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REGGEON CUTS IN A MULTIPARTICLE UNITARY MODEL,

II. FOUR-PARTICLE CASE

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A B S T R A C T

We investigate the Reggeon cuts in a model with four-particle unitarity in the t channel. The model, which was previously discussed by McCoy and Wu, is derived from ϕ^3 theory. Our analysis of asymptotic behaviour uses momentum space techniques. We investigate carefully the integral equation for the partial wave amplitude and use it to exhibit the origin of the various Reggeon cuts, which turn out to satisfy discontinuity formula consistent with Gribov's Reggeon calculus. We suggest there is a four-particle Regge pole to the right of all the cuts.

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1. INTRODUCTION

In a previous paper ¹⁾, (I), we analyzed the structure of the Reggeon cut which appears in a model with three-particle unitarity in the non-asymptotic t channel. The cut had a perfectly conventional structure and in particular the sign of its discontinuity was the same as that of the simple Mandelstam diagram. The model is somewhat artificial since the branch point, which is generated by a Reggeon and an off-shell particle, is at a fixed position in the angular momentum plane. Nevertheless it anticipates correctly some of the structure of the moving two-Reggeon cut we investigate in this paper.

Here we consider a model which is four-particle unitary in the t channel. As in I, our discussion parallels that of McCoy and Wu ²⁾ but differs from theirs by our use of momentum space techniques for the analysis of the asymptotic behaviour of Feynman amplitudes. It turns out that in the present case this leads to considerable simplification of the calculation since the amplitudes of a given order can be combined in such a way as to eliminate in advance the spurious leading behaviour of individual terms. In addition we learn about the momentum flow in the important regions of the integration space.

The outcome of our calculations is, of course, a perturbation series for the partial wave amplitude which must be summed up by a suitable integral equation. Our equation is essentially the same as that of McCoy and Wu ²⁾, but we make a deeper analysis of the Reggeon branch points. We find essentially that they satisfy discontinuity formulae consistent with Gribov's calculus ^{3),4)}. In addition, we are led to suggest the existence of a four-particle Regge pole lying to the right of all of the branch cuts.

In I, we discussed the fact that certain low order diagrams provide their contributions to the asymptotic behaviour in anomalous ways. The same kind of effect can be observed in the four-particle system. For reasons of space we do not discuss these effects in this paper but refer the reader to Nicoletopoulos and Prévost ⁵⁾. We will simply take it for granted that the low order terms in the series for the partial wave amplitude are generated correctly.

2. THE MODEL

The method of constructing the model is analogous to the three-particle case ¹⁾ (I), so we will give only a brief description. The principle behind the construction is that the resulting amplitude obeys four-particle unitarity below the five-particle threshold. We do not have a proof of this in the four-particle case, but we regard it as extremely plausible.

The model is extracted from a field theory with an interaction $-g\phi^3/3!$. We include in the first instance, all graphs which can be drawn in the form of Fig. 1, where the blob B stands for a subgraph in which four particles pass from one side to the other, interacting (or not) in any possible way. Some typical forms for B are shown in Fig. 2. The weight given to each graph depends of course on its symmetry properties. The Regge limit in which we are interested is $s \rightarrow \infty$ with t fixed (see Fig. 1).

The set of diagrams which results from this construction includes the Mandelstam diagram ⁶⁾, Fig. 3 and generalizations, Figs. 4 and 5. Out of all the diagrams in the model, we pick a dominant subset chosen on the basis of their asymptotic behaviour. The standard for asymptotic behaviour is set by the Mandelstam graph (Fig. 3). (Here and elsewhere A_n stands for the symmetrized two-particle amplitude with $n+1$ vertical rungs.) For example the asymptotic behaviour associated with Fig. 3 is

$$(g^2)^{m+n+\delta} (\ln s)^{m+n} / s^3, \quad (2.1)$$

where the quantity δ , which in I we called the coupling constant excess, has the value 4. The principle adopted by McCoy and Wu ²⁾ is to sum over all diagrams with the same coupling constant excess. Certain diagrams such as the AFS ⁷⁾ contribution in Fig. 6, have an excess greater than 4. Such diagrams we ignore. The remainder are diagrams discussed by McCoy and Wu ²⁾ which can all be drawn with typical crossed boxes at both ends of the diagram. Actually these remarks require some qualification in the case of certain low order diagrams and subdiagrams, as indicated in the introduction, we shall not pursue this point here.

The use of a symmetrized form for A_n is what allows us to avoid the spurious leading behaviour of individual graphs.

3. MANDELSTAM DIAGRAM

The Mandelstam ⁶⁾ diagram has been extensively analyzed using a Feynman parameter representation ⁸⁾. Its discontinuities have been discussed by Halliday and Sachrajda ⁹⁾ using momentum space techniques. We will present here a derivation of the asymptotic behaviour of the complete amplitude using these latter techniques as a convenient introduction to our notation and methods. Of course, it will be very close in spirit to Ref. 9).

As in I, we express momenta in terms of Sudakov ¹⁰⁾ parameters. If the initial state particles have momenta p_1 and p_2 (both with mass m) then we define

$$\begin{aligned}\bar{p}_1 &= p_1 - \frac{m^2}{s} p_2 \\ \bar{p}_2 &= p_2 - \frac{m^2}{s} p_1\end{aligned}\quad (3.1)$$

A general vector k is expressed as

$$k = \alpha \bar{p}_1 + \beta \bar{p}_2 + k_\perp \quad (3.2)$$

so that

$$k^2 = \alpha\beta s - k_\perp^2$$

and

$$d^4k = \frac{1}{2} s d\alpha d\beta d^2k_\perp \quad (3.3)$$

An inverse propagator is expressed as

$$\begin{aligned}k^2 - m^2 &= \alpha\beta s - \mu^2 \\ \mu^2 &= m^2 + k_\perp^2\end{aligned}\quad (3.4)$$

For convenience we will omit the $\perp$ suffix since no confusion is caused by this. If the final state momenta are p'_1 and p'_2 then

$$p'_1 - p_1 = p_2 - p'_2 = x \bar{p}_1 + y \bar{p}_2 + q \quad (3.5)$$

and $x, y \sim 1/s$ as $s \rightarrow \infty$ with the result that

$$t = (p'_1 - p_1)^2 \approx -q^2 \quad (3.6)$$

The amplitude corresponding to Fig. 3 is

$$M_{m,n} = g^\dagger \left(\frac{s}{2i(2\pi)^4} \right)^3 \int d\sigma_\perp J_{m,n}, \quad (3.7)$$

where $\int d\sigma_1$ represents the transverse integrals and (see Fig. 3 for the labelling of momenta)

$$J_{m,n} = \int d\alpha_1 d\beta_1 d\beta_2 d\alpha_3 d\beta_3 d\alpha_4 A_m A_n [P_1 P_2 P_a P_b P_3 P_4 P_c P_d]^{-1} \quad (3.8)$$

The inverse propagators are given by

$$\begin{aligned} P_1 &= \alpha_1 \beta_1 s - \mu_1^2 + i\varepsilon \\ P_2 &= (1 - \alpha_1 - \alpha_3 - \alpha_4) \beta_2 s - \mu_2^2 + i\varepsilon \\ P_a &= -(1 - \alpha_1) \beta_1 s - \mu_a^2 + i\varepsilon \\ P_b &= -(\alpha_1 + \alpha_3 + \alpha_4) \beta_2 s - \mu_b^2 + i\varepsilon \end{aligned} \quad (3.9)$$

The remaining propagators are determined by similar expressions.

The asymptotic behaviour we seek for $J_{m,n}$

$$J_{m,n} \sim (\ln s)^{m+n} / s^6 \quad (3.10)$$

Now, in an obvious notation, as s_m and $s_n \rightarrow \infty$ at fixed t_m and t_n *)

$$A_m(s_m, t_m) \approx \frac{i\pi g^2}{s_m} \left[\frac{g^2}{2\pi} K(-t_m) \right]^m \left[\ln \left(\frac{s_m}{P_b P_d} \right) \right]^{m-1} / (m-1)! \quad (3.11)$$

$$A_n(s_n, t_n) \approx \frac{i\pi g^2}{s_n} \left[\frac{g^2}{2\pi} K(-t_n) \right]^n \left[\ln \left(\frac{s_n}{P_a P_c} \right) \right]^{n-1} / (n-1)!$$

*) Our definition of $K(q^2)$ is the same as in I, namely

$$K(q^2) = \frac{1}{2} \int \frac{d^2 k}{(2\pi)^2} \frac{1}{(k^2 + m^2)((k-q)^2 + m^2)}.$$

We have included the factors $(P_b P_d)^{-1}$ and $(P_a P_c)^{-1}$ in the arguments of the logarithms in anticipation of a situation in which these "masses" also become large, while remaining nevertheless, much smaller than the corresponding energy s_m or s_n [see Ref. 9].

Since

$$\begin{aligned} s_m &\approx (\alpha_1 + \alpha_4)(1 + \beta_1 - \beta_3)S \\ s_n &\approx (1 - \alpha_1 + \alpha_3)(\beta_2 + \beta_3)S \end{aligned} \quad (3.12)$$

we see that the integrand in Eq. (3.8) provides us immediately with a behaviour

$$(\ln s)^{m+n-2} / s^2 \quad (3.13)$$

The remaining inverse powers of s arise from a neighbourhood in which four of the integration variables $\sim 1/s$. These must be chosen in such a way that the last two integrations each provide a power of $\ln s$. Guided by Ref. 9) we choose the following integration region

$$\begin{aligned} \alpha_1 &\gg \alpha_3, \alpha_4 \sim \frac{1}{s} \\ \beta_3 &\gg \beta_1, \beta_2 \sim \frac{1}{s} \end{aligned} \quad (3.14)$$

In this region

$$\begin{aligned} P_2 &\approx (1 - \alpha_1)\beta_2 S - \mu_2^2 + i\varepsilon \\ P_b &\approx -\alpha_1\beta_2 S - \mu_b^2 + i\varepsilon \\ s_m &\approx \alpha_1(1 - \beta_3)S \\ s_n &\approx (1 - \alpha_1)\beta_3 S \end{aligned} \quad (3.15)$$

and t_m and t_n are purely transverse in form.

Consider now the β_1 integration. The integrand only depends on β_1 through the propagators P_a^{-1} and P_1^{-1} and the mass dependences of A_m and A_n . These produce cuts in the β_1 plane. However, relative to the original integrand, the discontinuities across these cuts lose a power of $\ln s$. In manipulating the β_1 contour therefore, we need take account only of the propagator poles and are justified in ignoring these cuts. It is easily

verified that the propagator poles lie on opposite sides of the β_1 contour only when $0 < \alpha_1 < 1$. In this case we can evaluate the β_1 integral by closing the contour round the pole at $P_1=0$ and using Cauchy's theorem. The same strategy can be applied to the β_2, α_3 and α_4 integrals. For the latter, we must have $0 < \beta_3 < 1$. The momenta k_1, k_2, k_3 and k_4 are therefore all put on mass-shell and the result is

$$J_{m,n} \approx \left(\frac{-2\pi i}{s} \right)^{\dagger} \int \frac{d\alpha_1 d\beta_3}{\alpha_1(1-\alpha_1)\beta_3(1-\beta_3)} \frac{A_m A_n}{[P_a P_b P_c P_d]}, \quad (3.16)$$

where now, because of the mass shell conditions,

$$\begin{aligned} P_a &= -\frac{1-\alpha_1}{\alpha_1} \mu_1^2 - \mu_a^2 + i\varepsilon \\ P_b &= -\frac{\alpha_1}{1-\alpha_1} \mu_2^2 - \mu_b^2 + i\varepsilon \\ P_c &= -\frac{\beta_3}{1-\beta_3} \mu_4^2 - \mu_c^2 + i\varepsilon \\ P_d &= -\frac{1-\beta_3}{\beta_3} \mu_3^2 - \mu_d^2 + i\varepsilon \end{aligned} \quad (3.17)$$

The integration for α_1 and β_3 is the square in Fig. 7. Now it is not hard to see that each corner of the square will provide a contribution to the leading asymptotic behaviour. Consider for example the neighbourhood of $\alpha_1, \beta_3 \approx 0$. Here

$$\begin{aligned} P_a &\approx -\mu_1^2/\alpha_1 \\ P_b &\approx -\mu_b^2 \\ P_c &\approx -\mu_c^2 \\ P_d &\approx -\mu_3^2/\beta_3 \\ s_m &\approx \alpha_1 s \\ s_n &\approx \beta_3 s \end{aligned} \quad (3.18)$$

Note that the "masses" P_a and P_d become arbitrarily large. In that case we see from Eq. (3.10) that

$$A_m \approx \frac{i\pi g^2}{\alpha_1 s} \left[\frac{g^2}{2\pi} K(-t_m) \right]^m \left[\ln \alpha_1 \beta_3 s \right]^{m-1} / (m-1)! \quad (3.19)$$

$$A_n \approx \frac{i\pi g^2}{\beta_3 s} \left[\frac{g^2}{2\pi} K(-t_n) \right]^n \left[\ln \alpha_1 \beta_3 s \right]^{n-1} / (n-1)!$$

Putting these results into Eq. (3.16), we find

$$J_{m,n} \approx - \frac{\pi^2 g^4}{s^6} (2\pi i)^4 [\mu_1^2 \mu_b^2 \mu_3^2 \mu_c^2]^{-1} \times \left[\frac{g^2}{2\pi} K(-t_m) \right]^m \left[\frac{g^2}{2\pi} K(-t_n) \right]^n L_{m,n}, \quad (3.20)$$

where

$$L_{m,n} = \frac{1}{(m-1)!(n-1)!} \int \frac{d\alpha_1}{\alpha_1} \frac{d\beta_3}{\beta_3} (\ln \alpha_1 \beta_3 s)^{m+n-2} \quad (3.21)$$

The integration region in Eq. (3.21) is restricted by the requirement that the asymptotic behaviour Eq. (3.19) be applicable. That is $\alpha_1 \beta_3 s \gg 1$. We also have $0 < \alpha_1, \beta_3 < \epsilon$. As a consequence

$$L_{m,n} \approx \binom{m+n-2}{m-1} \frac{(\ln s)^{m+n}}{(m+n)!} \quad (3.22)$$

$M_{m,n}$ receives a contribution to its asymptotic behaviour

$$M_{m,n} \approx - \frac{i\pi^2 g^4}{2s^3} \binom{m+n-2}{m-1} \frac{(\ln s)^{m+n}}{(m+n)!} \times \int \frac{d^2 k}{(2\pi)^2} \left[\frac{g^2}{2\pi} K(k^2) \right]^{m+1} \left[\frac{g^2}{2\pi} K((q-k)^2) \right]^{n+1} \quad (3.23)$$

Here transverse propagators have provided extra powers of $K(-t_m)$ and $K(-t_n)$ and we used the transverse momentum flowing through A_n as the remaining loop integral. When we add the contributions of the other three corners of the square the transverse integral above is modified to

$$\int \frac{d^2 k}{(2\pi)^2} \left[\frac{g^2}{2\pi} K(k^2) \right]^m \left[\frac{g^2}{2\pi} K((q-k)^2) \right]^n \left[\frac{g^2}{2\pi} (K(k^2) + K((q-k)^2)) \right]^2 \quad (3.24)$$

It is actually more convenient for us to classify the contributions in powers of $K(k^2)$ and $K((q-k)^2)$. Combining the relevant terms from $M_{p-2,q}$, $M_{p-1,q-1}$ and $M_{p,q-2}$ we obtain a term

$$T_{pq} = - \frac{i\pi^2 g^4 (\ln s)}{2s^3} \frac{p+q-2}{(p+q-2)!} \binom{p+q-2}{p-1} \int \frac{d^2 k}{(2\pi)^2} \left[\frac{g^2}{2\pi} K(k^2) \right]^p \left[\frac{g^2}{2\pi} K((q-k)^2) \right]^q \quad (3.25)$$

Now, when $n=m$, Fig. 3 is completely symmetrical under the interchange of A_m and A_n . This means according to Feynman rules that a symmetry factor of $\frac{1}{2}$ should be included in (3.23) for this case. The possibility of interchanging the bubbles in Fig. 3 also means that we should keep $m \leq n$ in order not to overcount the diagrams. In turn, this implies that we should keep $p \leq q$ in Eq. (3.25) and include a factor of $\frac{1}{2}$ when $p=q$.

Finally we note that when m or n is zero, the symmetrized amplitude $A_0(s_0, t_0) \sim s_0^{-2}$ rather than s_0^{-1} . This means that the integration region we have investigated yields a zero contribution to the asymptotic behaviour. We should therefore examine other diagrams of the same order to see if they yield a relevant contribution ⁵⁾. As indicated in the Introduction, we shall not carry out this program but simply assume that all the terms T_{1q} , $q=1,2,\dots$ are correctly reproduced by the perturbation series.

4. GENERALIZED MANDELSTAM DIAGRAMS

In this section we shall consider the generalizations of Mandelstam's diagram in Figs. 4 and 5. Corresponding to Fig. 4 we have the Feynman amplitude

$$M = g^4 \left(\frac{s}{2i(2\pi)^4} \right)^5 \int d\sigma_{\perp} J, \quad (4.1)$$

where

$$J = \int d\alpha_1 d\beta_1 d\beta_2 d\alpha_3 d\beta_3 d\alpha_4 d\alpha_5 d\beta_5 d\alpha_6 d\beta_6 A_k A_l A_m A_n \times [P_1 P_2 P_3 P_4 P_5 P_6 P_a P_b P_c P_d P_e P_f]^{-1} \quad (4.2)$$

In writing down M we have ignored possible symmetry factors.

Guided by the previous section, we choose the integration region represented by

$$\alpha_1 \gg \alpha_5, \alpha_6 \gg \alpha_3, \alpha_4 \sim \frac{1}{5} \\ \beta_3 \gg \beta_5, \beta_6 \gg \beta_1, \beta_2 \sim \frac{1}{5} \quad (4.3)$$

Again the inequalities refer to absolute sizes. The subsequent discussion will be clearer if it is kept in mind that the strong inequalities above mean that the relevant integration region should be parametrized by, for example,

$$\alpha_i \sim 5^{\gamma_i} \quad (4.4)$$

where

$$0 \geq \gamma_1 > \gamma_5, \gamma_6 > \gamma_3, \gamma_4 = -1 \quad (4.5)$$

However, it is unnecessary to use this parametrization explicitly in evaluating the asymptotic behaviour, as we saw in the previous calculation. Further restrictions we impose on the integration region will be designed to elicit the most singular asymptotic behaviour.

In the region defined by Eqs. (4.3), (4.4) and (4.5) we find that P_1, P_2, P_a, P_b have the forms given in Eqs. (3.9) and (3.10). The quantities P_3, P_4, P_c, P_d also take the same form as in the previous section. The arguments used there allow us to reduce the integral for J to

$$J = \left(-\frac{2\pi i}{5}\right)^4 \int \frac{d\alpha_1 d\beta_3 d\alpha_5 d\beta_5 d\alpha_6 d\beta_6}{\alpha_1(1-\alpha_1) \beta_3(1-\beta_3)} A_k A_l A_m A_n \times [P_a P_b P_c P_d P_e P_f P_5 P_6]^{-1}, \quad (4.6)$$

where P_a, P_b, P_c, P_d have the forms given by Eq. (3.15). The (α_1, β_3) integration range is again the inside of the square in Fig. 7, and again each corner contributes to the complete asymptotic behaviour. As illustration we will consider the neighbourhood of $(\alpha_1, \beta_3) \approx (0,0)$.

With the above restrictions on the integration region, we find that the energies flowing through the bubbles A_k, A_l, A_m and A_n are respectively

$$\begin{aligned} S_k &\approx \alpha_5 S \\ S_l &\approx \beta_3 \alpha_6 S \\ S_m &\approx \alpha_1 \beta_6 S \\ S_n &\approx \beta_5 S \end{aligned} \quad (4.7)$$

The momentum transfers, t_k, t_l, t_m and t_n are purely transverse. The denominators P_a, P_b, P_c and P_d take the form in Eq. (3.18) while

$$\begin{aligned} P_5 &\approx \alpha_5 \beta_5 S - \mu_5^2 + i\varepsilon \\ P_6 &\approx \alpha_6 \beta_6 S - \mu_6^2 + i\varepsilon \\ P_e &\approx -\alpha_6 \beta_5 S - \mu_e^2 + i\varepsilon \\ P_f &\approx -\alpha_5 \beta_6 S - \mu_f^2 + i\varepsilon \end{aligned} \quad (4.8)$$

In discussing the remaining integrations, it is important to keep in mind that the asymptotic behaviour we seek for J is

$$J \sim (\ln s)^{k+l+m+n+2} / s^8, \quad (4.9)$$

while from the integrand in Eq. (4.6) we get directly, using the asymptotic forms for A_k , etc.

$$(\ln s)^{k+l+m+n-4} / s^8 \quad (4.10)$$

The remaining six integrations therefore must each produce a power of $\ln s$. Any contribution for which this does not happen can be neglected as not contributing to the leading asymptotic behaviour.

Consider now the α_5 integration. The singularities lying in the α_5 plane are reflections of the propagator poles $P_5=0$ or $P_f=0$, the mass singularities of A_k , etc., and the energy threshold singularities of A_k . In manipulating the α_5 contour we can ignore contributions from the propagator

poles since the resulting residue has one less integration variable but no compensating increase in the number of powers of $(\ln s)$. The mass singularities may be ignored since the discontinuity across the attached cuts has a lower power of $(\ln s)$ than the original integrand. This shows that only the A_k energy thresholds are significant. We are free therefore to sweep the left half α_5 contour through the lower half plane in order to wrap it around the A_k energy thresholds lying in the right half plane (see Fig. 8). The same operation can be performed on the β_5, α_6 and β_6 contours. The result is to replace the integral in Eq. (4.6) with a similar one in which $\alpha_5, \beta_5, \alpha_6$ and β_6 are integrated over positive ranges and the factors A_k , etc., are replaced by their normal threshold discontinuities which, since they are asymptotically imaginary, are just $2A_k$, etc.

Our discussion has shown that the leading asymptotic behaviour does not come from a region where $\alpha_5 \beta_5 s$ (for example) remains finite. It certainly cannot be associated with a region in which $\alpha_5 \beta_5 s$ becomes infinite since this would result in too severe a power fall-off. Consequently we must restrict the exponents γ_i in Eqs. (4.4) and (4.5), so that for large s ,

$$\alpha_5 \beta_5 s \rightarrow 0 \quad (4.11)$$

A similar discussion shows that we need

$$\alpha_5 \beta_6 s, \alpha_6 \beta_5 s, \alpha_6 \beta_6 s \rightarrow 0 \quad (4.12)$$

also. This implies that P_5, P_6, P_e, P_f acquire their transverse forms. We have then

$$\begin{aligned} J \approx & 2^4 \left(\frac{-2\pi i}{s} \right)^4 [\mu_1^2 \mu_3^2 \mu_b^2 \mu_c^2 \mu_5^2 \mu_6^2 \mu_e^2 \mu_f^2]^{-1} \\ & \times \left(\frac{i\pi g^2}{s} \right)^4 \left[\frac{g^2}{2\pi} K(-t_k) \right]^k \left[\frac{g^2}{2\pi} K(-t_\ell) \right]^\ell \left[\frac{g^2}{2\pi} K(-t_m) \right]^m \left[\frac{g^2}{2\pi} K(-t_n) \right]^n \\ & \times L / (k-1)! (\ell-1)! (m-1)! (n-1)! \end{aligned} \quad (4.13)$$

where

$$\begin{aligned}
 L = & \int \frac{d\alpha_1}{\alpha_1} \frac{d\beta_3}{\beta_3} \frac{d\alpha_5}{\alpha_5} \frac{d\beta_5}{\beta_5} \frac{d\alpha_6}{\alpha_6} \frac{d\beta_6}{\beta_6} \\
 & \times [\ln(\alpha_5 \beta_3 s)]^{k-1} [\ln(\alpha_6 \beta_3 s)]^{\ell-1} \\
 & \times [\ln(\alpha_1 \beta_6 s)]^{m-1} [\ln(\alpha_1 \beta_5 s)]^{n-1}
 \end{aligned} \tag{4.14}$$

In order to arrive at the expression for J in Eqs. (4.13) and (4.14), it is necessary to recall that, as in Section 3, P_a and P_d become arbitrarily large.

We show in the Appendix that the asymptotic behaviour of L is

$$L \approx \frac{(\ln s)^{\ell+k+m+n+2}}{(\ell+k+m+n+2)!} \frac{(n+m)! (k+\ell)!}{k \ell m n} \tag{4.15}$$

Putting these results together, we find for M that

$$\begin{aligned}
 M \approx & - \frac{i g^8}{8 s^3} \frac{(\ln s)^{\ell+k+m+n+2}}{(\ell+k+m+n+2)!} \binom{n+m}{m} \binom{\ell+k}{k} \\
 & \times \int \frac{d\tilde{\sigma}_I}{(2\pi)^6 \mu_s^2 \mu_b^2 \mu_c^2 \mu_f^2} \\
 & \times \left[\frac{g^2}{2\pi} K(-t_k) \right]^k \left[\frac{g^2}{2\pi} K(-t_\ell) \right]^{\ell+1} \left[\frac{g^2}{2\pi} K(-t_m) \right]^{m+1} \left[\frac{g^2}{2\pi} K(-t_n) \right]^n
 \end{aligned} \tag{4.16}$$

If we include the four corners of the (α_1, β_3) square, we obtain the result in Eq. (4.16) with the last factor in the integrand of the transverse integral replaced by

$$\begin{aligned} & \left[\frac{g^2}{2\pi} K(-t_k) \right]^k \left[\frac{g^2}{2\pi} K(-t_\ell) \right]^\ell \left[\frac{g^2}{2\pi} K(-t_m) \right]^m \left[\frac{g^2}{2\pi} K(-t_n) \right]^n \\ & \times \left[\frac{g^2}{2\pi} (K(-t_k) + K(-t_\ell)) \right] \\ & \times \left[\frac{g^2}{2\pi} (K(-t_m) + K(-t_n)) \right] . \end{aligned} \quad (4.17)$$

If we classify the terms according to powers of $K(-t_k)$, etc., then we obtain finally a set of contributions

$$T_{p,q;r,s} = - \frac{ig^8}{8s^3} \frac{(l_{ns})^{p+q+r+s}}{(p+q+r+s)!} \binom{p+q}{p} \binom{r+s}{r} \quad (4.18)$$

$$\begin{aligned} & \times \int \frac{d\tilde{\sigma}_\perp}{(2\pi)^6 \mu_s^2 \mu_b^2 \mu_e^2 \mu_f^2} \\ & \times \left[\frac{g^2}{2\pi} K(-t_k) \right]^r \left[\frac{g^2}{2\pi} K(-t_\ell) \right]^s \left[\frac{g^2}{2\pi} K(-t_m) \right]^p \left[\frac{g^2}{2\pi} K(-t_n) \right]^q \end{aligned}$$

Here k, ℓ, m and n are simply being used to label the bubbles.

In order not to include too many diagrams, we must impose the restrictions $p \leq q$ and $r \leq s$. In addition a symmetry factor of $\frac{1}{2}$ should be included in Eq. (4.18) each time $p=q$ or $r=s$.

The diagram in Fig. 5 can be treated by similar methods. We sketch the argument briefly, thus

$$M = g^4 \left(\frac{s}{2i(2\pi)^4} \right)^6 \int d\sigma_\perp J, \quad (4.19)$$

where

$$J = \int d\alpha_1 d\beta_1 d\beta_2 d\alpha_3 d\beta_3 d\alpha_4 d\alpha_5 d\beta_5 d\alpha_6 d\beta_6 d\beta_7 d\alpha_8$$

$$\times A_k A_\ell A_m A_n A_h$$

$$\times [P_1 P_2 \dots P_8 P_a P_b \dots P_f]^{-1} \quad (4.20)$$

Considerations analogous to those used above lead us to the integration region

$$\alpha_1 \gg \alpha_5, \alpha_7 \gg \alpha_6, \alpha_8 \gg \alpha_3, \alpha_4 \sim \frac{1}{s}$$

$$\beta_3 \gg \beta_6, \beta_8 \gg \beta_5, \beta_7 \gg \beta_1, \beta_2 \sim \frac{1}{s}, \quad (4.21)$$

as providing the dominant asymptotic behaviour. The first part of the calculation is the same as before, namely lines 1, 2, 3, 4 are put on mass shell and (α_1, β_3) restricted to the square in Fig. 7. Then we wrap the $\beta_5, \alpha_6, \beta_7, \alpha_8$ contours round the positive branch cuts corresponding to the normal thresholds of A_k, A_ℓ, A_m and A_n . That is, now these variables are restricted to positive values and A_k is replaced by $2A_k$, etc. In the interests of obtaining the maximum asymptotic behaviour, we further restrict the integration region so that P_e, P_f, P_5, P_6, P_7 and P_8 acquire transverse forms. If we evaluate these quantities in terms of Sudakov parameters, we see that the restrictions are that the integration variables should be scaled so that

$$\alpha_5 \beta_5 s, \alpha_6 \beta_5 s, \alpha_6 \beta_6 s, \alpha_7 \beta_7 s, \alpha_8 \beta_6 s, \alpha_8 \beta_7 s \quad (4.22)$$

all tend to zero as s becomes infinite.

If we compute the contribution to J from the corner $(\alpha_1, \beta_3) \approx (0, 0)$ we find

$$\begin{aligned}
 J = & 2^4 \left(\frac{2\pi i}{s} \right)^4 \left(\frac{i\pi g^2}{s} \right)^5 [\mu_1^2 \mu_b^2 \mu_3^2 \mu_c^2 \mu_e^2 \mu_f^2 \mu_s^2 \dots \mu_8^2]^{-1} \\
 & \times \left[\frac{g^2}{2\pi} K(-t_h) \right]^h \dots \left[\frac{g^2}{2\pi} K(-t_n) \right]^n \\
 & \times L
 \end{aligned} \tag{4.23}$$

where

$$\begin{aligned}
 L = & 4[(h-1)!(k-1)!(l-1)!(m-1)!(n-1)!]^{-1} \\
 & \times \int \frac{d\alpha_1 d\beta_3 d\alpha_5 d\beta_5 d\alpha_6 d\beta_6 dB_7 d\alpha_8}{\alpha_1 \beta_3 \alpha_5 \beta_5 \alpha_6 \beta_6 B_7 \alpha_8} \\
 & \times (\ln \alpha_5 \beta_6 s)^{h-1} (\ln \alpha_8 \beta_3 s)^{k-1} (\ln \alpha_6 \beta_3 s)^{l-1} \\
 & \times (\ln \alpha_1 \beta_7 s)^{m-1} (\ln \alpha_1 \beta_5 s)^{n-1}
 \end{aligned} \tag{4.24}$$

where we restrict α_5 and β_6 also to positive values and introduce a compensating factor of 4. By methods similar to those used to evaluate Eq. (4.14) we find

$$L \approx 4 \binom{m+n}{m} \binom{k+l}{l} \frac{(\ln s)^{h+k+l+m+n+3}}{(h+k+l+m+n+3)!} \tag{4.25}$$

If now we add in the contributions from the other corners of the (α_1, β_3) integration region, we obtain for M

$$\begin{aligned}
 M \approx & \frac{-i g^{10}}{16 \pi s^3} \frac{(l n s)^{h+k+l+m+n+3}}{(h+k+l+m+n+3)!} \binom{m+n}{m} \binom{k+l}{l} \\
 & \times \int \frac{d\tilde{\sigma}_1}{(2\pi)^8} \frac{\left[\frac{g^2}{2\pi} K(-t_n)\right]^h \left[\frac{g^2}{2\pi} K(-t_k)\right]^k \cdots \left[\frac{g^2}{2\pi} K(-t_n)\right]^n}{\mu_s^2 \mu_6^2 \mu_7^2 \mu_8^2 \mu_e^2 \mu_f^2} \quad (4.26) \\
 & \times \left[\frac{g^2}{2\pi} (K(-t_m) + K(-t_n)) \right] \left[\frac{g^2}{2\pi} (K(-t_k) + K(-t_l)) \right]
 \end{aligned}$$

When the various contributions are classified according to powers of $K(-t_k)$, etc., we find

$$\begin{aligned}
 T_{p,q;h;r,s} \approx & \frac{-i g^{10}}{16 \pi s^3} \frac{(l n s)^{h+p+q+r+s+1}}{(h+p+q+r+s+1)!} \binom{p+q}{p} \binom{r+s}{r} \\
 & \times \int \frac{d\tilde{\sigma}_1}{(2\pi)^8} \frac{\left[\frac{g^2}{2\pi} K(-t_h)\right]^h \left[\frac{g^2}{2\pi} K(-t_k)\right]^r \left[\frac{g^2}{2\pi} K(-t_l)\right]^s \left[\frac{g^2}{2\pi} K(-t_n)\right]^p \left[\frac{g^2}{2\pi} K(-t_n)\right]^q}{\mu_s^2 \mu_6^2 \mu_7^2 \mu_8^2 \mu_e^2 \mu_f^2} \quad (4.27)
 \end{aligned}$$

This time no symmetry factors are necessary and p, q, r, s range freely from one to infinity, while h can also take the value zero.

This concludes our calculation of sample diagrams. It is plausible that other more complicated diagrams can be computed in analogous ways with predictable results. We give the rules for these other contributions in the next section when we consider the partial wave amplitude.

5. PARTIAL WAVE AMPLITUDE

As in I, we define the partial wave amplitude to be $M(j)$ so that

$$M(s,t) = \frac{1}{2\pi i} \int dj M(j) \frac{s^j + (-s)^j}{2} \quad (5.1)$$

Now to leading order

$$\frac{1}{2\pi i} \int dj \frac{1}{(j+3)^{p+2}} \left(\frac{s^j + (-s)^j}{2} \right) \approx \frac{i\pi}{2s^3} \frac{(t/s)^p}{p!}, \quad (5.2)$$

so the Mandelstam term $T_{p,q}$ in Eq. (3.25) leads to the partial wave

$$T_{p,q} = - \frac{\pi g^4}{(j+3)^{p+q}} \binom{p+q-2}{p-1} \int \frac{d^2k}{(2\pi)^2} \left[\frac{g^2}{2\pi} K(k^2) \right]^p \left[\frac{g^2}{2\pi} K(k^2) \right]^q \quad (5.3)$$

We see that this term can be reproduced by the following rules. First we draw the diagram in Fig. 9 which represents two pairs of particles interacting $p-1$ and $q-1$ times, respectively. The amplitude in Eq. (5.3) is obtained from :

- i) a factor of $\lambda = g^2/2\pi(j+3)$ for each internal vertex;
 - ii) a factor of $(k^2+m^2)^{-1}$ for each internal line;
 - iii) $\int (d^2k/(2\pi)^2)$ for each internal loop;
 - iv) a factor $g^4/(2\pi(j+3))$ for each end vertex;
 - v) standard symmetry factors (that is $\frac{1}{2}$ for each bubble in Fig. 9);
 - vi) an over-all factor of $(-\pi)$;
 - vii) summing over all orders in which the internal vertices can appear.
- This rule yields the factor $\binom{p+q-2}{p-1}$ in Eq. (5.3).

It is not hard to verify that these rules also reproduce the terms in Eqs. (4.18) and (4.27) when applied to the diagrams in Figs. 10 and 11, respectively.

We shall take it then that the complete series for the partial wave amplitude is obtained by drawing all diagrams in which the four internal particles have point interactions in all possible ways. Implicit in this statement is the existence of the $O(g^8)$ diagram, Fig. 12, which yields a contribution

$$M(j) = -\frac{\pi}{4!} \left(\frac{g^4}{2\pi(j+3)} \right)^2 \int \frac{d\tilde{\sigma}_+}{(2\pi)^6 \mu_1^2 \mu_2^2 \mu_3^2 \mu_4^2} \quad (5.4)$$

We did not consider the corresponding Feynman diagrams but they do indeed yield a leading contribution corresponding to Eq. (5.4). Note that the symmetry factor is $1/4!$

6. PARTIAL WAVE INTEGRAL EQUATION

The integral equation which sums up the series for $M(j)$ can be conveniently expressed in terms of a four-particle partial wave Green's function [cf., Ref. 2) and our paper I]. We write

$$M(j) = \frac{-\pi}{(4!)^2} \left(\frac{g^4}{2\pi(j+3)} \right)^2 \int \prod_{i=1}^4 \left(\frac{d^2 k_i d^2 k'_i}{(2\pi)^4} \right) \delta_{q_{k_T}} \times \langle k_1 \dots k_4 | B | k'_1 \dots k'_4 \rangle, \quad (6.1)$$

where $k_T = \sum k_i$ and $\delta_{kk'} = (2\pi)^2 \delta^{(2)}(k-k')$. Here B is the four-particle Green's function. It satisfies the equation

$$B = B_0 + BV \quad (6.2)$$

where

$$\langle k_1 \dots k_4 | B_0 | k'_1 \dots k'_4 \rangle = [\mu_1^2 \mu_2^2 \mu_3^2 \mu_4^2]^{-1} \sum_{(\alpha, \beta, \gamma, \delta)} \delta_{k_1 k'_\alpha} \delta_{k_2 k'_\beta} \delta_{k_3 k'_\gamma} \delta_{k_4 k'_\delta} \quad (6.3)$$

the summation being over all permutations, $(\alpha, \beta, \gamma, \delta)$ of $(1, 2, 3, 4)$, and

$$V = \sum_{(\alpha, \beta)(\gamma, \delta)} V_{\alpha\beta, \gamma\delta} \quad (6.4)$$

the summation being over ordered pairs (α, β) and (γ, δ) [that is $(1, 3) \neq (3, 1)$] with

$$\begin{aligned} & \langle k_1 \dots k_4 | V_{\alpha\beta, \gamma\delta} | k'_1 \dots k'_4 \rangle \\ &= \frac{\lambda}{2 \times 4!} \delta_{k_1 k'_1} \delta_{k_2 k'_2} \delta_{k_3 k'_3} \delta_{k_4 k'_4} \frac{\mu_1'^2 \mu_2'^2}{\mu_1'^2 \mu_2'^2 \mu_3'^2 \mu_4'^2} \end{aligned} \quad (6.5)$$

If we develop the perturbation series for B from Eq. (6.2), we can easily check, especially for low order terms, that it reproduces the series of the previous section, when combined with Eq. (6.1).

We can solve for B thus

$$B = B_0 + B_0 W \quad (6.6)$$

where

$$W = V + V W \quad (6.7)$$

We can decompose W

$$W = \sum_{(\alpha, \beta)(\gamma, \delta)} W_{\alpha\beta, \gamma\delta} \quad (6.8)$$

where

$$W_{\alpha\beta, \gamma\delta} = V_{\alpha\beta, \gamma\delta} + \sum_{(\xi, \eta)(\sigma, \tau)} V_{\alpha\beta, \xi\eta} W_{\sigma\tau, \gamma\delta} \quad (6.9)$$

Now $W_{12, \gamma\delta}$ for example, corresponds to graphs in which the particles with momenta k_3 and k_4 interacted last. These graphs are of two kinds, those in which the momenta k_1 and k_2 leave from the same point and those in which they leave from separate points. The former give rise to terms which depend only on $k_1 + k_2$ and the latter to terms which depend on k_1 and k_2 separately. We are led then to the following form for $W_{\alpha\beta, \gamma\delta}$,

$$\langle k_1 \dots k_+ | W_{\alpha\beta, \gamma\delta} | k'_1 \dots k'_+ \rangle$$

$$= \frac{1}{2 \times 4!} \delta_{k_T k'_T} [C_{\gamma\delta}(k_\alpha, k_\beta) + D_{\gamma\delta}(k_\alpha + k_\beta)] \quad (6.10)$$

where we have suppressed the dependence of C and D on k_T and $(k'_1 \dots k'_4)$ for notational simplicity. Of course we are free to use the ansatz Eq. (6.10) without reference to the perturbation series. We find on substituting it into Eq. (6.9) that

$$C_{\gamma\delta}(k_\alpha, k_\beta) + D_{\gamma\delta}(k_\alpha + k_\beta)$$

$$= C_{\gamma\delta}^{(0)}(k_\alpha, k_\beta) + \frac{\lambda}{2 \times 4!} \sum_{(\xi, \eta)(\sigma, \tau)} \int \prod_{i=1}^4 \pi \left(\frac{d^2 k_i}{(2\pi)^2} \right) \frac{\mu_\xi'^2 \mu_\eta'^2 \delta_{k_\alpha k_\xi''} \delta_{k_\beta k_\eta''}}{\mu_1'^2 \mu_2'^2 \mu_3'^2 \mu_4'^2} \quad (6.11)$$

$$\times [C_{\gamma\delta}(k_\sigma'', k_\tau'') + D_{\gamma\delta}(k_\sigma'' + k_\tau'')]$$

where

$$C_{\gamma\delta}^{(0)}(k_\alpha, k_\beta) = \lambda \delta_{k_\alpha k_\gamma'} \delta_{k_\beta k_\delta'} \frac{\mu_\gamma'^2 \mu_\delta'^2}{\mu_1'^2 \mu_2'^2 \mu_3'^2 \mu_4'^2} \quad (6.12)$$

The terms in the sum in Eq. (6.11) may be divided into several groups :
 i) $(4!/2)$ terms in which $(\xi, \eta) = (\sigma, \tau)$; ii) $(4!/2)$ terms in which $(\xi, \eta) = (\tau, \sigma)$; iii) $4!$ terms in which (ξ, η) and (σ, τ) have no common element and iv) $4 \times 4!$ terms in which (ξ, η) and (σ, τ) have one member in common (without regard to order). From i) we obtain a contribution

$$\frac{\lambda}{2} K(k_T - k_\alpha - k_\beta) [C_{\gamma\delta}(k_\alpha, k_\beta) + D_{\gamma\delta}(k_\alpha + k_\beta)] \quad (6.13)$$

From ii)

$$\frac{\lambda}{2} K(k_T - k_\alpha - k_\beta) [C_{\gamma\delta}(k_\beta, k_\alpha) + D_{\gamma\delta}(k_\alpha + k_\beta)] \quad (6.14)$$

From iii)

$$\begin{aligned} & \lambda K(k_T - k_\alpha - k_\beta) D_{\gamma\delta}(k_T - k_\alpha - k_\beta) \\ & + \frac{\lambda}{2} \int \frac{d^2 k''}{(2\pi)^2} \frac{C_{\gamma\delta}(k'', k_T - k_\alpha - k_\beta - k'')}{[k''^2 + m^2][(k_T - k_\alpha - k_\beta - k'')^2 + m^2]} \end{aligned} \quad (6.15)$$

From iv)

$$\begin{aligned} & \lambda \int \frac{d^2 k''}{(2\pi)^2} \frac{1}{[k''^2 + m^2][(k_T - k_\alpha - k_\beta - k'')^2 + m^2]} \\ & \times \left\{ \frac{1}{2} (C_{\gamma\delta}(k_\alpha, k'') + C_{\gamma\delta}(k'', k_\alpha)) \right. \\ & \quad + \frac{1}{2} (C_{\gamma\delta}(k_\beta, k'') + C_{\gamma\delta}(k'', k_\beta)) \\ & \quad \left. + D_{\gamma\delta}(k_\alpha + k'') + D_{\gamma\delta}(k_\beta + k'') \right\} \end{aligned} \quad (6.16)$$

It is obvious that in Eqs. (6.15) and (6.16) only the symmetrical part

$$E_{\gamma\delta}(k_\alpha, k_\beta) = \frac{1}{2} (C_{\gamma\delta}(k_\alpha, k_\beta) + C_{\gamma\delta}(k_\beta, k_\alpha)) \quad (6.17)$$

enters and the same is true of the combination of Eqs. (6.13) and (6.14). It is convenient therefore to symmetrize Eq. (6.11) with respect to k_α and k_β . As a result, we find, on identifying the appropriate terms on both sides of the equation,

$$\begin{aligned} D_{\gamma\delta}(p) = & \lambda K(k_T - p) [D_{\gamma\delta}(p) + D_{\gamma\delta}(k_T - p)] \\ & + \frac{\lambda}{2} \int \frac{d^2 k''}{(2\pi)^2} \frac{E_{\gamma\delta}(k'', k_T - p - k'')}{[k''^2 + m^2][(k_T - p - k'')^2 + m^2]}, \end{aligned} \quad (6.18)$$

and

$$E_{\gamma\delta}(k, k') = E_{\gamma\delta}^{(0)}(k, k') + \lambda K(k_T - k - k') E_{\gamma\delta}(k, k') \\ + \lambda \int \frac{d^2 k''}{(2\pi)^2} \frac{[E_{\gamma\delta}(k, k'') + E_{\gamma\delta}(k'', k') + D_{\gamma\delta}(k+k'') + D_{\gamma\delta}(k'+k'')]}{[k''^2 + m^2][(k_T - k - k' - k'')^2 + m^2]} \quad (6.19)$$

$$E_{\gamma\delta}^{(0)}(k, k') = \frac{1}{2} [C_{\gamma\delta}^{(0)}(k, k') + C_{\gamma\delta}^{(0)}(k', k)] \quad (6.20)$$

Equation (6.19) is equivalent to Eq. (3.18) of Ref. 2).

A further symmetrization is useful, we introduce

$$F_{\gamma\delta}(p) = \frac{1}{2} [D_{\gamma\delta}(p) + D_{\gamma\delta}(k_T - p)] \quad (6.21)$$

From Eq. (6.18) we obtain after a little rearrangement (we drop the $\gamma\delta$ suffix from now on)

$$F(p) = \frac{1}{1 - \lambda K(p) - \lambda K(k_T - p)} \frac{\lambda}{4} \int \frac{d^2 k''}{(2\pi)^2} \times \\ \times \left\{ \frac{E(k'', k_T - p - k'')}{[k''^2 + m^2][(k_T - p - k'')^2 + m^2]} + \frac{E(k'', p - k'')}{[k''^2 + m^2][(p - k'')^2 + m^2]} \right\} \quad (6.22)$$

and Eq. (6.19) implies (use $k'' \rightarrow k_T - k - k' - k''$)

$$E(k, k') = E^{(0)}(k, k') + \lambda K(k_T - k - k') E(k, k') \\ + \lambda \int \frac{d^2 k''}{(2\pi)^2} \frac{\{E(k, k'') + E(k'', k') + E(k+k'') + F(k'+k'')\}}{[k''^2 + m^2][(k_T - k - k' - k'')^2 + m^2]} \quad (6.23)$$

We shall find that we only need the symmetrized quantities to compute $M(j)$. Equations (6.22) and (6.23) are coupled linear integral equations. In order to manipulate them it is convenient to express them in a more abstract notation. For example, Eq. (6.22) can be written

$$F = \Delta N E \quad (6.24)$$

where

$$\begin{aligned} \langle p | \Delta | p' \rangle &= \frac{\delta_{pp'}}{1 - \lambda K(p) - \lambda K(k_F - p)} \\ \langle p | N | k k' \rangle &= \frac{\lambda}{4} \left\{ \frac{\delta_{k+k', k_F - p} + \delta_{k+k', p}}{[k^2 + m^2][k'^2 + m^2]} \right\} \end{aligned} \quad (6.25)$$

In the same spirit we can write Eq. (6.23) as

$$E = E^{(0)} + M E + H F \quad (6.26)$$

We will discuss the form of M and H explicitly at an appropriate point.

It is clear that there are two matrix inversions to be carried out. First we must construct

$$\Sigma = (1 - M)^{-1}, \quad (6.27)$$

so that

$$E = \Sigma E^{(0)} + \Sigma H F. \quad (6.28)$$

On substituting in Eq. (6.24) we find

$$F = \Delta N \Sigma E^{(0)} + \Delta N \Sigma H F. \quad (6.29)$$

The final inversion is therefore of the matrix

$$1 - \Delta N \Sigma H \quad (6.30)$$

These two inversions correspond to distinct physical processes. The Born series for Σ is a sum over diagrams in which the particles interact in all possible ways except that of two separate interacting pairs. That is it involves graphs in which at each stage three-particles mutually interact and the fourth one is a spectator although the interacting triplet can change from time to time. The Born series for the inverse of Eq. (6.30) inserts those graphs in which particles interact in two distinct pairs. It is this inversion therefore which creates the two-Reggeon cut. In the next section, we study the structure of Σ , the analysis of the two-Reggeon cut is carried out in Section 8.

7. THE TWO-REGGEON IRREDUCIBLE AMPLITUDE

The most important property of this amplitude is that it does not contain the two-Reggeon branch point at

$$j = -3 + 2 \left[\frac{g^2}{2\pi} K\left(\frac{q^2}{4}\right) \right]. \quad (7.1)$$

It does have a branch cut at

$$j = -3 + \frac{g^2}{2\pi} K(0) \quad (7.2)$$

associated with states of one Reggeon and two off-shell particles.

McCoy and Wu ²⁾ showed that the three-particle system has a Regge pole in its angular momentum plane at

$$j = -2 + \frac{g^2}{2\pi} L(q^2) \quad (7.3)$$

and that

$$L(0) > 2K(0) \quad (7.4)$$

This pole gives rise to a branch point in the two-Reggeon irreducible amplitude at

$$j = -3 + \frac{g^2}{2\pi} L(0) \quad (7.5)$$

McCoy and Wu ²⁾ pointed out that Eq. (7.4) implies that near $t=0$ this singularity actually lies to the right of the two-Reggeon branch point. However, this will not prevent us analyzing the two-Reggeon cut discontinuity. In the absence of further singularities the branch point Eq. (7.5) will control the asymptotic behaviour. We are led to suggest, however, that there may be a four-particle Regge pole lying to the right of all cuts which is finally the dominant singularity.

The two-Reggeon irreducible amplitude is defined in terms of the matrix elements of Σ . Now a comparison of Eqs. (6.23) and (6.26) shows that the matrix elements of M have the form

$$\begin{aligned} \langle k k' | M | k'' k''' \rangle &= \lambda K(k_T - k - k') \delta_{k k''} \delta_{k' k'''} \\ &+ \frac{\lambda \delta_{k k''}}{[k'''^2 + m^2][(k_T - k - k' - k''')^2 + m^2]} + \frac{\lambda \delta_{k' k'''}{[k''^2 + m^2][(k_T - k - k' - k'')^2 + m^2]} \end{aligned} \quad (7.6)$$

An examination of the terms in the Born series for Σ suggests the following structure

$$\begin{aligned} \langle k k' | \Sigma | k'' k''' \rangle &= \frac{\delta_{k k''} \delta_{k' k'''}{1 - \lambda K(k_T - k - k')} \\ &+ \frac{1}{1 - \lambda K(k_T - k - k')} \left\{ \frac{X_{k_T - k}(k', k''') \delta_{k k''}}{[k'''^2 + m^2]} + \frac{X_{k_T - k'}(k, k'') \delta_{k' k'''}{[k''^2 + m^2]} \right. \\ &\left. + \frac{Y(k, k', k'', k''')}{[k''^2 + m^2][k'''^2 + m^2]} \right\} \frac{1}{1 - \lambda K(k_T - k'' - k''')} \end{aligned} \quad (7.7)$$

Using the fact that $(1-M)\Sigma=1$ we find that

$$\begin{aligned} X_q(k, k') &= \frac{\lambda}{[(q - k - k')^2 + m^2]} \\ &+ \lambda \int \frac{d^2 \bar{k}}{(2\pi)^2} \frac{X_q(\bar{k}, k')}{[(q - k - \bar{k})^2 + m^2][\bar{k}^2 + m^2][1 - \lambda K(q - \bar{k})]}, \end{aligned} \quad (7.8)$$

and that

$$\begin{aligned}
 Y(k, k', k'', k''') = & \frac{\lambda X_{k_T - k'''}(k, k'')}{[1 - \lambda K(k_T - k - k''')] [(k_T - k - k' - k''')^2 + m^2]} \\
 & + \frac{\lambda X_{k_T - k''}(k', k''')}{[1 - \lambda K(k_T - k' - k'')] [(k_T - k - k' - k'')^2 + m^2]} \\
 & + \lambda \int \frac{d^2 \bar{k}}{(2\pi)^2} \frac{1}{[\bar{k}^2 + m^2]} \left\{ \frac{Y(k, \bar{k}, k'', k''')}{[(k_T - k - k' - \bar{k})^2 + m^2] [1 - \lambda K(k_T - k - \bar{k})]} \right. \\
 & \left. + \frac{Y(\bar{k}, k', k'', k''')}{[(k_T - k - k' - \bar{k})^2 + m^2] [1 - \lambda K(k_T - \bar{k} - k')]} \right\}
 \end{aligned} \tag{7.9}$$

Apart from the necessary angular momentum shift $j+2 \rightarrow j+3$, $X_q(k, k')$ is just the Reggeon particle scattering amplitude defined by Eq. (8.21) in paper I. It satisfies

$$X_q(k, k') = X_q(k', k), \tag{7.10}$$

and has the branch point in Eq. (7.2). Equation (7.9) guarantees that this singularity is passed on to Y , reinforced by singularities of the kernel. The three-Reggeon pole in Eq. (7.2) appears in X_q shifted to the left by one unit.

The amplitude Y has the symmetry properties

$$Y(k, k', k'', k''') = Y(k', k, k''', k'') \tag{7.11}$$

and

$$Y(k'', k''', k, k') = Y(k, k', k'', k'''). \tag{7.12}$$

The former follows from Eq. (7.9) while the latter can be established either by examining the terms of the Born series for Σ or by noting that M [see Eq. (7.6)] can be symmetrized by a suitable similarity transformation. It follows from (7.10) and (7.12) that if we define V so that

$$\begin{aligned}
 & \frac{\langle p|V|p' \rangle}{[1-\lambda K(k_T-p)][1-\lambda K(k_T-p')]} \\
 &= \frac{\lambda}{4} \int \frac{d^2k d^2k' d^2k'' d^2k'''}{(2\pi)^8} \frac{\delta_{k+k', p} \delta_{k''+k''', p'}}{[k^2+m^2][k'^2+m^2]} \langle k, k' | \hat{\Sigma} | k'', k''' \rangle
 \end{aligned} \tag{7.13}$$

where $\hat{\Sigma}$ is Σ without its diagonal term $(1-\lambda K)^{-1}$, then

$$\langle p|V|p' \rangle = \langle p'|V|p \rangle. \tag{7.14}$$

The amplitude $\langle p|V|p' \rangle$ is the two-Reggeon irreducible amplitude.

In computing Σ it is slightly simpler algebraically to put

$$1-M = (1-M_1-M_2)(1-\lambda K), \tag{7.15}$$

so that

$$\Sigma = (1-\lambda K)^{-1} (1-M_1-M_2)^{-1}, \tag{7.16}$$

where

$$\begin{aligned}
 & \langle k, k' | M_1 | k'', k''' \rangle \\
 &= \frac{\lambda \delta_{k k''}}{[k''^2+m^2][k_T^2-k-k'-k''^2+m^2][1-\lambda K(k_T-k-k'')] }
 \end{aligned} \tag{7.17}$$

The matrix elements for M_2 are obtained by the simultaneous interchanges $k \rightleftharpoons k'$, $k'' \rightleftharpoons k'''$. In fact it is convenient to introduce a linear operator C , which performs these interchanges, thus

$$\begin{aligned}
 C |k, k' \rangle &= |k', k \rangle \\
 C^2 &= 1
 \end{aligned} \tag{7.18}$$

It follows that

$$M_2 = C M_1 C. \tag{7.19}$$

Now we write

$$(1 - M_1 - M_2)^{-1} = 1 + N_1 + N_2 \quad (7.20)$$

where

$$N_2 = C N_1 C \quad (7.21)$$

Equations (7.20) and (7.21) are satisfied if we require

$$N_1 = M_1 + M_1 (N_1 + N_2) \quad (7.22)$$

$$N_2 = M_2 + M_2 (N_1 + N_2)$$

We find

$$N_2 = B_2 + B_2 N_1 \quad (7.23)$$

$$N_1 = B_1 + B_1 N_2$$

where

$$B_i = (1 - M_i)^{-1} M_i \quad (7.24)$$

$$B_2 = C B_1 C$$

and, ultimately

$$N_1 = B_1 + B_1 C B_1 C + B_1 C B_1 C N_1 \quad (7.25)$$

That is

$$N_1 = (1 - (B_1 C)^2)^{-1} (B_1 + (B_1 C)^2) \quad (7.26)$$

Equation (7.21) gives N_2 . Now

$$\hat{\Sigma} = (1 - \lambda K)^{-1} (N_1 + N_2) \quad (7.27)$$

From which one deduces that

$$\begin{aligned} \frac{\langle P | V | P' \rangle}{1 - \lambda K(k_T - p')} &= \frac{\lambda}{2} \int \frac{d^3 k \dots d^3 k'''}{(2\pi)^8} \frac{\delta_{k+k', p} \delta_{k''+k''', p'}}{(k^2 + m^2)(k'^2 + m^2)} \\ &\quad \times \langle k k' | N_1 | k'' k''' \rangle \end{aligned} \quad (7.28)$$

Since C acting on state in Eq. (7.28) can be absorbed by a change of integration variables we see that N_1 given by Eq. (7.26) can be replaced by

$$\bar{N}_1 = (1 - (B_1 C)^2)^{-1} (B_1 C + (B_1 C)^2) = (1 - B_1 C)^{-1} B_1 C, \quad (7.29)$$

from which it follows that the important inversion is that of $(1 - B_1 C)$. We have equivalently that

$$\bar{N}_1 = B_1 C + B_1 C \bar{N}_1 \quad (7.30)$$

Now it is easily checked that

$$\langle k k' | B_1 C | k'' k''' \rangle = \frac{\delta_{k k'''} X_{k_T - k}(k', k'')}{[k''^2 + m^2][1 - \lambda K(k_T - k - k'')]} \quad (7.31)$$

so the three-particle Regge pole [Eq. (7.3)] gives rise to a contribution to $B_1 C$ of the form

$$\langle k k' | B_1 C | k'' k''' \rangle \approx \frac{\delta_{k k'''} r(k') r(k'')}{[k''^2 + m^2][1 - \lambda K(k_T - k - k'')][j + 3 - \frac{g^2}{2\pi} L(k_T - k)]} \quad (7.32)$$

where $r(k')r(k'')$ is the factoring residue of $X_{k_T - k}(k', k'')$. The pole in Eq. (7.32) generates the cut attached to the branch point Eq. (7.5) in $\bar{N}_1$ and finally in V . We can deduce a formula for the discontinuity of V across this cut using standard methods. We find that

$$\Delta \langle p | V | p' \rangle = -2\pi i \int \frac{d^2 k}{(2\pi)^2} \psi^{(+)}(p, k) \delta[j + 3 - \frac{g^2}{2\pi} L(k_T - k)] \psi^{(-)}(p', k) \quad (7.33)$$

where

$$\begin{aligned} \psi(p, k) = & \int \frac{d^2 k'' d^2 k''' d^2 \bar{k}}{(2\pi)^6} \frac{r(\bar{k})}{[\bar{k}^2 + m^2][1 - \lambda K(k_T - \bar{k} - k)]} \\ & \times \langle \bar{k} k | (1 - B_1 C)^{-1} | k'' k''' \rangle \delta_{k'' + k''', p} \end{aligned} \quad (7.34)$$

Here $(\pm)$ refer to the wave functions being evaluated above and below the cut. Equation (7.33) will allow us to obtain a formula for the corresponding discontinuity in $M(j)$.

Finally we note, using the trace approximation that [see Eq. (7.31)]

$$\det(1-B, C) \approx 1 - \int \frac{d^2k}{(2\pi)^2} \frac{X_{k_T-k}(k, k)}{[k^2 + m^2][1 - \lambda K(k-2k)]} \quad (7.35)$$

Using Eq. (7.32) we deduce that the determinant has a logarithmic branch point

$$\det(1-B, C) \approx 1 + \beta \ln(j+3 - \frac{g^2}{2\pi} L(1)) \quad (7.36)$$

where β is a positive constant. We argued in I that under certain circumstances the trace approximation is good enough for a qualitative understanding of its logarithmic branch point. If these arguments hold good here then we see that the negative character of the logarithm near the branch point guarantees the presence of a zero in $\det(1-B, C)$ and therefore a pole in $\langle p|V|p' \rangle$ to the right of the branch point in Eq. (7.5).

8. THE TWO-REGGEON GREEN'S FUNCTION

The problem of constructing the two-Reggeon Green's function is the same as that of inverting the matrix in Eq. (6.30). It is therefore necessary to investigate the kernel $\langle p|N\bar{N}|p' \rangle$. The form of $\langle p|N|kk' \rangle$ is given by Eq. (6.25) while a comparison of Eqs. (6.23) and (6.26) shows that

$$\langle kk'|H|p' \rangle = \lambda \left\{ \frac{1}{[(p'-k)^2 + m^2][(k_T - k' - p')^2 + m^2]} + \frac{1}{[(p'-k')^2 + m^2][(k_T - k - p')^2 + m^2]} \right\} \quad (8.1)$$

Note that

$$\langle k k' | H | k_T - p' \rangle = \langle k k' | H | p' \rangle \quad (8.2)$$

Now it is easy to verify using Eqs. (7.6) and (8.1), that

$$\langle k k' | \Sigma H | p' \rangle = \int \frac{d^2 k'' d^2 k'''}{(2\pi)^4} \langle k k' | \Sigma \hat{M} | k'' k''' \rangle \delta_{k+k'', p'}^{k''', p'} \quad (8.3)$$

where

$$1 - M = 1 - \lambda K - \hat{M} \quad (8.4)$$

Using the fact that Σ is the inverse of $(1-M)$ we find

$$\langle k k' | \Sigma H | p' \rangle = \int \frac{d^2 k'' d^2 k'''}{(2\pi)^4} \langle k k' | \Sigma (1 - \lambda K) | k'' k''' \rangle \delta_{k+k'', p'}^{k''', p'} \quad (8.5)$$

From Eq. (7.13) for V we have then

$$\frac{\langle p | V | p' \rangle}{1 - \lambda K(k_T - p)} = \frac{\lambda}{4} \int \frac{d^2 k d^2 k'}{(2\pi)^4} \frac{\delta_{k+k', p}}{[k^2 + m^2][k'^2 + m^2]} \langle k k' | \Sigma H | p' \rangle \quad (8.6)$$

Equations (8.2) and (8.6) together with the symmetry of V imply that

$$\langle p | V | p' \rangle = \langle p | V | k_T - p' \rangle = \langle k_T - p | V | p' \rangle \quad (8.7)$$

It is now easy to deduce that

$$\langle p | N \Sigma H | p' \rangle = \left(\frac{1}{1 - \lambda K(k_T - p)} + \frac{1}{1 - \lambda K(p)} \right) \langle p | V | p' \rangle \quad (8.8)$$

If we introduce R_0 such that

$$\langle p | R_0 | p' \rangle = \frac{\delta_{pp'}}{1 - \lambda K(k_T - p) - \lambda K(p)} \left(\frac{1}{1 - \lambda K(k_T - p)} + \frac{1}{1 - \lambda K(p)} \right) \quad (8.9)$$

then the matrix to be inverted is

$$1 - R_0 V \quad (8.10)$$

This form has the advantage that both R_0 and V are symmetrical. It leads us to define a two-Reggeon Green's function R , by

$$R = R_0 + R_0 V R. \quad (8.11)$$

Obviously the branch points of V are transferred to R . In addition R acquires the two-Reggeon branch point because of the vanishing of the denominator in the first factor in Eq. (8.9). As for the four-particle Regge pole in V , the fact that it has a factorizing residue means that it should survive subject only to an inevitable displacement. To prove these remarks would require of course a more careful analysis.

We could use R and V therefore to solve Eq. (6.24) and (6.26) for E and F . However, it is simpler and more convenient to see first how they enter into the construction of the partial wave amplitude $M(j)$.

9. THE DISCONTINUITIES OF $M(j)$

Neglecting the contribution of B_0 itself it follows from Eqs. (6.1) to (6.10) that

$$M(j) = -\frac{\pi}{8} \left(\frac{g^4}{2\pi(j+3)} \right)^2 \int \prod_{i=1}^4 \left(\frac{d^3 k_i d^3 k'_i}{(2\pi)^4} \right) \frac{\delta_{2k_T} \delta_{k_T k'_T}}{\mu_1^2 \mu_2^2 \mu_3^2 \mu_4^2} \quad (9.1)$$

$$\times \left[C_{12}(k_1, k_2) + D_{12}(k_1 + k_2) \right].$$

In Eq. (9.1) we have taken account of the multiplicity of identical contributions. It is not hard to see that in the above C_{12} and D_{12} may be replaced by their symmetrized forms $E(k_1, k_2)$ and $F(k_1 + k_2)$. We are concerned then with the quantities

$$f(p) = \int \prod_{i=1}^4 \frac{d^2 k_i'}{(2\pi)^2} \delta_{k_T k_T'} F(p)$$

$$e(k_1, k_2) = \int \prod_{i=1}^4 \frac{d^2 k_i'}{(2\pi)^2} \delta_{k_T k_T'} E(k_1, k_2) \quad (9.2)$$

From Eq. (6.29) we deduce that

$$f(p) = f^{(0)}(p) + \int \frac{d^2 p'}{(2\pi)^2} \langle p | R_0 V | p' \rangle f(p') \quad (9.3)$$

where

$$f^{(0)}(p) = \frac{\lambda}{1 - \lambda K(k_T - p) - \lambda K(p)} \int \prod_{i=1}^4 \left(\frac{d^2 k_i'}{(2\pi)^2} \right) \frac{d^2 k d^2 k'}{(2\pi)^4} \delta_{k_T k_T'} \times \langle p | N \Sigma | k k' \rangle \frac{\delta_{k k'} \delta_{k' k'_2}}{\mu_3'^2 / \mu_4'^2} \quad (9.4)$$

After some manipulation we find

$$f^{(0)}(p) = \int \frac{d^2 p'}{(2\pi)^2} \langle p | R_0 | p' \rangle u(p') \quad (9.5)$$

where

$$u(p) = \lambda^2 K(p) K(k_T - p) + \int \frac{d^2 p'}{(2\pi)^2} \langle p | V | p' \rangle w(p') \quad (9.6)$$

and we have introduced

$$w(p) = \frac{\lambda K(k_T - p)}{1 - \lambda K(k_T - p)} + \frac{\lambda K(p)}{1 - \lambda K(p)} \quad (9.7)$$

It follows from Eqs. (9.3) and (9.5) then, that

$$f(p) = \int \frac{d^2 p'}{(2\pi)^2} \langle p | R | p' \rangle u(p') \quad (9.8)$$

From Eq. (6.28) we find

$$e(k_1, k_2) = e^{(0)}(k_1, k_2) + \int \frac{d^2 p'}{(2\pi)^2} \langle k_1, k_2 | \sum H | p' \rangle f(p'), \quad (9.9)$$

where

$$e^{(0)}(k_1, k_2) = \lambda \int \prod_{i=1}^4 \left(\frac{d^2 k_i'}{(2\pi)^2} \right) \frac{d^2 k d^2 k'}{(2\pi)^4} \delta_{k_T, k'_T} \times \langle k_1, k_2 | \sum | k k' \rangle \frac{\delta_{k k'} \delta_{k' k_2'}}{\mu_3'^2 \mu_4'^2} \quad (9.10)$$

Eventually we find

$$e(k_1, k_2) = \frac{2\lambda K(k_T - k_1, -k_2)}{1 - \lambda K(k_T - k_1, -k_2)} + \int \frac{d^2 k d^2 k' d^2 p'}{(2\pi)^6} \langle k_1, k_2 | \hat{\Sigma} (1 - \lambda K) | k k' \rangle \times \delta_{k+k', p'} (f(p') + w(p')) \quad (9.11)$$

The functions $f(k_1, k_2)$ and $e(k_1, k_2)$ are now used [recall Eqs. (9.1) and (9.2)] to construct the partial wave amplitude. The result is

$$M(j) = -\frac{\pi g^4}{2} \left\{ \lambda^2 \int \frac{d^2 p}{(2\pi)^2} K(p) K(q-p) w(p) + \int \frac{d^2 p d^2 p'}{(2\pi)^4} w(p) \langle p | V | p' \rangle w(p') \right. \\ \left. + \int \frac{d^2 p d^2 p'}{(2\pi)^4} u(p) \langle p | R | p' \rangle u(p') \right\} \quad (9.12)$$

The two-Reggeon branch point is contained in the last term in Eq. (9.12). Standard reasoning leads to the result for the discontinuity across the attached cut

$$\Delta M(j) = -\frac{\pi g^4}{2} \int \frac{d^2 p d^2 p'}{(2\pi)^4} U^{(+)}(p) \langle p | \Delta R_0 | p' \rangle U^{(-)}(p') \quad (9.13)$$

where

$$U(p) = u(p) + \int \frac{d^2 p'}{(2\pi)^2} \langle p | V R_0 | p' \rangle U(p') \quad (9.14)$$

$$\langle p | \Delta R_0 | p' \rangle = -2\pi i \delta \left[j+3 - \frac{g^2}{2\pi} K(p) - \frac{g^2}{2\pi} K(q-p) \right] \phi(p) \delta_{pp'} \quad (9.15)$$

and

$$\phi(p) = (j+3) \left(\frac{1}{1 - \lambda K(q-p)} + \frac{1}{1 - \lambda K(p)} \right) \quad (9.16)$$

The superfixes $(\pm)$ indicate the limits above and below the two-Reggeon cut shown in Fig. 13. The difference between these two boundary values is $\Delta M(j)$. Note that both limits onto the two-Reggeon cut lie on the same side of the cut attached to the Reggeon-particle branch point, Eq. (7.5).

If now we make the identifications

$$C_{\alpha_1, \alpha_2}(j) = g^2 (8\phi)^{1/2} U \quad (9.17)$$

$$\alpha_i = \alpha(t_i) = -1 + \frac{g^2}{2\pi} K(-t_i)$$

then Eqs. (9.13) to (9.16) lead to the discontinuity result, Eq. (6.10) of Ref. 4). If we were able to ignore the Reggeon-particle cut, then this discontinuity would have a definite sign, and provide a negative contribution to the cross-section. This situation would occur if for sufficiently large (g^2/m^2) the two-Reggeon cut overtakes the particle-Reggeon cut. To this extent we feel that the model justifies the discontinuity formulae used in the Gribov calculus³⁾. A remaining point which should be checked is that the identification Eq. (9.17) is consistent with $C_{\alpha_1, \alpha_2}(j)$ defined by a suitable partial wave projection. However, we leave this to a future analysis.

The discontinuity across the most prominent particle-Reggeon cut has the same form as the corresponding cut in the three-particle problem. To investigate this cut, we note that it originates in V but is absent from R_0 . We can rearrange Eq. (9.12) so that

$$\begin{aligned}
 M(j) = & -\frac{\pi g^4}{2} \left\{ \int \frac{d^2 p}{(2\pi)^2} v(p) w(p) \right. \\
 & + \int \frac{d^2 p d^2 p'}{(2\pi)^4} v(p) \langle p | R_0 | p' \rangle v(p') \\
 & \left. + \int \frac{d^2 p d^2 p'}{(2\pi)^4} \chi(p) \langle p | \mathcal{V} | p' \rangle \chi(p') \right\} \quad (9.18)
 \end{aligned}$$

where for convenience we have put

$$\chi(p) = w(p) + \int \frac{d^2 p'}{(2\pi)^2} \langle p | R_0 | p' \rangle v(p') \quad (9.19)$$

and

$$v(p) = \lambda^2 K(q-p) K(p) \quad (9.20)$$

The operator $\mathcal{V}$ is defined by

$$\mathcal{V} = V + V R_0 \mathcal{V} \quad (9.21)$$

so it obeys equations like

$$\begin{aligned}
 \mathcal{V} &= V + V R V \\
 R V &= R_0 \mathcal{V}
 \end{aligned} \quad (9.22)$$

which are useful in carrying out the above rearrangement. The Reggeon particle cut is contained only in the last term of Eq. (9.18), so the usual reasoning leads to the result for the discontinuity across this cut

$$\Delta M(j) = -\frac{\pi g^4}{2} \int \frac{d^2 p d^2 p'}{(2\pi)^4} \gamma^{(+)}(p) \langle p | \Delta V | p' \rangle \gamma^{(-)}(p') \quad (9.23)$$

where

$$\gamma(p) = \chi(p) + \int \frac{d^2 p'}{(2\pi)^2} \langle p | R_0 V | p' \rangle \gamma(p') \quad (9.24)$$

and again $(\pm)$ refer to appropriate limits onto the cut. Now ΔV is given by Eq. (7.33). Inserting this into Eq. (9.23) we have finally

$$\Delta M(j) = -\frac{\pi g^4}{2} (-2\pi i) \int \frac{d^2 k}{(2\pi)^2} \bar{\Psi}^{(+)}(k) \delta[j+3-\frac{g^2}{2\pi} L(k,k)] \Psi^{(+)}(k) \quad (9.25)$$

which is the standard form for such a discontinuity [compare Eq. (8.17) in paper I]. Here

$$\Psi(k) = \int \frac{d^2 p}{(2\pi)^2} \psi(p, k) \eta(p) \quad (9.26)$$

Unfortunately the discontinuity across the Reggeon two-particle cut Eq. (7.2) is difficult to analyze starting from either Eq. (9.12) or Eq. (9.18). It is influenced by singularities in $w(p)$, R_0 and V . This does not necessarily mean that it has a particularly complicated structure, but we have not found a form for $M(j)$ which exhibits this structure in a convenient way.

10. CONCLUSIONS

In this paper we have investigated the structure of Reggeon cuts in a model which exhibits four-particle unitarity. We calculated the asymptotic behaviour of the perturbation theory contributions to the elastic amplitude $M(s,t)$ in the leading logarithm approximation, using momentum space techniques. This shows that the leading behaviour comes from generalized multiperipheral configurations of the internal momenta in which the big components (i.e., those in the beam direction) are strongly ordered. Our results coincide, of course, with those of McCoy and Wu ²⁾ who used Feynman parameter representations to calculate the asymptotic behaviour. We feel that our method of using momenta is physically more appealing particularly since it allows us to avoid dealing with the spuriously large asymptotic behaviour of individual Feynman diagrams.

The outcome of the calculation is a perturbation series for the partial wave amplitude which must be summed up by a suitable integral equation. Our integral equation is essentially identical to that of McCoy and Wu, but we investigate its structure more deeply. We are thus able to exhibit clearly the origin of the two-Reggeon, the Reggeon-particle and Reggeon-two-particle

cuts. Furthermore, we derive discontinuity formulae for the first two branch cuts and show that they are entirely analogous to the standard discontinuity formulae used as a basis for the Reggeon calculus.

In carrying out the above analysis, we defined various Reggeon scattering amplitudes including a two-Reggeon irreducible amplitude. Unfortunately as was discovered by McCoy and Wu ²⁾ the two-Reggeon branch point is not the right-most singularity. This irreducible amplitude is not real in the weak coupling limit (i.e., leading logarithm approximation). The model cannot therefore be used to comment on the sign of this important coupling. An incompleting link in the chain of arguments is the relationship of the amplitudes we have defined for coupling the two-Reggeon system either to itself or to external particles, to similar amplitudes defined by partial wave projections of suitable production amplitudes. We hope our model can be used to illuminate and extend previous analyses of this question ^{11),12)}.

By admittedly non-rigorous arguments, we were led to suggest the existence of a four-particle Regge pole which finally controls the asymptotic behaviour of $M(s,t)$. It would be interesting to know whether such a pole is always present and whether its origin, as our analysis suggests, is always associated with the existence of the three-particle Reggeon scattering on the remaining particle.

We did not find a convenient way of analyzing the Reggeon two-particle cut. This would be an interesting thing to do because the branch point is a model for a three-Reggeon singularity in which two of the Reggeons have zero slope.

A P P E N D I X

In this Appendix we evaluate the asymptotic behaviour of the quantity L defined in Eq. (4.14). We introduce the variables

$$\begin{aligned} u_i &= \ln \alpha_i s / \ln s \\ v_i &= \ln \beta_i s / \ln s \end{aligned} \quad (A.1)$$

We have then

$$\begin{aligned} L &= (\ln s)^{k+l+m+n+2} \int du_1 dv_3 du_5 dv_5 du_6 dv_6 \\ &\quad \times [u_5 + v_3 - 1]^{k-1} [u_6 + v_3 - 1]^{l-1} \\ &\quad \times [u_1 + v_6 - 1]^{m-1} [u_1 + v_5 - 1]^{n-1}. \end{aligned} \quad (A.2)$$

The integration region is fixed in part by requiring that the arguments of the logarithms in Eq. (4.14) become infinite as $s \rightarrow \infty$. That is u_i and v_i must cover

$$\begin{aligned} u_1 + v_5 - 1 &> 0 \\ v_3 + u_5 - 1 &> 0 \\ u_1 + v_6 - 1 &> 0 \\ u_6 + v_3 - 1 &> 0 \end{aligned} \quad (A.3)$$

Further conditions are that P_5, P_b, P_e, P_f become transverse. That is that $\alpha_5 \beta_5 s, \alpha_6 \beta_6 s, \alpha_5 \beta_6 s$ and $\alpha_6 \beta_5 s \rightarrow 0$ as $s \rightarrow \infty$. We have then

$$\begin{aligned} u_5 + v_5 &< 1 \\ u_6 + v_6 &< 1 \\ u_5 + v_6 &< 1 \\ u_6 + v_5 &< 1 \end{aligned} \quad (A.4)$$

Finally the restrictions on α_1 and β_3 imply that

$$\begin{aligned} 0 &< u_1 < 1 \\ 0 &< v_3 < 1 \end{aligned} \quad (A.5)$$

The integration volume is conveniently split into four pieces by the conditions

$$u_6 \geq u_5, \quad v_6 \geq v_5 \quad (A.6)$$

As illustration we consider $u_5 < u_6$ and $v_5 < v_6$. It follows that we need only the condition $u_5 + v_5 < 1$ from Eq. (A.4). The integral for L may now be easily evaluated by repeated integration. The result for the region under consideration is

$$\frac{1}{lm} \frac{(n+m-1)! (l+k-1)!}{(l+k+m+n+2)!} \quad (A.7)$$

If we add in the other three regions specified by the conditions (A.6), the result is

$$\frac{1}{klmn} \frac{(n+m)! (l+k)!}{(l+k+m+n+2)!} \quad (A.8)$$

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FIG.1

FIG.2

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FIG. 4

FIG. 5

FIG. 6

FIG.7

FIG.8

FIG.9

FIG. 10

FIG. 11

FIG. 12

FIG. 13