_P(0)|^2$ we take 0.01 (GeV)⁵; these values are taken from Ref. 7.
- 12) See: N. Barash-Schmidt et al., Tables of Particle Properties, April 1974
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Figure captions

Fig. 1 : Feynman graphs for the two-gluon annihilation.

Fig. 2 : Two-gluon production mechanism of C-even states in pp collisions.

Fig. 1

Fig. 2