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INFRA - RED SLAVERY AND QUARK CONFINEMENT

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ABSTRACT

We pose the question if the so-called infra-red slavery mechanism as, e.g., being manifest in non-Abelian gauge theories, necessarily confines quarks. Making a specific ansatz for the long range forces, we solve the Schwinger-Dyson equation for the quark Green's function. Besides having a confining solution, it appears that quarks may by-pass the long range forces and be produced.

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1. INTRODUCTION

During the last couple of years, much hope has been set on non-Abelian gauge theories since they realize asymptotic freedom at short distances ¹⁾ and open the possibility of quark confinement. It appears that the infra-red divergences inherent in such theories could make the rate for producing quarks vanish, i.e., might generate quark confinement potentials which has led to the notion of infra-red slavery ²⁾.

The structure of such potentials may be inferred from the properties of the β function, and is most transparently expressed in terms of the effective coupling constant $\bar{g}(u)$:

$$\frac{d}{du} \bar{g}(u) = \beta(\bar{g}(u)), \quad \bar{g}(0) = g, \quad (1.1)$$

where g is the renormalized coupling constant, $u = \ln(t/t_0)$ and t being the momentum transfer. For particular assumptions on β which are consistent with asymptotic freedom and exact gauge symmetry, one may obtain an effective potential which behaves as R^λ , $\lambda > 0$ for large quark separations and as $(R \ln R)^{-1}$ at small distances.

Two-dimensional gauge theories ³⁾, as well as the lattice gauge theory ⁴⁾, seem to favour a large distance behaviour of the effective potential proportional to R . In four-dimensional covariant theories, no particular large distance behaviour appears to be distinguished. Here one may, however, be guided by speculations or various mundane models ⁵⁾ suggesting $\lambda = 1$ and $\lambda = 2$.

So far, the quark confinement character of such potentials has been little questioned. Kogut and Susskind ⁶⁾ have speculated that the infra-red divergences in non-Abelian gauge theories most economically undergo the Schwinger mechanism as in two-dimensional quantum electrodynamics, and hence prevent quarks from being produced in electron-positron annihilation. Otherwise, the lattice gauge theory ⁴⁾ is always quoted as an instance which, in fact, develops quark binding in the strong coupling and large m_0 limit (m_0 being the bare quark mass).

On general grounds it cannot, however, be excluded that other geometries are energetically favoured over the Schwinger mechanism or, at least, are equally privileged. Moreover, it is not clear at all if a similar mechanism also may prevent quarks or quark lumps from being produced in hadron-hadron collisions. Later on we will indeed see that the infra-red divergences may resolve through a different scheme. The lattice gauge theory, on

the other hand, gives very little insight into the problem as to whether quarks undergo a binding mechanism in any case, since it can only be solved for a particular configuration where quark confinement happens to occur. Here a more detailed analysis needed to be performed allowing for small m_0 .

In this work we shall attempt a more extensive answer to the question as to what extent the infra-red slavery phenomenon does confine quarks.

The key expression to be evaluated is the quark Green's function $\Delta_F(k)$. The vacuum polarization tensor $\Pi_{\mu\nu}(k)$, as often referred to, may lead to a treacherous conclusion as we shall see. The quark Green's function can be derived from the Bethe-Salpeter (BS) kernel, i.e., the sum over all two-quark (to be precise, quark-antiquark) irreducible diagrams which now is the legitimate expression for the effective potential paraphrased at the beginning.

For our aims, the short distance behaviour of the potential should not be important. At large distances we shall assume a harmonic oscillator type of behaviour ⁷⁾, i.e., $\lambda = 2$. This simplifies the calculations somewhat. For the question of quark confinement, the exact power should, however, not be of primary importance as long as the confining forces are not too weak.

The BS approach to quark confinement also offers an alternative point of view. It is well known that the static and electromagnetic properties of the two-quark (quark-antiquark) system such as the spectrum, form factors ⁸⁾, deep inelastic structure functions ⁹⁾, etc., are completely determined by the BS kernel. This makes the BS framework an attractive laboratory for constructing phenomenological bag models. Rather than going through Lagrangian field theory, the BS kernel may be viewed as the basic ingredient of the model.

The paper is organized as follows. In Section 2 we present the basic BS framework in which the question of quark confinement will be discussed. The R^2 interaction being considered here gives rise to a non-linear differential equation for the quark Green's function which is investigated in Section 3 and the Appendix. The net result is that there exists a quark confining and a non-confining solution. In Section 4 we shall give a brief discussion of our achievements.

2. BETHE-SALPETER FRAMEWORK

Let us consider a world of mesons and spin-zero quarks. The mesonic bound states are then described by the BS amplitudes ¹⁰⁾

$$\phi_p(k) = (2\pi)^{\frac{3}{2}} \int d^4x e^{ikx} \langle 0 | T(\psi(\frac{x}{2}) \bar{\psi}^+(-\frac{x}{2})) | \phi_p \rangle \quad (2.1)$$

which satisfy the homogeneous BS equation

$$\phi_p(k) = \Delta_F(\frac{1}{2}P+k) \Delta_F(\frac{1}{2}P-k) i \int d^4k' K(k, k', P) \phi_p(k'). \quad (2.2)$$

The normalization of the bound state wave function is determined by the accessory condition ¹¹⁾

$$(2\pi)^4 i \int d^4k \phi_p(k) \left[\frac{\partial}{\partial P^2} \Delta_F^{-1}(\frac{1}{2}P+k) \Delta_F^{-1}(\frac{1}{2}P-k) - \frac{\partial}{\partial P^2} K(k, k, P) \right] \phi_p(k) = 1. \quad (2.3)$$

The BS kernel $K(k, k', P)$ stands for the sum over all two-particle irreducible diagrams. This has to be kept in mind for our later derivations.

The (dressed) quark Green's function

$$\Delta_F(k) = \int d^4x e^{ikx} \langle 0 | T(\psi(x) \bar{\psi}^+(0)) | 0 \rangle \quad (2.4)$$

entering into the BS equation (2.2) is the first quantity to be calculated once the BS kernel is specified. As is well known, $\Delta_F(k)$ is determined by the integral equation ¹²⁾

$$\Delta_F^{-1}(k) = Z \Delta_0^{-1}(k) - i \int d^4k' K(k, k', P=0) \Delta_F(k') \quad (2.5)$$

as graphically verified in Fig. 1. Here

$$\Delta_0(k) = \frac{1}{(k^2 - m_0^2)} \quad (2.6)$$

has the meaning of the bare propagator with m_0 being the bare mass and Z being the wave function renormalization constant.

In the limit of quark confinement, $\Delta_P(k)$ is expected to undergo a drastic change as compared to the standard form ¹³⁾. It will be the main task of the next section to solve Eq. (2.5) for what is believed to be a confining kernel.

Kogut and Susskind ^{3),6)} restrict the discussion of quark shielding to the vacuum polarization, which we think is misleading. It may, e.g., happen that quarks or quark lumps are shielded from being produced in electron-positron annihilation, but are still present in the spectrum of $q, q^+q, q\bar{q}q$, etc., i.e., appear among the complete set of states. Let us illustrate this a little further.

The vacuum polarization tensor may be expressed in the form

$$\Pi_{\mu\nu}(k) = i \int d^4x e^{ikx} \langle 0 | T(j_\mu(x) j_\nu(0)) | 0 \rangle \quad (2.7)$$

where

$$j_\mu(x) = Z Q [g^+(x), \overleftrightarrow{\partial}_\mu g(x)] \quad (2.8)$$

The singularities of the vacuum polarization tensor can be read off from the imaginary part of Eq. (2.7), i.e.,

$$\begin{aligned} \text{Im } \Pi_{\mu\nu}(k) &= \int d^4x e^{ikx} \langle 0 | j_\mu(x) j_\nu(0) | 0 \rangle \\ &= (2\pi)^4 \sum_P \langle 0 | j_\mu(0) | P \rangle \langle P | j_\nu(0) | 0 \rangle \delta^4(k-P) \end{aligned} \quad (2.9)$$

Here $\{|P\rangle\}$ represents a complete set of states. For a bound state $|\phi_P\rangle$ coupling to the photon, we find

$$\begin{aligned} \langle 0 | j_\mu(0) | P \rangle &= Z Q \langle 0 | [z^\dagger(0), \overleftrightarrow{\mathcal{D}}_\mu z(0)] | P \rangle \\ &= Z Q \int d^4 k' \phi_T(k') (2k'_\mu + k_\mu) . \end{aligned} \quad (2.10)$$

If the bound states themselves now define a complete set of states in the q^+q^- sector as, e.g., can be proved for the harmonic oscillator¹⁴⁾ (not including quantum corrections), then, obviously, the quark-antiquark cut will be absent from $\Pi_{\mu\nu}(k)$. In particular, if the bound states described by the BS kernel of the next section constitute a complete set of states (what we expect), there will be no zero mass structure in $\Pi_{\mu\nu}(k)$ though the spectrum of q^- extends from zero to infinity. This is the analogue of the Schwinger mechanism in the BS language which, as our example shows, alone does not allow any conclusion as to whether quarks are confined or not.

3. THE QUARK GREEN'S FUNCTION

Let us now concentrate on Eq. (2.5) and evaluate the quark Green's function $\Delta_F(k)$. For the BS kernel we make the ansatz in (Euclidean) configuration space^{*})

$$K = \beta R^2, \quad (3.1)$$

where

$$R^2 = (\vec{x} - \vec{x}')^2 + (x_4 - x_4')^2, \quad (3.2)$$

i.e., a harmonic oscillator potential¹⁴⁾. In (Minkowskian) momentum space, Eq. (3.1) corresponds to

$$\begin{aligned} K(k, k', P) &= -i\beta \square_k \delta^4(k - k') \\ \square_k &= \frac{\partial^2}{\partial k_0^2} - \frac{\partial^2}{\partial k_1^2} - \frac{\partial^2}{\partial k_2^2} - \frac{\partial^2}{\partial k_3^2} \end{aligned} \quad (3.3)$$

which, when inserted into Eq. (2.5), leads to the differential equation for Δ_F

^{*}) Strictly speaking, we needed to make this ansatz for $P=0$ only.

$$\Delta_F^{-1}(k) = Z(k^2 - m_0^2) - \beta \square_k \Delta_F(k) . \quad (3.4)$$

Since $\Delta_F(k)$ is a function of k^2 only and, as one can easily check,

$$\square_k f(k^2) = 8 f'(k^2) + 4 k^2 f''(k^2) , \quad (3.5)$$

Eq. (3.4) can furthermore be written

$$\Delta_F(k) \left(k^2 \frac{d^2}{d(k^2)^2} + 2 \frac{d}{d k^2} \right) \Delta_F(k) - \frac{Z}{4\beta} (k^2 - m_0^2) \Delta_F(k) + \frac{1}{4\beta} = 0 . \quad (3.6)$$

Equation (3.6) can be solved explicitly for the most interesting cases as we shall see later on. In the first place we have, however, to find out the full variety of solutions of Eq. (3.6) in order to be able to cut down the infinity of parameters (Z and m_0) to a few physically relevant cases at all.

The first observation we make is that $\Delta_F(k)$, as determined by Eq. (3.6), cannot have poles in the complex plane, apart from $k^2 = 0$. It must even be bounded in the finite plane excluding the origin and possible essential singularities. In order to demonstrate this, let us suppose that $\Delta_F(k)$ is infinite at some $k^2 = k_0^2$. If the infinity is not due to an essential singularity^{*)}, Eq. (3.6) can be written in the neighbourhood of k_0^2

$$\frac{d^2}{d(k^2)^2} (k^2 \Delta_F(k)) = h(k^2) , \quad (3.7)$$

where $h(k^2)$ is a well behaved function. From Eq. (3.7) now follows, by integration, that $\Delta_F(k)$ must be finite at $k^2 = k_0^2$ which is a contradiction to our assumption. Essential singularities will be excluded from our discussion since they cause serious difficulties (such as violation of causality, etc.) and are not compatible with conventional field theory.

*) In this form it also includes accumulations of soft cuts.

So far we have excluded $k^2=0$. As can be inferred from Eq. (3.7), $\Delta_F(k)$ may have a simple pole at $k^2=0$. This is, in fact, the only singularity $\Delta_F(k)$ is allowed to have in the finite plane (not counting possible soft cuts), as we shall see immediately. Following the same line of reasoning as before, we conclude that $\Delta_F(k)$ cannot be more singular (even by logarithms) than $(k^2)^{-1}$ at $k^2=0$. On the other hand, suppose that $\Delta_F(k) \simeq (k^2)^{-\gamma}$, $0 < \gamma < 1$ as $k^2 \rightarrow 0$. Then we would get

$$\frac{d^2}{d(k^2)^2} (k^2 \Delta_F(k)) \simeq (k^2)^{-1-\gamma} \quad k^2 \rightarrow 0$$

which is inconsistent with Eq. (3.7) since $h(k^2)$ is finite at $k^2=0$.

The solutions of Eq. (3.6) can now be divided into two classes:

- i) those having a pole at $k^2=0$ and being finite elsewhere;
- ii) those being bounded everywhere in the finite plane.

This constitutes a physical classification of the different solutions as we shall see later on.

Let us first concentrate on the solutions of class (i). Here it is understood that $Z > 0$, since Z measures the probability of finding a single quark state in the spectrum of the quark field operator.

For $m_0=0$,

$$\Delta_F(k) = \frac{1}{k^2} \quad (3.8)$$

is a solution of Eq. (3.6) where we have set $Z=1$ for consistency [note that the general solution is $\Delta_F(k) = (Zk^2)^{-1}$]. The Jost-Schroer theorem, however, tells us that the quark Green's function (3.8) corresponds to a free theory¹⁵⁾ which means that the long range forces become ineffective in this case. If the BS kernel (3.3) now is considered to define the complete theory, solution (3.8) is, of course, meaningless. Nevertheless, it demonstrates that the infra-red divergences resolve trivially for zero-mass quarks.

*) The proof goes through in the same way for a non-power behaviour such as $(k^2)^{-1} \log k^2$, etc.

We have also searched for other solutions (still $m_0^2 = 0$) using Padé techniques. All other solutions appear to have unphysical singularities (such as left-hand cuts, etc.), so that they have to be rejected.

For $m_0^2 > 0$ it is convenient to write

$$\Delta_F(k) = \frac{\rho(k^2)}{k^2}, \quad \rho(0) = 1 \quad (3.9)$$

which leads to the differential equation for $\rho(k^2)$

$$\rho \rho'' - \frac{2}{4\beta} (k^2 - m_0^2) \rho + \frac{1}{4\beta} k^2 = 0. \quad (3.10)$$

We cannot solve Eq. (3.10) in a closed form. Since we know the solution of Eq. (3.10) for $m_0^2 = 0$ (and it seems that the bare quark mass is small, if one thinks of applications), it is useful to expand $\rho(k^2)$ in terms of powers of m_0^2 , i.e.,

$$\rho = \rho^{(0)} + m_0^2 \rho^{(1)} + m_0^4 \rho^{(2)} + \dots, \quad (3.11)$$

where now

$$\begin{aligned} \rho^{(0)} &= \frac{1}{2} \\ \rho^{(n)}'' - \frac{2}{4\beta} k^2 \rho^{(n)} &= -2 \sum_{i=1}^{n-1} \rho^{(i)} \rho^{(n-i)}'' \\ &\quad - \frac{2}{4\beta} \rho^{(n-1)}, \quad n = 1, 2, \dots \end{aligned} \quad (3.12)$$

This has the great advantage that the non-linear problem reduces to a set of linear differential equations which can be solved successively. Note that the Green's function of Eq. (3.12) is even the same for any n and that the right-hand side of Eq. (3.12) only involves lower order solutions, i.e., $\rho^{(n-1)}$, $\rho^{(n-2)}$, etc.

On the real axis we then have

$$\begin{aligned} \rho^{(1)}(k^2) &= \int_{-\infty}^{+\infty} dk'^2 G(k^2, k'^2) \left(-\frac{Z}{4\beta} \right), \\ \rho^{(2)}(k^2) &= \int_{-\infty}^{+\infty} dk'^2 G(k^2, k'^2) \left(-Z \rho^{(1)}(k'^2) \rho^{(1)''}(k'^2) - \frac{Z^2}{4\beta} \rho^{(1)}(k'^2) \right), \\ \rho^{(3)}(k^2) &= \int_{-\infty}^{+\infty} dk'^2 G(k^2, k'^2) \left(-Z \rho^{(1)}(k'^2) \rho^{(2)''}(k'^2) \right. \\ &\quad \left. - Z \rho^{(2)}(k'^2) \rho^{(1)''}(k'^2) - \frac{Z^2}{4\beta} \rho^{(2)}(k'^2) \right), \\ &\text{etc.} \end{aligned} \tag{3.13}$$

The Green's function G and any further particulars of Eq. (3.12) are given in the Appendix. Equation (3.13) can be continued into the complex plane without difficulties.

If the series (3.11) now converges everywhere in the finite plane, the analytic structure of the components $\rho^{(n)}(k^2)$ (see Appendix) tells us that $\rho(k^2)$ can only have an essential singularity at infinity. In the more general case, cuts may develop in the finite plane ^{*}), but the essential singularity at infinity will persist. This can be seen as follows. From Eqs. (3.11) and (3.13), we derive the (equivalent) integral equation for $\rho(k^2)$

$$\begin{aligned} \rho(k^2) - \rho^{(0)}(k^2) &= \int_{-\infty}^{+\infty} dk'^2 G(k^2, k'^2) \left[-Z (\rho(k'^2) - \rho^{(0)}(k'^2)) \right. \\ &\quad \left. \times \rho''(k'^2) - m_0^2 \frac{Z^2}{4\beta} \rho(k'^2) \right] \end{aligned} \tag{3.14}$$

which is certainly inconsistent with not having an essential singularity at infinity ^{**}).

^{*}) This has been found solving Eq. (3.10) ($m_0^2 > 0$) numerically with Padé approximants.

^{**}) If, e.g., $\rho(k^2)$ is power behaved at infinity, Eq. (3.10) tells us that $\rho(k^2) \approx 1/Z$. This, however, is obviously inconsistent with Eq. (3.14).

Since conventional field theory cannot live with an essential singularity at infinity, we conclude on that basis that $m_0^2 = 0$ (note that m_0^2 is a priori a free parameter). This brings us back to Eq. (3.8), i.e., the free solution.

Let us now discuss the solutions of class (ii). Here we are tempted to set $Z=0$ since there is no single quark state in the spectrum of the quark field operator. On the other hand, $Z \neq 0$ would always lead to an essential singularity at infinity, as can be inferred from Eqs. (3.9) and (3.14), but $\rho(0) = 0$ now^{*)}. This has the effect that the differential equation (3.6) simplifies to a form which we can solve explicitly.

For m_0^2 finite, one easily verifies that

$$\Delta_F(k) = \sqrt{\frac{4}{3}} \frac{1}{4\beta} \sqrt{-k^2} \quad (3.15)$$

is a solution of Eq. (3.6) (with $Z=0$ now). This solution allows quarks only to exist as coherent quark-gluon, etc., states. Nevertheless, one should observe quark lumps in analogy to the ρ , Δ , etc. We like to call Eq. (3.15) the bootstrap solution¹⁸⁾ for obvious reasons.

There are also other solutions of Eq. (3.6) in this case ($Z=0$). For example,

$$\Delta_F(k) = -\sqrt{\frac{4}{3}} \frac{1}{4\beta} \sqrt{-k^2}, \quad (3.16)$$

i.e., the negative of Eq. (3.15) solves the differential Eq. (3.6). This solution has, however, to be rejected since it corresponds to ghost states. For the general situation see Ref. 16), p. 560 [or p. 570 if dealing with Eq. (3.10)].

Instead of keeping m_0^2 finite, we can also let m_0^2 go to infinity such that

$$Z m_0^2 \xrightarrow[Z \rightarrow 0, m_0^2 \rightarrow \infty]{} \delta, \quad (3.17)$$

*) For $Z=0$, this is, however, not true any more since the expansion which led to Eq. (3.14) becomes meaningless in this case.

where δ is an arbitrary, but finite, constant. In this limit Eq. (3.6) can also be solved explicitly. One solution is

$$\Delta_F(x) = -\delta^{-1}, \quad (3.18)$$

i.e., a constant. This means that the quarks cannot propagate, i.e., have infinite mass.

Such a Green's function has also been obtained in Wilson's lattice gauge theory ⁴⁾ in the strong coupling limit. Moreover, a constant propagator has been promoted already for a couple of years in the massive quark model ¹⁹⁾.

So much for the mathematical situation. We will leave any further discussion of the different solutions for the next section.

4. DISCUSSION

Apart from the free solution (3.8), we found two different solutions for the quark Green's function, Eqs. (3.15) and (3.18). Both solutions belong to class (ii), i.e., correspond to $Z=0$. The first solution certainly is not consistent with quark confinement, whereas the second solution is. In the context of our approach, both solutions are equally privileged, so that the quarks either succumb to the infra-red slavery mechanism or dress up with gluons so as to become zero mass quanta, which means that they can escape on the light cone, where the R^2 potential becomes ineffective (the latter possibility is, of course, the more intelligent one). In the realistic case of coloured quarks it may, however, happen that solution (3.18) is distinguished by the requirement of local gauge invariance as one might infer from the lattice gauge theory ⁴⁾.

If the BS kernel (3.3) is modified as to include some short range interaction, it is likely that Eq. (3.8) will acquire a cut in addition to the pole so that the Jost-Schroer theorem does not apply any more. This means that there would even be single quark states in the spectrum of asymptotic states.

In order to complete the discussion of the Schwinger phenomenon in four dimensions, one had to calculate the bound state spectrum of solution (3.15). If this turns out to be complete, quarks will be shielded from being produced in electron-positron annihilation into hadrons, but are not confined.

Our results now pave the way for a quark model calculation of the static and electromagnetic quantities of hadrons [i.e., the spectrum, form factors, deep inelastic structure functions, $\sigma(e^+e^- \rightarrow \text{hadrons})$, etc.] comprising quark confinement. As has already been stated in the Introduction, we consider the BS approach to be particularly suited for this task.

So far we only considered a harmonic oscillator-type of interaction. As for phenomenological calculations, more general BS kernels being consistent with quark confinement, can be inferred from Eq. (2.5).

Finally, we learned that propagator (quantum) corrections become very important for long range forces. That is to say, that model calculations based on the BS equation with free propagators and on the Dirac- or Klein-Gordon equation with quasi-free constituent motion²⁰⁾ have to be regarded with reservation.

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APPENDIX

As can easily be verified,

$$\sqrt{-k^2} J_{\frac{1}{3}} \left(\frac{2}{3} \sqrt{\frac{z^2}{4\beta}} (-k^2)^{\frac{3}{2}} \right), \sqrt{-k^2} Y_{\frac{1}{3}} \left(\frac{2}{3} \sqrt{\frac{z^2}{4\beta}} (-k^2)^{\frac{3}{2}} \right) \quad (A.1)$$

are two independent solutions of the homogeneous equation (3.12)¹⁶⁾. Hence, the corresponding Green's function reads (by abuse of terminology, we shall not impose any boundary condition upon the Green's function)

$$\begin{aligned} G(k^2, k'^2) = & \frac{\Theta(k^2 - k'^2)}{W} \sqrt{-k^2} \sqrt{-k'^2} \left[Y_{\frac{1}{3}} \left(\frac{2}{3} \sqrt{\frac{z^2}{4\beta}} (-k^2)^{\frac{3}{2}} \right) J_{\frac{1}{3}} \left(\frac{2}{3} \sqrt{\frac{z^2}{4\beta}} (-k'^2)^{\frac{3}{2}} \right) \right. \\ & \left. - J_{\frac{1}{3}} \left(\frac{2}{3} \sqrt{\frac{z^2}{4\beta}} (-k^2)^{\frac{3}{2}} \right) Y_{\frac{1}{3}} \left(\frac{2}{3} \sqrt{\frac{z^2}{4\beta}} (-k'^2)^{\frac{3}{2}} \right) \right] \\ & + \sqrt{-k^2} J_{\frac{1}{3}} \left(\frac{2}{3} \sqrt{\frac{z^2}{4\beta}} (-k^2)^{\frac{3}{2}} \right) f(k'^2) + \sqrt{-k^2} Y_{\frac{1}{3}} \left(\frac{2}{3} \sqrt{\frac{z^2}{4\beta}} (-k^2)^{\frac{3}{2}} \right) g(k'^2), \end{aligned} \quad (A.2)$$

where the Wronskian determinant is

$$\begin{aligned} W = & - \sqrt{\frac{z^2}{4\beta}} (-k'^2)^{\frac{3}{2}} \left[Y_{\frac{1}{3}} \left(\frac{2}{3} \sqrt{\frac{z^2}{4\beta}} (-k'^2)^{\frac{3}{2}} \right) J_{\frac{1}{3}}' \left(\frac{2}{3} \sqrt{\frac{z^2}{4\beta}} (-k'^2)^{\frac{3}{2}} \right) \right. \\ & \left. - J_{\frac{1}{3}} \left(\frac{2}{3} \sqrt{\frac{z^2}{4\beta}} (-k'^2)^{\frac{3}{2}} \right) Y_{\frac{1}{3}}' \left(\frac{2}{3} \sqrt{\frac{z^2}{4\beta}} (-k'^2)^{\frac{3}{2}} \right) \right] \\ = & - \frac{3}{\pi}. \end{aligned} \quad (A.3)$$

In Eq. (A.2), f and g are arbitrary functions which are only constrained by the boundary condition $\rho(0) = 1$. The convergence of the integrals in Eq. (3.13) is guaranteed even for $f = g = 0$.

The analytic structure of the various components $\rho^{(n)}(k^2)$, as given by Eq. (3.13), can now be read off from the properties of the Bessel functions (A.1). The latter are regular in the finite k^2 plane and have an essential singularity at infinity. For $-2/3 \pi < \arg(-k^2) < 2/3 \pi$ they have the asymptotic expansions ¹⁷⁾

$$\left(\frac{\pi}{3} \sqrt{\frac{z^2}{4\beta}}\right)^{-\frac{1}{2}} (-k^2)^{-\frac{1}{4}} \cos \left[\frac{2}{3} \sqrt{\frac{z^2}{4\beta}} (-k^2)^{\frac{3}{2}} - \frac{5}{12} \pi \right] \quad (\text{A.4})$$

and

$$\left(\frac{\pi}{3} \sqrt{\frac{z^2}{4\beta}}\right)^{-\frac{1}{2}} (-k^2)^{-\frac{1}{4}} \sin \left[\frac{2}{3} \sqrt{\frac{z^2}{4\beta}} (-k^2)^{\frac{3}{2}} - \frac{5}{12} \pi \right] \quad (\text{A.5})$$

respectively, while on the positive real k^2 axis, one derives

$$\begin{aligned} \sqrt{-k^2} J_{\pm \frac{1}{3}} \left(\frac{2}{3} \sqrt{\frac{z^2}{4\beta}} (-k^2)^{\frac{3}{2}} \right) &= \frac{\sqrt{3}}{2} \left(\frac{2}{3} \sqrt{\frac{z^2}{4\beta}} \right)^{-\frac{1}{3}} \left[\frac{\sqrt{3}}{\pi} \int_0^{\infty} dt \cos \left(\frac{1}{3} t^3 \right. \right. \\ &+ \left. \left. \left(\frac{2}{3} \sqrt{\frac{z^2}{4\beta}} \right)^{\frac{2}{3}} k^2 t \right) + \frac{1}{\pi} \int_0^{\infty} dt \left[\exp \left(-\frac{1}{3} t^3 + \left(\frac{2}{3} \sqrt{\frac{z^2}{4\beta}} \right)^{\frac{2}{3}} k^2 t \right) \right. \right. \\ &\left. \left. + \sin \left(\frac{1}{3} t^3 + \left(\frac{2}{3} \sqrt{\frac{z^2}{4\beta}} \right)^{\frac{2}{3}} k^2 t \right) \right] \right], \end{aligned}$$

$$Y_{\frac{1}{3}} = \frac{1}{\sqrt{3}} (J_{\frac{1}{3}} - 2 J_{-\frac{1}{3}}). \quad (\text{A.6})$$

This means that any $\rho^{(n)}(k^2)$ is regular in the finite k^2 plane and has an essential singularity at infinity.

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Fig. 1 : Diagrammatical derivation of the Schwinger-Dyson equation for the quark Green's function.
 K stands for the BS kernel.

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