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POLARIZED LEPTON - HADRON SCATTERING
IN ASYMPTOTICALLY FREE GAUGE THEORIES

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A B S T R A C T

We analyze polarized deep inelastic electron and neutrino scattering in asymptotically free gauge theories of the strong interactions. We compute the logarithmic deviations from Bjorken scaling in the moment sum rules for the structure functions due to the leading twist 2 and 3 operators that control the light cone expansion. We also consider continuations to the elastic limit and to the Regge region.

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1. - INTRODUCTION

Asymptotically free non-Abelian gauge theories provide a natural field-theoretic explanation of scaling up to calculable logarithmic corrections. Spin-averaged electroproduction and neutrino production have already been analyzed within the framework of such theories ¹⁾. Electron positron annihilation into hadrons has also been studied ²⁾. The basic tools for such studies have been developed earlier by Christ et al. ³⁾ using the renormalization group or Callan-Symanzik equations ⁴⁾. In this paper we analyze spin-dependent electroproduction and neutrino production. A novel feature here is the presence of twist 3 besides twist 2 operators in the light cone (LC) expansion ⁵⁾. In Section 2, we treat polarized electroproduction and derive moment sum rules for the structure functions. The Bjorken ⁶⁾ and Burkhardt-Cottingham ⁷⁾ sum rules arise as special cases. We compute the anomalous dimensions of the twist 2 and twist 3 operators that control the LC expansion. We also discuss continuations to the elastic limit and to the Regge region. Section 3 is devoted to the analysis of polarized inelastic neutrino scattering. Moment sum rules are derived and as special cases several interesting sum rules for the structure functions are obtained, some of which are the analogues of the Burkhardt-Cottingham sum rule for the electroproduction structure function G_2 . Section 4 is devoted to some conclusions. A brief summary of our results on inelastic spin-dependent electron scattering has already appeared ^{8),9)}. In this paper we present the details as well as a study of spin-dependent inelastic neutrino scattering.

2. - POLARIZED ELECTROPRODUCTION

In spin-dependent electron scattering we are concerned with the product of two electromagnetic currents between polarized nucleon states. The forward off-shell Compton amplitude is given by

$$\begin{aligned}
 W_{\mu\nu}(p, q, s) &= \frac{1}{2\pi} \int d^4x e^{iq \cdot x} \langle p, s | [\bar{J}_\mu^{e.m.}(x), J_\nu^{e.m.}(0)] | p, s \rangle \\
 &\equiv W_{\mu\nu}^{[5]} + i W_{\mu\nu}^{[A]}
 \end{aligned}
 \tag{2.1}$$

where p, s, q are the target four-momentum, covariant polarization and virtual photon four-momentum, respectively. The symmetric tensor $W_{\mu\nu}^{[S]}$ is composed of the usual structure functions W_1 and W_2 describing spin averaged electroproduction :

$$W_{\mu\nu}^{[S]} = -(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) W_1(p, q, q^2) + \frac{1}{M^2} (p_\mu - \frac{p \cdot q}{q^2} q_\mu) (p_\nu - \frac{p \cdot q}{q^2} q_\nu) W_2(p, q, q^2) \quad (2.2)$$

where M is the target mass. The antisymmetric tensor $W_{\mu\nu}^{[A]}$ is composed of two structure functions G_1 and G_2 that describe polarized electroproduction

$$W_{\mu\nu}^{[A]} = M \epsilon_{\mu\nu\lambda\sigma} q^\lambda s^\sigma G_1(p, q, q^2) + \frac{1}{M} \epsilon_{\mu\nu\lambda\sigma} q^\lambda (q \cdot p s^\sigma - q \cdot s p^\sigma) G_2(p, q, q^2) \quad (2.3)$$

The LC expansion for the product of two electromagnetic currents reads

$$\begin{aligned} J_\mu^{e.m.}(x) J_\nu^{e.m.}(0) &\simeq (-g_{\mu\nu} \square + \partial_\mu \partial_\nu) x^2 \frac{1}{x^2 - i\epsilon x_0} \sum_{n=0}^{\infty} \sum_{i \text{ (even)}} C_{i,1}^n(x^2 - i\epsilon x_0, g) \\ &O^{i\mu_1 \dots \mu_n}(0) x_{\mu_1} \dots x_{\mu_n} - (g_{\mu\lambda} g_{\nu\sigma} \square - g_{\mu\lambda} \partial_\nu \partial_\sigma - g_{\nu\sigma} \partial_\mu \partial_\lambda + g_{\mu\nu} \partial_\lambda \partial_\sigma) \\ &\sum_{n=0}^{\infty} \sum_{i \text{ (even)}} C_{i,2}^n(x^2 - i\epsilon x_0, g) O^{i\lambda\sigma\mu_1 \dots \mu_n}(0) x_{\mu_1} \dots x_{\mu_n} - i\epsilon_{\mu\nu\lambda\sigma} \partial^\lambda \\ &\frac{1}{(x^2 - i\epsilon x_0)} \sum_{n=0}^{\infty} \sum_{i \text{ (odd)}} E_{i,1}^n(x^2 - i\epsilon x_0, g) R_1^{i\sigma\mu_1 \dots \mu_n}(0) x_{\mu_1} \dots x_{\mu_n} \\ &- i(\epsilon_{\mu\rho\lambda\sigma} \partial_\nu \partial^\rho - \epsilon_{\nu\rho\lambda\sigma} \partial_\mu \partial^\rho - \epsilon_{\mu\nu\lambda\sigma} \square) \sum_{n=1}^{\infty} \sum_{i \text{ (odd)}} E_{i,2}^n(x^2 - i\epsilon x_0, g) \\ &R_2^{i\lambda\sigma\mu_1 \dots \mu_n}(x^2 - i\epsilon x_0, g) x_{\mu_1} \dots x_{\mu_n} \quad (2.4) \end{aligned}$$

The i summation takes cognizance of the fact that there may be several operators with the same internal quantum numbers as the product of two currents. The first two terms in the expansion describe the unpolarized case. The anomalous dimensions of the relevant operators $O^{i\mu_1 \dots \mu_n}$ and the coefficient functions $C_{i,k}^n(x^2 - i\epsilon x_0, g)$ were calculated in Ref. 1).

The last two terms in Eq. (2.4) describe the polarized case and we shall study them in detail below. That the indicated forms are the most general has been shown by Heimann ⁵⁾. The operator $R_1^{i\sigma\mu_1\cdots\mu_n}$ is symmetric in its Lorentz indices while $R_2^{i\lambda\sigma\mu_1\cdots\mu_n}$ is antisymmetric in the first two and symmetric in the rest. Their matrix elements between nucleon states are defined by

$$\begin{aligned} \langle p, s | R_1^{i\sigma\mu_1\cdots\mu_n}(0) | p, s \rangle &= (i)^n a_n^i \{ s^{\sigma\mu_1} \cdots p^{\mu_n} \}, \\ \langle p, s | R_2^{i\lambda\sigma\mu_1\cdots\mu_n}(0) | p, s \rangle &= (i)^{n+1} d_n^i \frac{1}{2} (p^\sigma s^\lambda - p^\lambda s^\sigma) p^{\mu_1} \cdots p^{\mu_n} \end{aligned} \quad (2.5)$$

where $\{ \dots \}$ denotes symmetrization of the indices and division by $n+1$.

To proceed with the analysis we consider, as usual, the amplitude $T_{\mu\nu}$, the absorptive part of which is the tensor $W_{\mu\nu}$:

$$\begin{aligned} T_{\mu\nu} &= i \int d^4x e^{iq \cdot x} \langle p, s | T^* (J_\mu^{e.m}(x) J_\nu^{e.m}(0)) | p, s \rangle \\ &= T_{\mu\nu}^{[S]} + i T_{\mu\nu}^{[A]} \end{aligned} \quad (2.6)$$

$T_{\mu\nu}$ is expanded on the same tensor basis as before in terms of invariant amplitudes T_i , $i=1, \dots, 4$. Then we have ($v=p \cdot q$)

$$\begin{aligned} G_1(\nu, q^2) &= \frac{1}{\pi} \int_{\nu_0}^\infty d\nu' T_3(\nu', q^2) \\ G_2(\nu, q^2) &= \frac{1}{\pi} \int_{\nu_0}^\infty d\nu' T_4(\nu', q^2) \end{aligned} \quad (2.7)$$

Substitution of the LC expansion (2.4) into Eq. (2.6) and using Eq. (2.5) then leads to asymptotic expansions for T_3 and T_4 in the Bjorken limit in terms of the scaling variable $\omega = -(2\nu/q^2)$. As is well known the series do not converge in the physical region $1 < \omega < \infty$. However, they can be used to define T_i as functions of complex ω which are analytic with branch points at $\omega = \pm 1$. This information about the T_i is then converted into moment sum rules for the structure functions G_i upon noting the crossing properties.

$$\begin{aligned} G_1(-\nu, q^2) &= -G_1(\nu, q^2) \\ G_2(-\nu, q^2) &= G_2(\nu, q^2) \end{aligned} \quad (2.8)$$

The sum rules read

$$\int_1^\infty d\omega \omega^{-n-2} \Gamma_\nu G_1(\omega, q^2) = \sum_{\substack{i \\ n \text{ even}}} a_n^i \tilde{E}_{i,1}^{(n)}(q^2, g) \quad (2.9)$$

$$\begin{aligned} \int_1^\infty d\omega \omega^{-n-1} \frac{\nu^2}{\Gamma} G_2(\omega, q^2) &= - \sum_i \left\{ \frac{n-1}{n} a_{n-1}^i \tilde{E}_{i,1}^{n-1}(q^2, g) \right. \\ &\quad \left. + \frac{1}{2} a_{n-2}^i \tilde{E}_{i,2}^{n-2}(q^2, g) \right\}, \quad n \text{ odd} \end{aligned} \quad (2.10)$$

where

$$\begin{aligned} \tilde{E}_{i,1}^n(q^2, g) &= -\frac{1}{4}(-q^2)^{n+1} \left(\frac{\partial}{\partial q^2} \right)^n \int d^4x e^{iq \cdot x} \frac{1}{x^2 - i\epsilon} E_{i,1}^n(x^2 - i\epsilon, g) \\ \tilde{E}_{i,2}^n(q^2, g) &= -\frac{1}{4}(-q^2)^{n+2} \left(\frac{\partial}{\partial q^2} \right)^n \int d^4x e^{iq \cdot x} E_{i,2}^n(x^2 - i\epsilon, g) \end{aligned} \quad (2.11)$$

As announced in the Introduction, our theoretical framework for the calculation of the functions $\tilde{E}_{i,k}^{(n)}(q^2, g)$ is the quark model with the strong interactions being mediated by non-Abelian colour gluons. More specifically the theory is described by the Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + i\bar{\psi} \not{D} \psi \quad (2.12)$$

where

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c \quad (2.13)$$

and D_μ is the covariant derivative on spinors : $D_\mu \psi = (\partial_\mu + ig A_\mu^a \sigma^a) \psi$. f^{abc} are the structure constants of the non-Abelian semi-simple gauge group G and σ^a are the matrices of the group generators in the fermion representation. In writing (2.12) we have omitted the gauge fixing and ghost terms needed to properly define $\mathcal{L}$. In the "standard" model the gauge group of strong interactions is taken to be colour $SU(N)$, with three colours being the most popular choice, and with the quarks entering in triplets or quartets. In this model then the leading operators contributing to $R_2^{i\sigma\mu_1 \dots \mu_n}$ are twist 2 and are

$$F_{R_1}^{i\sigma\mu_1 \dots \mu_n}(0) = (i)^n S \bar{\psi} \gamma_5 \gamma^\sigma D^{\mu_1} \dots D^{\mu_n} \frac{1}{2} \lambda^i \psi$$

$$F_{R_1}^{0\sigma\mu_1 \dots \mu_n}(0) = (i)^n S \bar{\psi} \gamma_5 \gamma^\sigma D^{\mu_1} \dots D^{\mu_n} \psi$$

$$V_{R_1}^{0\sigma\mu_1 \dots \mu_n}(0) = (i)^n S \frac{1}{2} \epsilon^{\sigma\alpha\beta\gamma} \text{Tr} F_{\beta\gamma} D^{\mu_1} \dots D^{\mu_{n-1}} F_{\alpha}^{\mu_n} \quad (2.14)$$

with $F_{\mu\nu} = F_{\mu\nu}^a \tau^a$, τ^a being the representation matrices of the vectors. The covariant derivative on the vector gauge field is given by $D_\mu = \partial_\mu + ig \tau^a A_\mu^a$. The λ^i , $i=1, \dots, 8$, are the standard Gell-Mann matrices and the a summation takes values $a=1, \dots, N^2-1$ corresponding to colour $SU(N)$. S denotes symmetrization over $\sigma\mu_1 \dots \mu_n$.

The leading operators contributing to $R_2^{i\sigma\mu_1 \dots \mu_n}$ are twist 3 and are given by

$$\begin{aligned} F_{R_2}^{i\lambda\sigma\mu_1 \dots \mu_n} &= (i)^{n+1} S' \bar{\psi} \gamma_5 \gamma^\lambda D^\sigma D^{\mu_1} \dots D^{\mu_n} \frac{1}{2} \lambda^i \psi \\ F_{R_2}^{0\lambda\sigma\mu_1 \dots \mu_n} &= (i)^{n+1} S' \bar{\psi} \gamma_5 \gamma^\lambda D^\sigma D^{\mu_1} \dots D^{\mu_n} \psi \\ V_{R_2}^{0\lambda\sigma\mu_1 \dots \mu_n} &= (i)^{n+1} S' \epsilon^{\sigma\alpha\beta\gamma} F_{\beta\gamma} D^{\mu_1} \dots D^{\mu_n} F_{\alpha}^{\lambda} \end{aligned} \quad (2.15)$$

where S' denotes symmetrization on the indices $\sigma\mu_1\dots\mu_n$ and antisymmetrization on the indices $\lambda\sigma$.

The Wilson coefficient functions $\tilde{E}_{i,k}^n$ obey the renormalization group equation ^{1),3)}

$$\left(\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g}\right) \tilde{E}_{i,k}^n(q^2, g) = \sum_j \gamma_j^i(g) \tilde{E}_{j,k}^n(q^2, g) \quad (2.16)$$

Here μ is the subtraction point introduced to perform the renormalization and g is the strong gauge coupling constant introduced above. The anomalous dimensions γ_j^i form a matrix γ defined by

$$\left(\gamma\right)_j^i = \left(Z^{-1} \mu \frac{\partial}{\partial \mu} Z \right)_j^i \quad (2.17)$$

where Z is the renormalization matrix of the Wilson operators $R_1^{\sigma\mu_1\dots\mu_n}$ or $R_2^{i\lambda\sigma\mu_1\dots\mu_n}$ defined earlier and the differentiation is performed with the bare coupling constant and the cut-off held fixed. In order to compute the γ matrix to second order in g we must express the renormalized Wilson operators in terms of the bare ones. This is done by taking matrix elements between vector and fermion states. Denoting by F_R^n and V_R^n any of the operators introduced above, we are thus led to compute the logarithmically divergent pieces of the matrix elements of F_R^n and V_R^n with the same twist and tensor structure as the bare operators. This then leads to the determination of the wave function renormalization matrix Z and hence via Eq. (2.17) to the γ matrix. The computations are quite lengthy and require the evaluation of the set of graphs shown in Fig. 1. The vertices corresponding to the insertion of Wilson operators on two or three-point functions are listed in the Appendix. Note that in computing γ we ignore composite operators formed from Faddeev-Popov ghost fields ¹⁰⁾.

For the operator R_1 , in the $SU(3)$ singlet case (n even) we obtain

$$\gamma_{FF}^F = \frac{g^2 C_2(R)}{8\pi^2} \left\{ 1 - \frac{2}{(n+1)(n+2)} + \sum_{j=2}^{n+1} \frac{4}{j} \right\} \quad (2.18)$$

$${}^n_1 \gamma_{VV}^F = - \frac{g^2 T(R)}{8\pi^2} \left\{ \frac{4n}{(n+1)(n+2)} \right\} \quad (2.19)$$

$${}^n_1 \gamma_{FF}^V = - \frac{g^2 C_2(R)}{8\pi^2} \left\{ \frac{2(n+3)}{(n+1)(n+2)} \right\} \quad (2.20)$$

$${}^n_1 \gamma_{VV}^V = \frac{g^2}{8\pi^2} \left[C_2(G) \left\{ \frac{4}{3} + \frac{2(n-2)}{(n+1)(n+2)} + \sum_{j=0}^{n-2} \left(\frac{2}{j+3} + \frac{2}{j+2} \right) \right\} + \frac{4}{3} T(R) \right] \quad (2.21)$$

Here $C_2(G)$ and $C_2(R)$ are the quadratic Casimir operators of the gauge group evaluated for the adjoint representation and the Fermion representation R , respectively. $T(R)$ is defined by

$$\text{Tr}(\sigma^a \sigma^b) = T(R) \delta_{ab} \quad (2.22)$$

For the operator R_2 , in the $SU(3)$ singlet case we obtain
(n odd)

$${}^n_2 \gamma_{FF}^F = \frac{g^2 C_2(R)}{8\pi^2} n \left\{ \frac{1}{(n+2)} + \frac{1}{(n+1)} \sum_{j=2}^{n+2} \frac{4}{j} \right\} \quad (2.23)$$

$${}^n_2 \gamma_{VV}^F = 0 \quad (2.24)$$

$${}^n_2 \gamma_{FF}^V = \frac{g^2 C_2(R)}{8\pi^2} \frac{4}{(n+2)} \quad (2.25)$$

$${}^n_2 \gamma_{VV}^V = \frac{g^2}{8\pi^2} \left[C_2(G) \left\{ \frac{1}{3} + \frac{2(2n+1)}{(n+2)(n+3)} + \sum_{j=0}^{n-1} \left(\frac{2}{j+3} + \frac{2}{j+2} \right) \right\} + \frac{4}{3} T(R) \right] \quad (2.26)$$

The renormalization group Eq. (2.16) may be solved for the dimensionless functions $\tilde{E}_{i,k}^{(n)}$

$$\tilde{E}_{i,k}^{(n)}\left(-\frac{q^2}{\mu^2}, g\right) = \sum_j \tilde{E}_{j,k}^{(n)}(1, \bar{g}(t, g)) \exp\left\{-\int_0^t \sum_k \gamma_k^j(\bar{g}(t', g)) dt'\right\} \quad (2.27)$$

where $t = \frac{1}{2} \ln(-q^2/\mu^2)$ and $\bar{g}(t, g)$ is the effective coupling constant defined (in lowest order) by

$$\frac{d\bar{g}^2}{dt} = -b_0 \bar{g}^4 \quad ; \quad \bar{g}(0, g) = g \quad (2.28)$$

and b_0 is given by

$$b_0 = \frac{1}{8\pi^2} \left[\frac{11}{3} C_2(G) - \frac{4}{3} T(R) \right] \quad (2.29)$$

Equation (2.28) for the effective coupling constant yields

$$\bar{g}^2(t, g) = \frac{g^2}{1 + b_0 g^2 t} \quad (2.30)$$

For $SU(3)$ singlet operators the leading behaviour of the $\tilde{E}_{i,k}^{(n)}$ is found by diagonalizing the matrix γ_k^n and picking the lowest eigenvalue γ_k^n . For $SU(3)$ octet operators only γ_{FF}^F contributes and is given by Eqs. (2.18) and (2.23).

For both singlet and octet operators $\gamma_1^0 = 0$ corresponding to conserved axial vector currents. For $n \neq 0$ the off-diagonal elements vanish like $1/n$ and we may, to a good approximation, set them equal to zero. Then for both singlet and octet operators we have

$$\gamma_k^n = \text{Min} \left\{ \gamma_{FF}^F, \gamma_{VV}^V \right\} \quad (2.31)$$

Using Eqs. (2.27), (2.29) and (2.30) we find for the leading behaviour of $\tilde{E}_{i,j}^{(n)}$

$$\tilde{E}_{i,j}^n \xrightarrow{-q^2 \rightarrow \infty} C_j^n (1+g^2 b_0 t)^{-n_j \delta / b_0 g^2} \quad (2.32)$$

where the C_j^n are constants.

Since $\frac{n-1}{2} \gamma < \frac{n}{1} \gamma$ we see from Eq. (2.10) that $\tilde{E}_{i,2}^{(n)}$ governs the leading behaviour of the moments of G_2 . Thus from Eqs. (2.9), (2.10) and (2.32) we have

$$\int_1^\infty dw w^{-n-2} M \gamma G_1(w, q^2) \sim_{-q^2 \rightarrow \infty} (1+g^2 b_0 t)^{-A_n} \quad (2.33)$$

$$\int_1^\infty dw w^{-n-1} \frac{\gamma^2}{M} G_2(w, q^2) \sim_{-q^2 \rightarrow \infty} (1+g^2 b_0 t)^{-B_n} \quad (2.34)$$

where

$$A_n = G \left\{ -0.69 + 4 \ln(n+1) \right\} \quad (2.35)$$

and

$$B_n = G \left\{ -0.69 + \frac{4(n-2)}{(n-1)} \ln n \right\} \quad (2.36)$$

with

$$G = \frac{3C_2(R)}{11C_2(G) - 4T(R)}$$

being totally determined by the gauge group and the representation of the quarks. If the group is colour $SU(3)$ and the theory contains three quark triplets, then $G = 4/27$. Increasing the number of triplets increases G thus leading to larger violations of scaling. E.g., if there are three more coloured charmed quarks then $G = 4/25$. The interpolation used above for $\frac{n}{k} \gamma$ is valid to $O(1/n)$.

It is seen from Eqs. (2.33) and (2.34) that in general sum rules for G_1 and G_2 pick up logarithmic suppressions. However, since the axial vector current has no anomalous dimension, the integral

$$\int_1^{\infty} d\omega \omega^{-2} M \nu G_1(\omega, q^2)$$

if convergent, must scale. Similarly the Bjorken sum rule

$$\int_1^{\infty} d\omega \omega^{-2} M \nu [G_1^p(\omega, q^2) - G_1^n(\omega, q^2)] = \frac{1}{6} \frac{G_A}{G_V} \quad (2.37)$$

has no anomalous dimensions. Next since $d_{-1}^i = 0$ we obtain the Burkhardt-Cottingham sum rule ⁷⁾

$$\int_1^{\infty} d\omega \omega^{-2} \frac{\nu^2}{M} G_2(\omega, q^2) = 0 \quad (2.38)$$

as in the free quark model ^{5), 11)}. In this sum rule the leading Regge singularities must be subtracted to make the sum rule convergent. ¹²⁾

It is of interest to determine the rate of approach of these sum rules to their asymptotic limit. For example, in the Bjorken sum rule, the leading contribution to the right-hand side comes from the operators $\bar{\psi} \gamma_5 \gamma^{\frac{1}{2} \lambda} \psi$, with Wilson coefficients computed in the free quark model. However, there are also terms in the Wilson coefficients of $O(\bar{g}^2)$ and above coming from the exchange of colour vector gluons. Thus the approach to the free quark model value of $1/6 G_A/G_V$ is logarithmic, the correction terms being $O(\bar{g}^2) \equiv O(1/\ln q^2)$.

At first sight the Burkhardt-Cottingham sum rule looks different ⁹⁾. Since the operator corresponding to d_{-1} does not exist it appears that only trace terms and less singular terms in the operator product expansion can contribute, corresponding to a $1/Q^2$ rate of approach to the zero limit. This is true even in the presence of terms of order $\bar{g}^2$ and above since, although these terms can introduce powers of $1/x^2$ and can enhance non-leading operator contributions, these corrections can occur only in the form $(x_\alpha x_\beta)/x^2$. Due to the numerator, this term cannot contribute to the lowest moment and the Burkhardt-Cottingham sum rule remains true. However, explicit calculation of the $O(\bar{g}^2)$ corrections to G_2 yields the form

$$\nu^2 G_2(\omega) \propto \bar{g}^2 \left[\delta\left(\frac{1}{\omega}\right) - \frac{1}{2} \theta\left(1 - \frac{1}{\omega^2}\right) \right]$$

Due to the cancellation between the first and second terms the Burkhardt-Cottingham sum rule remains true. However, the $\delta(1/\omega)$ singularity is unphysical and will never be observed experimentally. Thus, in practice, the Burkhardt-Cottingham sum rule will appear to be approached logarithmically, with a correction of $O(\bar{g}^2)$ coming from the term $\bar{g}^2 \theta(1 - (1/\omega^2))$.

The situation is analogous to the Schwinger term sum rule where to order $\bar{g}^2$ it was found that ¹³⁾ $F_L(\omega)$ is given by

$$\omega^2 F_L(\omega) = \frac{-\bar{g}^2}{\pi} \left[\delta\left(\frac{1}{\omega}\right) - \frac{1}{2} \theta\left(1 - \frac{1}{\omega^2}\right) \right]$$

so that the Schwinger sum rule strictly remains true

$$\int_1^\infty d\omega F_L(\omega) = 0$$

Again for the experimentally observed structure function the approach to the zero asymptotic value will only be logarithmic due to the presence only of the second term $+(\bar{g}^2/2\pi)\theta(1-1/\omega^2)$.

The implications of Eqs. (2.33) and (2.34) for the structure functions G_1 and G_2 themselves require a knowledge of the constants of proportionality in these equations. This means in general an analysis of the type suggested by Parisi and by Gross ¹⁴⁾ is necessary to relate the structure functions at one value of q^2 to another. Let us here consider what can be said for small and large values of ω .

Near $\omega = 1$ the high n moments dominate the structure functions ¹⁵⁾. For large n the forms given by Eqs. (2.33) and (2.34) together with Eqs. (2.35) and (2.36) are good approximations for $G = SU(3)'$. If $F(\omega, t')$ (where $F \equiv \nu G_1$ or $\nu^2 G_2$) has a power behaviour near threshold

$$F(\omega, t') \sim (\omega - 1)^d, \quad \omega \rightarrow 1 \quad (2.39)$$

Then at $t > t'$ ¹⁵⁾ (for the three triplet model)

$$\frac{F(\omega, t)}{F(\omega, t')} \xrightarrow{\omega \rightarrow 1} \left(\frac{t}{t'} \right)^{0.102} \frac{\Gamma(d+1)}{\Gamma(d+1+P)} (\ln \omega)^P \quad (2.40)$$

where $P = 0.592 \ln(t/t')$.

On a more speculative note we may use this result to investigate the behaviour of the proton form factor. If we assume Bloom-Gilman type duality, the structure function near $\omega = 1$ is related to the proton form factor as described in Ref. 13). This leads to the prediction that the form factors fall faster than any power of q^2 for large q^2 . In the resonance region the higher order terms in $\frac{1}{g^2}$ are important ¹⁴⁾ and this result must be viewed with caution. However, in the unpolarized case a direct evaluation of these terms showed them to be small in the quark model with three colours and with $10 < Q^2 < 50$ if $\omega \geq 1.3$ ¹⁶⁾.

For large ω the nearest singularity in n dominates the structure functions ¹⁷⁾. These occur at $n = -1$ in γ_1^n and $n = -2$ in γ_2^n . If one assumes that the C_j^n 's have no singularities in $\text{Re } n$ to the right of those in γ_{kr}^n then the following bounds apply for large ω

$$G_1(\omega, q^2) \underset{\omega \rightarrow \infty, q^2 \text{ fixed}}{<} \omega^{-1}, \quad G_2(\omega, q^2) \underset{\omega \rightarrow \infty, q^2 \text{ fixed}}{<} \omega^{-2} \quad (2.41)$$

Regge-lore would suggest G_1 fall off slightly faster than ω^{-1} and G_2 like some inverse power of $\ln \omega$. This would suggest the singularity at $n = -1$ in γ_1^n could dominate. We assume C_1^n has this singularity in the form

$$C_1^n = \Gamma(n+1) K^{\frac{a}{n+1}} \quad (2.42)$$

where

$$\gamma_1^n \xrightarrow{n \rightarrow -1} -\frac{a}{(n+1)} + b$$

and K is a constant > 1 . $M(n+1)$ is regular at $n = -1$. Then

$$G_1(\omega, q^2) \xrightarrow{\omega \rightarrow \infty} \frac{1}{2\sqrt{\pi}} \lambda^{-b} M\left(\sqrt{\frac{z}{\ln \omega}}\right) \left(\frac{z}{\ln^3 \omega}\right)^{1/4} \exp(2\sqrt{z \ln \omega}) \quad (2.43)$$

where

$$\lambda = \frac{\ln q^2/\mu^2}{\ln q_0^2/\mu^2}$$

and $z = a \ln(K\lambda)$ and q_0^2 denotes the onset of scaling.

The terms of higher order in $\bar{g}^2$ are expected also to have the singularity at $n = -1$ and the corrections will become important unless ¹⁶⁾

$$\frac{g^2}{4\pi^2} \left(\frac{\ln \omega}{a \ln K\lambda} \right)^{3/2} \ll 1$$

Clearly this condition cannot be satisfied for very large ω and the form of Eq. (2.42) cannot there be justified. However, there will be a range of ω , depending on the unknown constant K , for which the form should be a good approximation.

For $G_2(\omega, q^2)$ we expect the Regge singularity to dominate. This would have to correspond to an $n=0$ singularity in C_1^n or an $n=1$ singularity in C_2^n . Since $\frac{n}{1} \gamma(n=0) = 0$ the leading q^2 dependence will be zero

$$G_2(\omega, q^2) \underset{\omega \rightarrow \infty}{\sim} (\ln \omega)^{-2} \quad (2.44)$$

3. - POLARIZED NEUTRINO SCATTERING

We consider the process $\nu p \rightarrow \text{anything}$. In the cross-section there enters the hadronic tensor

$$W_{\mu\nu}^{\nu}(p, q, s) = \frac{1}{2\pi} \int d^4x e^{iq \cdot x} \langle p, s | [J_{\mu}^{\dagger}(x), J_{\nu}(0)] | p, s \rangle \quad (3.1)$$

where the superscript ν signifies the neutrino reaction and J_{μ} is the weak current. The antineutrino reaction is described by

$$W_{\mu\nu}^{\bar{\nu}}(p, q, s) = \frac{1}{2\pi} \int d^4x e^{iq \cdot x} \langle p, s | [J_{\mu}(x), J_{\nu}^{\dagger}(0)] | p, s \rangle \quad (3.2)$$

We decompose $W_{\mu\nu}^{\nu}$ as follows :

$$\begin{aligned} W_{\mu\nu}^{\nu} = & -g_{\mu\nu} W_1^{\nu} + \frac{p_{\mu} p_{\nu}}{M^2} W_2^{\nu} + \frac{i}{2M^2} \epsilon_{\mu\nu\lambda\sigma} q^{\lambda} p^{\sigma} W_3^{\nu} \\ & + \frac{i}{M} \epsilon_{\mu\nu\lambda\sigma} q^{\lambda} s^{\sigma} G_1^{\nu} + \frac{i}{M^3} \epsilon_{\mu\nu\lambda\sigma} q^{\lambda} (q \cdot p s^{\sigma} - q \cdot s p^{\sigma}) G_2^{\nu} \\ & + \frac{1}{M} g_{\mu\nu} q \cdot s X_1^{\nu} + \frac{1}{M^3} p_{\mu} p_{\nu} q \cdot s X_2^{\nu} + \frac{1}{M} (s_{\mu} p_{\nu} + p_{\mu} s_{\nu}) X_3^{\nu} \end{aligned} \quad (3.3)$$

where we have omitted terms proportional to q_{μ} and q_{ν} . The invariant amplitudes are functions of q^2 , $\nu = p \cdot q$ as usual. Next define the amplitude

$$T_{\mu\nu}^{\nu}(p, q, s) = i \int d^4x e^{iq \cdot x} \langle p, s | T^*(J_{\mu}^{\dagger}(x) J_{\nu}(0)) | p, s \rangle \quad (3.4)$$

the absorptive part of which is $W_{\mu\nu}^{\nu}$. We decompose $T_{\mu\nu}^{\nu}$ with respect to the same tensor basis given in Eq. (3.3) thus introducing invariant amplitudes T_1^{ν} , $T_1^{V,\nu}$ and $T_1^{A,\nu}$ such that

$$\begin{aligned} W_1^{\nu} &= \frac{1}{\pi} \text{Im } T_1^{\nu} \\ G_1^{\nu} &= \frac{1}{\pi} \text{Im } T_1^{V,\nu} \\ X_1^{\nu} &= \frac{1}{\pi} \text{Im } T_1^{A,\nu} \end{aligned} \quad (3.5)$$

The crossing property

$$W_{\mu\nu}^{\nu}(t, -q, s) = -W_{\nu\mu}^{\bar{\nu}}(t, q, s)$$

reads in terms of the invariant amplitudes

$$\begin{aligned} W_i^{\nu}(-\nu, q^2) &= -W_i^{\bar{\nu}}(\nu, q^2) \quad ; \quad i=1,2,3 \\ G_1^{\nu}(-\nu, q^2) &= -G_1^{\bar{\nu}}(\nu, q^2) \\ G_2^{\nu}(-\nu, q^2) &= G_2^{\bar{\nu}}(\nu, q^2) \\ X_j^{\nu}(-\nu, q^2) &= X_j^{\bar{\nu}}(\nu, q^2) \quad ; \quad j=1,2 \\ X_3^{\nu}(-\nu, q^2) &= -X_3^{\bar{\nu}}(\nu, q^2) \end{aligned} \quad (3.6)$$

It is convenient to define amplitudes

$$G_i^{+} = G_i^{\nu} + G_i^{\bar{\nu}} \quad , \quad G_i^{-} = G_i^{\nu} - G_i^{\bar{\nu}} \quad \text{etc.}$$

that go into $\pm$ themselves under crossing. Then

$$\begin{aligned} W_i^{\pm}(-\nu, q^2) &= \mp W_i^{\pm}(\nu, q^2) \quad ; \quad i=1,2,3 \\ G_1^{\pm}(-\nu, q^2) &= \mp G_1^{\pm}(\nu, q^2) \\ G_2^{\pm}(-\nu, q^2) &= \pm G_2^{\pm}(\nu, q^2) \\ X_j^{\pm}(-\nu, q^2) &= \pm X_j^{\pm}(\nu, q^2) \quad ; \quad j=1,2 \\ X_3^{\pm}(-\nu, q^2) &= \mp X_3^{\pm}(\nu, q^2) \end{aligned} \quad (3.7)$$

where on the right-hand side the upper sign refers to the (+) amplitudes and the lower sign to the (-) amplitudes.

Next we write the LC expansion for the product of two weak currents

$$\begin{aligned}
 & T_{\mu}^+(x) T_{\nu}(0) : (-g_{\mu\nu} \square + \partial_{\mu} \partial_{\nu}) \frac{1}{x^2 - i\epsilon x_0} \sum_n \sum_i C_{i,1}^n(x^2 - i\epsilon x_0, g) \\
 & O^{i\mu_1 \dots \mu_n}(0) x_{\mu_1} \dots x_{\mu_n} - (g_{\mu\lambda} g_{\nu\sigma} \square - g_{\mu\lambda} \partial_{\nu} \partial_{\sigma} - g_{\nu\sigma} \partial_{\mu} \partial_{\lambda} \\
 & + g_{\mu\nu} \partial_{\lambda} \partial_{\sigma}) \sum_n \sum_i C_{i,2}^n(x^2 - i\epsilon x_0, g) O^{i\lambda\sigma\mu_1 \dots \mu_n}(0) x_{\mu_1} \dots x_{\mu_n} \\
 & - i\epsilon_{\mu\nu\lambda\sigma} \partial^{\lambda} \frac{1}{x^2 - i\epsilon x_0} \sum_n \sum_i C_{i,3}^n(x^2 - i\epsilon x_0, g) O^{i\sigma\mu_1 \dots \mu_n}(0) x_{\mu_1} \dots x_{\mu_n} \\
 & - i\epsilon_{\mu\nu\lambda\sigma} \partial^{\lambda} \frac{1}{x^2 - i\epsilon x_0} \sum_n \sum_i E_{i,1}^n(x^2 - i\epsilon x_0, g) R_1^{i\sigma\mu_1 \dots \mu_n}(0) x_{\mu_1} \dots x_{\mu_n} \\
 & - i(\epsilon_{\mu\rho\lambda\sigma} \partial_{\nu} \partial^{\rho} - \epsilon_{\nu\rho\lambda\sigma} \partial_{\mu} \partial^{\rho} - \epsilon_{\mu\nu\lambda\sigma} \square) \sum_n \sum_i E_{i,2}^n(x^2 - i\epsilon x_0, g) \\
 & R_2^{i\lambda\sigma\mu_1 \dots \mu_n}(0) x_{\mu_1} \dots x_{\mu_n} + i(-g_{\mu\nu} \square + \partial_{\mu} \partial_{\nu}) \frac{1}{x^2 - i\epsilon x_0} \sum_n \sum_i \\
 & \bar{E}_{i,1}^{n-1}(x^2 - i\epsilon x_0, g) R_1^{i\mu_1 \dots \mu_n}(0) x_{\mu_1} \dots x_{\mu_n} - i(g_{\mu\lambda} g_{\nu\sigma} \square \\
 & - g_{\mu\lambda} \partial_{\nu} \partial_{\sigma} - g_{\nu\sigma} \partial_{\mu} \partial_{\lambda} + g_{\mu\nu} \partial_{\lambda} \partial_{\sigma}) \sum_n \sum_i \bar{E}_{i,2}^{n+1}(x^2 - i\epsilon x_0, g) \\
 & R_1^{i\lambda\sigma\mu_1 \dots \mu_n}(0) x_{\mu_1} \dots x_{\mu_n} \quad (3.8)
 \end{aligned}$$

Finally we note that translational invariance imposes some restrictions upon the various terms in the expansion (3.8). For $v + \bar{v}$ ($v - \bar{v}$) these are :

- the series involving $R_1^{i\sigma\mu_1 \dots \mu_n}$ and $R_2^{i\lambda\sigma\mu_1 \dots \mu_n}$ contain only odd (even) spin operators ;
- those involving $R_1^{i\mu_1 \dots \mu_n}$ and $R_1^{i\lambda\sigma\mu_1 \dots \mu_n}$ contain only even (odd) spin operators.

We are now ready to derive the sum rules. Using the LC expansion (3.8) in Eq. (3.4) we derive asymptotic expansions for the invariant amplitudes T_i^v , $T_i^{V,v}$ and $T_i^{A,v}$ valid in the Bjorken limit. With the aid of crossing and translational invariance properties these are turned into sum rules for the structure functions. We omit from consideration the sum rules for W_i since these have already been discussed in the literature ¹⁾. The sum rules for the remaining structure functions read

$$\int_1^\infty \omega^{-n-1} \frac{\nu}{M} G_1^\pm(\omega, q^2) d\omega = 2 \sum_i a_{n-1}^i \tilde{E}_{i,1}^{\nu, n-1}; n = \begin{cases} \text{odd} \\ \text{even} \end{cases} \quad (3.9)$$

$$\int_1^\infty \omega^{-n-1} \frac{\nu^2}{M^3} G_2^\pm(\omega, q^2) d\omega = - \sum_i d_{n-2}^i \tilde{E}_{i,2}^{\nu, n-2} - 2 \sum_i \frac{(n-1)}{n} a_{n-1}^i \tilde{E}_{i,1}^{\nu, n-1};$$

$$n = \begin{cases} \text{odd} \\ \text{even} \end{cases} \quad (3.10)$$

$$\int_1^\infty \omega^{-n-1} \frac{\nu}{M} X_1^\pm(\omega, q^2) d\omega = \sum_i a_{n-1}^i [4 \tilde{E}_{i,1}^{\nu, n} + \tilde{E}_{i,2}^{\nu, n-2}]; n = \begin{cases} \text{even} \\ \text{odd} \end{cases} \quad (3.11)$$

$$\int_1^\infty \omega^{-n-1} \frac{\nu^2}{M^3} X_2^\pm(\omega, q^2) d\omega = -2 \sum_i \frac{n-1}{n+1} a_n^i \tilde{E}_{i,2}^{\nu, n-1}; n = \begin{cases} \text{odd} \\ \text{even} \end{cases} \quad (3.12)$$

$$\int_1^\infty \omega^{-n-1} \frac{\nu}{M} X_3^\pm(\omega, q^2) d\omega = -2 \sum_i \left(\frac{1}{n+1} \right) a_n^i \tilde{E}_{i,2}^{\nu, n-1}; n = \begin{cases} \text{odd} \\ \text{even} \end{cases} \quad (3.13)$$

where $\tilde{E}_{i,k}^n$ are defined just as $\tilde{E}_{i,k}^n$ in Eq. (2.11) with $E_{i,k}^n$ replaced by $\tilde{E}_{i,k}^n$. On the right-hand side the top entry for n refers to the (+) sum rules and the lower one for the (-) sum rules. The functions $\tilde{E}_{i,k}^{(n)}$ have been defined in the previous sections. Now, however, we need to know them for both even and odd values of n . This entails knowledge of the anomalous dimensions of the operators R_1 and R_2 for all values of n . Note, however, that since the (-) structure functions are the absorptive parts of the $\nu-\bar{\nu}$ amplitudes they are determined by the difference of the two-current products

$$\mathcal{T}_\mu^+(x) \mathcal{T}_\nu(0) - \mathcal{T}_\mu(x) \mathcal{T}_\nu^+(0) = 2i [\mathcal{T}_{1\mu}(x) \mathcal{T}_{2\nu}(0) - \mathcal{T}_{2\mu}(x) \mathcal{T}_{1\nu}(0)] \quad (3.14)$$

Now since the singlet operators contribute equally to $J_{i\mu}(x)J_{j\nu}(0)$, $i,j=1,2$, it follows that the sum rules (3.9) and (3.11)-(3.13) for the (-) structure functions have a single q^2 dependence coming only from one operator, namely the fermionic octet operator $F_{R_1}^1$, while in the sum rule (3.10), an additional q^2 dependence from the octet operator $F_{R_2}^1$ also arises. Furthermore we have argued in the previous section that the asymptotic behaviour is governed by the exponents A_n and B_n and that these, to a good approximation, are determined by the fermionic operators. For these operators we easily verify that the formulae for the anomalous dimensions $\frac{n}{1}F_{FF}$ and $\frac{n}{2}F_{FF}$ derived in the previous section hold for both even and odd values of n .

We now take a closer look at the sum rules (3.9)-(3.13). As expected in general logarithmic violations of scaling occur. Also for selected values of n we obtain some interesting sum rules. Thus setting $n=1$ in Eqs. (3.9) and (3.11) and $n=0$ in Eq. (3.12) we conclude that

$$\int_1^\infty \omega^{-2} \frac{\nu}{\pi} G_1^+(\omega, q^2) \quad (3.15)$$

and

$$\int_1^\infty \omega^{-2} \frac{\nu}{\pi} X_1^-(\omega, q^2) \quad (3.16)$$

and

$$\int_1^\infty \omega^{-1} \frac{\nu^2}{\pi^3} X_2^-(\omega, q^2) \quad (3.17)$$

should scale, corresponding to the fact that the axial vector current has zero anomalous dimension. One can determine the scaling form of the sum rules by appealing to the LC algebra which is, of course, valid in this theory. Thus, for example, consider the sum rule (3.15) involving G_1^+ . The LC algebra gives for the antisymmetric part of the commutator

$$\begin{aligned} [J_\mu^+(x), J_\nu(0)]_A &= \frac{2i}{\pi} \partial^\rho (\epsilon(x) \delta(x^2)) \epsilon_{\mu\rho\nu\sigma} \left[\frac{1}{\sqrt{3}} S_8^{5\sigma}(x|0) \right. \\ &\quad \left. + \sqrt{\frac{2}{3}} S_0^{5\sigma}(x|0) - A_3^{5\sigma}(x|0) \right] \end{aligned} \quad (3.18)$$

where $S_i^{5\sigma}(x|0)$ is the symmetrized bilocal operator

$$S_i^{5\sigma}(x|0) = \frac{1}{2} \left[\bar{\psi}(x) \gamma^{\nu} \gamma_5 \gamma_i \psi(0) + \bar{\psi}(0) \gamma^{\nu} \gamma_5 \gamma_i \psi(x) \right] \quad (3.19)$$

and $A_i^{5\sigma}(x|0)$ is the antisymmetrized operator. In the familiar way, Eqs. (3.18) and (3.19) lead to the sum rule

$$\int_1^{\infty} \omega^{-2} \frac{\nu}{M} G_1^+(\omega, q^2) d\omega = -\frac{2}{\sqrt{3}} b_8 - 2\sqrt{\frac{2}{3}} b_0 \quad (3.20)$$

where b_8 and b_0 are defined by the nucleon matrix elements of the axial currents $S_8^{5\sigma}(0|0)$ and $S_0^{5\sigma}(0|0)$:

$$\langle p, s | S_{0,8}^{5\sigma} | p, s \rangle = 2 S^{\sigma} b_{0,8} \quad (3.21)$$

What our analysis tells us is that the sum rule (3.20) is approached logarithmically. We can eliminate the constants b_0 and b_8 by taking the proton-neutron difference thus obtaining the sum rule

$$\int_1^{\infty} \omega^{-2} \frac{\nu}{M} \left[G_1^{\nu, P-N}(\omega, q^2) + G_1^{\bar{\nu}, P-N}(\omega, q^2) \right] d\omega = 0 + O\left(\frac{1}{\ln q^2}\right) \quad (3.22)$$

Next, since $a_{-1}^1 = 0$, by setting $n=0$ in Eq. (3.9) we obtain the sum rule

$$\int_1^{\infty} \omega^{-1} \frac{\nu}{M} G_1^-(\omega, q^2) d\omega = 0 + O\left(\frac{1}{\ln q^2}\right) \quad (3.23)$$

provided it converges. Similarly since $d_{-2}^1 = d_{-1}^1 = 0$ we obtain from Eq. (3.10)

$$\int_1^{\infty} \omega^{-1} \frac{\nu^2}{M^2} G_2^-(\omega, q^2) d\omega = 0 + O\left(\frac{1}{\ln q^2}\right) \quad (3.24)$$

$$\int_1^{\infty} \omega^{-2} \frac{\nu^2}{M^2} G_2^+(\omega, q^2) d\omega = 0 + O\left(\frac{1}{\ln q^2}\right) \quad (3.25)$$

Furthermore from Eq. (3.11) we obtain

$$\int_1^{\infty} \omega^{-1} \frac{\nu}{M} X_1^+(\omega, q^2) d\omega = 0 + O\left(\frac{1}{\ln - q^2}\right) \quad (3.26)$$

and from (3.12) we have

$$\int_1^{\infty} \omega^{-2} \frac{\nu^2}{M^3} X_2^+(\omega, q^2) d\omega = 0 + O\left(\frac{1}{\ln - q^2}\right) \quad (3.27)$$

The sum rules (3.23)-(3.27) that we have derived for the polarized neutrino structure functions are the analogues of the Burkhardt-Cottingham sum rule (2.38) for the polarized electroproduction structure function G_2 . For the same reasons as we discussed in the electroproduction case, we believe that for the physically measured structure functions the rate of approach to the asymptotic limit should, in all cases, be logarithmic in q^2 .

4. - SUMMARY AND CONCLUSIONS

We have computed the anomalous dimensions for all the operators contributing in leading order to polarized lepton-hadron scattering. For $SU(3)$ singlet structure functions both quark and gluon operators may contribute but we find that the leading dependence is given, to a good approximation, by the quark operators. Observation of these anomalous dimensions will ultimately be a crucial test of AFGT.

Sum rules will, in general, pick up logarithmic suppressions due to these anomalous dimensions. However, those sum rules with contributions coming only from the axial vector current will have zero anomalous dimension and asymptotically will scale. Due to contributions of $O(\bar{g}^2)$ and above the approach to the asymptotic limit is only logarithmic. Another class of sum rule, in electroproduction the Burkhardt-Cottingham sum rule, is obtained by looking at moments which have no contribution from the leading, twist two, operators. Although $O(\bar{g}^2)$ and above terms cannot give leading order contributions to these sum rules we still do not expect

a power law approach to the asymptotic (zero) value. This is due to the appearance at $O(\bar{g}^2)$ and above of δ for singularities in the structure functions which cannot be experimentally observed. Moments of the observed structure functions will therefore appear to approach the sum rule value only logarithmically.

Thus the sum rules obtained are those of the free quark model but the rate of approach in an AFGT is characteristically logarithmic. This also should provide a crucial test of AFGT¹⁸⁾.

The implications for the structure functions themselves required a knowledge of the operator reduced matrix elements. This, in general, can only be obtained from a complicated numerical analysis. However, for $\omega \rightarrow 1$ the high n moments dominate and for $\omega \rightarrow \infty$ the nearest singularity in n dominates. Using this we obtained the behaviour of the electro-production structure functions in these regions. However, the presence of possibly important non-leading terms means that the results of this analysis must be handled with care.

APPENDIX

Here we list the vertices corresponding to the insertions of the operators R_1 and R_2 on two and three-point functions ¹⁹⁾. We start with the twist 2 $SU(3)$ singlet operator R_1 . Symmetrization over a set of $n+1$ indices of $\sigma_{\mu_1 \dots \mu_n}$ is effected by contraction with $\Delta^{\sigma} \Delta^{\mu_1} \dots \Delta^{\mu_n}$ where Δ^{μ} is an arbitrary four-vector. The index σ is freed by differentiation with respect to Δ_{σ} and dividing by $n+1$. All momenta are taken to flow inwards.

i) $F_{R_1}{}^{\sigma\sigma\mu_1 \dots \mu_n} = (i)^n S \bar{\psi} \gamma_5 \gamma^{\sigma} D^{\mu_1} \dots D^{\mu_n} \psi ; n \text{ even}$

$$\frac{1}{(n+1)} \frac{\partial}{\partial \Delta_{\sigma}} \left\{ \gamma_5 \not{\Delta} (\Delta \cdot p)^n \right\}$$

$$\frac{1}{(n+1)} \frac{\partial}{\partial \Delta_{\sigma}} \left\{ g^{\sigma c} \gamma_5 \not{\Delta} \Delta^{\mu} \sum_{j=0}^{n-1} (-1)^j (\Delta \cdot p_1)^j (\Delta \cdot p_2)^{n-j-1} \right\}$$

ii) $V_{R_1}{}^{\sigma\sigma\mu_1 \dots \mu_n} = (i)^n S \frac{1}{2} \epsilon^{\sigma\alpha\beta\gamma} \text{Tr} F_{\beta\gamma} D^{\mu_1} \dots D^{\mu_{n-1}} F_{\alpha}{}^{\mu_n} ; n \text{ even}$

$$\frac{1}{n+1} \frac{\partial}{\partial \Delta_{\sigma}} \left\{ 2i \delta_{ab} \epsilon^{\mu\nu\alpha\beta} \Delta_{\alpha} k_{\beta} (\Delta \cdot k)^n \right\}$$

$$\frac{1}{n+1} \frac{\partial}{\partial \Delta_{\sigma}} \left[ig f_{abc} \left\{ \epsilon^{\alpha\mu\nu\beta} \Delta^{\lambda} \Delta_{\alpha} \left[p_{1\beta} (\Delta \cdot p_1)^{n-1} + p_{2\beta} (\Delta \cdot p_2)^{n-1} \right] + \epsilon^{\alpha\lambda\mu\beta} \Delta^{\nu} \Delta_{\alpha} \left[p_{3\beta} (\Delta \cdot p_3)^{n-1} \right. \right. \right. \right.$$

$$\left. \left. + p_{1\beta} (\Delta \cdot p_1)^{n-1} \right] + \epsilon^{\alpha\nu\lambda\beta} \Delta^{\mu} \Delta_{\alpha} \left[p_{2\beta} (\Delta \cdot p_2)^{n-1} + p_{3\beta} (\Delta \cdot p_3)^{n-1} \right] \right\}$$

$$\begin{aligned}
 & -igf_{abc} \sum_{j=0}^{n-2} (-1)^j \left\{ \epsilon^{\gamma\alpha\beta\mu} \Delta_\gamma \Delta^\nu \Delta^\lambda (\Delta \cdot p_1)^j \left[(\Delta \cdot p_2)^{n-j-2} + (\Delta \cdot p_3)^{n-j-2} \right] \right. \\
 & \quad + \epsilon^{\gamma\alpha\beta\nu} \Delta_\gamma \Delta^\lambda \Delta^\mu (\Delta \cdot p_2)^j \left[(\Delta \cdot p_1)^{n-j-2} + (\Delta \cdot p_3)^{n-j-2} \right] \\
 & \quad \left. + \epsilon^{\gamma\alpha\beta\lambda} \Delta_\gamma \Delta^\mu \Delta^\nu (\Delta \cdot p_3)^j \left[(\Delta \cdot p_1)^{n-j-2} + (\Delta \cdot p_2)^{n-j-2} \right] \right\} p_{1\alpha} p_{2\beta} \\
 & -igf_{abc} \sum_{j=0}^{n-2} (-1)^j \left\{ \epsilon^{\gamma\lambda\mu\beta} \Delta_\gamma \Delta^\nu \left[p_{1\beta} (\Delta \cdot p_3)^{n-j-1} (\Delta \cdot p_1)^j \right. \right. \\
 & \quad + p_{3\beta} (\Delta \cdot p_1)^{n-j-1} (\Delta \cdot p_3)^j \left. \right] + \epsilon^{\gamma\mu\nu\beta} \Delta_\gamma \Delta^\lambda \left[p_{2\beta} (\Delta \cdot p_1)^{n-j-1} (\Delta \cdot p_2)^j \right. \\
 & \quad + p_{1\beta} (\Delta \cdot p_2)^{n-j-1} (\Delta \cdot p_1)^j \left. \right] + \epsilon^{\gamma\nu\lambda\beta} \Delta_\gamma \Delta^\mu \left[p_{3\beta} (\Delta \cdot p_2)^{n-j-1} (\Delta \cdot p_3)^j \right. \\
 & \quad \left. + p_{2\beta} (\Delta \cdot p_3)^{n-j-1} (\Delta \cdot p_2)^j \right] \left. \right\} -igf_{abc} \left\{ \epsilon^{\gamma\mu\nu\alpha} \Delta_\gamma \Delta^\lambda p_{3\alpha} (\Delta \cdot p_3)^{n-1} \right. \\
 & \quad + \epsilon^{\gamma\nu\lambda\alpha} \Delta_\gamma \Delta^\mu p_{1\alpha} (\Delta \cdot p_1)^{n-1} + \epsilon^{\gamma\lambda\mu\alpha} \Delta_\gamma \Delta^\nu p_{2\alpha} (\Delta \cdot p_2)^{n-1} \\
 & \quad \left. - \epsilon^{\gamma\mu\nu\lambda} \Delta_\gamma \left[(\Delta \cdot p_1)^n + (\Delta \cdot p_2)^n + (\Delta \cdot p_3)^n \right] \right\}
 \end{aligned}$$

The vertices for the $SU(3)$ octet operator $F_{R_1}^{i\sigma\mu_1 \dots \mu_n}$ are obtained from those of $F_{R_1}^{0\sigma\mu_1 \dots \mu_n}$ by inserting the matrix $\frac{1}{2\lambda} i$.

Next we turn to the twist 3 operator R_2 .

$$\text{iii)} \quad F_{R_2}^{\sigma\lambda\sigma'\mu_1\ldots\mu_n} = (i)^{n+1} S' \bar{\psi} \gamma_5 \gamma^\lambda D^\sigma D^{\mu_1} \ldots D^{\mu_n} \psi; n \text{ odd}$$

$$\gamma_5 (\gamma^\lambda p^\sigma - \gamma^\sigma p^\lambda) (\Delta \cdot p)^n$$

$$\frac{1}{(n+1)} g^{\sigma\lambda} \gamma_5 \left(\gamma^\lambda \frac{\partial}{\partial \Delta_\sigma} - \gamma^\sigma \frac{\partial}{\partial \Delta_\lambda} \right) \cdot$$

$$\left[\Delta^\mu \sum_{j=0}^n (-1)^j (\Delta \cdot p_1)^j (\Delta \cdot p_2)^{n-j} \right]$$

$$\text{iv)} \quad v_{R_2}^{\sigma\lambda\sigma'\mu_1\ldots\mu_n} = (i)^n S' \frac{1}{2} \epsilon^{\sigma\alpha\beta\gamma} F_{\beta\gamma} D^{\mu_1} \ldots D^{\mu_n} F_\alpha^\lambda; n \text{ odd}$$

$$-2i \delta_{ab} \frac{1}{(n+1)} \left\{ \frac{\partial}{\partial \Delta_\sigma} \left[\epsilon^{\mu\nu\beta\alpha} k_\beta k^\lambda \Delta_\alpha (\Delta \cdot k)^n \right] - \frac{\partial}{\partial \Delta_\lambda} \left[\epsilon^{\mu\nu\beta\alpha} k_\beta k^\sigma \Delta_\alpha (\Delta \cdot k)^n \right] \right\}$$

$$\frac{1}{(n+1)} \frac{\partial}{\partial \Delta_\sigma} \left[\left\{ \epsilon^{\gamma\nu\beta\mu} \Delta_\gamma g^{\lambda\mu} \left[p_{1\beta} (\Delta \cdot p_1)^n + p_{2\beta} (\Delta \cdot p_2)^n \right] + \epsilon^{\gamma\mu\beta\nu} \Delta_\gamma g^{\lambda\mu} \left[p_{2\beta} (\Delta \cdot p_2)^n \right. \right. \right. \right.$$

$$\left. \left. + p_{3\beta} (\Delta \cdot p_3)^n \right] + \epsilon^{\gamma\mu\beta\nu} \Delta_\gamma g^{\lambda\mu} \left[p_{3\beta} (\Delta \cdot p_3)^n + p_{1\beta} (\Delta \cdot p_1)^n \right] \right\}$$

$$-ig f_{abc} \sum_{j=0}^{n-1} (-1)^j \left\{ \epsilon^{\sigma\alpha\beta\mu} \Delta_\gamma (\Delta \cdot p_1)^j \left[\Delta^\nu g^{\lambda\mu} (\Delta \cdot p_3)^{n-j-1} \right. \right. \right.$$

$$\begin{aligned}
 & + \Delta^\tau g^{\lambda\gamma} (\Delta \cdot p_2)^{n-j-1} \Big] + \epsilon^{\gamma\alpha\beta\nu} \Delta_\gamma (\Delta \cdot p_2)^j \Big[\Delta^\tau g^{\lambda\mu} (\Delta \cdot p_1)^{n-j-1} \\
 & + \Delta^\mu g^{\lambda\tau} (\Delta \cdot p_3)^{n-j-1} \Big] + \epsilon^{\gamma\alpha\beta\tau} \Delta_\gamma (\Delta \cdot p_3)^j \Big[\Delta^\mu g^{\lambda\nu} (\Delta \cdot p_2)^{n-j-1} \\
 & + \Delta^\nu g^{\lambda\mu} (\Delta \cdot p_1)^{n-j-1} \Big] \Big\} p_{1\alpha} p_{2\beta} \\
 & - i g f_{abc} \sum_{j=0}^{n-1} (-1)^j \Big\{ \epsilon^{\tau\mu\nu\rho} \Delta_\gamma \Delta^\nu \Big[p_{1\beta} p_3^\lambda (\Delta \cdot p_1)^j (\Delta \cdot p_3)^{n-j-1} + p_{3\beta} p_1^\lambda \\
 & (\Delta \cdot p_3)^j (\Delta \cdot p_1)^{n-j-1} \Big] + \epsilon^{\gamma\mu\nu\rho} \Delta_\gamma \Delta^\nu \Big[p_{2\beta} p_1^\lambda (\Delta \cdot p_2)^j (\Delta \cdot p_1)^{n-j-1} \\
 & + p_{1\beta} p_2^\lambda (\Delta \cdot p_1)^j (\Delta \cdot p_2)^{n-j-1} \Big] + \epsilon^{\gamma\nu\tau\rho} \Delta_\gamma \Big[p_{3\beta} p_2^\lambda (\Delta \cdot p_3)^j (\Delta \cdot p_2)^{n-j-1} \\
 & + p_{2\beta} p_3^\lambda (\Delta \cdot p_2)^j (\Delta \cdot p_3)^{n-j-1} \Big] \Big\} \\
 & - i g f_{abc} \Big\{ \epsilon^{\gamma\mu\nu\alpha} g^{\lambda\tau} \Delta_\gamma p_{3\alpha} (\Delta \cdot p_3)^n + \epsilon^{\gamma\nu\tau\alpha} g^{\lambda\mu} \Delta_\gamma p_{1\alpha} \\
 & (\Delta \cdot p_1)^n + \epsilon^{\gamma\tau\mu\alpha} g^{\lambda\nu} \Delta_\gamma p_{2\alpha} (\Delta \cdot p_2)^n - \Big[\epsilon^{\gamma\mu\nu\tau} \Delta_\gamma p_3^\lambda \\
 & (\Delta \cdot p_3)^n + \epsilon^{\gamma\nu\tau\mu} \Delta_\gamma p_1^\lambda (\Delta \cdot p_1)^n + \epsilon^{\gamma\tau\mu\nu} \Delta_\gamma p_2^\lambda (\Delta \cdot p_2)^n \Big] \Big\} \\
 & - (\lambda \leftrightarrow \sigma)
 \end{aligned}$$

Again the vertices for the twist 3 $SU(3)$ octet operator $F_{R1\sigma\mu_1\ldots\mu_n}$ are obtained from those of $F_{R2}^{O\sigma\mu_1\ldots\mu_n}$ by the insertion of $\frac{1}{2\lambda} i$.

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FIGURE CAPTION

- Figure 1
- a) The matrix element of F_{R_k} between spinor states.
 - b) The matrix element of F_{R_k} between vector states.
 - c) The matrix element of V_{R_k} between vector states.
 - d) The matrix element of V_{R_k} between spinor states.

(a)

(b)

(c)

(d)

FIG.1