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ANALYSIS OF PCAC CONSTRAINTS IN A DISPERSIVE APPROACH
TO OFF-MASS SHELL $\pi\pi$ SCATTERING

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A B S T R A C T

A one-parameter set of $\pi\pi$ amplitudes fulfilling general on-shell constraints is extrapolated off the mass shell of one pion by means of the dispersive method suggested by Basdevant and Hayot. The ability of PCAC to restrict this class of amplitudes is investigated within some given criteria for the smoothness in the off-shell mass dependence. It is found in particular that the requirement of maximal smoothness selects some neighbourhood of Weinberg's solution ($0 \lesssim a_0^0 \lesssim 0.4$).

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1. - INTRODUCTION

It appears ¹⁾⁻⁴⁾ at the moment that all the known constraints, both experimental and theoretical, do not allow a complete phenomenological determination of the $\pi\pi$ amplitudes below 2 GeV. The major ambiguities lie in the threshold region ($E_{\pi\pi} \lesssim 500$ MeV) and are, roughly speaking, described in all isospins by one low energy parameter, which is conveniently chosen as a_0 , the $I=0$ S wave scattering length. Then, unless a definite choice is made among the various low energy data sets, one can simply say that a_0 has to be somewhere between 0 and 1.

A possible way to reduce this ambiguity can be searched in the application, in addition to the previous general tools, of the current algebra (CA) or PCAC theorems which involve off-shell amplitudes. Such a programme has been attempted for a long time, by Weinberg ⁵⁾ especially, he predicts $a_0 \simeq 0.2$, a value compatible with some of the present data ³⁾ and remarkable by its stability upon refinements and changes of its original derivation ⁶⁾.

In these models, however, the off-shell continuation is rather arbitrary and the work of Basdevant and Hayot ⁷⁾ gives deeper insight into the external mass dependence of the amplitudes. They have shown in particular that CA theorems can be fulfilled without constraining the on-shell amplitudes. This is a natural consequence of their dispersive approach which allows a model-independent extrapolation on very general grounds, but which introduces some unknowns like subtractions or high energy terms.

These unwanted ambiguities are usually skipped through some specific parametrization thanks to the "smoothness" assumption in the external mass dependence. As this assumption is, in any way, needed to ensure some predictive power to CA, the purpose of this paper is to report some of its consequences in the framework of the dispersive method, where it has to be stated explicitly.

We shall proceed in two steps. First, we construct the off-shell extrapolation of a whole spectrum of physical amplitudes which correspond to all admissible values of a_0 ($0 \leq a_0 \leq 1.4$). For each value of a_0 , this extrapolation introduces two degrees of freedom. Thus, in Section 3, we study some possibilities linked to high energy smoothness to fix these unknown before applying zero mass pion theorems, which then allow the restriction of a_0 within some range.

2. - DEFINING EQUATIONS FOR THE OFF-MASS SHELL AMPLITUDES

We denote by $F^I(s, t, m^2)$ the $\pi\pi$ amplitude of isospin I in the s channel with one pion off its mass shell, as shown on Fig. 1. The physical pion mass is set equal to unity ($p_2^2 = p_3^2 = p_4^2 = 1$) and m is the mass of the off-shell one ($p_1^2 = m^2$). The normalization is such that on-shell $F^I(s, t, m^2 = 1)$ coincides with the amplitudes of Ref. 2).

The s channel scattering angle θ is given by

$$\cos \theta = (t - u) / 4pq = (2t + s - \Sigma) / 4pq \quad (2.1)$$

where p and q are the initial and final momenta

$$\begin{aligned} p &= \left[(s - s_1)(s - s_2) / 4s \right]^{\frac{1}{2}} \\ q &= \left[(s - 4) / 4 \right]^{\frac{1}{2}} \\ s_1 &= (m + 1)^2 \quad s_2 = (m - 1)^2 \end{aligned} \quad (2.2)$$

We shall now give a short derivation of the equations which define the off-shell amplitudes in the dispersive method of Ref. 7), where details can be found. The basic properties attributed to $F^I(s, t, m^2)$ are the following :

- 1) Mandelstam analyticity (at least for $0 \leq m^2 \leq 1$ which is the case we are interested in) ;
- 2) crossing symmetry as on-shell (since only one pion is off-shell) ;
- 3) linear continuation of elastic unitarity

$$\text{Im } f_e^I(s, m^2) = \sqrt{\frac{s-4}{s}} \left[\frac{s-4}{4} \right]^{\frac{1}{2}} f_e^I(s, m^2) \overline{f_e^I(s, m^2 = 1)} \quad (2.3)$$

for $4 \leq s \leq s_I$, where s_I is the first inelastic threshold and $f_e^I(s, m^2)$ the reduced off-shell partial waves (RPW)

$$f_e^I(s, m^2) = \frac{1}{2(pq)} \int_{-1}^1 F^I(s, t, m^2) P_e(\cos \theta) d(\cos \theta) \quad (2.4)$$

These very general properties enable a dispersive approach if some control is put on the amplitudes at high energy. This behaviour has to be consistent with CA constraints - for example positivity and strong asymptotic decrease are not compatible with PCAC since through unsubtracted dispersion relations they imply that the $\pi^0\pi^0$ and $\pi^+\pi^0$ amplitudes never vanish ⁴⁾ - but otherwise is left free. However, since Eq. (2.3) can be used up to 1 GeV approximately ($K\bar{K}$ production is the first important inelastic threshold), we can assume that above this energy the variations due to $\Delta m^2 \simeq 1$ are negligible, which leads to the following convenient hypothesis :

- 4) high energy smoothness assumptions ; two subtractions are sufficient for off-shell dispersion relation (DR) and if $s' \geq s_M$, $s_M \lesssim s(K\bar{K})$, $0 \leq m^2 \leq 1$, $\cos \theta$ in the Lehmann ellipse then

$$R^I(s', t, m^2) \simeq R^I(s', t, m^2) \Big|_{m^2=1} \quad (2.5)$$

where $A^I(s', t, m^2)$ is the absorptive part of $F^I(s', t, m^2)$.

The amplitudes fulfilling properties 1 to 4 can be constructed in the following way. First, a twice subtracted fixed t DR gives

$$F^I(s, t, m^2) = L_J^I(s, t, m^2) C^J(m^2) + \int_A^\infty K_J^I(s', s, t, m^2) R^J(s', t, m^2) ds' \quad (2.6)$$

where summation on repeated indices is understood, K_J^I are known kernels and $L_J^I C^J$ the subtraction polynomials. The subtraction points are chosen to be

$$\begin{aligned} s &= s_0 = 4 \\ t &= u = \frac{s - s_0}{2} = \frac{m^2 - 1}{2} \end{aligned} \quad (2.7)$$

then

$$\begin{aligned} c^0(m^2) &= a_0(m^2) \\ c^1(m^2) &= 0 \\ c^2(m^2) &= a_2(m^2) \end{aligned} \quad (2.8)$$

where $a_J(m^2)$ are the off-shell S wave scattering lengths.

In the dispersive part of Eq. (2.6), if $4 \leq s' \leq s_M$ only the contribution of the S and P waves is retained, if $s' > s_M$ Eq. (2.5) is used. The resulting expression is then projected to obtain the S and P waves for $s < s_M$:

$$f_I(s, m^2) = L_I^J(s, m^2) c^J(m^2) + f_I^\infty(s, m^2) + \frac{1}{\pi} \int_4^{s_M} \tilde{Q}_I^J(s', s, m^2) \text{Im} f_J(s', m^2) ds' \quad (2.9)$$

(From now on we omit the subscript ℓ since we are interested only in the S and P waves.) In this expression L_I^J and $\tilde{Q}_I^J$ are known (they can be found in the Appendix), and the meaning of each term is obvious. For example, $f_I^\infty(s, m^2)$ comes from the projection of

$$\int_{s_M}^{\infty} K_J^I(s', s, t, m^2) H^J(s', t, m^2=1) ds' \quad (2.10)$$

It is clear from their derivation that the on-shell restriction of Eqs. (2.3) and (2.9) leads to the non-linear singular system which serves as a basic tool for present $\pi\pi$ phenomenology (Roy's equations). As a result of our previous work of Ref. 2), we have at our disposal a one-parameter set of amplitudes $F^I(s, t, m^2=1)$ "solutions" of this system which fulfill some additional requirements (mainly experimental). To extrapolate this family we thus only have to solve Eqs. (2.3) and (2.9) for non-physical values of m^2 , using in (2.10) the high energy terms defined in Ref. 2), according to assumption (2.5).

Off-shell, this system is linear, and we have simply to turn it into a regular (Fredholm) form. We then isolate the components of the RPW singular on the elastic cut :

$$f_I(s, m^2) = f_I^R(s, m^2) + \frac{1}{\pi} \int_4^{s_M} \text{Im} f_I(s', m^2) \frac{ds'}{s' - s} \quad (2.11)$$

where

$$f_I^R(s, m^2) = L_I^J c^J(m^2) + f_I^\infty(s, m^2) + \frac{1}{\pi} \int_4^{s_M} Q_I^J(s', s, m^2) \text{Im} f_J(s', m^2) ds' \\ Q_I^J(s', s, m^2) = \tilde{Q}_I^J(s', s, m^2) - \delta_I^J / (s' - s) \quad (2.12)$$

As the singular parts vanish at infinity like $1/s$, there is no CDD ambiguity and standard methods ⁸⁾ give the following expression for the discontinuities across the elastic cut :

$$\frac{\text{Im} f_I(s, m^2)}{\sin \delta_I(s)} = \cos \delta_I(s) f_I^R(s, m^2) - \frac{1}{\pi |D_I(s)|} \int_4^{s_M} f_I^R(s', m^2) \frac{\text{Im} D_I(s')}{s' - s} ds' \quad (2.13)$$

where $D_I(s)$ are the Omnès functions constructed from the elastic phase shifts $\delta_I(s)$ taken from $F^I(s, t, m^2=1)$:

$$D_I(s) = \exp \left[- \frac{s-4}{\pi} \int_4^{s_M} \frac{\delta_I(x)}{(x-4)(x-s)} dx \right] \quad (2.14)$$

[s_M is fixed at $\sqrt{s_M} = 950$ MeV, then $|\delta_I(s)| < \pi$ and $D_I(s)$ have only logarithmic end-point singularities].

Inserting representation (2.12) for $f_I^R(s, m^2)$ into (2.13), we obtain, after a change in the order of integrations, a Fredholm system for the three unknowns $\text{Im} f_I(s, m^2)$ ($4 \leq s < s_M$) :

$$\text{Im} f_I(s, m^2) = X_I(s, m^2) + \frac{1}{\pi} \int_4^{s_M} F_I^J(s', s, m^2) \text{Im} f_J(s', m^2) ds' \quad (2.15)$$

with

$$X_I(s, m^2) = \sin \delta_I(s) O_I \left\{ s; f_I^\infty(x, m^2) + L_I^J c^J(m^2) \right\} \\ F_I^J(s', s, m^2) = O_I \left\{ s; Q_I^J(s', x, m^2) \right\} \quad (2.16)$$

where O_I is a linear operator constructed from δ_I according to

$$O_I \{ s; f(x, y) \} = \cos \delta_I(s) f(s, y) - \frac{1}{\pi |D_I(s)|} \int_4^{s_M} \frac{f(x, y)}{x - s} \text{Im} D_I(x) dx \quad (2.17)$$

Solving this system (a 60×60 matrix inversion was used) for $\text{Im } f_I(s, m^2)$, putting back the result in (2.6) and taking into account the linearity of all these operations, we obtain the off-shell amplitudes as

$$F^I(s, t, m^2) = H_\infty^I(s, t, m^2) + H_J^I(s, t, m^2) c^J(m^2) \quad (2.18)$$

3. - IMPLICATIONS OF SMOOTHNESS AND CA CONSTRAINTS

Let us recall the content of representation (2.18). When $m^2 = 1$, it is a complicated way to rewrite the spectrum of physical amplitudes given in Ref. 2). (This, we have numerically checked.) When $0 \leq m^2 < 1$, as H_∞^I and H_J^I are explicitly known functionals of a_0 (through the phase shifts δ_I which are themselves functionals of a_0) and of high energy terms (only H_∞^I), it gives the off-shell continuation of this spectrum fulfilling requirements 1 to 4 of the previous section. But, as announced, this extrapolation contains two degrees of freedom, $c^J(m^2)$. As pointed out in Ref. 7), these unknowns can be fixed at $m^2 = 0$ by imposing Adler and Adler-Weisberger relations, this leaving a_0 completely free.

We thus first follow this programme. The Adler's condition which reads (its $I=0$ component is sufficient due to crossing symmetry)

$$F^0(1, 1, 0) = H_\infty^0\{a_0\} + c^0(m^2=0) H_0^0\{a_0\} + c^2(m^2=0) H_2^0\{a_0\} = 0 \quad (3.1)$$

and the Adler sum rule :

$$\frac{1}{f_\pi^2} = \frac{1}{\pi} \int_4^\infty \frac{ds}{s-1} \left[\sigma_T^{+-}(m^2=0) - \sigma_T^{++}(m^2=0) \right] \simeq 2.25 \quad (3.2)$$

leads to a (2×2) linear (regular) system for $c^I(0)$.

Let us for the moment simply look at (3.1) [nothing better comes from (3.2)]. It is numerically found that

$$H_2^0\{a_0\} \simeq 2 H_0^0\{a_0\} \quad (3.3)$$

So, the neutral combination $c_N = c_N(m^2=0)$ with

$$c_N(m^2) = F^0(4, \frac{m^2-1}{2}, m^2) + 2 F^2(4, \frac{m^2-1}{2}, m^2) \quad (3.4)$$

is given by PCAC according to

$$c_N \equiv c_N^A(a_0) = - H_\infty^0\{a_0\} / H_0^0\{a_0\} \quad (3.5)$$

This quantity is plotted on Fig. 2a as a function of a_0 with its on-shell analogue a_N

$$a_N = a_0 + 2 a_2 \quad (3.6)$$

since the discrepancy $|c_N^A(a_0) - a_N|$ gives a measure of the m^2 dependence implied by PCAC. Also shown on this Fig. 2a are estimates of the uncertainties under high energy variations (for fixed a_0). The error δa_N comes from the width of a "universal band" ²⁾ ($\simeq 0.015$) combined with its displacement ³⁾ ($\simeq 0.025$) :

$$\delta a_N = 2 \delta a_2(a_0) \simeq 0.08 \quad (3.7)$$

The corresponding variation δc_N^A (the high energy arbitrary m^2 dependence being neglected) is

$$\frac{\delta c_N^A}{c_N^A} = \frac{\delta H_\infty^0\{a_0\}}{H_\infty^0\{a_0\}} \simeq 20\% \quad (3.8)$$

Thus, neglecting the m^2 dependence wherever it is unknown (i.e., in high energy terms and off-shell scattering lengths) leads us to select, by PCAC, an area around the Weinberg value which cannot safely be restricted by more than

$$0.1 \leq a_0 \leq 0.25 \quad (3.9)$$

Compared to the high energy smoothness assumption, the requirement $c_N^A \simeq a_N$ seems an arbitrarily strong constraint ["naturally" implemented in polynomial expansions around threshold of the on- and off-shell amplitudes ; for example at third order ⁹⁾ $a_N(m^2=0) = 9/8 a_N$].

However, the combination of high energy smoothness with on-shell Regge asymptotic behaviour can bring some estimate of the off-shell scattering lengths. This behaviour implies the fulfilment by $F^I(s, t, m^2=1)$ [checked in Ref. 2] of some minimally subtracted fixed t dispersion relations which, off-shell read :

$$F^I(s, t, m^2) = \frac{1}{\pi} \int_4^\infty \tilde{K}_J^I(s', s, t, m^2) A^J(s', t, m^2) ds' \quad (3.10)$$

When the low energy absorptive parts in (3.10) are taken from the solution of the Fredholm system (2.15), this can be rewritten as :

$$F^I(s, t, m^2) = \tilde{H}_\omega^I(s, t, m^2) + \tilde{H}_J^I(s, t, m^2) c^J(m^2) \quad (3.11)$$

in close analogy with (2.18). In that case, however, (3.11) does not reduce to an identity at the subtraction point and give a linear system for the off-shell scattering lengths :

$$c^I(m^2) = \tilde{H}_\omega^I(4, \frac{m^2-1}{2}, m^2) + \tilde{H}_J^I(4, \frac{m^2-1}{2}, m^2) c^J(m^2) \quad (3.12)$$

[On-shell (3.12) is a self-consistency relation fulfilled by a_I within some uncertainties ²⁾.]

The resolution of this system with the Regge and high energy terms given in Ref. 2) leads to the following results.

1) - For all a_0, m^2 $0 \leq a_0 < 1.5$ $0 \leq m^2 \leq 1$

$$c^I(m^2) \simeq a_I \quad (3.13)$$

(the discrepancy is of the order of the numerical precision) and also $f_I(s, m^2) \simeq f_I(s, 1)$ (except in the vicinity of off-shell kinematical singularities like $s=0$).

2) - As the correspondence between on- and off-mass shell amplitudes is one-to-one for all a_0 , the zero mass pion theorems are constraints on a_0 that we display on Fig. 2b. They are found perfectly consistent and select the Weinberg value

$$a_0 \simeq 0.16 \quad a_2 \simeq -0.055 \pm 0.015 \quad (3.14)$$

However $c^I(m^2)$ are sensitive to the high energy data through minimally subtracted DR. Again we neglect the m^2 dependence of the high energy term ; only experimental uncertainties between 1 and 2 GeV are considered. Then

$$\delta c^I(m^2) \simeq \delta c^I(m^2=1) \quad (3.15)$$

thus

$$\delta c_N(0) \simeq \delta a_N \Big|_{a_0 \text{ fixed}} + \delta a_0 \leq 0.2$$

(3.16)

$[\delta a_0 \simeq 0.1 \text{ from Ref. 2}]$.

Combining this result with PCAC $[c_N(0) = c_N^A]$, it can be read on Fig. 2a that

$$a_0 = 0.2 \pm 0.2 \quad (3.17)$$

4. - CONCLUDING REMARKS

A) - First, a technical point. It has been conjectured, and realized in some models ^{9),10)}, that strong unitarity threshold or sub-threshold effects, like virtual bound states, for example, imply a non-smooth extrapolation in the Adler's point vicinity. (In fact this is one of the reasons why we start with high energy smoothness only.) This does not happen, since even for $a_0 \simeq 1.4$, the on- and off-shell amplitudes can be made very similar when all unitarity effects are properly taken into account.

B) - About the predictive power of PCAC for the scattering lengths. As stated in Ref. 7), it is worthless if not implemented with some specific m^2 dependence. In case of smoothness, to reach model-independent predictions, one has to state something about high energy scattering, and in the best situation (where we are not too wrong with high energy estimates) this puts a limit $a_0 = 0.2 \pm 0.2$ on its practical present use.

C) - How the value $a_0 = 0.2 \pm 0.2$ fits with present data. Fits to various data sets with the spectrum used here (or equivalently with one of Basdevant et al. ^{1),3)}) give the following estimates :

Baillon et al. ¹¹⁾	(1972)	$0.2 \leq a_0 \leq 0.8$
Baton et al. ¹²⁾	(1970)	$-0.05 \leq a_0 \leq 0.3$
Bensinger et al. ¹³⁾	(1971)	$0.3 \leq a_0 \leq 0.8$
Estabrooks-Martin ¹⁴⁾	(1974)	$0.2 \leq a_0 \leq 0.8$
Protopopescu et al. ¹⁵⁾	(1973)	$0.2 \leq a_0 \leq 0.5$
Villet et al. ¹⁶⁾	(1973)	$0.5 \leq a_0 \leq 1.0$
Basile et al. ¹⁷⁾ (Ke4)	(1971)	$0.3 \leq a_0 \leq 1.5$
Beier et al. ¹⁸⁾ (Ke4)	(1972)	$0.3 \leq a_0 \leq 0.55$ ³⁾

As is well known, the intersection is empty. Nevertheless, $a_0 = 0.2 \pm 0.2$ is compatible with all the data, except those of Ref. 16). The outliving value is then $a_0 \simeq 0.3$ and, as said here, the shift from the Weinberg's value is not significant. However, the selection of the experiments which give the data closest to threshold [Refs. 13), 16), 17), 18)] induces a best fit ^{2),3)} of $a_0 \simeq 0.5$. If such a value is borne out by future experiments (especially by the Ke4 forthcoming results), PCAC needs a quite strong dependence in m^2 of the off-shell scattering lengths. A possible test of this hypothesis would then be to look at τ decays in the pion pole dominance model, where m^2 sits at the K or η mass.

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APPENDIX - OFF-SHELL KERNELS AND SUBTRACTION TERMS

s_0 and t_0 are the subtraction points, such that $s_0 + 2t_0 = \Sigma = 3 + m^2$ the subtraction term $L_I^J(s, m^2) c^J(m^2)$ introduced in (2.9) is

$$L_I^J(s, m^2) c^J(m^2) = \begin{bmatrix} 1 + 2(s-s_0)/3(s_0-t_0), & -5(s-s_0)/3(s_0-t_0) \\ 4/9(s_0-t_0), & -10/9(s_0-t_0) \\ -(s-s_0)/3(s_0-t_0), & 1 + 5(s-s_0)/6(s_0-t_0) \end{bmatrix} \begin{bmatrix} c^0(m^2) \\ c^1(m^2) \\ c^2(m^2) \end{bmatrix}$$

If we define η as $\eta(s', s) = (2s' + s - \Sigma)/4p(s)q(s)$ then Q_0' and Q_1' are

$$Q_0' = Q_0(\eta)/4p(s)q(s) \quad Q_1' = Q_1(\eta)/16(p(s)q(s))^2$$

where Q_0 and Q_1 are the second kind Legendre functions. The regular kernels of Eq. (2.12) are then given by

$$A = (s + s_0 - 2s')/(s' - s_0)(s' - t_0)$$

$$B = (s_0 + t_0 - s - s')/(s' - s_0)(s' - t_0)$$

$$Q_0^0 = (4Q_0' + A)/3 + B$$

$$Q_0^1 = 3(2s + s' - \Sigma)Q_0' - 3(3s + 2s' - \Sigma - 2t_0)/4(s - t_0)$$

$$Q_0^2 = 5(4Q_0' + A)/3$$

$$Q_1^0 = 16Q_1'/3 - 4/9(s' - s_0)(s' - t_0)$$

$$Q_1^1 = 6(2s + s' - \Sigma)Q_1' - 3/2(s' - t_0)$$

$$Q_1^2 = 10/9(s' - s_0)(s' - t_0) - 40Q_1'/3$$

$$Q_2^0 = 4Q_0'/3 + A/3$$

$$Q_2^1 = -Q_0^1/2$$

$$Q_2^2 = 2Q_0^2/3 + A/6 + B$$

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FIGURE CAPTIONS

Figure 1 Kinematics for off-shell amplitude.

Figure 2a Variation with a_0 of c_N^A (band 1) and a_N (band 2).
 c_N^A is the off-shell scattering length given by PCAC,
 a_N the on-shell one.

Figure 2b Zero mass pion theorems when $c_I(0)$ are central solutions
of (3.12). Curves 1) and 2) display the a_0 dependence of
 $F^I(s=1, t=1, m^2=0)$ for $I=0$ and 2, respectively.
Curve 3) shows the corresponding variation of

$$\frac{1}{2} \left[\frac{1}{\pi} \int_4^\infty \frac{ds}{s-1} \left\{ \sigma_T^{+-} - \sigma_T^{++} \right\} - \frac{1}{f_\pi^2} \right] ; \frac{1}{f_\pi^2} = 2.25$$

PCAC and Adler's sum rules are fulfilled at the common
intersection W.

$$\begin{aligned}
 s &= (p_1 + p_2)^2 \\
 t &= (p_1 - p_3)^2 \\
 u &= (p_1 - p_4)^2 \\
 s + t + u &= \Sigma = 3 + m^2
 \end{aligned}$$

FIG. 1

FIG. 2 a

FIG. 2b