Ref.TH.2131-CERN

COMPLETE AND EFFECTIVELY COMPLETE INCLUSIVE EXPERIMENTS

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A B S T R A C T

The experiments required to measure all the independent amplitudes in any single particle inclusive process with experimentally available incident particles are determined, and the amplitude (or combination of amplitudes) obtained from each measurement is given. Various kinematic regions are considered, and the measurements which determine the subset of amplitudes expected to be important in each region are found. The prospects of a meaningful separation of exchanged naturalities are also investigated and found to be difficult, due to a kinematic suppression of certain amplitudes.

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Ref.TH.2131-CERN

12 February 1976

1. INTRODUCTION

As high-energy inclusive experiments begin to yield more detailed information, it becomes important to have a general model-independent reference framework for use as a guide both in planning experiments and in extracting information from experiments. Further, polarization is important in inclusive processes since, unlike the situation in exclusive reactions, polarization effects need not disappear with increasing energy (e.g., in $p + p \rightarrow \Lambda + X$ one would expect the Λ polarization to tend to a non-zero asymptotic value or, in other words, to scale). It is now feasible to measure spin density matrix elements of resonances produced in inclusive reactions and to perform experiments with polarized incident beams as well as targets. If these experiments are to be efficiently planned and utilized, it is necessary to know in advance what measurements are needed to determine particular amplitudes -- and conversely what amplitudes (or combinations of amplitudes) can be determined by a given experiment; and it is useful to know what constitutes a complete experiment, determining all independent amplitudes. In addition, it is important to know whether any amplitudes can be expected to be suppressed in particular kinematic regions, allowing one to concentrate initial experimental and theoretical efforts on easily measured effects.

Initial investigations of inclusive polarization ^{1),2)} were concerned with specific reactions or kinematic regions. More general work is that of Ader, Meyers and Salin ³⁾, who concentrated on naturality separation and defining observables, and that of Goldstein and Owens (GO) ⁴⁾, who considered a range of single and double polarization measurements and also pointed out the presence and importance of the half-angle factors arising from angular momentum conservation.

In this paper we consider all possible polarization measurements for the inclusive process $a + b \rightarrow c + X$, where:

- (a) the beam has $J^P = 0^-, \frac{1}{2}^+, \text{ or } 1^-$ (massless);
- (b) the target has $J^P = \frac{1}{2}^+$;
- (c) the produced particle is any single particle or resonance.

We first determine the experiments required to measure all the independent amplitudes. Unlike the situation in two-to-two reactions ⁵⁾, we find that there are few redundant measurements and that, in general, triple polarization measurements are necessary to perform a complete experiment. Next we determine sets of "effectively complete" experiments. These are experiments which measure all the independent amplitudes which are expected to be

important in a particular kinematic region, based on the assumption that the dominant exchange in the $a\bar{a}$ and $b\bar{b}$ channels is non-flip at high energies. We then consider in detail the effects of the half-angle factors and find that they greatly reduce the utility of polarization measurements and, in particular, make the complete separation of naturalities impractical (in the sense that it is difficult to obtain any real dynamical information) in all but a small kinematic region. Inclusive polarization predictions are considered next, and we conclude with a summary of our results and a few comments.

Section 2 is devoted to notation and other preliminaries. In Section 3 we consider complete experiments, and in Section 4 the more realistic case of effectively complete experiments. The question of kinematic suppression is covered in Section 5, and Section 6 consists of a discussion of some model predictions, followed by a summary and comments. In the Appendix the specific reactions $p+p \rightarrow \Lambda + X$ and $\gamma + p \rightarrow \pi + X$ are considered in detail.

2. PRELIMINARIES

2.1 Optical theorem and notation

As noted in Refs. 1) and 2), the optical theorem used to relate inclusive differential cross-sections to the discontinuity in a forward three-to-three amplitudes ⁶⁾ can be extended to include polarization measurements by considering three-to-three amplitudes with final helicities different from initial helicities. Thus, terms arising from the interference between different helicity amplitudes can be written

$$\sum_K \int \prod_{i=1}^m \frac{d^3 q_i}{(2\pi)^3 2E_i} (2\pi)^4 \delta^4 \left(\sum_{i=1}^m q_i + p_c - p_a - p_b \right) \times \sum_K \int_{\lambda_c K; \lambda_a \lambda_b} f_{\lambda'_a K; \lambda'_a \lambda'_b}^* = \frac{1}{2i} \text{Disc}_{m^2} \int_{\lambda'_c \lambda'_a \lambda'_b; \lambda_a \lambda_b \lambda_c} \quad (2.1)$$

where the discontinuity is evaluated in the forward direction, taking care to be on the correct side of cuts in the subenergies ⁷⁾. In particular, one wishes to be above the cut in $s_{ab} = (p_a + p_b)^2$ and below the cut in $s_{a'b'}$.

Since we will be using equations such as (2.1) extensively in this paper, we need a less cumbersome notation. We shall represent the summations, integrals, phase space factors, and delta function collectively as $\mathcal{F}$, and the right-hand side will be written as $\begin{pmatrix} \lambda'_a & \lambda'_c & \lambda'_b \\ \lambda_a & \lambda_c & \lambda_b \end{pmatrix}$.

In this notation Eq. (2.1) becomes

$$\sum_{\lambda_c \kappa; \lambda_a \lambda_b} f_{\lambda_c \kappa; \lambda_a \lambda_b} f_{\lambda_c' \kappa; \lambda_a' \lambda_b'}^* = \begin{pmatrix} \lambda_a' & \lambda_c' & \lambda_b' \\ \lambda_a & \lambda_c & \lambda_b \end{pmatrix} \quad (2.1a)$$

Throughout this paper we work with s channel helicity amplitudes. Transversity amplitudes have proven very useful in investigations of complete sets of exclusive experiments ⁵⁾, but for complete inclusive experiments there is no great advantage in using them, and they are less familiar than helicity amplitudes.

All the initial states considered contain at least one $\frac{1}{2}^+$ particle; this is chosen as the target particle (b) and is "type 2" in the notation of Jacob and Wick ⁸⁾, whose phase conventions we use. The parity relation of the helicity amplitudes is then

$$f_{-\lambda_c - \kappa; -\lambda_a - \lambda_b}(-\phi_i) = \frac{\eta_c \eta_x}{\eta_a \eta_b} (-i)^{S_c + S_x - S_a - S_b + \kappa - \lambda_c - \lambda_a - \lambda_b} f_{\lambda_c \kappa; \lambda_a \lambda_b}(\phi_i) \quad (2.2)$$

where the ϕ_i are angles of momenta of particles in X and need not concern us since we will be integrating over all of them. [See Ref. 3) for a detailed discussion -- but note that a different phase convention is used.] The more relevant form of the parity constraint for this work is

$$\sum_{\lambda_c \kappa; \lambda_a \lambda_b} f_{\lambda_c \kappa; \lambda_a \lambda_b} f_{\lambda_c' \kappa; \lambda_a' \lambda_b'}^* = (-1)^{(\lambda_a - \lambda_a') + (\lambda_b - \lambda_b') + (\lambda_c - \lambda_c')} \sum_{-\lambda_c \kappa; -\lambda_a - \lambda_b} f_{-\lambda_c \kappa; -\lambda_a - \lambda_b} f_{-\lambda_c' \kappa; -\lambda_a' - \lambda_b'}^* \quad (2.3a)$$

or

$$\begin{pmatrix} \lambda_a & \lambda_c & \lambda_b \\ \lambda_a' & \lambda_c' & \lambda_b' \end{pmatrix} = (-1)^{(\lambda_a - \lambda_a') + (\lambda_b - \lambda_b') + (\lambda_c - \lambda_c')} \begin{pmatrix} -\lambda_a - \lambda_c - \lambda_b \\ -\lambda_a' - \lambda_c' - \lambda_b' \end{pmatrix} \quad (2.3b)$$

which is valid irrespective of which of the particles are bosons or fermions. Equations (2.3) then also imply an equivalent parity relation among the three-to-three amplitudes, via Eq. (2.1a).

Our co-ordinate system is shown in Fig. 1. The z axis is in the direction of the incident beam, $\hat{z} = \hat{p}_a$; the y axis is out of the a - c plane, $\hat{y} = \vec{p}_a \times \vec{p}_c / |\vec{p}_a \times \vec{p}_c|$; and $\hat{x}$ is in the plane, $\hat{x} = \hat{y} \times \hat{z}$. Polarizations of all particles are with respect to this co-ordinate system. The spin density matrices for the initial beam particles are

$$\rho_a = \frac{1}{2} \begin{pmatrix} 1+P_z & P_x-iP_y \\ P_x+iP_y & 1-P_z \end{pmatrix}, \frac{1}{2} \begin{pmatrix} 1 & -P_L e^{-2i\phi} \\ -P_L e^{2i\phi} & 1 \end{pmatrix}, \frac{1}{2} \begin{pmatrix} 1+P_c & 0 \\ 0 & 1-P_c \end{pmatrix} \quad (2.4)$$

for, respectively, $\frac{1}{2}^+$ with polarization $\vec{P}$, 1^- with linear polarization P_L at an angle ϕ with respect to the x axis, and 1^- with circular polarization P_c . The target particle's spin density matrix is

$$\rho_b = \frac{1}{2} \begin{pmatrix} 1-P_z & P_x+iP_y \\ P_x-iP_y & 1+P_z \end{pmatrix} \quad (2.5)$$

for polarization $\vec{P}$. The signs of P_y and P_z change as a consequence of the target being a "type 2" particle.

Throughout the paper we assume the beam energy is sufficiently high that the concepts of central and fragmentation regions have some meaning ^{*)}. Also, over-all multiplicative factors are all absorbed into the $\mathcal{F}$.

2.2 Independent amplitudes

Parity restrictions halve the number of independent amplitudes. We implement these (and the Hermiticity constraints which follow) by restrictions on the spin density matrix elements of the produced particle (unless it is spinless, in which case parity is used to reduce the number of initial helicities we must consider). For weak processes, the parity constraint would, of course, be omitted. The Hermiticity requirement is

$$\begin{pmatrix} \lambda_a & \lambda_c & \lambda_b \\ \lambda'_a & \lambda'_c & \lambda'_b \end{pmatrix} = \begin{pmatrix} \lambda'_a & \lambda'_c & \lambda'_b \\ \lambda_a & \lambda_c & \lambda_b \end{pmatrix}^* \quad (2.6)$$

which follows from Eq. (2.1), or from Hermiticity of the three-to-three transition matrix. Time-reversal invariance does not reduce the number of amplitudes. This is because it relates one amplitude to another amplitude

^{*)} This is not necessary for Section 3.

whose boundary conditions are such that it does not fall into the set of amplitudes we are considering. In the diagrammatic notation of Eden et al.⁹⁾, time-reversal invariance requires

$$\left(\begin{array}{c} a \rightarrow \\ b \rightarrow \\ c' \rightarrow \end{array} \begin{array}{c} \oplus \\ \ominus \end{array} \begin{array}{c} c \\ b' \\ a' \end{array} \right) = e^{i\phi} \left(\begin{array}{c} a' \rightarrow \\ b' \rightarrow \\ c \rightarrow \end{array} \begin{array}{c} \ominus \\ \oplus \end{array} \begin{array}{c} c' \\ b \\ a \end{array} \right) \quad (2.7)$$

and the diagram on the right is not one of the sets of amplitudes under consideration. In this notation the Hermiticity condition takes the form

$$\left(\begin{array}{c} a \rightarrow \\ b \rightarrow \\ c' \rightarrow \end{array} \begin{array}{c} \oplus \\ \ominus \end{array} \begin{array}{c} c \\ b' \\ a' \end{array} \right) = \left(\begin{array}{c} a' \rightarrow \\ b' \rightarrow \\ c \rightarrow \end{array} \begin{array}{c} \oplus \\ \ominus \end{array} \begin{array}{c} c' \\ b \\ a \end{array} \right)^* \quad (2.8)$$

and it can be seen that this does lead to relations within the one set of amplitudes.

The number of three-to-three helicity amplitudes is $4 \times N_a^2 \times N_c^2$ for a spin $\frac{1}{2}$ target, where N_a is the number of helicity states of particle a ($2s_a + 1$ for massive particles). There are, therefore, $8N_a^2 N_c^2$ numbers to be determined. Parity and Hermiticity reduce this to $2N_a^2 N_c^2$ different measurements required to determine all the independent amplitudes (assuming the energy is not so low that the unobserved X must be a single particle state).

3. COMPLETE EXPERIMENTS

3.1 $0^- + \frac{1}{2}^+ \rightarrow c + X$

We consider the spinless beam first because it is the simplest and can be used to illustrate the manner in which the results are obtained. The most important examples of this group are pion and kaon beams incident on nucleon targets.

The initial spin density matrix for arbitrary target polarization is given by Eq. (2.5). Consequently, for a given initial polarization $\vec{P}$,

$$\begin{aligned}
 E_c \frac{d^3\sigma}{d^3p_c} \rho_{\lambda\lambda'}^c(\vec{P}) &= \sum_{\sigma, \sigma'} \int_{\lambda K; 0 \sigma} \rho_{\sigma \sigma'} \int_{\lambda' K; 0 \sigma'}^* \\
 &= \sum \left\{ \left(\int_{\lambda K; 0 \frac{1}{2}} \int_{\lambda' K; 0 \frac{1}{2}}^* + \int_{\lambda K; 0 -\frac{1}{2}} \int_{\lambda' K; 0 -\frac{1}{2}}^* \right) - P_z \left(\int_{\lambda K; 0 \frac{1}{2}} \int_{\lambda' K; 0 \frac{1}{2}}^* \right. \right. \\
 &\quad \left. \left. - \int_{\lambda K; 0 -\frac{1}{2}} \int_{\lambda' K; 0 -\frac{1}{2}}^* \right) + P_x \left(\int_{\lambda K; 0 \frac{1}{2}} \int_{\lambda' K; 0 -\frac{1}{2}}^* + \int_{\lambda K; 0 -\frac{1}{2}} \int_{\lambda' K; 0 \frac{1}{2}}^* \right) \right. \\
 &\quad \left. + i P_y \left(\int_{\lambda K; 0 \frac{1}{2}} \int_{\lambda' K; 0 -\frac{1}{2}}^* + \int_{\lambda K; 0 -\frac{1}{2}} \int_{\lambda' K; 0 \frac{1}{2}}^* \right) \right\} \\
 &= \left[\begin{pmatrix} 0 & \lambda' + \\ 0 & \lambda + \end{pmatrix} + \begin{pmatrix} 0 & \lambda' - \\ 0 & \lambda - \end{pmatrix} \right] - P_z \left[\begin{pmatrix} 0 & \lambda' + \\ 0 & \lambda + \end{pmatrix} - \begin{pmatrix} 0 & \lambda' - \\ 0 & \lambda - \end{pmatrix} \right] \\
 &\quad + P_x \left[\begin{pmatrix} 0 & \lambda' - \\ 0 & \lambda + \end{pmatrix} + \begin{pmatrix} 0 & \lambda' + \\ 0 & \lambda - \end{pmatrix} \right] + i P_y \left[\begin{pmatrix} 0 & \lambda' - \\ 0 & \lambda + \end{pmatrix} - \begin{pmatrix} 0 & \lambda' + \\ 0 & \lambda - \end{pmatrix} \right] \quad (3.1)
 \end{aligned}$$

where the $+, -$ represent target helicities of $+\frac{1}{2}$ and $-\frac{1}{2}$, respectively. This shows the information accessible with each orientation of target polarization. Table 1 summarizes the polarizations needed for each amplitude and the amplitude measured by each polarization. In this and subsequent tables, polarizations marked by an asterisk multiply combinations of amplitudes which are not observable if particle c is spinless. It is clear that, in general, all three orientations of target polarization are required in order to determine all the independent amplitudes. In addition, of course, the density matrix of the produced particle must be measured. If the decay is parity-conserving, only the combinations $\rho_{\lambda\lambda'} + (-1)^{\lambda-\lambda'} \rho_{-\lambda, -\lambda'}$ are observable³⁾. In this case the polarization measurements only measure imaginary parts of discontinuities^{*)} of amplitudes, and unpolarized initial particles allow measurement only of real parts. The expression for the observables in the parity-conserving decay of a particle with spin is

*) The discontinuities need be real only if $\lambda'_a, \lambda'_b, \lambda'_c = \lambda_a, \lambda_b, \lambda_c$.

$$\begin{aligned}
 E_c \frac{d^3\sigma}{d p_c^3} (\rho_{\lambda\lambda'} + (-1)^{\lambda-\lambda'} \rho_{-\lambda'-\lambda}) &= \text{Re} \left[\begin{pmatrix} 0 & \lambda' & + \\ 0 & \lambda & + \end{pmatrix} + \begin{pmatrix} 0 & \lambda' & - \\ 0 & \lambda & - \end{pmatrix} \right] \\
 -i P_z \int_m \left[\begin{pmatrix} 0 & \lambda' & + \\ 0 & \lambda & + \end{pmatrix} - \begin{pmatrix} 0 & \lambda' & - \\ 0 & \lambda & - \end{pmatrix} \right] &+ i P_x \int_m \left[\begin{pmatrix} 0 & \lambda' & - \\ 0 & \lambda & + \end{pmatrix} + \begin{pmatrix} 0 & \lambda' & + \\ 0 & \lambda & - \end{pmatrix} \right] \\
 - P_y \int_m \left[\begin{pmatrix} 0 & \lambda' & - \\ 0 & \lambda & + \end{pmatrix} - \begin{pmatrix} 0 & \lambda' & + \\ 0 & \lambda & - \end{pmatrix} \right] & \quad (3.2)
 \end{aligned}$$

For the special case of particle c spinless, Eq. (3.1) becomes

$$E_c \frac{d^3\sigma}{d p_c^3} = \begin{pmatrix} 0 & 0 & + \\ 0 & 0 & + \end{pmatrix} - P_y \int_m \begin{pmatrix} 0 & 0 & - \\ 0 & 0 & + \end{pmatrix} \quad (3.2a)$$

The situation can be summarized as follows. If particle c is spinless a complete experiment is possible and requires only polarization out of the a - c plane, in addition to the unpolarized cross-section. If $J_c \neq 0$ and c is stable or decays weakly, a complete experiment is again possible, but now requires all three orientations of initial polarization and measurement of all independent density matrix elements for each. If $J_c \neq 0$ and c conserves parity in its decay, a complete experiment is not possible. To obtain all the information which is available all three initial polarizations are necessary, and one must measure the observable [see Eq. (3.2)] density matrix elements in each case.

3.2 $\frac{1}{2}^+ + \frac{1}{2}^+ \rightarrow c + X$

As in the previous Section, starting from the initial density matrix one can obtain an expression for $E_c(d^3\sigma/d p_c^3) \rho_{\lambda\lambda'}^c(\vec{P}_a, \vec{P}_b)$. The result is rather lengthy but informative, in that it shows the amplitudes measured with each combination of initial polarizations,

$$\begin{aligned}
 E_c \frac{d^3\sigma}{d p_c^3} \rho_{\lambda\lambda'}^c(\vec{P}_a, \vec{P}_b) &= \left[\begin{pmatrix} + & \lambda' & + \\ + & \lambda & + \end{pmatrix} + \begin{pmatrix} + & \lambda' & - \\ + & \lambda & - \end{pmatrix} + \begin{pmatrix} - & \lambda' & + \\ - & \lambda & + \end{pmatrix} + \begin{pmatrix} - & \lambda' & - \\ - & \lambda & - \end{pmatrix} \right] \\
 + P_{az} \left[\begin{pmatrix} + & \lambda' & + \\ + & \lambda & + \end{pmatrix} + \begin{pmatrix} + & \lambda' & - \\ + & \lambda & - \end{pmatrix} - \begin{pmatrix} - & \lambda' & + \\ - & \lambda & + \end{pmatrix} - \begin{pmatrix} - & \lambda' & - \\ - & \lambda & - \end{pmatrix} \right] &- P_{bz} \left[\begin{pmatrix} + & \lambda' & + \\ + & \lambda & + \end{pmatrix} - \begin{pmatrix} + & \lambda' & - \\ + & \lambda & - \end{pmatrix} + \begin{pmatrix} - & \lambda' & + \\ - & \lambda & + \end{pmatrix} - \begin{pmatrix} - & \lambda' & - \\ - & \lambda & - \end{pmatrix} \right]
 \end{aligned}$$

$$\begin{aligned}
& -P_{ax}P_{bz} \left[\begin{pmatrix} +\lambda' + \\ +\lambda + \end{pmatrix} - \begin{pmatrix} +\lambda' - \\ +\lambda - \end{pmatrix} - \begin{pmatrix} -\lambda' + \\ -\lambda + \end{pmatrix} + \begin{pmatrix} -\lambda' - \\ -\lambda - \end{pmatrix} \right] + P_{ax} \left[\begin{pmatrix} -\lambda' + \\ +\lambda + \end{pmatrix} + \begin{pmatrix} -\lambda' - \\ +\lambda - \end{pmatrix} \right. \\
& \left. + \begin{pmatrix} +\lambda' + \\ -\lambda + \end{pmatrix} + \begin{pmatrix} +\lambda' - \\ -\lambda - \end{pmatrix} \right] - i P_{ay} \left[\begin{pmatrix} -\lambda' + \\ +\lambda + \end{pmatrix} + \begin{pmatrix} -\lambda' - \\ +\lambda - \end{pmatrix} - \begin{pmatrix} +\lambda' + \\ -\lambda + \end{pmatrix} - \begin{pmatrix} +\lambda' - \\ -\lambda - \end{pmatrix} \right] \\
& - P_{ax}P_{bz} \left[\begin{pmatrix} -\lambda' + \\ +\lambda + \end{pmatrix} - \begin{pmatrix} -\lambda' - \\ +\lambda - \end{pmatrix} + \begin{pmatrix} +\lambda' + \\ -\lambda + \end{pmatrix} - \begin{pmatrix} +\lambda' - \\ -\lambda - \end{pmatrix} \right] + i P_{ay}P_{bz} \left[\begin{pmatrix} -\lambda' + \\ +\lambda + \end{pmatrix} - \begin{pmatrix} -\lambda' - \\ +\lambda - \end{pmatrix} \right. \\
& \left. - \begin{pmatrix} +\lambda' + \\ -\lambda + \end{pmatrix} + \begin{pmatrix} +\lambda' - \\ -\lambda - \end{pmatrix} \right] + P_{bx} \left[\begin{pmatrix} +\lambda' - \\ +\lambda + \end{pmatrix} + \begin{pmatrix} -\lambda' - \\ -\lambda + \end{pmatrix} + \begin{pmatrix} +\lambda' + \\ +\lambda - \end{pmatrix} + \begin{pmatrix} -\lambda' + \\ -\lambda - \end{pmatrix} \right] \\
& + i P_{by} \left[\begin{pmatrix} +\lambda' - \\ +\lambda + \end{pmatrix} + \begin{pmatrix} -\lambda' - \\ -\lambda + \end{pmatrix} - \begin{pmatrix} +\lambda' + \\ +\lambda - \end{pmatrix} - \begin{pmatrix} -\lambda' + \\ -\lambda - \end{pmatrix} \right] + P_{bx}P_{az} \left[\begin{pmatrix} +\lambda' - \\ +\lambda + \end{pmatrix} - \begin{pmatrix} -\lambda' - \\ -\lambda + \end{pmatrix} \right. \\
& \left. + \begin{pmatrix} +\lambda' + \\ +\lambda - \end{pmatrix} - \begin{pmatrix} -\lambda' + \\ -\lambda - \end{pmatrix} \right] + i P_{by}P_{az} \left[\begin{pmatrix} +\lambda' - \\ +\lambda + \end{pmatrix} - \begin{pmatrix} -\lambda' - \\ -\lambda + \end{pmatrix} - \begin{pmatrix} +\lambda' + \\ +\lambda - \end{pmatrix} + \begin{pmatrix} -\lambda' + \\ -\lambda - \end{pmatrix} \right] \\
& + P_{ax}P_{bx} \left[\begin{pmatrix} -\lambda' - \\ +\lambda + \end{pmatrix} + \begin{pmatrix} -\lambda' + \\ +\lambda - \end{pmatrix} + \begin{pmatrix} +\lambda' - \\ -\lambda + \end{pmatrix} + \begin{pmatrix} +\lambda' + \\ -\lambda - \end{pmatrix} \right] + P_{ay}P_{by} \left[\begin{pmatrix} -\lambda' - \\ +\lambda + \end{pmatrix} - \begin{pmatrix} -\lambda' + \\ +\lambda - \end{pmatrix} \right. \\
& \left. - \begin{pmatrix} +\lambda' - \\ -\lambda + \end{pmatrix} + \begin{pmatrix} +\lambda' + \\ -\lambda - \end{pmatrix} \right] + i P_{by}P_{ax} \left[\begin{pmatrix} -\lambda' - \\ +\lambda + \end{pmatrix} - \begin{pmatrix} -\lambda' + \\ +\lambda - \end{pmatrix} + \begin{pmatrix} +\lambda' - \\ -\lambda + \end{pmatrix} - \begin{pmatrix} +\lambda' + \\ -\lambda - \end{pmatrix} \right] \\
& - i P_{ay}P_{bx} \left[\begin{pmatrix} -\lambda' - \\ +\lambda + \end{pmatrix} + \begin{pmatrix} -\lambda' + \\ +\lambda - \end{pmatrix} - \begin{pmatrix} +\lambda' - \\ -\lambda + \end{pmatrix} - \begin{pmatrix} +\lambda' + \\ -\lambda - \end{pmatrix} \right] \tag{3.3}
\end{aligned}$$

where the left(right) $+, -$ refer to beam (target) helicities $+\frac{1}{2}, -\frac{1}{2}$. Again it is evident that, in order to determine all independent amplitudes (for $J_c \neq 0$), one needs all orientations of original polarizations, unless the two incident particles are identical. If they are identical, one need not use both P_{ai} and P_{bi} or both $P_{ai}P_{bj}$ and $P_{aj}P_{bi}$ ($i \neq j$) since the two sets of amplitudes are no longer independent.

Referring to Eq. (3.3) one sees that the general pattern is the same as in the preceding subsection; P_z allows separation of $(+)$ from $(-)$, P_x and P_y allow measurement of $(+) \pm (-)$. Table 2 gives the measurements necessary to determine each amplitude. As in the preceding case, if particle c has spin and conserves parity in its decay, each polarization measures only the real or imaginary part of the combination of amplitudes

multiplying it in Eq. (3.3). This can be seen by computing the observables $E_c (d^3\sigma/dp_c^3)^{1/2} [\rho_{\lambda\lambda'} + (-1)^{\lambda-\lambda'} \rho_{-\lambda, -\lambda'}]$ and using parity and Hermiticity. In the terminology of Ref. 3), the "Bohr antisymmetric" terms are imaginary. Explicitly, the result is

$$\begin{aligned}
 E_c \frac{d^3\sigma}{dp_c^3} (\rho_{\lambda\lambda'} + (-1)^{\lambda-\lambda'} \rho_{-\lambda, -\lambda'}) = & \operatorname{Re} \left[\begin{pmatrix} + & \lambda' & + \\ + & \lambda & + \end{pmatrix} + \begin{pmatrix} + & \lambda' & - \\ + & \lambda & - \end{pmatrix} + \begin{pmatrix} - & \lambda' & + \\ - & \lambda & + \end{pmatrix} + \begin{pmatrix} - & \lambda' & - \\ - & \lambda & - \end{pmatrix} \right] \\
 & + i P_{az} \Im \left[\begin{pmatrix} + & \lambda' & + \\ + & \lambda & + \end{pmatrix} + \begin{pmatrix} + & \lambda' & - \\ + & \lambda & - \end{pmatrix} - \begin{pmatrix} - & \lambda' & + \\ - & \lambda & + \end{pmatrix} - \begin{pmatrix} - & \lambda' & - \\ - & \lambda & - \end{pmatrix} \right] - i P_{bz} \Im \left[\begin{pmatrix} + & \lambda' & + \\ + & \lambda & + \end{pmatrix} - \begin{pmatrix} + & \lambda' & - \\ + & \lambda & - \end{pmatrix} \right. \\
 & \left. + \begin{pmatrix} - & \lambda' & + \\ - & \lambda & + \end{pmatrix} - \begin{pmatrix} - & \lambda' & - \\ - & \lambda & - \end{pmatrix} \right] + i P_{ax} \Im \left[\begin{pmatrix} - & \lambda' & + \\ + & \lambda & + \end{pmatrix} + \begin{pmatrix} - & \lambda' & - \\ + & \lambda & - \end{pmatrix} + \begin{pmatrix} + & \lambda' & + \\ - & \lambda & + \end{pmatrix} + \begin{pmatrix} + & \lambda' & - \\ - & \lambda & - \end{pmatrix} \right] \\
 & - i P_{bx} \Im \left[\begin{pmatrix} + & \lambda' & - \\ + & \lambda & + \end{pmatrix} + \begin{pmatrix} - & \lambda' & - \\ - & \lambda & + \end{pmatrix} + \begin{pmatrix} + & \lambda' & + \\ + & \lambda & - \end{pmatrix} + \begin{pmatrix} - & \lambda' & + \\ - & \lambda & - \end{pmatrix} \right] + P_{ay} \Im \left[\begin{pmatrix} - & \lambda' & + \\ + & \lambda & + \end{pmatrix} + \begin{pmatrix} - & \lambda' & - \\ + & \lambda & - \end{pmatrix} \right. \\
 & \left. - \begin{pmatrix} + & \lambda' & + \\ - & \lambda & + \end{pmatrix} - \begin{pmatrix} + & \lambda' & - \\ - & \lambda & - \end{pmatrix} \right] - P_{by} \Im \left[\begin{pmatrix} + & \lambda' & - \\ + & \lambda & + \end{pmatrix} + \begin{pmatrix} - & \lambda' & - \\ - & \lambda & + \end{pmatrix} - \begin{pmatrix} + & \lambda' & + \\ + & \lambda & - \end{pmatrix} - \begin{pmatrix} - & \lambda' & + \\ - & \lambda & - \end{pmatrix} \right] \\
 & - P_{az} P_{bz} \operatorname{Re} \left[\begin{pmatrix} + & \lambda' & + \\ + & \lambda & + \end{pmatrix} - \begin{pmatrix} + & \lambda' & - \\ + & \lambda & - \end{pmatrix} - \begin{pmatrix} - & \lambda' & + \\ - & \lambda & + \end{pmatrix} + \begin{pmatrix} - & \lambda' & - \\ - & \lambda & - \end{pmatrix} \right] - P_{ax} P_{bz} \operatorname{Re} \left[\begin{pmatrix} - & \lambda' & + \\ + & \lambda & + \end{pmatrix} - \begin{pmatrix} - & \lambda' & - \\ + & \lambda & - \end{pmatrix} \right. \\
 & \left. + \begin{pmatrix} + & \lambda' & + \\ - & \lambda & + \end{pmatrix} - \begin{pmatrix} + & \lambda' & - \\ - & \lambda & - \end{pmatrix} \right] + P_{bx} P_{az} \operatorname{Re} \left[\begin{pmatrix} + & \lambda' & - \\ + & \lambda & + \end{pmatrix} - \begin{pmatrix} - & \lambda' & - \\ - & \lambda & + \end{pmatrix} + \begin{pmatrix} + & \lambda' & + \\ + & \lambda & - \end{pmatrix} - \begin{pmatrix} - & \lambda' & + \\ - & \lambda & - \end{pmatrix} \right] \\
 & + i P_{ay} P_{bz} \operatorname{Re} \left[\begin{pmatrix} - & \lambda' & + \\ + & \lambda & + \end{pmatrix} - \begin{pmatrix} - & \lambda' & - \\ + & \lambda & - \end{pmatrix} - \begin{pmatrix} + & \lambda' & + \\ - & \lambda & + \end{pmatrix} + \begin{pmatrix} + & \lambda' & - \\ - & \lambda & - \end{pmatrix} \right] + i P_{by} P_{az} \operatorname{Re} \left[\begin{pmatrix} + & \lambda' & - \\ + & \lambda & + \end{pmatrix} - \begin{pmatrix} - & \lambda' & - \\ - & \lambda & + \end{pmatrix} \right. \\
 & \left. - \begin{pmatrix} + & \lambda' & + \\ + & \lambda & - \end{pmatrix} + \begin{pmatrix} - & \lambda' & + \\ - & \lambda & - \end{pmatrix} \right] + P_{ax} P_{bx} \operatorname{Re} \left[\begin{pmatrix} - & \lambda' & - \\ + & \lambda & + \end{pmatrix} + \begin{pmatrix} - & \lambda' & + \\ + & \lambda & - \end{pmatrix} + \begin{pmatrix} + & \lambda' & - \\ - & \lambda & + \end{pmatrix} + \begin{pmatrix} + & \lambda' & + \\ - & \lambda & - \end{pmatrix} \right] \\
 & + P_{ay} P_{by} \operatorname{Re} \left[\begin{pmatrix} - & \lambda' & - \\ + & \lambda & + \end{pmatrix} - \begin{pmatrix} - & \lambda' & + \\ + & \lambda & - \end{pmatrix} - \begin{pmatrix} + & \lambda' & - \\ - & \lambda & + \end{pmatrix} + \begin{pmatrix} + & \lambda' & + \\ - & \lambda & - \end{pmatrix} \right] + i P_{ax} P_{by} \operatorname{Re} \left[\begin{pmatrix} - & \lambda' & - \\ + & \lambda & + \end{pmatrix} \right. \\
 & \left. - \begin{pmatrix} - & \lambda' & + \\ + & \lambda & - \end{pmatrix} + \begin{pmatrix} + & \lambda' & - \\ - & \lambda & + \end{pmatrix} - \begin{pmatrix} + & \lambda' & + \\ - & \lambda & - \end{pmatrix} \right] - i P_{ay} P_{bx} \operatorname{Re} \left[\begin{pmatrix} - & \lambda' & - \\ + & \lambda & + \end{pmatrix} + \begin{pmatrix} - & \lambda' & + \\ + & \lambda & - \end{pmatrix} - \begin{pmatrix} + & \lambda' & - \\ - & \lambda & + \end{pmatrix} - \begin{pmatrix} + & \lambda' & + \\ - & \lambda & - \end{pmatrix} \right]
 \end{aligned} \tag{3.4}$$

If particle c is spinless, Eq. (3.3) simplifies to

$$\begin{aligned}
 E_c \frac{d^3\sigma}{d\vec{p}_c^3} = & \left[\begin{pmatrix} + & 0 & + \\ + & 0 & + \end{pmatrix} + \begin{pmatrix} + & 0 & - \\ + & 0 & - \end{pmatrix} \right] - P_{az} P_{bz} \left[\begin{pmatrix} + & 0 & + \\ + & 0 & + \end{pmatrix} - \begin{pmatrix} + & 0 & - \\ + & 0 & - \end{pmatrix} \right] \\
 & - i P_{ay} \left[\begin{pmatrix} - & 0 & + \\ + & 0 & + \end{pmatrix} + \begin{pmatrix} - & 0 & - \\ + & 0 & - \end{pmatrix} \right] - P_{ax} P_{bz} \left[\begin{pmatrix} - & 0 & + \\ + & 0 & + \end{pmatrix} - \begin{pmatrix} - & 0 & - \\ + & 0 & - \end{pmatrix} \right] \\
 & + i P_{by} \left[\begin{pmatrix} + & 0 & - \\ + & 0 & + \end{pmatrix} + \begin{pmatrix} - & 0 & - \\ - & 0 & + \end{pmatrix} \right] + P_{bx} P_{az} \left[\begin{pmatrix} + & 0 & - \\ + & 0 & + \end{pmatrix} - \begin{pmatrix} - & 0 & - \\ - & 0 & + \end{pmatrix} \right] \\
 & + P_{ax} P_{bx} \left[\begin{pmatrix} - & 0 & - \\ + & 0 & + \end{pmatrix} + \begin{pmatrix} - & 0 & + \\ + & 0 & - \end{pmatrix} \right] + P_{ay} P_{by} \left[\begin{pmatrix} - & 0 & - \\ + & 0 & + \end{pmatrix} - \begin{pmatrix} - & 0 & + \\ + & 0 & - \end{pmatrix} \right] \quad (3.5)
 \end{aligned}$$

and we see that polarization in the a - c plane is useless unless both initial particles have polarization in this plane.

The conclusions are similar to those of the previous Section. For spinless particles some polarizations lead to no new information, but a complete experiment is possible using the polarizations which appear in Eq. (3.5). For particles with spin which conserve parity in their decay, only the real or imaginary part of each amplitude can be measured, and a complete experiment is not possible. For parity-violating decays, complete experiments are possible, in principle, but require all possible polarization measurements (see Table 2). Similarly, for stable particles the complete determination of the spin density matrix (for each initial polarization) is required for a complete experiment.

3.3 $1^- + \frac{1}{2}^+ \rightarrow c + X$

The third example we consider is a photon beam. For linearly polarized photons (2.4) incident upon a target with polarization $\vec{P}$,

$$\begin{aligned}
 E_c \frac{d^3\sigma}{d\vec{p}_c^3} \rho_{\lambda\lambda'}^c(P_L, \phi, \vec{P}) = & \left[\begin{pmatrix} + & \lambda' & + \\ + & \lambda & + \end{pmatrix} + \begin{pmatrix} + & \lambda' & - \\ + & \lambda & - \end{pmatrix} + \begin{pmatrix} - & \lambda' & + \\ - & \lambda & + \end{pmatrix} + \begin{pmatrix} - & \lambda' & - \\ - & \lambda & - \end{pmatrix} \right] \\
 & - P_z \left[\begin{pmatrix} + & \lambda' & + \\ + & \lambda & + \end{pmatrix} - \begin{pmatrix} + & \lambda' & - \\ + & \lambda & - \end{pmatrix} + \begin{pmatrix} - & \lambda' & + \\ - & \lambda & + \end{pmatrix} - \begin{pmatrix} - & \lambda' & - \\ - & \lambda & - \end{pmatrix} \right] + P_x \left[\begin{pmatrix} + & \lambda' & - \\ + & \lambda & + \end{pmatrix} + \begin{pmatrix} - & \lambda' & - \\ - & \lambda & + \end{pmatrix} \right]
 \end{aligned}$$

$$\begin{aligned}
& + \left(\begin{smallmatrix} +\lambda' + \\ +\lambda - \end{smallmatrix} \right) + \left(\begin{smallmatrix} -\lambda' + \\ -\lambda - \end{smallmatrix} \right) \Big] + i P_y \left[\left(\begin{smallmatrix} +\lambda' - \\ +\lambda + \end{smallmatrix} \right) + \left(\begin{smallmatrix} -\lambda' - \\ -\lambda + \end{smallmatrix} \right) - \left(\begin{smallmatrix} +\lambda' + \\ +\lambda - \end{smallmatrix} \right) - \left(\begin{smallmatrix} -\lambda' + \\ -\lambda - \end{smallmatrix} \right) \right] \\
& - P_L \left[e^{-2i\phi} \left\{ \left(\begin{smallmatrix} -\lambda' + \\ +\lambda + \end{smallmatrix} \right) + \left(\begin{smallmatrix} -\lambda' - \\ +\lambda - \end{smallmatrix} \right) \right\} + e^{2i\phi} \left\{ \left(\begin{smallmatrix} +\lambda' + \\ -\lambda + \end{smallmatrix} \right) + \left(\begin{smallmatrix} +\lambda' - \\ -\lambda - \end{smallmatrix} \right) \right\} \right] \\
& + P_L P_z \left[e^{-2i\phi} \left\{ \left(\begin{smallmatrix} -\lambda' + \\ +\lambda + \end{smallmatrix} \right) - \left(\begin{smallmatrix} -\lambda' - \\ +\lambda - \end{smallmatrix} \right) \right\} + e^{2i\phi} \left\{ \left(\begin{smallmatrix} +\lambda' + \\ -\lambda + \end{smallmatrix} \right) - \left(\begin{smallmatrix} +\lambda' - \\ -\lambda - \end{smallmatrix} \right) \right\} \right] \\
& - P_L P_x \left[e^{-2i\phi} \left\{ \left(\begin{smallmatrix} -\lambda' - \\ +\lambda + \end{smallmatrix} \right) + \left(\begin{smallmatrix} -\lambda' + \\ +\lambda - \end{smallmatrix} \right) \right\} + e^{2i\phi} \left\{ \left(\begin{smallmatrix} +\lambda' - \\ -\lambda + \end{smallmatrix} \right) + \left(\begin{smallmatrix} +\lambda' + \\ -\lambda - \end{smallmatrix} \right) \right\} \right] \\
& - i P_L P_y \left[e^{-2i\phi} \left\{ \left(\begin{smallmatrix} -\lambda' - \\ +\lambda - \end{smallmatrix} \right) - \left(\begin{smallmatrix} -\lambda' + \\ +\lambda + \end{smallmatrix} \right) \right\} + e^{2i\phi} \left\{ \left(\begin{smallmatrix} +\lambda' - \\ -\lambda - \end{smallmatrix} \right) - \left(\begin{smallmatrix} +\lambda' + \\ -\lambda + \end{smallmatrix} \right) \right\} \right]
\end{aligned} \tag{3.6}$$

where the $+, -$ on the left refers to the photon helicity of $+1, -1$. From the fact that the four amplitudes in the first two brackets appear nowhere else in Eq. (3.6), it is evident that even with measurement of the complete density matrix of particle c , one cannot perform a complete experiment with linearly polarized photons alone. Circular polarization is also required. (This is true even if c is spinless, as we shall see.) For a circularly polarized beam,

$$\begin{aligned}
E_c \frac{d^3\sigma}{d\vec{p}_c^3} \rho_{\lambda\lambda'}^c(P_c, \vec{P}) = & \left[\left(\begin{smallmatrix} +\lambda' + \\ +\lambda + \end{smallmatrix} \right) + \left(\begin{smallmatrix} +\lambda' - \\ +\lambda - \end{smallmatrix} \right) + \left(\begin{smallmatrix} -\lambda' + \\ -\lambda + \end{smallmatrix} \right) + \left(\begin{smallmatrix} -\lambda' - \\ -\lambda - \end{smallmatrix} \right) \right] \\
& + P_c \left[\left(\begin{smallmatrix} +\lambda' + \\ +\lambda + \end{smallmatrix} \right) + \left(\begin{smallmatrix} +\lambda' - \\ +\lambda - \end{smallmatrix} \right) - \left(\begin{smallmatrix} -\lambda' + \\ -\lambda + \end{smallmatrix} \right) - \left(\begin{smallmatrix} -\lambda' - \\ -\lambda - \end{smallmatrix} \right) \right] - P_z \left[\left(\begin{smallmatrix} +\lambda' + \\ +\lambda + \end{smallmatrix} \right) - \left(\begin{smallmatrix} +\lambda' - \\ +\lambda - \end{smallmatrix} \right) \right. \\
& + \left. \left(\begin{smallmatrix} -\lambda' + \\ -\lambda + \end{smallmatrix} \right) - \left(\begin{smallmatrix} -\lambda' - \\ -\lambda - \end{smallmatrix} \right) \right] - P_x \left[\left(\begin{smallmatrix} +\lambda' + \\ +\lambda + \end{smallmatrix} \right) - \left(\begin{smallmatrix} +\lambda' - \\ +\lambda - \end{smallmatrix} \right) - \left(\begin{smallmatrix} -\lambda' + \\ -\lambda + \end{smallmatrix} \right) + \left(\begin{smallmatrix} -\lambda' - \\ -\lambda - \end{smallmatrix} \right) \right] \\
& + P_x \left[\left(\begin{smallmatrix} +\lambda' - \\ +\lambda + \end{smallmatrix} \right) + \left(\begin{smallmatrix} -\lambda' - \\ -\lambda + \end{smallmatrix} \right) + \left(\begin{smallmatrix} +\lambda' + \\ +\lambda - \end{smallmatrix} \right) + \left(\begin{smallmatrix} -\lambda' + \\ -\lambda - \end{smallmatrix} \right) \right] + i P_y \left[\left(\begin{smallmatrix} +\lambda' - \\ +\lambda + \end{smallmatrix} \right) + \left(\begin{smallmatrix} -\lambda' - \\ -\lambda + \end{smallmatrix} \right) \right. \\
& - \left. \left(\begin{smallmatrix} +\lambda' + \\ +\lambda - \end{smallmatrix} \right) - \left(\begin{smallmatrix} -\lambda' + \\ -\lambda - \end{smallmatrix} \right) \right] + P_x P_c \left[\left(\begin{smallmatrix} +\lambda' - \\ +\lambda + \end{smallmatrix} \right) - \left(\begin{smallmatrix} -\lambda' - \\ -\lambda + \end{smallmatrix} \right) + \left(\begin{smallmatrix} +\lambda' + \\ +\lambda - \end{smallmatrix} \right) - \left(\begin{smallmatrix} -\lambda' + \\ -\lambda - \end{smallmatrix} \right) \right] \\
& + i P_c P_y \left[\left(\begin{smallmatrix} +\lambda' - \\ +\lambda + \end{smallmatrix} \right) - \left(\begin{smallmatrix} -\lambda' - \\ -\lambda + \end{smallmatrix} \right) - \left(\begin{smallmatrix} +\lambda' + \\ +\lambda - \end{smallmatrix} \right) + \left(\begin{smallmatrix} -\lambda' + \\ -\lambda - \end{smallmatrix} \right) \right]
\end{aligned} \tag{3.7}$$

This, in conjunction with Eq. (3.6) is sufficient to determine all amplitudes, if all independent density matrix elements are measured. Table 3 lists the polarizations needed for each amplitude. If the decay conserves parity, the observables are again $[\rho_{\lambda\lambda'}^c + (-1)^{\lambda-\lambda'} \rho_{-\lambda, -\lambda'}^c] E_c (d^3\sigma/dp_c^3)$ and involve only the real or imaginary part of each amplitude. Explicitly, for a linearly polarized beam

$$\begin{aligned}
 (\rho_{\lambda\lambda'}^c + (-1)^{\lambda-\lambda'} \rho_{-\lambda, -\lambda'}^c) E_c \frac{d^3\sigma}{d p_c^3} (P_L, \phi, \vec{P}) = & \text{Re} \left[\begin{pmatrix} +\lambda' + \\ +\lambda + \end{pmatrix} + \begin{pmatrix} +\lambda' - \\ +\lambda - \end{pmatrix} + \begin{pmatrix} -\lambda' + \\ -\lambda + \end{pmatrix} + \begin{pmatrix} -\lambda' - \\ -\lambda - \end{pmatrix} \right] \\
 & + i P_x q_{\text{Im}} \left[\begin{pmatrix} +\lambda' - \\ +\lambda + \end{pmatrix} + \begin{pmatrix} -\lambda' - \\ -\lambda + \end{pmatrix} + \begin{pmatrix} +\lambda' + \\ +\lambda - \end{pmatrix} + \begin{pmatrix} -\lambda' + \\ -\lambda - \end{pmatrix} \right] - P_y q_{\text{Im}} \left[\begin{pmatrix} +\lambda' - \\ +\lambda + \end{pmatrix} + \begin{pmatrix} -\lambda' - \\ -\lambda + \end{pmatrix} \right. \\
 & \left. - \begin{pmatrix} +\lambda' + \\ +\lambda - \end{pmatrix} - \begin{pmatrix} -\lambda' + \\ -\lambda - \end{pmatrix} \right] - i P_z q_{\text{Im}} \left[\begin{pmatrix} +\lambda' + \\ +\lambda + \end{pmatrix} - \begin{pmatrix} +\lambda' - \\ +\lambda - \end{pmatrix} + \begin{pmatrix} -\lambda' + \\ -\lambda + \end{pmatrix} - \begin{pmatrix} -\lambda' - \\ -\lambda - \end{pmatrix} \right] \\
 & - P_L \left\{ e^{-2i\phi} \text{Re} \left[\begin{pmatrix} -\lambda' + \\ +\lambda + \end{pmatrix} + \begin{pmatrix} -\lambda' - \\ +\lambda - \end{pmatrix} \right] + e^{2i\phi} \text{Re} \left[\begin{pmatrix} +\lambda' + \\ -\lambda + \end{pmatrix} + \begin{pmatrix} +\lambda' - \\ -\lambda - \end{pmatrix} \right] \right\} \\
 & + i P_L P_z \left\{ e^{-2i\phi} q_{\text{Im}} \left[\begin{pmatrix} -\lambda' + \\ +\lambda + \end{pmatrix} - \begin{pmatrix} -\lambda' - \\ +\lambda - \end{pmatrix} \right] + e^{2i\phi} q_{\text{Im}} \left[\begin{pmatrix} +\lambda' + \\ -\lambda + \end{pmatrix} - \begin{pmatrix} +\lambda' - \\ -\lambda - \end{pmatrix} \right] \right\} \\
 & - i P_L P_x \left\{ e^{-2i\phi} q_{\text{Im}} \left[\begin{pmatrix} -\lambda' - \\ +\lambda + \end{pmatrix} + \begin{pmatrix} -\lambda' + \\ +\lambda - \end{pmatrix} \right] + e^{2i\phi} q_{\text{Im}} \left[\begin{pmatrix} +\lambda' - \\ -\lambda + \end{pmatrix} + \begin{pmatrix} +\lambda' + \\ -\lambda - \end{pmatrix} \right] \right\} \\
 & + P_L P_y \left\{ e^{-2i\phi} q_{\text{Im}} \left[\begin{pmatrix} -\lambda' - \\ +\lambda + \end{pmatrix} - \begin{pmatrix} -\lambda' + \\ +\lambda - \end{pmatrix} \right] + e^{2i\phi} q_{\text{Im}} \left[\begin{pmatrix} +\lambda' - \\ -\lambda + \end{pmatrix} - \begin{pmatrix} +\lambda' + \\ -\lambda - \end{pmatrix} \right] \right\}
 \end{aligned} \tag{3.8}$$

and for a circularly polarized beam

$$\begin{aligned}
 (\rho_{\lambda\lambda'}^c + (-1)^{\lambda-\lambda'} \rho_{-\lambda, -\lambda'}^c) E_c \frac{d^3\sigma}{d p_c^3} (P_L, \vec{P}) = & \text{Re} \left[\begin{pmatrix} +\lambda' + \\ +\lambda + \end{pmatrix} + \begin{pmatrix} +\lambda' - \\ +\lambda - \end{pmatrix} + \begin{pmatrix} -\lambda' + \\ -\lambda + \end{pmatrix} + \begin{pmatrix} -\lambda' - \\ -\lambda - \end{pmatrix} \right] \\
 & + i P_L q_{\text{Im}} \left[\begin{pmatrix} +\lambda' + \\ +\lambda + \end{pmatrix} + \begin{pmatrix} +\lambda' - \\ +\lambda - \end{pmatrix} - \begin{pmatrix} -\lambda' + \\ -\lambda + \end{pmatrix} - \begin{pmatrix} -\lambda' - \\ -\lambda - \end{pmatrix} \right] + i P_z q_{\text{Im}} \left[\begin{pmatrix} +\lambda' + \\ +\lambda + \end{pmatrix} - \begin{pmatrix} +\lambda' - \\ +\lambda - \end{pmatrix} \right. \\
 & \left. + \begin{pmatrix} -\lambda' + \\ -\lambda + \end{pmatrix} - \begin{pmatrix} -\lambda' - \\ -\lambda - \end{pmatrix} \right] - P_L P_z \text{Re} \left[\begin{pmatrix} +\lambda' + \\ +\lambda + \end{pmatrix} - \begin{pmatrix} +\lambda' - \\ +\lambda - \end{pmatrix} - \begin{pmatrix} -\lambda' + \\ -\lambda + \end{pmatrix} + \begin{pmatrix} -\lambda' - \\ -\lambda - \end{pmatrix} \right] \\
 & + i P_x q_{\text{Im}} \left[\begin{pmatrix} +\lambda' - \\ +\lambda + \end{pmatrix} + \begin{pmatrix} -\lambda' - \\ -\lambda + \end{pmatrix} + \begin{pmatrix} +\lambda' + \\ +\lambda - \end{pmatrix} + \begin{pmatrix} -\lambda' + \\ -\lambda - \end{pmatrix} \right] - P_y q_{\text{Im}} \left[\begin{pmatrix} +\lambda' - \\ +\lambda + \end{pmatrix} + \begin{pmatrix} -\lambda' - \\ -\lambda + \end{pmatrix} \right. \\
 & \left. - \begin{pmatrix} +\lambda' + \\ +\lambda - \end{pmatrix} - \begin{pmatrix} -\lambda' + \\ -\lambda - \end{pmatrix} \right] + P_L P_x \text{Re} \left[\begin{pmatrix} +\lambda' - \\ +\lambda + \end{pmatrix} - \begin{pmatrix} -\lambda' - \\ -\lambda + \end{pmatrix} + \begin{pmatrix} +\lambda' + \\ +\lambda - \end{pmatrix} - \begin{pmatrix} -\lambda' + \\ -\lambda - \end{pmatrix} \right] \\
 & + i P_L P_y \text{Re} \left[\begin{pmatrix} +\lambda' - \\ +\lambda + \end{pmatrix} - \begin{pmatrix} -\lambda' - \\ -\lambda + \end{pmatrix} - \begin{pmatrix} +\lambda' + \\ +\lambda - \end{pmatrix} + \begin{pmatrix} -\lambda' + \\ -\lambda - \end{pmatrix} \right]
 \end{aligned} \tag{3.9}$$

For the case of $J_c = 0$, for linear polarization one has

$$\begin{aligned} \frac{1}{2} E_c \frac{d^3 \sigma}{d p_c^3} = & \left[\begin{pmatrix} + & 0 & + \\ + & 0 & + \end{pmatrix} + \begin{pmatrix} + & 0 & - \\ + & 0 & - \end{pmatrix} \right] - P_L \cos 2\phi \left[\begin{pmatrix} - & 0 & + \\ + & 0 & + \end{pmatrix} + \begin{pmatrix} - & 0 & - \\ + & 0 & - \end{pmatrix} \right] \\ & - i P_L P_2 \sin 2\phi \left[\begin{pmatrix} - & 0 & + \\ + & 0 & + \end{pmatrix} - \begin{pmatrix} - & 0 & - \\ + & 0 & - \end{pmatrix} \right] + i P_y \left[\begin{pmatrix} + & 0 & - \\ + & 0 & + \end{pmatrix} + \begin{pmatrix} - & 0 & - \\ - & 0 & + \end{pmatrix} \right] \\ & + i P_L P_2 \sin 2\phi \left[\begin{pmatrix} - & 0 & - \\ + & 0 & + \end{pmatrix} + \begin{pmatrix} - & 0 & + \\ + & 0 & - \end{pmatrix} \right] - i P_L P_y \cos 2\phi \left[\begin{pmatrix} - & 0 & - \\ + & 0 & + \end{pmatrix} - \begin{pmatrix} - & 0 & + \\ + & 0 & - \end{pmatrix} \right] \quad (3.10) \end{aligned}$$

and for circular polarization

$$\begin{aligned} \frac{1}{2} E_c \frac{d^3 \sigma}{d p_c^3} = & \left[\begin{pmatrix} + & 0 & + \\ + & 0 & + \end{pmatrix} + \begin{pmatrix} + & 0 & - \\ + & 0 & - \end{pmatrix} \right] - P_2 P_c \left[\begin{pmatrix} + & 0 & + \\ + & 0 & + \end{pmatrix} - \begin{pmatrix} + & 0 & - \\ + & 0 & - \end{pmatrix} \right] \\ & + i P_y \left[\begin{pmatrix} + & 0 & - \\ + & 0 & + \end{pmatrix} + \begin{pmatrix} - & 0 & - \\ - & 0 & + \end{pmatrix} \right] + P_c P_x \left[\begin{pmatrix} + & 0 & - \\ + & 0 & + \end{pmatrix} - \begin{pmatrix} - & 0 & - \\ - & 0 & + \end{pmatrix} \right] \quad (3.11) \end{aligned}$$

Thus, both types of polarization are needed for a complete experiment even when particle c is spinless.

The general situation is summarized in Table 3. Both linear and circular polarization, in addition to a polarized target, are needed for a complete pseudoscalar production experiment or to extract maximum information from production of resonances with parity-conserving decays. And, of course, for weak decays of particle c , if you want to know everything, you have to use all polarizations.

Regarding the possibility of high-energy photons with circular polarization, there are two ways they can be obtained. One is to backscatter laser light from a high-energy polarized electron beam, as at SLAC. The other is to use slightly virtual photons (small $|k^2/\bar{k}^2|$), which if obtained from a polarized electron beam contain a component of circular polarization ¹⁰⁾. This component can be very large, even at rather small angles between the photon and lepton beam, if the electron is almost stopped. For example, for $|\vec{k}^2/\bar{k}^2| = (25)^2$ and an angle of 5° , the circular polarization is 0.96.

4. EFFECTIVELY COMPLETE EXPERIMENTS

4.1 General

Due to the almost prohibitive difficulty involved in performing most complete inclusive experiments, it is useful to look for kinematic regions in which only a subset of all the independent amplitudes is expected to be important. A smaller number of polarization measurements would then be required to determine only these important amplitudes, and it could be feasible to do "effectively complete" experiments (ECE), i.e., experiments which determine all the amplitudes expected to be important in a particular kinematic region. Or, even if there is no hope of performing an ECE, one can determine which polarization measurements can be expected to be useful.

The definition of an ECE is clearly not unique, but depends on how one decides which amplitudes should be dominant. Our assumption is that for large $|p_i \cdot k_j / p_1 \cdot p_2|$ (for all i, j), where the amplitude can be represented by diagrams of the form shown in Fig. 2a, the leading singularity in the $b\bar{b}$ channel preserves s channel helicity and couples equally to each helicity. Abarbanel and Gross ¹⁾ have shown this is true for factorizing poles, and Salin and Soffer ¹¹⁾ have shown it follows from exchange of a definite naturality. It appears to be the case in two-body scattering ¹²⁾. We assume it holds in this case as well. We also assume that we are at sufficiently high (sub) energy that the leading singularity dominates such diagrams ^{*)}. With these assumptions, the important amplitudes in a region where Fig. 2a is valid are of the form $\begin{pmatrix} a' \lambda' b \\ a \lambda b \end{pmatrix} = \begin{pmatrix} a' \lambda' b' \\ a \lambda b' \end{pmatrix}$, and we obtain the results of Refs. 1), 2) that beam/target polarization has little effect outside the beam/target fragmentation region.

Kinematic suppression also can affect which amplitudes are important. This is considered in Section 5.

4.2 Fragmentation region

Figure 2a does represent the three-to-three amplitude in the beam fragmentation region, where $p_a \cdot p_b$ and $p_b \cdot p_c$ are much larger than $p_a \cdot p_c$ or the particle masses. Therefore, amplitudes in which the helicity of the target particle flips can be neglected. The number of amplitudes to be determined is further reduced by the relation $\begin{pmatrix} a' \lambda' + \\ a \lambda + \end{pmatrix} = \begin{pmatrix} a' \lambda' - \\ a \lambda - \end{pmatrix}$. For πp type reactions, in the π fragmentation region, Eq. (3.1) becomes

$$E_c \frac{d^3\sigma}{d^3p_c} \rho_{\lambda\lambda'}^c(\vec{P}) = \begin{pmatrix} 0 & \lambda' + \\ 0 & \lambda + \end{pmatrix} \quad (4.1)$$

*) Even if the Pomeron does not dominate, the next leading trajectory, the f_0 , is also believed to be predominantly non-flip.

and no initial polarization is needed for an ECE. For $J_c \neq 0$ there is still the problem of determining all the independent $\rho_{\lambda\lambda'}$'s. In terms of which initial polarizations are useful in this region, to a first approximation no polarization gives anything new. If one feels capable of measuring small effects, target polarization out of the plane ($P_y \neq 0$) is probably the most useful since the most common fragments of the pion (kaon) will be spinless, rendering P_x and P_z useless [see Eq. (3.2a)]. For the same initial state, but with particle c a fragment of the target, none of the amplitudes in (3.1) can be neglected. The situation is therefore the same as considered in Section 3.

For a spin- $\frac{1}{2}$ beam, we consider only beam fragmentation, since target fragmentation expressions will not be any different. All terms in Eq. (3.3) with the target helicity flipping are neglected, those with $(-)$ are taken equal to $(+)$, and we obtain

$$E_c \frac{d^3\sigma}{d^3p_c} \rho_{\lambda\lambda'}^c(\vec{P}_a, \vec{P}_b) = \left[\begin{pmatrix} +\lambda' \\ +\lambda \end{pmatrix} + \begin{pmatrix} -\lambda' \\ -\lambda \end{pmatrix} \right] + P_{az} \left[\begin{pmatrix} +\lambda' \\ +\lambda \end{pmatrix} - \begin{pmatrix} -\lambda' \\ -\lambda \end{pmatrix} \right] \\ + P_{ax} \left[\begin{pmatrix} -\lambda' \\ +\lambda \end{pmatrix} + \begin{pmatrix} +\lambda' \\ -\lambda \end{pmatrix} \right] - i P_{ay} \left[\begin{pmatrix} -\lambda' \\ +\lambda \end{pmatrix} - \begin{pmatrix} +\lambda' \\ -\lambda \end{pmatrix} \right] \quad (4.2)$$

This is similar to the expression for $np \rightarrow cX$ in the proton fragmentation region, Eq. (3.1), expressing the fact that (with our assumptions) a target fragment does not know about the spin of the beam. Sign differences between Eq. (4.2) and Eq. (3.1) arise because one is for a polarized target, the other for a polarized beam. If particle c decays non-weakly only the real or imaginary part of each amplitude is measurable [cf. discussion following Eq. (3.3)]; and if $J_c = 0$, P_{az} and P_{ax} are useless.

For photon fragmentation, linear photon polarization yields the expression

$$E_c \frac{d^3\sigma}{d^3p_c} \rho_{\lambda\lambda'}^c(P_L, \phi, \vec{P}) = \left[\begin{pmatrix} +\lambda' \\ +\lambda \end{pmatrix} + \begin{pmatrix} -\lambda' \\ -\lambda \end{pmatrix} \right] \\ - P_L \left[e^{-2i\phi} \begin{pmatrix} -\lambda' \\ +\lambda \end{pmatrix} + e^{2i\phi} \begin{pmatrix} +\lambda' \\ -\lambda \end{pmatrix} \right] \quad (4.3)$$

and circular polarization leads to

$$E_c \frac{d^3\sigma}{d^3p_c} \rho_{\lambda\lambda'}^c(P_c, \vec{P}) = \left[\begin{pmatrix} +\lambda' \\ +\lambda \end{pmatrix} + \begin{pmatrix} -\lambda' \\ -\lambda \end{pmatrix} \right] + P_c \left[\begin{pmatrix} +\lambda' \\ +\lambda \end{pmatrix} - \begin{pmatrix} -\lambda' \\ -\lambda \end{pmatrix} \right] \quad (4.4)$$

both independent of target polarization. Thus circular polarization and two linear polarizations (in addition to complete determination of $\rho_{\lambda\lambda'}^c$, for each polarization) are needed for an ECE in the photon fragmentation region, unless c is spinless. The P_c term could be written $[(+\lambda' +) - (-\lambda' -)]$, which vanishes by parity for $\lambda' = \lambda = 0$.

Table 4 summarizes the results for fragmentation experiments (as well as central experiments, which will be discussed next). For each amplitude, we list the polarizations needed to measure it in each region. An entry of 0 indicates that the amplitude is small in that region. As in previous tables, an asterisk marks polarizations which do not affect observables if $J_c = 0$. Because the amplitudes are not all pure real or pure imaginary when $J_c \neq 0$, an ECE is not possible if particle c has spin and decays strongly or electromagnetically.

The results of this subsection also apply to the triple-Regge region, which is contained in the fragmentation region.

4.3 Central region

In the central region, defined by $p_a \cdot p_c$ and $p_b \cdot p_c$ large, one has the diagram of Fig. 2b, and neither of the initial particles can flip helicity. This has the effect of making the central cross-section independent of the initial polarizations, and thus there is only the density matrix of c to be measured. If one wanted to look for polarization effects, long-range correlations between produced particle and incident polarization, P_y would be needed if the produced particle were spinless. Results for the central region are summarized in Table 4.

5. KINEMATIC SUPPRESSION

5.1 General

The presence and importance of half-angle factors, forcing certain helicity amplitudes to vanish when particle c is collinear with beam or target, has been pointed out by GO. They show that

$$\begin{pmatrix} a' & \lambda' & b' \\ a & \lambda & b \end{pmatrix} \propto \begin{pmatrix} \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{pmatrix} \begin{pmatrix} |a-b-\lambda-a'+b'+\lambda'| & |a-b+\lambda-a'+b'-\lambda'| \end{pmatrix} \quad (5.1)$$

and point out that since $\sin \theta = p_{c\perp}/p_c$, either $\sin \theta/2$ or $\cos \theta/2$ will be very small for typical transverse momenta unless $x = p_{c\perp}/p_{L\max}$ is very small. The limited $p_{\perp}$ distribution causes most events to occur very near

the forward or backward directions except near $x=0$, i.e., in the central region. In this Section we systematically study the effects of these half-angle factors on the ECE, determining which amplitudes will be small due to this kinematic constraint and which polarizations should therefore be useful. We also look into the practicability of naturality separation, the framework for which has been constructed by Ader et al.³⁾.

5.2 Effectively complete experiments revisited

The first point is that the kinematic suppression does not occur for small $p_{c\perp}$, and consequently central region results are unchanged. We need consider only the fragmentation regions of the three different spin beams.

For spinless beams we have, from Eq. (4.1)

$$E_c \frac{d^3\sigma}{dp_c^3} \rho_{\lambda\lambda'}^c = \begin{pmatrix} 0 & \lambda' + \\ 0 & \lambda + \end{pmatrix} \propto \left(\sin \frac{\theta}{2} \right)^{|\lambda - \lambda'|} \quad (5.2)$$

Thus only the diagonal elements of $\rho_{\lambda\lambda'}^c$ can be expected to be important in the beam fragmentation region. This will have important consequences when we consider naturality separation.

For spin- $\frac{1}{2}$ beams, the terms in Eq. (4.2) have the behaviour

$$\left[\begin{pmatrix} + & \lambda' + \\ + & \lambda + \end{pmatrix} + \begin{pmatrix} - & \lambda' + \\ - & \lambda + \end{pmatrix} \right] + P_{az} \left[\begin{pmatrix} + & \lambda' + \\ + & \lambda + \end{pmatrix} - \begin{pmatrix} - & \lambda' + \\ - & \lambda + \end{pmatrix} \right] \propto \left(\sin \frac{\theta}{2} \right)^{|\lambda - \lambda'|} \quad (5.3)$$

$$(P_{ax} - i P_{ay}) \begin{pmatrix} - & \lambda' + \\ + & \lambda + \end{pmatrix} \propto \left(\sin \frac{\theta}{2} \right)^{|\lambda - \lambda' - 1|} \quad (5.4)$$

$$(P_{ax} + i P_{ay}) \begin{pmatrix} + & \lambda' + \\ - & \lambda + \end{pmatrix} \propto \left(\sin \frac{\theta}{2} \right)^{|\lambda - \lambda' + 1|} \quad (5.5)$$

Therefore, beams which are polarized along the beam direction or not at all will measure only (to leading order in $p_{c\perp}/p_c$) diagonal elements of $\rho_{\lambda\lambda'}$; off-diagonal elements will be suppressed. Polarization perpendicular to the beam direction will have all amplitudes suppressed except those with $|\lambda - \lambda'| = 1$. Consequently the diagonal elements of $\rho_{\lambda\lambda'}$ will be unaffected, and transverse initial polarization will have only a small effect on the cross-section for any particle in the fragmentation region of the polarized (spin- $\frac{1}{2}$) particle.

For photon fragmentation one might expect similar results, and indeed this proves to be the case. The amplitudes in Eqs. (4.3) and (4.4) obey

$$\left[\begin{pmatrix} +\lambda' + \\ +\lambda + \end{pmatrix} + \begin{pmatrix} -\lambda' + \\ -\lambda + \end{pmatrix} \right] + \mathcal{P}_c \left[\begin{pmatrix} +\lambda' + \\ +\lambda + \end{pmatrix} - \begin{pmatrix} -\lambda' + \\ -\lambda + \end{pmatrix} \right] \propto \left(\sin \frac{\theta}{2} \right)^{|\lambda - \lambda'|} \quad (5.6)$$

$$\mathcal{P}_L e^{-2i\phi} \begin{pmatrix} -\lambda' + \\ +\lambda + \end{pmatrix} \propto \left(\sin \frac{\theta}{2} \right)^{|\lambda - \lambda' - 2|} \quad (5.7)$$

$$\mathcal{P}_L e^{2i\phi} \begin{pmatrix} +\lambda' + \\ -\lambda + \end{pmatrix} \propto \left(\sin \frac{\theta}{2} \right)^{|\lambda - \lambda' + 2|} \quad (5.8)$$

Therefore, linear polarization effects on diagonal terms (and other terms with $|\lambda - \lambda'| \neq 2$) will be small, as will unpolarized or circularly polarized off-diagonal terms. In particular, linear polarization should have little effect on the cross-section for any particle in the fragmentation region; and for an unpolarized beam, vector or tensor mesons produced in the fragmentation region should have density matrices which are very nearly diagonal in the s channel helicity frame.

In general then, unpolarized beams or beams polarized along the beam direction will have fragments with nearly diagonal density matrices. Polarizations transverse to the beam direction will affect only the elements with $|\lambda - \lambda'| = 1, 2$ for protons, photons, respectively. These results are included in Table 4. We now consider the implications for naturality separation.

5.3 Naturality separation

This subsection deals with the naturality in the t channel, i.e., the $a\bar{c}$ (or $b\bar{c}$) channel. It is also possible to consider the naturality in the $b\bar{b}$ or $a\bar{a}$ channel, as Salin and Soffer¹⁰⁾ did, but once we assume the $b\bar{b}$ vertex is non-flip the half-angle factors are not relevant to such considerations. Ader et al.³⁾ have constructed combinations of density matrix elements which receive contributions from only one naturality to leading order in $s/p_a \cdot p_c$, leading to the hope that the dynamics of the $a\bar{c}$ channel can be studied in much the same way one studies quasi-two-body reactions at lower energies. These naturality conserving combinations are of the form

$$E_{\lambda\lambda'}(\pm) = \left[\begin{pmatrix} a' & \lambda' & b' \\ a & \lambda & b \end{pmatrix} + (-1)^{\lambda-\lambda'} \begin{pmatrix} a' & -\lambda' & b' \\ a & -\lambda & b \end{pmatrix} \right] \\ \pm \eta \left[\begin{pmatrix} a' & -\lambda' & b' \\ a & \lambda & b \end{pmatrix} + (-1)^{\lambda+\lambda'} \begin{pmatrix} a' & \lambda' & b' \\ a & \lambda & b \end{pmatrix} \right] \quad (5.9)$$

$$O_{\lambda\lambda'}(\pm) = \left[\begin{pmatrix} a' & \lambda' & b' \\ a & \lambda & b \end{pmatrix} - (-1)^{\lambda-\lambda'} \begin{pmatrix} a' & -\lambda' & b' \\ a & -\lambda & b \end{pmatrix} \right] \\ \pm \eta \left[\begin{pmatrix} a' & -\lambda' & b' \\ a & \lambda & b \end{pmatrix} - (-1)^{\lambda+\lambda'} \begin{pmatrix} a' & \lambda' & b' \\ a & \lambda & b \end{pmatrix} \right] \quad (5.10)$$

In the fragmentation region of a , amplitudes with $(a-a'-b+b'-\lambda+\lambda') \neq 0$ will be suppressed for small $p_{c\perp}/p_c$. Therefore (considering diagonal elements) if the first bracket in $E_{\lambda\lambda'}(\pm)$ or $O_{\lambda\lambda'}(\pm)$ is not suppressed, the second must be, unless $\lambda=0$. Thus the kinematics will force the two naturality contributions to be equal (for $\lambda \neq 0$) and one will learn little about the dynamics.

Are there any large kinematic regions where one can obtain dynamic information from the naturality separation? Unfortunately the answer is no, such regions being rather small. For the naturality separation to be valid one needs $p_a \cdot p_c / s \simeq \mu_{\perp}^2 / 4p_{\perp}^m$ to be small, but to avoid the kinematic suppression one needs $p_{c\perp}/p_c$ not small. Figure 3 illustrates the situation. Below the solid line $p_a \cdot p_c / s$ is small (< 0.1) and the naturality separation should be valid. Above the dashed line $p_{c\perp}/p_c > 1/3$ and the kinematic suppression is not overwhelming. The cross-hatched area is the region in which naturality separation is possible and gives information about dynamics. It can be seen to be only a small region at small p_c/p_{inc} .

Two simple, practical examples are $\pi N \rightarrow \rho(\omega)X$ and $\gamma N \rightarrow \pi X$. In the first case the naturality conserving quantities are

$$\sigma_{11}^{\pm} = \frac{1}{2} \sigma(\rho_{11} \pm \rho_{-1}) \quad \sigma_{00}^{-} = \sigma \rho_{00} \quad (5.11)$$

In the pion fragmentation region $\rho_{1-1} \propto (\sin \theta/2)^2 \simeq 0$, and consequently $\sigma_{11}^{+} \simeq \sigma_{11}^{-}$, quite independent of dynamics. For $\gamma N \rightarrow \pi X$ with linearly polarized photons, the two naturalities are isolated by [see Eq. (4.3)]

$$\sigma^+ = \sigma_{\perp} = \begin{pmatrix} + & 0 & + \\ + & 0 & + \end{pmatrix} - \begin{pmatrix} - & 0 & + \\ + & 0 & + \end{pmatrix} \quad (5.12)$$

$$\sigma^- = \sigma_{\parallel} = \begin{pmatrix} + & 0 & + \\ + & 0 & + \end{pmatrix} + \begin{pmatrix} - & 0 & + \\ + & 0 & + \end{pmatrix} \quad (5.13)$$

where we have used parity and the assumptions of Section 4. The second term of Eq. (5.12) is suppressed by $(\sin \theta/2)^2$ in the photon fragmentation region and therefore $\sigma^+ \simeq \sigma^- \simeq \frac{1}{2}\sigma$, and the dynamics is veiled.

6. COMMENTS AND CONCLUSIONS

6.1 Model predictions

There are not a great number of specific models for inclusive polarization. One of the more general predictions or expectations has been incorporated in the analysis already, the prediction of a factorizable pole model that the bb' vertex does not flip helicity for central or beam fragment production. Assuming this is the case we are dealing with the ECE of Section 4, and the interesting quantities are the $(aa'cc')$ blob in the fragmentation region and the (cc') structure in the central region. There are a few model predictions for these for some reactions.

The reaction $\pi^{\pm}p \rightarrow \rho^0 X$ has been studied both in dual models and in the multiperipheral model. Dual models with π exchange ¹³⁾ predict that ρ_{00} will be the dominant (Gottfried-Jackson frame) density matrix element, both in the π fragmentation and central regions. If the A_2 trajectory is used in the $\pi\rho^0$ channel, ρ_{11}^{GJ} dominates in these two regions ¹⁴⁾. In a multiperipheral model ¹⁵⁾ ρ_{00}^{GJ} is predicted to be large in the central region.

Craigie et al. ¹⁶⁾ have considered inclusive pion photoproduction in a model including cuts. They predict appreciable differences between $\sigma_{\perp}$ and $\sigma_{\parallel}$, except at $p_{\perp} = 0$. However, their predictions are for low energies ($E_{\gamma} \leq 6$ GeV). As the incident energy increases $\sigma_{\perp}$ and $\sigma_{\parallel}$ will remain equal out to increasingly large values of $p_{\perp}$ due to the half-angle factor.

There is a certain intuitive appeal to the idea that for small M_X^2/s (with c a fragment of a) polarizations for $ab \rightarrow cX$ will resemble the polarizations for $ab \rightarrow cb$ (at $s_{\text{excl}}/m_b^2 = s_{\text{incl}}/M_X^2$). This, however, is not required unless the mechanism responsible for the polarization factorizes, which is not the case for most exclusive models. Predictions of a factorizing triple-Regge model are given by GO.

6.2 Summary

We have shown, in general, which polarization measurements determine which s channel helicity amplitudes in inclusive processes. These results are tabulated in Tables 1-3. We then considered effectively complete experiments (ECE), i.e., ones which would measure all the amplitudes expected to be important in certain kinematic regions. The ECE results are found in Table 4. We also pointed out the difficulties in attempts at naturality separation.

For the first generation of polarization experiments, the most feasible would appear to be:

- a) unpolarized initial state, measuring the density matrix of the produced particle in both the central and the fragmentation regions;
- b) polarized beam/target, looking for effects in the beam/target fragmentation region.

The experiments with a polarized beam/target will also have to measure final state polarizations in order to measure unsuppressed amplitudes, or they will have to look at large $p_{\perp}$, thus evading the $p_{c\perp}/p_c$ suppression. If the experiments can be done at non-small values of $p_{c\perp}/p_c$, it will be interesting to study the behaviour of, for example, the polarized target asymmetry as a function of the rapidity as one moves out of the fragmentation region. To determine which polarization measurements would be unsuppressed at larger $p_{\perp}$, refer to Table 4 or Section 4.

ACKNOWLEDGEMENTS

The authors are grateful to Ian Barber, Maurice Jacob and John Storrow for helpful comments and discussions.

Table 1

$J_a = 0$, complete experiments

Amplitude to be determined	Polarization needed †)	$J_c = 0$	Polarization used	Amplitude measured	
				General	$J_c \neq 0$ non-weak decay
$\begin{bmatrix} 0 & \lambda' + \\ 0 & \lambda + \end{bmatrix}$	u, P_z^*	u	u	$\begin{bmatrix} 0 & \lambda' + \\ 0 & \lambda + \end{bmatrix} + \begin{bmatrix} 0 & \lambda' - \\ 0 & \lambda - \end{bmatrix}$	$\text{Re} \begin{bmatrix} 0 & \lambda' + \\ 0 & \lambda + \end{bmatrix} + \begin{bmatrix} 0 & \lambda' - \\ 0 & \lambda - \end{bmatrix}$ $\begin{bmatrix} 0 & 0 + \\ 0 & 0 + \end{bmatrix}$
$\begin{bmatrix} 0 & \lambda' - \\ 0 & \lambda - \end{bmatrix}$	u, P_z^*	u	P_z	$\begin{bmatrix} 0 & \lambda' + \\ 0 & \lambda + \end{bmatrix} - \begin{bmatrix} 0 & \lambda' - \\ 0 & \lambda - \end{bmatrix}$	$\text{Im} \begin{bmatrix} 0 & \lambda' + \\ 0 & \lambda + \end{bmatrix} - \begin{bmatrix} 0 & \lambda' - \\ 0 & \lambda - \end{bmatrix}$ 0
$\begin{bmatrix} 0 & \lambda' - \\ 0 & \lambda + \end{bmatrix}$	P_x^*, P_y	P_y	P_x	$\begin{bmatrix} 0 & \lambda' - \\ 0 & \lambda + \end{bmatrix} + \begin{bmatrix} 0 & \lambda' + \\ 0 & \lambda - \end{bmatrix}$	$\text{Im} \begin{bmatrix} 0 & \lambda' - \\ 0 & \lambda + \end{bmatrix} + \begin{bmatrix} 0 & \lambda' + \\ 0 & \lambda - \end{bmatrix}$ 0
$\begin{bmatrix} 0 & \lambda' + \\ 0 & \lambda - \end{bmatrix}$	P_x^*, P_y	P_y	P_y	$\begin{bmatrix} 0 & \lambda' - \\ 0 & \lambda + \end{bmatrix} - \begin{bmatrix} 0 & \lambda' + \\ 0 & \lambda - \end{bmatrix}$	$\text{Im} \begin{bmatrix} 0 & \lambda' - \\ 0 & \lambda + \end{bmatrix} - \begin{bmatrix} 0 & \lambda' + \\ 0 & \lambda - \end{bmatrix}$ $\begin{bmatrix} 0 & 0 - \\ 0 & 0 + \end{bmatrix}$

†) u = unpolarized.

*) Indicates not observable for $J_c = 0$.

Table 2

$J_a = \frac{1}{2}$, complete experiments

Amplitude to be determined	Polarization needed	Observable for non-weak decay
$\begin{bmatrix} \sigma & \lambda' & \sigma' \\ \sigma & \lambda & \sigma' \end{bmatrix}$	$u, P_{az}^*, P_{bz}^*, P_{az} P_{bz}$	$\begin{aligned} &\text{Re} \left[\begin{bmatrix} \sigma & \lambda' & \sigma' \\ \sigma & \lambda & \sigma' \end{bmatrix} + \begin{bmatrix} -\sigma & \lambda' & -\sigma' \\ -\sigma & \lambda & -\sigma' \end{bmatrix} \right] \\ &\text{Im} \left[\begin{bmatrix} \sigma & \lambda' & \sigma' \\ \sigma & \lambda & \sigma' \end{bmatrix} - \begin{bmatrix} -\sigma & \lambda' & -\sigma' \\ -\sigma & \lambda & -\sigma' \end{bmatrix} \right] \end{aligned}$
$\begin{bmatrix} -\sigma & \lambda' & \sigma' \\ \sigma & \lambda & \sigma' \end{bmatrix}$	$P_{ax}^*, P_{ay}, P_{ax} P_{bz}, P_{ay} P_{bz}^*$	$\begin{aligned} &\text{Im} \left[\begin{bmatrix} -\sigma & \lambda' & \sigma' \\ \sigma & \lambda & \sigma' \end{bmatrix} + \begin{bmatrix} -\sigma & \lambda' & -\sigma' \\ \sigma & \lambda & -\sigma' \end{bmatrix} \right] \\ &\text{Re} \left[\begin{bmatrix} -\sigma & \lambda' & \sigma' \\ \sigma & \lambda & \sigma' \end{bmatrix} - \begin{bmatrix} -\sigma & \lambda' & -\sigma' \\ \sigma & \lambda & -\sigma' \end{bmatrix} \right] \end{aligned}$
$\begin{bmatrix} \sigma & \lambda' & -\sigma' \\ \sigma & \lambda & \sigma' \end{bmatrix}$	$P_{bx}^*, P_{by}, P_{bx} P_{az}, P_{by} P_{az}^*$	$\begin{aligned} &\text{Im} \left[\begin{bmatrix} \sigma & \lambda' & -\sigma' \\ \sigma & \lambda & \sigma' \end{bmatrix} + \begin{bmatrix} -\sigma & \lambda' & -\sigma' \\ -\sigma & \lambda & -\sigma' \end{bmatrix} \right] \\ &\text{Re} \left[\begin{bmatrix} \sigma & \lambda' & -\sigma' \\ \sigma & \lambda & \sigma' \end{bmatrix} - \begin{bmatrix} -\sigma & \lambda' & -\sigma' \\ -\sigma & \lambda & -\sigma' \end{bmatrix} \right] \end{aligned}$
$\begin{bmatrix} -\sigma & \lambda' & -\sigma' \\ \sigma & \lambda & \sigma' \end{bmatrix}$	$P_{ax} P_{bx}, P_{ay} P_{by}, P_{ax} P_{by}^*, P_{ay} P_{bx}^*$	$\text{Re} \left[\begin{bmatrix} -\sigma & \lambda' & -\sigma' \\ \sigma & \lambda & \sigma' \end{bmatrix} \right]$

*) Indicates not observable for $J_c = 0$.

Table 3

$J_a = 1$, complete experiments

Amplitude to be determined	Polarization needed	Observable for non-weak decay
$\begin{bmatrix} e & \lambda' & \sigma \\ e & \lambda & \sigma \end{bmatrix}$	$u, P_z^*, P_c^*, P_z P_c$	$\text{Re} \left[\begin{bmatrix} e & \lambda' & \sigma \\ e & \lambda & \sigma \end{bmatrix} + \begin{bmatrix} -e & \lambda' & -\sigma \\ -e & \lambda & -\sigma \end{bmatrix} \right]$ $\text{Im} \left[\begin{bmatrix} e & \lambda' & \sigma \\ e & \lambda & \sigma \end{bmatrix} - \begin{bmatrix} -e & \lambda' & -\sigma \\ -e & \lambda & -\sigma \end{bmatrix} \right]$
$\begin{bmatrix} -e & \lambda' & \sigma \\ e & \lambda & \sigma \end{bmatrix}$	$P_L \cos 2\phi, P_L \sin 2\phi^*,$ $P_z P_L \cos 2\phi^*, P_z P_L \sin 2\phi$	$\text{Re} \left[\begin{bmatrix} -e & \lambda' & \sigma \\ e & \lambda & \sigma \end{bmatrix} + \begin{bmatrix} -e & \lambda' & -\sigma \\ e & \lambda & -\sigma \end{bmatrix} \right]$ $\text{Im} \left[\begin{bmatrix} -e & \lambda' & \sigma \\ e & \lambda & \sigma \end{bmatrix} - \begin{bmatrix} -e & \lambda' & -\sigma \\ e & \lambda & -\sigma \end{bmatrix} \right]$
$\begin{bmatrix} e & \lambda' & -\sigma \\ e & \lambda & \sigma \end{bmatrix}$	$P_x^*, P_y, P_x P_c, P_y P_c^*$	$\text{Im} \left[\begin{bmatrix} e & \lambda' & -\sigma \\ e & \lambda & \sigma \end{bmatrix} + \begin{bmatrix} -e & \lambda' & -\sigma \\ -e & \lambda & \sigma \end{bmatrix} \right]$ $\text{Re} \left[\begin{bmatrix} e & \lambda' & -\sigma \\ e & \lambda & \sigma \end{bmatrix} - \begin{bmatrix} -e & \lambda' & -\sigma \\ -e & \lambda & \sigma \end{bmatrix} \right]$
$\begin{bmatrix} -e & \lambda' & -\sigma \\ e & \lambda & \sigma \end{bmatrix}$	$P_x P_L \cos 2\phi^*, P_y P_L \cos 2\phi,$ $P_x P_L \sin 2\phi, P_y P_L \sin 2\phi^*$	$\text{Im} \left[\begin{bmatrix} -e & \lambda' & -\sigma \\ e & \lambda & \sigma \end{bmatrix} \right]$

*) Indicates not observable for $J_c = 0$.

Table 4

Effectively complete experiments

	Amplitude to be determined	Polarization needed		
		Beam fragment	Central	Target fragment
$J_a = 0$	$\begin{bmatrix} 0 & \lambda' \sigma \\ 0 & \lambda \sigma \end{bmatrix}$	$u \ (\lambda' - \lambda = 0)$	u	$u, P_z^* \ (\lambda' - \lambda = 0)$
	$\begin{bmatrix} 0 & \lambda' - \sigma \\ 0 & \lambda \sigma \end{bmatrix}$	0	0	$P_x^*, P_y \ (\lambda' - \lambda = 1)$
$J_a = \frac{1}{2}$	$\begin{bmatrix} \sigma & \lambda' \sigma \\ \sigma & \lambda \sigma \end{bmatrix}$	$u, P_{az}^* \ (\lambda' - \lambda = 0)$	u	$u, P_{bz}^* \ (\lambda' - \lambda = 0)$
	$\begin{bmatrix} -\sigma & \lambda' \sigma' \\ \sigma & \lambda \sigma' \end{bmatrix}$	$P_{ax}^*, P_{ay} \ (\lambda' - \lambda = 1)$	0	0
	$\begin{bmatrix} \sigma & \lambda' - \sigma' \\ \sigma & \lambda \sigma' \end{bmatrix}$	0	0	$P_{bx}^*, P_{by} \ (\lambda - \lambda' = 1)$
	$\begin{bmatrix} -\sigma & \lambda' - \sigma' \\ \sigma & \lambda \sigma' \end{bmatrix}$	0	0	0
$J_a = 1$	$\begin{bmatrix} e & \lambda' \sigma \\ e & \lambda \sigma \end{bmatrix}$	$u, P_c^* \ (\lambda - \lambda' = 0)$	u	$u, P_z^* \ (\lambda' - \lambda = 0)$
	$\begin{bmatrix} -e & \lambda' \sigma \\ e & \lambda \sigma \end{bmatrix}$	$P_L \cos 2\phi, P_L \sin 2\phi^* \ (\lambda' - \lambda = 2)$	0	0
	$\begin{bmatrix} e & \lambda' - \sigma \\ e & \lambda \sigma \end{bmatrix}$	0	0	$P_x^*, P_y \ (\lambda - \lambda' = 1)$
	$\begin{bmatrix} -e & \lambda' - \sigma \\ e & \lambda \sigma \end{bmatrix}$	0	0	0

*) Indicates no observable for $J_c = 0$.Conditions on $(\lambda' - \lambda)$ indicate which amplitudes are not suppressed kinematically.

APPENDIX

Two explicit examples are considered as illustrations of the results. The first is inclusive pion production by polarized photons and the second is inclusive lambda production by polarized protons, both from a polarized target. A complete determination of all amplitudes is possible in principle for both examples, in the first case because the produced particle is spinless and in the second case because the produced particle decays weakly so that all the density matrix elements can be obtained.

A.1. - $\gamma p \rightarrow \pi X$

Since $J_c = 0$ the relevant equations are (3.10) and (3.11), and there are eight numbers to be determined. Eight independent real amplitudes A_i ($i=1, \dots, 8$) can be defined as follows.

$$\begin{aligned} A_1 &= \begin{pmatrix} + & 0 & + \\ + & 0 & + \end{pmatrix} & A_2 &= \begin{pmatrix} + & 0 & - \\ + & 0 & - \end{pmatrix} \\ A_3 &= \begin{pmatrix} - & 0 & + \\ + & 0 & + \end{pmatrix} + \begin{pmatrix} - & 0 & - \\ + & 0 & - \end{pmatrix} & iA_4 &= \begin{pmatrix} - & 0 & + \\ + & 0 & + \end{pmatrix} - \begin{pmatrix} - & 0 & - \\ + & 0 & - \end{pmatrix} \\ A_5 &= \begin{pmatrix} + & 0 & - \\ + & 0 & + \end{pmatrix} - \begin{pmatrix} - & 0 & - \\ - & 0 & + \end{pmatrix} & iA_6 &= \begin{pmatrix} + & 0 & - \\ + & 0 & + \end{pmatrix} + \begin{pmatrix} - & 0 & - \\ - & 0 & + \end{pmatrix} \\ iA_7 &= \begin{pmatrix} - & 0 & - \\ + & 0 & + \end{pmatrix} & iA_8 &= \begin{pmatrix} - & 0 & + \\ + & 0 & - \end{pmatrix} \end{aligned}$$

That all other helicity amplitudes appearing in Eqs. (3.10) and (3.11) are linear combinations of these A_i 's and that the A_i 's are indeed real can easily be checked using Eqs. (2.3) and (2.6).

For linearly polarized photons incident on a polarized target we have

$$\begin{aligned} E_c \frac{d^3\sigma}{d^3p_c} (P_L, \phi, \vec{P}) &= (A_1 + A_2) - P_L \cos 2\phi (A_3) \\ &+ P_L P_z \sin 2\phi (A_4) - P_y (A_6) \\ &- P_L P_x \sin 2\phi (A_7 + A_8) + P_L P_y \cos 2\phi (A_7 - A_8) \end{aligned} \quad (A.1)$$

and for circularly polarized photons incident on a polarized target we have

$$E_c \frac{d^3\sigma}{d^3p_c}(\vec{P}_c, \vec{P}) = (A_1 + A_2) - P_c P_z (A_1 - A_2) - P_y (A_6) + P_c P_x (A_5) \quad (\text{A.2})$$

It is obvious from Eqs. (A.1) and (A.2) that linear polarization does not suffice to determine all the amplitudes and that a circularly polarized beam is necessary to separate A_1 and A_2 and to determine A_5 .

This general result no longer holds if we restrict ourselves to the photon fragmentation region. In this region, the imposition of our assumption of negligible spin flip at the proton vertex removes all amplitudes except A_1 , A_2 and A_3 and requires that $A_2 = A_1$. In this case, the result for linearly polarized photons reduces to

$$E_c \frac{d^3\sigma}{d^3p_c}(\vec{P}_L, \phi) = 2A_1 - P_L \cos 2\phi A_3 \quad (\text{A.3})$$

and that for circularly polarized photons to

$$E_c \frac{d^3\sigma}{d^3p_c}(\vec{P}_c) = 2A_1 \quad (\text{A.4})$$

Consequently in the photon fragmentation region all the information available can be obtained with linear polarization alone, and without a polarized target.

A.2. - pp → AX

In this case the relevant equation is (3.3) and there are thirty two numbers to be determined, the independent real amplitudes A_i ($i=1, \dots, 32$) being defined as follows.

Again using Eqs. (2.3) and (2.6) it is not difficult to show that all other helicity amplitudes appearing in Eq. (3.3) are linear combinations of these and that these are indeed real.

It is sufficient to consider the results for ρ_{++}^c and ρ_{-+}^c , and they are

$$\begin{aligned} E_c \frac{d^3\sigma}{d\beta_c^3} \rho_{++}^c(\vec{P}_a, \vec{P}_b) = & (A_1 + A_2 + A_3 + A_4) + P_{az}(A_1 + A_2 - A_3 - A_4) \\ & - P_{bz}(A_1 - A_2 + A_3 - A_4) - P_{az}P_{bz}(A_1 - A_2 - A_3 + A_4) \\ & + P_{ax}(A_5 + A_7) + P_{ay}(A_6 + A_8) - P_{ax}P_{bz}(A_5 - A_7) \\ & - P_{ay}P_{bz}(A_6 - A_8) + P_{bx}(A_9 + A_{11}) - P_{by}(A_{10} + A_{12}) \\ & + P_{bx}P_{az}(A_9 - A_{11}) - P_{by}P_{az}(A_{10} - A_{12}) + P_{ax}P_{bx}(A_{13} + A_{15}) \\ & + P_{ay}P_{by}(A_{13} - A_{15}) - P_{by}P_{ax}(A_{14} - A_{16}) + P_{ay}P_{bx}(A_{14} + A_{16})^{(A.5)} \end{aligned}$$

and

$$\begin{aligned} E_c \frac{d^3\sigma}{d\beta_c^3} \rho_{-+}^c(\vec{P}_a, \vec{P}_b) = & i(A_{18} + A_{20}) + P_{az}(A_{17} + A_{19}) - P_{bz}(A_{17} - A_{19}) \\ & - iP_{az}P_{bz}(A_{18} - A_{20}) + P_{ax}(A_{21} + A_{23}) - iP_{ay}(A_{21} - A_{23}) \\ & - iP_{ax}P_{bz}(A_{22} + A_{24}) - P_{ay}P_{bz}(A_{22} - A_{24}) + P_{bx}(A_{25} + A_{27}) \\ & + iP_{by}(A_{25} - A_{27}) + iP_{bx}P_{az}(A_{26} + A_{28}) - P_{by}P_{az}(A_{26} - A_{28}) \\ & + iP_{ax}P_{bx}(A_{29} + A_{30} + A_{31} + A_{32}) + iP_{ay}P_{by}(A_{29} - A_{30} - A_{31} + A_{32}) \\ & - P_{by}P_{ax}(A_{29} - A_{30} + A_{31} - A_{32}) + P_{ay}P_{bx}(A_{29} + A_{30} - A_{31} - A_{32})^{(A.6)} \end{aligned}$$

Equations (A.5) and (A.6) show explicitly that all the independent amplitudes can be obtained in principle.

In the region of beam proton fragmentation, our usual assumption requires that A_9 to A_{16} inclusive, A_{21} to A_{32} inclusive and A_{22} all vanish, $A_2 = A_1$, $A_4 = A_3$, $A_7 = A_5$, $A_8 = A_6$, $A_{19} = A_{17}$ and $A_{20} = A_{18}$. We are thus left with only eight independent amplitudes and Eqs. (A.5) and (A.6) reduce to

$$\begin{aligned} E_c \frac{d^3\sigma}{d^3p_c} \rho_{++}^c(\vec{p}_a) &= 2(A_1 + A_3) + 2P_{az}(A_1 - A_3) \\ &+ 2P_{ax}(A_5) + 2P_{ay}(A_6) \end{aligned} \quad (A.7)$$

and

$$\begin{aligned} E_c \frac{d^3\sigma}{d^3p_c} \rho_{-+}^c(\vec{p}_a) &= 2i(A_{18}) + 2P_{xz}(A_{17}) + P_{ax}(A_{21} + A_{23}) \\ &- i P_{ay}(A_{21} - A_{23}) \end{aligned} \quad (A.8)$$

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FIGURE CAPTIONS

- Figure 1 Co-ordinate system in s channel centre-of-mass.
- Figure 2a Diagram for fragmentation region.
- Figure 2b Diagram for central region.
- Figure 3 Naturality separation yields dynamical information in
cross-hatched area.

FIG.1

FIG.2

FIG.3