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SEMI-LEPTONIC DECAYS OF CHARMED BARYONS

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In our calculation of semi-leptonic decays of charmed baryons (CERN Preprint TH. 2142) we have not noticed that in the equations of Ref. 11) the magnetic form factors which we have used in the calculation of $\frac{1}{2}^+ \rightarrow \frac{1}{2}^+$ transitions have wrong ^{*}) normalization ($F_2^p = \mu_p^2$, $F_2^n = \mu_n^2$). Consequently the form factors $H_2^{(6)}$ and $H_2^{(3^*)}$ as given by (3.12) and (3.13) are incorrect and should be replaced by

$$H_2^{(6)} = F_2^p(s) + 2 F_2^n(s)$$

and

$$H_2^{(3^*)} = F_2^p(s)$$

We have repeated all our calculations using the form factors given above. ⁺⁾ It turns out that whereas the decays of the members of 3^* representation are essentially unaltered, the rates for the decays of the members of 6 representation into $\frac{1}{2}^+$ non-charmed states have been substantially decreased. As a result the decays of the multiplet 6 turn out to be dominated by $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+$ transitions, contrary to our previous result.

In the following we present our corrected numerical results, which replace Sections 5 and 6 as well as the figures of Ref. TH. 2142.

5. - NUMERICAL RESULTS

In this section we present and discuss the numerical results which we have obtained by utilizing the formulae established in the previous sections. As can be seen from these formulae the results depend on the masses of the decaying baryons as well as on the masses of vector mesons m_{D^*} and m_{F^*} which enter the form factors. Since these masses are not known we have investigated how the calculated quantities depend on their values. The branching ratios for all charmed baryons in question are presented in subsection 5.2. Their partial lifetimes are given in subsection 5.3, whereas the various $M(\text{Be}^+)$ distributions, $\langle M(\text{Be}^+) \rangle$ and $\langle \omega \rangle$ are the subject of subsection 5.4.

^{*}) Printing error.

⁺⁾ The values $F=0.45$ and $D=0.78$ have been used [see Eq. (3.14)].

Before we begin our presentation we want to discuss the following problem. It is clear that the results given here can only be of relevance if the charmed baryons are stable against strong and radiative decays. Therefore let us first discuss the situation in which weak decays are dominant.

5.1. - Stability Against Strong and Radiative Decays

First the charmed baryons may undergo strong decays into a non-charmed baryon and the meson D or F, unless the following conditions are satisfied

$$\begin{aligned}
 m_{C_0} &< m_N + m_D, \\
 m_{C_1} &< m_N + m_D, \\
 m_A &< \min(m_\Lambda + m_D, m_N + m_F), \\
 m_S &< \min(m_\Lambda + m_D, m_N + m_F), \\
 m_T &< \min(m_\Xi + m_D, m_\Lambda + m_F).
 \end{aligned} \tag{5.1}$$

Choosing, for instance, $m_D = 1.9$ GeV and $m_F = 1.95$ GeV, we obtain

$$\begin{aligned}
 m_{C_0}, m_{C_1} &\lesssim 2.84 \text{ GeV}, \\
 m_A, m_S &\lesssim 2.9 \text{ GeV}, \\
 m_T &\lesssim 3.06 \text{ GeV}.
 \end{aligned}$$

Even if conditions (5.1) are satisfied, charmed baryons may undergo strong decays into other charmed baryons unless the following inequalities hold

$$\begin{aligned}
 |m_{C_1} - m_{C_0}| &< m_\pi, \\
 |m_{C_1} - m_A| &< m_K, \\
 |m_T - m_A| &< m_K, \\
 |m_S - m_A| &< m_\pi, \\
 |m_{C_0} - m_S| &< m_K.
 \end{aligned} \tag{5.2}$$

There can also be radiative decays of C_1^+ , S^+ and S^0 into C_0^+ , A^+ and A^0 , respectively, or vice versa, depending on the mass assignment. These decays being much stronger than the weak ones will be dominant if conditions (5.1) and (5.2) are satisfied. Thus there will be always at least three baryons which will dominantly undergo other than weak decays.

Finally there is a question of weak non-leptonic transitions. Although one expects them to be stronger than the semi-leptonic ones by an order of magnitude ¹¹⁾, they are probably much more difficult to detect than the transitions discussed in this paper.

In the calculations below we have varied the masses of the baryons $C_1^{++,+0}$ and C_0^+ in the range from 2.0 GeV to 2.8 GeV, the masses of $S^{+,0}$ and $A^{+,0}$ in the range from 2.0 GeV to 2.9 GeV and the mass of T^0 in the range from 2.0 GeV to 3 GeV. If the mass of the D or the F meson is not smaller than 1.9 GeV this choice of masses satisfies conditions (5.1). Since it may not satisfy all conditions (5.2) and because of the presence of the radiative decays as discussed above, we should keep in mind while analyzing our results that some of the charmed baryons may dominantly undergo non-weak transitions.

5.2. - Branching Ratios

The branching ratios for the semi-leptonic decays of charmed baryons into $\frac{1}{2}^+$ and $\frac{3}{2}^+$ non-charmed baryons can be estimated by means of the formulae (3.22), (3.23) and (4.11), (4.12), respectively. We first discuss the dependence of Γ on the dipole parameters m_{D^*} and m_{F^*} . These parameters control the enhancement of the Γ due to form factors. With the increase of m_{D^*} or m_{F^*} the enhancement is decreased. Clearly the effect depends on Δ . The larger Δ , the stronger the effect ^{*)}. We have found that this combined dependence of Γ on the dipole parameter m_{a^*} and Δ can be very well described for $2 \leq m_{a^*} \leq 2.3$ and $0.8 \leq \Delta \leq 1.8$ by the formula

) Notice that if conditions (5.1) are satisfied and $m_{D^}, m_{F^*} > m_D$ as we assume, then the pole of the form factor is never reached.

$$\Gamma(\Delta, \Sigma, M_1, m_{a*}) = K(\Delta, m_{a*}) L(\Delta, \Sigma, M_1) \quad (5.3)$$

with $K(\Delta, m_{a*})$ given by

$$K(\Delta, m_{a*}) = \left(1 - \frac{\Delta^2}{m_{a*}^2}\right)^{-1.8} \quad (5.4)$$

and $L()$ independent of m_{a*} . In Fig. 1 we have plotted $K(\Delta, m_{a*})$ as function of m_{a*} for various values of Δ . For illustration purposes $K(\Delta, m_{a*})$ has been renormalized so that $K(\Delta, 2.15) = 1$. All the effects discussed above are clearly seen. We observe that whereas for $\Delta \leq 1.2$ GeV Γ is practically independent of m_{a*} , for $\Delta > 1.6$ GeV it changes by a factor of 3 as m_{a*} is varied through the range considered.

It turns out that the branching ratios are practically independent of m_{a*} and that the partial lifetimes depend much more strongly on the mass of the decaying baryon than on the dipole parameter. Therefore for simplicity we present our results for fixed intermediate values of m_{D*} and m_{F*} , $m_{D*} = 2.15$ GeV and $m_{F*} = 2.2$ GeV.

We next turn to the presentation of various branching ratios. These have been collected in Table III. Since the branching ratios depend only weakly on the masses of the decaying baryons we present the results for the intermediate value of M_1 , $M_1 = 2.5$ GeV. Only electron modes have been given since the muonic ones are smaller by less than 2%. This independence of the branching ratios on the lepton mass is a direct consequence of the high values of Δ ^{*)}. We recall that this is not the case of the usual decays of $\frac{1}{2}^+$ non-charmed baryons.

The main conclusion from Table III is that whereas the members of the multiplet $\underline{3}^*$, C_0^+ , $A_0^{+,-}$ dominantly decay into $\frac{1}{2}^+$ non-charmed baryons, the decays of the members of the $\underline{6}$ multiplet are dominated by the $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+$ transitions. This result can be understood as follows. The members of the multiplet $\underline{3}^*$ cannot decay into $\frac{3}{2}^+$ non-charmed baryons which belong to the multiplet $\underline{10}$, since the decomposition of $\underline{3}^* \otimes \underline{3}$

*) In our calculations the dependence on the lepton mass comes only through the phase space, since the induced pseudoscalar term which gives a contribution proportional to the lepton mass has been neglected.

into irreducible representations obviously does not contain 10. Thus from two possibilities, $\frac{3}{2}^+$ and $\frac{1}{2}^+$, only the latter remains for these baryons. On the other hand the dominance of $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+$ transitions for the multiplet 6 may reflect the number of final spin states. The dominance of $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+$ transitions is particularly pronounced in the case of the baryon T^0 , since there is no $\frac{1}{2}^+$ state with three s quarks, to which it could go through F transition ($\sim \cos^2 \theta$). On the other hand in the case of baryons $C_1^{+, ++, 0}$ the dominance of $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+$ transition is not clearly seen. In fact for these baryons the modes $C_1^{+, ++, 0} \rightarrow \Sigma^{+, 0, -}$ and $C_1^{+, ++, 0} \rightarrow \Sigma^{*, 0, -}$ are comparable. This is due to a larger Clebsch-Gordan coefficient (factor 6) for the former transitions relative to the latter ones as can be seen in Table II.

We next notice that the decays of C_0^+ , $C_1^{+, ++, 0}$, $A^{+, 0}$, $S^{+, 0}$ and T^0 are characterized by the dominant decays into Λ^0 , $\Sigma^{+, 0, -}$ (or $\Sigma^{*, 0, -}$), $\Xi^{0, -}$, $\Xi^{*, 0, -}$ and Ω^- , respectively. In the case of discovery of a charmed baryon this observation could be helpful in the identification of the baryon in question. In particular, recalling our discussion on radiative decays in Section 5.1, observation of Λ^0 would indicate the inequality $m_{C_0^+} < m_{C_1^+}$, whereas appearance of Σ^0 would signal $m_{C_1^+} < m_{C_0^+}$. Similar arguments can be applied to the baryons $S^{0, +}$ and $A^{0, +}$.

We finally notice that since in the decay modes discussed D^- transitions correspond to larger phase space as compared to F^- transitions and consequently also to stronger enhancement through form factors the Cabibbo suppression is not as clearly seen as one would expect from the ratio $\sin^2 \theta / \cos^2 \theta \approx 0.06$. We have, for instance,

$$\frac{\Gamma(C_0^+ \xrightarrow{D^-} N^0 e^+ \nu)}{\Gamma(C_0^+ \xrightarrow{F^-} \Lambda^0 e^+ \nu)} \approx \frac{1}{4},$$

$$\frac{\Gamma(C_1^{++} \xrightarrow{D^-} N^+ e^+ \nu)}{\Gamma(C_1^{++} \xrightarrow{F^-} \Sigma^+ e^+ \nu)} \approx \frac{1}{4},$$

and

$$\frac{\Gamma(C_1^+ \xrightarrow{D^-} \Delta^0 e^+ \nu)}{\Gamma(C_1^+ \xrightarrow{F^-} \Sigma^{*0} e^+ \nu)} \approx \frac{1}{3}.$$

5.3. - The partial Lifetimes *)

In Fig. 2a we have plotted the partial lifetimes of the baryons $C_1^{++,+0}$ and C_0^+ as functions of their masses for the intermediate values of m_{D^*} and m_{F^*} , $m_{D^*} = 2.15$ GeV and $m_{F^*} = 2.20$ GeV. The partial lifetimes of $S^{0,+}$, $A^{0,+}$ and T^0 as functions of their masses are shown in Fig. 2b. We observe a very strong dependence of the lifetimes in question on the masses of the decaying baryons. Generally the partial lifetimes change from 10^{-10} sec to 10^{-14} sec as the baryon masses are varied from 2 to 2.8 GeV. Consequently we expect partial lifetimes to be much more subject to SU(4) breaking than the branching ratios, where very weak dependence on the masses of the decaying baryons has been observed.

Needless to say, the above estimates can only be at best the upper limits for the lifetimes of the baryons in question. They are probably strongly affected by the weak non-leptonic transitions and are meaningless if radiative and strong decays are present.

5.4. - The $M(Be^+)$ Distributions, $\langle \omega \rangle$ and $\langle M(B^+ \ell) \rangle$

The $(1/\Gamma)(d\Gamma/dM(Be^+))$ distributions can be estimated by means of formulae (3.26)-(3.28). Only the distributions for the important $\frac{1}{2}^+ \rightarrow \frac{1}{2}^+$ transitions are presented here, since the calculation of the corresponding distributions for $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+$ transitions is less reliable. In Fig. 3 we have plotted $(1/\Gamma)(d\Gamma/dM(Be^+))$ for the decays $C_0^+ \rightarrow \Lambda^0 e^+ \nu$ and $C_1^{++} \rightarrow \Sigma^+ e^+ \nu$ for $M_{C_0} = M_{C_1} = 2.5$ GeV and $m_{F^*} = 2.2$ GeV. These are typical mass distributions for the members of the $\underline{3}^*$ and $\underline{6}$ representations, respectively. We observe that the mass distributions for the decays in question are similar in shape, but the distribution for the decay of C_1^{++} is shifted towards higher values of $M()$. As the mass of the decaying baryon increases the maximum of the mass distributions shifts to higher values of $M(Be^+)$. Since the distributions are roughly symmetric the dependence of the position of the maximum on the baryon mass can be well represented by plotting $\langle M(Be^+) \rangle$ versus M_1 . This has been done in Fig. 4, where also the decay $A^0 \rightarrow \Xi^- e^+ \nu$ has been

*) We recall that only semi-leptonic transitions into $\frac{1}{2}^+$ and $\frac{3}{2}^+$ non-charmed states have been taken into account.

included. We observe that the dependence of $\langle M(\text{Be}^+) \rangle$ on M_1 is almost linear in the range of masses discussed. The difference in slopes for the $\underline{3}^*$ and $\underline{6}$ representations is a direct consequence of different form factors corresponding to these representations as can be seen in Eqs. (3.12) and (3.13).

Ending this section we would like to point out that the plots of Fig 4 can be helpful in the determination of the mass of the decaying baryon, once the value of $\langle M(\text{Be}^+) \rangle$ is known. In this respect also the knowledge of $\langle \omega \rangle$ could be of help. The relation between $\langle \omega \rangle$ and the mass of the decaying baryon as given by our approach has been presented in Fig. 5.

6. - SUMMARY

In this paper we have presented estimates of the semi-leptonic decays of the charmed $\frac{1}{2}^+$ baryons into ordinary $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons. The form factors and general formulae used in our calculations are collected in Sections 3 and 4. The relevant parameters entering these formulae are to be found in Table II. Our main results, details of which occupy Section 5, are as follows.

- i) The baryons C_0^+ , $C_1^{++,+,-}$, $A^{+,0}$, $S^{+,0}$ and T^0 decay dominantly into Λ^0 , $\Sigma^{*+,0,-}$ (or $\Sigma^{+,0,-}$), $\Xi^{0,-}$, $\Xi^{*0,-}$ and Ω^- , respectively. All branching ratios for $\frac{1}{2}^+ \rightarrow \frac{1}{2}^+$ and $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+$ transitions are to be found in Table III.
- ii) The partial lifetimes depend very strongly on the masses of the decaying baryons and are $O(10^{-10} - 10^{-14})\text{sec}$ as the masses in question are varied from 2 to 2.9 GeV.
- iii) It is pointed out that the measurements of $\langle M(\text{Be}^+) \rangle$ or $\langle \omega \rangle$ may give an idea about the masses of the decaying charmed baryons.
- iv) The Cabibbo suppression of D^- transitions relative to F^- ones is in many cases not clearly seen due to phase space effects and enhancement through form factors both being stronger for the former transitions.

We have also discussed the conditions under which the semi-leptonic decays are dominant. At best only six baryons out of nine discussed in this paper can be stable against strong and radiative decays. There will be always three baryons which will be dominated by other than weak decays. This fact as discussed in Section 5.2 may give an idea about the relative masses of some baryons.

We hope that the results of this paper will be helpful to both experimentalists and theorists in their search for charmed particles.

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Decaying baryon	Baryon in the final state	Branching ratios %	Partial lifetimes [sec]
C_0^+	Λ^0 N^0	80 20	$5 \cdot 10^{-12} \rightarrow 3 \cdot 10^{-14}$
C_1^{++}	Σ^+ N^+ Σ^{*+} Δ^+	33 8 47 12	$8 \cdot 10^{-12} \rightarrow 4 \cdot 10^{-14}$
C_1^+	Σ^0 N^0 Σ^{*0} Δ^0	31 4 48 17	$8 \cdot 10^{-12} \rightarrow 4 \cdot 10^{-14}$
C_1^0	Σ^- Σ^{*-} Δ^-	32 47 21	$8 \cdot 10^{-12} \rightarrow 4 \cdot 10^{-14}$
A^+	Λ^0 Σ^0 Ξ^0	3 6 91	$1 \cdot 10^{-11} \rightarrow 3 \cdot 10^{-14}$
A^0	Σ^- Ξ^-	11 89	$1 \cdot 10^{-11} \rightarrow 3 \cdot 10^{-14}$
S^+	Ξ^0 Σ^0 Λ^0 Ξ^{*0} Σ^{*0}	15 1 5 71 8	$2 \cdot 10^{-11} \rightarrow 5 \cdot 10^{-14}$
S^0	Ξ^- Σ^- Ξ^{*-} Σ^{*-}	16 2 70 12	$2 \cdot 10^{-11} \rightarrow 5 \cdot 10^{-14}$
T^0	Ξ^- Ω^- Ξ^{*-}	5 91 4	$1 \cdot 10^{-13} \rightarrow 1 \cdot 10^{-10}$

TABLE III : Branching ratios (%) and partial lifetimes (for $2 \leq M_1 \leq 2.8$) for the semi-leptonic decays of the charmed $\frac{1}{2}^+$ baryons into $\frac{1}{2}^+$ and $\frac{3}{2}^+$ non-charmed states.

FIGURE CAPTIONS

Figure 1 The dependence of $K(\Delta, m_{a^*})$ [see, e.g., (5.4)] on the dipole parameter m_{a^*} for $\Delta = 0.8, 1, 1.2, 1.4, 1.6$ and 1.8 GeV.

Figure 2 The partial lifetimes τ of the charmed baryons as functions of their masses (M_1) ;
a) the partial lifetimes of $C_1^{++},^0$ and C_0^+ ;
b) the partial lifetimes of $A^{+,0}, S^{+,0}$ and T^0 .

Figure 3 The normalized mass distributions $1/\Gamma(d\Gamma/dM(Be^+))$ at fixed $M_1 = 2.5$ GeV and $m_{P^*} = 2.2$ GeV for the reactions
a) $C_1^{++} \rightarrow \Sigma^+ e^+ \nu$ and $C_0^+ \rightarrow \Lambda^0 e^+ \nu$.

Figure 4 The dependence of $\langle M(Be^+) \rangle$ on the mass M_1 for the reactions $C_0^+ \rightarrow \Lambda^0 e^+ \nu$, $A^0 \rightarrow \Xi^- e^+ \nu$ and $C_1^{++} \rightarrow \Sigma^+ e^+ \nu$.

Figure 5 The dependence of $\langle w \rangle$ on the mass M_1 for the reactions $C_0^+ \rightarrow \Lambda^0 e^+ \nu$, $A^0 \rightarrow \Xi^- e^+ \nu$ and $C_1^{++} \rightarrow \Sigma^+ e^+ \nu$.

FIG.1

FIG.2 a.

FIG. 2b.

FIG. 3

FIG. 4

FIG. 5

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SEMI-LEPTONIC DECAYS OF CHARMED BARYONS

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A B S T R A C T

We present estimates of semi-leptonic decays of the charmed $\frac{1}{2}^+$ baryons, members of the $SU(3)$ 3^* and 6 multiplets, into ordinary $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons. We give general expressions for decay rates and the mass $M(B\ell^+)$ spectra. We relate the relevant form factors for the $\frac{1}{2}^+ \rightarrow \frac{1}{2}^+$ transitions to the electromagnetic ones by means of CVC and $SU(4)$. The form factors for $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+$ transitions are related through $SU(4)$ to those measured in Δ production. We discuss the dependence of the results on the masses of charmed baryons. The $\frac{1}{2}^+ \rightarrow \frac{1}{2}^+$ transitions turn out to be dominant for all baryons but T^0 . The partial lifetimes are $O(10^{-10} - 10^{-14})$ sec as the masses of the decaying baryons are varied from 2 to 2.9 GeV. The mass spectra turn out to be useful in distinguishing between 3^* and 6 multiplets. Possible ways of getting an idea about relative masses of some of the charmed baryons are given.

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1. - INTRODUCTION

One of the simplest possible explanations of the existence of the narrow resonances $J/\psi(3.1)$ and $\psi'(3.7)$ is charm^{1),2)}. Introduced on theoretical grounds in order to suppress the neutral strangeness changing currents in gauge theories of weak interactions³⁾, charm evolved in the last year to something maybe closely related to reality. This is due to such experimental findings (in addition to those mentioned above) as the observation of an anomaly in γ distribution in antineutrino deep inelastic scattering⁴⁾, dilepton events seen in neutrino and antineutrino reactions^{5),6)} and an apparent $\Delta S = -\Delta Q$ event observed in the Brookhaven bubble chamber experiment. All seem to support the charm picture. Clearly in order to prove existence of charm, the discovery of a particle carrying this new quantum number is needed. In this context estimates of cross-sections for charm production and of decay rates of charmed particles could be helpful. There exist already in the literature theoretical estimates of cross-sections for charm production in deep inelastic neutrino reactions⁷⁾, e^+e^- annihilation⁸⁾, diffractive elastic neutrino reactions⁹⁾ and quasi-elastic reactions induced by neutrinos^{10),11)}. Also estimates of strong and non-leptonic weak decays of both charmed mesons and baryons have been made¹²⁾. Calculations of semi-leptonic decays of charmed particles concentrated on mesons¹⁾. On the other hand, to our best knowledge, no such estimates exist for baryons. Although $SU(4)$ mass formulae^{1),13)} seem to indicate that most of the charmed baryons will predominantly undergo charm conserving strong decays, it is quite possible that at least the lowest lying charmed baryons may be stable against strong decays. In such a case estimates of semi-leptonic decays of the baryons in question will be of great relevance.

In this paper we present calculations of semi-leptonic decay rates of the $\frac{1}{2}^+$ baryon with charm $C=1$ into non-charmed baryons with spin $\frac{1}{2}^+$ and $\frac{3}{2}^+$. We have calculated neither $(C=2) \rightarrow (C=1)$ nor charm conserving $(C=1) \rightarrow (C=1)$ transitions. The former, although possibly comparable in magnitude to those calculated in this paper, are of academic nature at present. The latter having much smaller phase space are much smaller than those discussed by us.

The paper is organized as follows. In Section 2 we recall for the convenience of readers $SU(4)$ properties of the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons and of the charm changing currents involved. We calculate the relevant $SU(4)$ Clebsch-Gordan coefficients and list various relations between decay rates arising from $SU(2)$.

In Section 3 we present general expressions for Γ , $d\Gamma/dM_{(B\ell)}$ and $d\Gamma/dE_\gamma$ for the $\frac{1}{2}^+ \rightarrow \frac{1}{2}^+$ case. We relate the relevant form factors to the electromagnetic ones.

In Section 4 we derive general expression for Γ for the $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+$ case. We relate the relevant form factors to those measured in N^* production using $SU(4)$ ^{*)}. Section 5 contains numerical results based on formulae of Sections 3 and 4. We finally summarize and discuss our results in Section 6. Some of the formulae and their derivations have been collected in Appendices A and B.

2. - SYMMETRY PROPERTIES OF $\frac{1}{2}^+$, $\frac{3}{2}^+$ BARYONS AND CHARM CHANGING CURRENTS. RELATIONS BETWEEN DECAY RATES

2.1. - Baryons and Currents

The $\frac{1}{2}^+$ baryons form an irreducible representation 20 of $SU(4)$ ¹⁴⁾. The $SU(3)$ multiplets are 8 ($C=0$), $\underline{6} + \underline{3}^*$ ($C=1$) and $\underline{3}$ ($C=2$). Quantum numbers of charmed baryons with $C=1$, as well as their quark content are collected in Table I.

The $\frac{3}{2}^+$ decuplet of the non-charmed baryons belongs to the representation 20' of $SU(4)$.

The two hadronic charged currents which we use in our calculations are given in terms of quark fields as follows ^{**) (2.1)}

$$J_{F^-}^\mu = \bar{s} \gamma^\mu (1 - \gamma_5) c \cos \theta_c, \quad (2.1)$$

$$J_{D^-}^\mu = -\bar{d} \gamma^\mu (1 - \gamma_5) c \sin \theta_c. \quad (2.2)$$

^{*)} The reader interested only in the results may skip Sections 3 and 4.

^{**) $J_{F^-}^\mu + J_{D^-}^\mu$ is just the charm changing part of the GIM current ³⁾.}

They have the same $SU(4)$ transformation properties as mesons F^- and D^- , respectively. These mesons belong to the representation 15 of $SU(4)$ and are members of the multiplet 3 of $SU(3)$. We recall that $J_{F^-}^\mu$ describes decays characterized by

$$\Delta S = \Delta Q = \Delta C = -1, \quad \Delta I = 0 \quad (2.3)$$

while $J_{D^-}^\mu$ corresponds to transitions

$$\Delta S = 0, \quad \Delta Q = \Delta C = -1, \quad \Delta I = \frac{1}{2}. \quad (2.4)$$

2.2. - Decay Modes of Charmed Spin $\frac{1}{2}^+$ Baryons

Having established symmetry properties and quantum numbers of baryons and currents in question, it is straightforward to find all semi-leptonic decay modes of $\frac{1}{2}^+$ baryons into $\frac{1}{2}^+$ and $\frac{3}{2}^+$ states. We list them in Table II together with relevant parameters which are defined below.

We write (suppressing Lorentz indices) the matrix elements of the currents (2.1) and (2.2) in terms of the reduced matrix elements as follows *)

$$\langle \frac{1}{2}^+ | J_a | \frac{1}{2}^+, \alpha \rangle = A(\alpha) \left\{ K_4^{(\alpha)} \langle 20_1 \| 15 \| 20 \rangle + \bar{K}_4^{(\alpha)} \langle 20_2 \| 15 \| 20 \rangle \right\}, \quad (2.5)$$

$$\langle \frac{3}{2}^+ | J_a | \frac{1}{2}^+, \alpha \rangle = B(\alpha, \alpha) \langle 20' \| 15 \| 20 \rangle. \quad (2.6)$$

Here a stands for F^- or D^- and α denotes the $SU(3)$ multiplet to which the decaying baryon belongs. Since the 20 representation of $SU(4)$ occurs twice in the decomposition 15 $\otimes$ 20 into irreducible representations, there are two reduced matrix elements in the $\frac{1}{2}^+ \rightarrow \frac{1}{2}^+$ case. We denote the corresponding $SU(3)$ singlet factors by $K_4^{(\alpha)}$ and $\bar{K}_4^{(\alpha)}$,

*) $SU(2)$ indices and the Cabibbo factor have been suppressed.

respectively. $A(a)$ stands for the product of the $SU(2)$ Clebsch-Gordan coefficient and the isoscalar factor, whereas $B(a,\alpha)$ denotes the full $SU(4)$ Clebsch-Gordan coefficient.

The values of $A(a)$ and $B(a,\alpha)$ listed in Table II have been found using Ref. 15). The relevant $K_4^{(\alpha)}$ and $\bar{K}_4^{(\alpha)}$ are as follows

$$K_4^{(6)} = \left(\begin{array}{cc|c} 20 & 15 & 20_1 \\ 6 & 3 & 8 \end{array} \right) = -\frac{17}{4\sqrt{38}} \quad , \quad \bar{K}_4^{(6)} = \left(\begin{array}{cc|c} 20 & 15 & 20_2 \\ 6 & 3 & 8 \end{array} \right) = -\sqrt{\frac{2}{13}} \quad (2.7)$$

for the decay of the multiplet 6 and

$$K_4^{(3^*)} = \left(\begin{array}{cc|c} 20 & 15 & 20_1 \\ 3^* & 3 & 8 \end{array} \right) = \frac{3\sqrt{3}}{4\sqrt{26}} \quad , \quad \bar{K}_4^{(3^*)} = \left(\begin{array}{cc|c} 20 & 15 & 20_2 \\ 3^* & 3 & 8 \end{array} \right) = -\sqrt{\frac{2}{13}} \quad (2.8)$$

for the decay of the multiplet 3*.

We finally notice an important fact : the baryons $A^{0,+}$, C_0^+ belonging to the 3* representation cannot decay into $\frac{3}{2}^+$ states.

2.3. - $SU(2)$ Relations for the Widths

Assuming $SU(2)$ symmetry to be exact we obtain the following relations

i) for F^- transitions

$$\Gamma(C_1^{++} \rightarrow \Sigma^+(\Sigma^{+*})) = \Gamma(C_1^+ \rightarrow \Sigma^0(\Sigma^{0*})) = \Gamma(C_1^0 \rightarrow \Sigma^-(\Sigma^{-*})) ,$$

$$\Gamma(S^+ \rightarrow \Xi^0(\Xi^{0*})) = \Gamma(S^0 \rightarrow \Xi^-(\Xi^{-*})) , \quad (2.9)$$

$$\Gamma(A^+ \rightarrow \Xi^0) = \Gamma(A^0 \rightarrow \Xi^-)$$

ii) for D^- transitions

$$\begin{aligned}
 \Gamma(C_1^{++} \rightarrow N^+) &= 2 \Gamma(C_1^+ \rightarrow N^0), \\
 \Gamma(C_1^+ \rightarrow \Delta^0) &= 2 \Gamma(C_1^{++} \rightarrow \Delta^+), \\
 \Gamma(C_1^0 \rightarrow \Delta^-) &= 3 \Gamma(C_1^{++} \rightarrow \Delta^+), \\
 \Gamma(H^0 \rightarrow \Sigma^-) &= 2 \Gamma(H^+ \rightarrow \Sigma^0), \\
 \Gamma(S^0 \rightarrow \Sigma^-(\Sigma^{*-})) &= 2 \Gamma(S^+ \rightarrow \Sigma^0(\Sigma^{*0})).
 \end{aligned} \tag{2.10}$$

Thus from 30 decay modes only 17 are to be calculated. The remaining can be found from Eqs. (2.9) and (2.10).

3. - $\frac{1}{2}^+ \rightarrow \frac{1}{2}^+$ TRANSITIONS

3.1. - Notations and Kinematics

Let a particle Z with mass M , four-momentum p and energy E be characterized by the symbol $Z(M, p, E)$. Then our notation can be summarized ^{*)} by the following equation

$$\begin{aligned}
 B_1(M_1, p_1, M_1) &\rightarrow B_2(M_2, p_2, E) + L^+(m, k_1, \omega) \\
 &+ \nu(0, k_2, E_\nu).
 \end{aligned} \tag{3.1}$$

The invariants of interest for us are

$$s = q^2 = (p_2 - p_1)^2 = (k_1 + k_2)^2 \tag{3.2}$$

and

$$W = (p_2 + k_1)^2 \equiv M^2(BL^+). \tag{3.3}$$

^{*)} We work in the rest frame of the decaying particle. Our metric in this section is that of Feynman.

Their range of variation is given ¹⁶⁾ by

$$s_- \leq s \leq s_+, \quad (m + M_2)^2 \leq W \leq M_1^2 \quad (3.4)$$

with

$$s_{\pm} = m^2 - \frac{1}{2W} \left\{ (W - M_1^2)(W + m^2 - M_2^2) \mp \lambda^{\frac{1}{2}}(W, M_1^2, 0) \lambda^{\frac{1}{2}}(W, m^2, M_2^2) \right\}. \quad (3.5)$$

Here $\lambda()$ are the usual triangle functions.

Finally, we introduce the variables

$$\Sigma = M_1 + M_2 \quad \text{and} \quad \Delta = M_1 - M_2. \quad (3.6)$$

3.2. - Form Factors

The invariant amplitudes for the decays (3.1) are given by

$$M = \frac{G}{\sqrt{2}} c \bar{v}_\nu \gamma_\lambda (1 - \gamma_5) v_L \langle \frac{1}{2}^+ | J_a^\lambda | \frac{1}{2}^+, \alpha \rangle \quad (3.7)$$

where $G = 1.02 \times 10^{-5}/m_N^2$ and c is the Cabibbo factor, $-\sin\theta$ and $\cos\theta$ for D^- and F^- transitions, respectively. $\langle \frac{1}{2}^+ | J_a^\lambda | \frac{1}{2}^+, \alpha \rangle$ is defined by Eqs. (2.5).

We want to express the matrix elements $\langle \frac{1}{2}^+ | J_a^\lambda | \frac{1}{2}^+, \alpha \rangle$ through the electromagnetic form factors. To this end we follow Ref. 11). We first notice the $SU(4)$ relations

$$\langle 20_1 || 15 || 20 \rangle^\lambda = \frac{8}{\sqrt{13}} \mathcal{D}^\lambda \quad (3.8a)$$

and

$$\langle 20_2 || 15 || 20 \rangle^\lambda = \frac{5\sqrt{3}}{6\sqrt{13}} \mathcal{D}^\lambda - \frac{\sqrt{39}}{2} F^\lambda \quad (3.8b)$$

where F^λ and D^λ are the reduced matrix elements of the Cabibbo current between $C=0$ $SU(3)$ octet states.

By inserting (3.8) into (2.5) and utilizing (2.7) and (2.8) we obtain

$$\langle \frac{1}{2}^+ | J_a^\lambda | \frac{1}{2}^+, 6 \rangle = A(\alpha) \sqrt{\frac{3}{2}} (F - D)^\lambda \quad (3.9)$$

and

$$\langle \frac{1}{2}^+ | J_a^\lambda | \frac{1}{2}^+, 3^* \rangle = A(\alpha) \sqrt{\frac{3}{2}} (F + \frac{D}{3})^\lambda. \quad (3.10)$$

Since F_λ and D_λ can be expressed through the electromagnetic form factors and the usual parameters F and D [see Ref. 11) for details] we finally obtain ^{*)}

$$\langle \frac{1}{2}^+ | J_a^\lambda | \frac{1}{2}^+, \alpha \rangle = A(\alpha) \sqrt{\frac{3}{2}} \bar{u}(p_2) \left[H_1^{(\alpha)} \gamma^\lambda + H_2^{(\alpha)} \frac{i \sigma^{\lambda r}}{\Sigma} q_r + H_3^{(\alpha)} \gamma^\lambda \gamma_5 \right] u(p_1) \quad (3.11)$$

with

$$\begin{aligned} H_1^{(6)} &= F_1^P(s) + 2 F_1^n(s), & H_2^{(6)} &= \mu_p F_2^P(s) + 2 \mu_n F_2^n(s), \\ H_3^{(6)} &= (D - F) F_A(s) \end{aligned} \quad (3.12)$$

and

$$\begin{aligned} H_1^{(3^*)} &= F_1^P(s), & H_2^{(3^*)} &= \mu_p F_2^P(s), \\ H_3^{(3^*)} &= - (F + \frac{D}{3}) F_A(s). \end{aligned} \quad (3.13)$$

Here

$$\begin{aligned} F &= 0.78, & D &= 0.45 \\ \mu_p &= 1.79, & \mu_n &= -1.91. \end{aligned} \quad (3.14)$$

^{*)} We neglect second class currents, the $i \sigma^{\mu\nu} q_\nu \gamma_5$ term and the induced pseudoscalar term $q^\mu \gamma_5$.

The various form factors are parametrized as follows

$$\begin{aligned}
 F_1^P(s) &= G_E^P(s) \frac{(\Sigma^2 - s - s\mu_p)}{\Sigma^2 - s} \\
 F_2^P(s) &= G_E^P(s) \frac{\Sigma^2 \mu_p}{\Sigma^2 - s} \\
 F_1^n(s) &= -G_E^P(s) \frac{s\mu_n}{\Sigma^2 - s} \\
 F_2^n(s) &= G_E^P(s) \frac{\Sigma^2 \mu_n}{\Sigma^2 - s}
 \end{aligned} \tag{3.15}$$

with

$$G_E^P(s) = F_A(s) = \left(1 - \frac{s}{m_{a^*}^2}\right)^{-2} \tag{3.16}$$

m_{a^*} stands for m_{D^*} or m_{F^*} . This is the parametrization used in Ref. 11). Notice that in writing (3.16) the dominance of both vector and axial vector part of J_a^λ by the charmed mesons a^* has been assumed.

3.3. - Decay Rate

The rate for the process (3.1) is

$$\Gamma = \frac{1}{(2\pi)^5} \frac{1}{2M_1} \int d\phi_3 \frac{1}{2} \sum_{\text{spins}} |M|^2 \tag{3.17}$$

where ϕ_3 is the three-particle phase space and

$$\sum_{\text{spins}} |M|^2 = \frac{G^2}{2} \mathcal{L}^{\mu\nu} B_{\mu\nu}. \tag{3.18}$$

Here $\mathcal{L}^{\mu\nu}$ is the leptonic trace given by

$$\mathcal{L}^{\mu\nu} = 8 \left\{ k_2^\mu k_1^\nu + k_1^\mu k_2^\nu - (k_1 \cdot k_2) g^{\mu\nu} + i k_{2s} k_{1\tau} \epsilon^{\delta\tau\mu\nu} \right\} \tag{3.19}$$

The baryon trace $B_{\mu\nu}$ can be calculated using standard techniques. The result is given in Appendix A.

In order to evaluate (3.17) we first integrate over lepton momenta. This gives ¹⁷⁾

$$\int \frac{d^3 k_1}{2\omega} \frac{d^3 k_2}{2E_\nu} \mathcal{L}^{\mu\nu} = \frac{2\pi}{3} \frac{(\tau_4^-)^2}{s^2} \left\{ 2\tau_5 \frac{q^\mu q^\nu}{s} - \tau_6 q^{\mu\nu} \right\} \quad (3.20)$$

with $r_\perp$ given by (A.3).

We next use the formula

$$\int \frac{d^3 p_2}{2E} f(q^2) = \frac{\pi}{2M_1} \int_{m^2}^{\Delta^2} ds [\tau_1 \tau_2]^{1/2} f(s) \quad (3.21)$$

to obtain, from (3.17), (3.18), (3.20) and (A.6), the final result

$$\Gamma = \int_{m^2}^{\Delta^2} ds N(s) \quad (3.22)$$

where

$$\begin{aligned} N(s) = & \frac{1}{256} \frac{G^2 c^2 H^2}{\pi^3 M_1^3} [\tau_1 \tau_2]^{1/2} \frac{(\tau_4^-)^2}{s^2} \\ & \times \left\{ H_1^2 \left[\Delta^2 (4s - m^2) + 2 \Sigma^2 \Delta^2 \frac{\tau_5}{s} - (\Sigma^2 + 2s) \tau_6 \right] + \right. \\ & H_3^2 \left[\Sigma^2 (4s - m^2) + 2 \Sigma^2 \Delta^2 \frac{\tau_5}{s} - (\Delta^2 + 2s) \tau_6 \right] + \\ & \left. H_2^2 (2\Sigma^2 + s) \frac{\tau_1 \tau_6}{\Sigma^2} + 6 H_1 H_2 \tau_1 \tau_6 \right\} \quad (3.23) \end{aligned}$$

This expression agrees with that given in Ref. 17). Thus we can calculate all decay rates of the type $\frac{1}{2}^+ \rightarrow \frac{1}{2}^+$ by means of the formulae (3.12)-(3.16) and (3.22), (3.23). The parameters A and c are given in Table II.

3.4. - $M(B\ell^+)$ Distributions

The $M(B\ell^+)$ distribution is given by

$$\frac{d\Gamma}{dM(B\ell^+)} = \frac{M(B\ell^+)}{(2\pi)^5} \frac{\pi^2}{4M_1^3} \int_{s_-}^{s_+} ds \frac{1}{2} \sum_{\text{spins}} |M|^2 \quad (3.24)$$

with $s_{\pm}$ defined by (3.5).

It is convenient to introduce functions $R_i^{\pm}(s, W)$ defined as follows

$$\begin{aligned} R_1^{\pm}(s, W) &= \left\{ (W - M_2^2 - m^2)(M_1^2 - W) \pm (s + W - M_2^2)(M_1^2 + m^2 - s - W) \right\}, \\ R_2^{\pm}(s, W) &= (s + W - M_1^2 - m^2) \tau_4^+ \pm (M_2^2 + m^2 - W) \tau_4^-, \\ R_3^{\pm}(s, W) &= (W - M_1^2) \tau_4^+ \pm (M_2^2 - s - W) \tau_4^- \end{aligned} \quad (3.25)$$

where $W \equiv M^2(B\ell^+)$ and the parameters r_i are given by (A.3).

By using (A.5), (A.6), (3.19), (3.18) and (3.24) we obtain

$$\frac{d\Gamma}{dM(B\ell^+)} = 3 \frac{G^2 A^2 c^2 M(B\ell^+)}{128 \pi^3 M_1^3} \int_{s_-}^{s_+} ds N(s, W) \quad (3.26)$$

where

$$N(s, W) = \sum_{i,j=1}^3 H_i^{(\alpha)} H_j^{(\alpha)} \mathcal{D}_{ij}(s, W) \quad (3.27)$$

and $\mathcal{D}_{ij}(s, W)$ are given as follows

$$\mathcal{D}_{11} = 2 R_1^+ - \tau_4^- (\tau_2 - \tau_1) ,$$

$$\mathcal{D}_{22} = \left\{ \tau_3^- R_3^+ - \tau_3^+ R_2^+ - s \tau_4^- (\tau_2 - \tau_1) - m^2 \tau_4^- \tau_2 - 2 s R_1^+ \right\} / \Sigma^2$$

$$\mathcal{D}_{33} = 2 R_1^+ + \tau_4^- (\tau_2 - \tau_1)$$

$$\mathcal{D}_{12} = \{ 2 M_2 R_3^+ - 2 M_1 R_2^+ + 2 \sum \tau_4^- \tau_1 \} / \Sigma$$

$$\mathcal{D}_{13} = 4 R_1^-$$

$$\mathcal{D}_{23} = 4 M_1 R_2^- + 4 M_2 R_3^- \quad (3.28)$$

Thus we can calculate all $M(B\ell^+)$ distributions of the type $\frac{1}{2}^+ \rightarrow \frac{1}{2}^+$ by means of the formulae (3.12)-(3.16) and (3.26)-(3.28).

3.5. - Energy Spectrum of the Neutrino

Since

$$E_\nu = \frac{M_1^2 - W}{2 M_1} \quad (3.29)$$

we have in the rest frame of the decaying particle

$$\frac{d\Gamma}{dE_\nu} = 2 M_1 \frac{d\Gamma}{dW} (W = M_1^2 - 2 M_1 E_\nu). \quad (3.30)$$

These distributions may be of use to experimentalists who want to know how much energy is carried away by the invisible neutrino. Therefore we present them here for completeness together with $\langle E_\nu \rangle$ as function of the mass of the decaying charmed baryon.

4. - $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+$ TRANSITIONS

In this section we derive the general expression for the decay rate of the process

$$B_1\left(\frac{1}{2}^+\right) \rightarrow B_2\left(\frac{3}{2}^+\right) + l^+ + \nu \quad (4.1)$$

To our knowledge no such calculations exist in the literature. Fortunately there exist similar calculations of Δ production and so our calculations, although done in a different kinematical region, can be checked at least partially.

In this respect, a very useful calculation for us was that of Albright and Liu¹⁸⁾. We have used their metric, which is that of Pauli.

4.1. - The Invariant Amplitudes

The invariant amplitudes for the decays (4.1) are given by

$$M = c \frac{G}{\sqrt{2}} \langle \frac{3}{2}^+ | J_{a\mu}^{(\lambda)} | \frac{1}{2}^+, \alpha \rangle \bar{v}_\nu \gamma_\mu (1 + \gamma_5) u_l \quad (4.2)$$

where c is the Cabibbo factor as in Eq. (3.7) and the hadronic matrix element is given by (2.6). Index λ appearing in Eq. (4.2) is the polarization index of particle with spin $\frac{3}{2}$, which is described by the Rarita-Schwinger operator $\psi_{\mu}^{(\lambda)*}$.

The hadronic matrix element in Eq. (4.2) can be written in terms of four-vector and four-axial vector form factors as follows

$$\langle \frac{3}{2}^+ | J_{a\mu}^{(\lambda)} | \frac{1}{2}^+, \alpha \rangle = 3 B(a, \alpha) \bar{\psi}_{\mu}^{(\lambda)}(p_2) S_{\mu\mu} u(p_1) \quad (4.3)$$

with **)

*) Here μ is the vector index.

**) Our notation is given by Eq. (3.1).

$$\begin{aligned}
 S_{\kappa\mu} = & \delta_{\kappa\mu} (F_1^A + F_1^V \gamma_5) + i \frac{P_1 \kappa \gamma_\mu}{\Sigma} (F_2^A + F_2^V \gamma_5) \\
 & + \frac{P_1 \kappa (P_1 + P_2)_\mu}{\Sigma^2} (F_3^A + F_3^V \gamma_5) + \frac{P_1 \kappa (P_1 - P_2)_\mu}{\Sigma^2} (F_4^A + F_4^V \gamma_5) \quad (4.4)
 \end{aligned}$$

Here the form factors F_i^A are those defined for the reaction $\nu n \rightarrow \mu^- \Delta^+$. In writing (4.3) we have used the SU(4) relation

$$\tilde{F}_i^{V,A} = 3 B(\alpha, \alpha) F_i^{V,A} \quad (4.5)$$

where $\tilde{F}_i^{V,A}$ denotes form factors relevant for the process (4.1). Before we go on to calculate the decay rates we want to say a few words about the form factors.

4.2. - The Form Factors

The form factors $F_i^{A,V}$ which we have found to be convenient for our calculation can be expressed in terms of the form factors $C_i^{A,V}$ ($i=3\dots 6$), which are frequently used in Δ production estimates. In the latter estimates one usually takes

$$C_5^V = C_6^V = C_3^A = C_6^A = 0 \quad (4.6)$$

The vanishing of C_5^V and C_3^A is taken on phenomenological grounds. $C_6^V = 0$ and $C_6^A = 0$ come from the conservation of vector current and the neglect of the lepton mass, respectively.

After utilization of Eqs. (4.6) and the relations between $F_i^{A,V}$ and $C_i^{A,V}$ as given in Ref. 19), we are left with the following five form factors

$$\begin{aligned}
 F_1 &= F_1^V = \frac{\Sigma}{M_1} C_3^V + \frac{\tau_3^-}{2 M_1^2} C_4^V, \\
 F_2 &= \frac{F_2^V}{\Sigma} = - \frac{C_3^V}{M_1}, \\
 F_3 &= \frac{F_3^V}{\Sigma^2} = - \frac{F_4^V}{\Sigma^2} = - \frac{C_4^V}{2 M_1^2}, \quad (4.7)
 \end{aligned}$$

$$G_1 \equiv F_1^A = - \frac{\tau_3^-}{2M_1^2} C_4^A - C_5^A$$

and

$$G_2 \equiv \frac{F_3^A}{\Sigma^2} = - \frac{F_4^A}{\Sigma^2} = \frac{C_4^A}{2M_1^2}$$

Notice that $F_2^A = 0$. r_3^- is given by (A.3). Following Ref. 11), we parametrize the form factors $C_i^{V,A}$ as follows :

$$C_i^{V,A}(s) = C_i^{V,A}(0) \left(1 - \frac{s}{m_{a^*}^2}\right)^{-2} \quad (4.8)$$

Here m_{a^*} stands for m_{D^*} or m_{F^*} as in Eqs. (3.16). The values of $C_i^{V,A}(0)$ are *)

$$\begin{aligned} C_3^V(0) &= 2.0, & C_4^V(0) &= - \frac{M_1}{\Sigma} C_3^V(0) \\ C_4^A(0) &= 0.3, & C_5^A(0) &= -1.2. \end{aligned} \quad (4.9)$$

In summary, Eqs. (4.7)-(4.9) give the form factors which we have used in our calculations.

4.3. - Decay Rate

The calculation of the decay rate for the process (4.1) follows the steps of Section (3.1). Only the formulae are much longer. The calculation of the lepton and baryon traces is given in Appendix B. Here we quote the final result for Γ .

In writing the expression for Γ it is useful to utilize variables r_i defined by (A.3) and introduce three auxiliary functions

$$\begin{aligned} X_1(s) &= \frac{(\tau_3^-)^2}{2s} \tau_5 - M_2^2 \tau_6, & X_2(s) &= - \frac{\tau_3^- \tau_3^+ \tau_5}{s} - \tau_6 \tau_7, \\ X_3(s) &= \frac{(\tau_3^+)^2}{2s} \tau_5 - M_1^2 \tau_6. \end{aligned} \quad (4.10)$$

*) These values give a good description of Δ production data ¹⁹⁾.

Then

$$\Gamma = \frac{G^2 B^2 c^2}{64 \pi^3 M_1^3 M_2^2} \int_{m^2}^{\Delta^2} ds \frac{(\tau_4^-)^2}{s^2} [\tau_1 \tau_2]^{1/2} K(s) \quad (4.11)$$

where

$$\begin{aligned} K(s) = & 3s \left\{ 2M_2^2 \tau_1 F_1^2 + \frac{\tau_1 \tau_2^2}{2} F_2^2 + M_2 \tau_1 \tau_2 F_1 F_2 + 2M_2^2 \tau_2 G_1^2 \right\} \\ & + X_1(s) \left\{ \tau_1 F_1^2 + \tau_2 \tau_1^2 F_3^2 + \frac{M_1}{2} (3\tau_1 + \tau_2) F_1 F_2 - 2\tau_1 \tau_2 F_1 F_3 \right. \\ & \quad \left. - 2M_1 \tau_1 \tau_2 F_2 F_3 + \tau_2 G_1^2 + \tau_1 \tau_2^2 G_2^2 - 2\tau_1 \tau_2 G_1 G_2 \right\} \\ & + X_2(s) \left\{ \frac{\tau_1 \tau_2}{2} F_2^2 - 2M_1 M_2^2 F_1 F_2 + 2\tau_1 M_2^2 F_1 F_3 + M_2 \tau_1 \tau_2 F_2 F_3 \right. \\ & \quad \left. + 2M_2^2 \tau_2 G_1 G_2 \right\} \\ & + 2M_1^3 X_3(s) F_1 F_2 \end{aligned} \quad (4.12)$$

Thus all the decay rates of the type $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+$ can be calculated by means of Eqs. (4.7)-(4.12) and the parameters B and c of Table II.

5. - NUMERICAL RESULTS

In this section we present and discuss the numerical results which we have obtained by utilizing the formulae established in the previous sections. As can be seen from these formulae the results depend on the masses of the decaying baryons as well as on the masses of vector mesons m_{D^*} and m_{F^*} which enter the form factors. Since these masses are not known we have plotted the calculated quantities as functions of them. The branching ratios for all charmed baryons in question are presented in subsection 5.2. Their partial lifetimes are given in subsection 5.3, whereas the various $M(\text{Be}^+)$ distributions and neutrino energy spectra are the subject of subsections 5.4 and 5.5.

Before we begin our presentation we want to discuss the following problem. It is clear that the results given here can only be of relevance if the charmed baryons are stable against strong and radiative decays. Therefore let us first discuss the situation in which weak decays are dominant.

5.1. - Stability Against Strong and Radiative Decays

First the charmed baryons may undergo strong decays into a non-charmed baryon and the meson D or F, unless the following conditions are satisfied

$$\begin{aligned}
 m_{C_0} &< m_N + m_D, \\
 m_{C_1} &< m_N + m_D, \\
 m_A &< \min(m_\Lambda + m_D, m_N + m_F), \\
 m_S &< \min(m_\Lambda + m_D, m_N + m_F), \\
 m_T &< \min(m_\Xi + m_D, m_\Lambda + m_F).
 \end{aligned} \tag{5.1}$$

Choosing, for instance, $m_D = 1.9$ GeV and $m_F = 1.95$ GeV, we obtain

$$\begin{aligned}
 m_{C_0}, m_{C_1} &\lesssim 2.84 \text{ GeV}, \\
 m_A, m_S &\lesssim 2.9 \text{ GeV}, \\
 m_T &\lesssim 3.06 \text{ GeV}.
 \end{aligned}$$

Even if conditions (5.1) are satisfied, charmed baryons may undergo strong decays into other charmed baryons unless the following inequalities hold

$$\begin{aligned}
 |m_{C_1} - m_{C_0}| &< m_\pi, \\
 |m_{C_1} - m_A| &< m_K, \\
 |m_T - m_A| &< m_K, \\
 |m_S - m_A| &< m_\pi, \\
 |m_{C_0} - m_S| &< m_K.
 \end{aligned} \tag{5.2}$$

There can also be radiative decays of C_1^+ , S^+ and S^0 into C_0^+ , A^+ and A^0 , respectively, or vice versa, depending on the mass assignment. These decays being much stronger than the weak ones will be dominant if conditions (5.1) and (5.2) are satisfied. Thus there will be always at least three baryons which will dominantly undergo other than weak decays.

Finally there is a question of weak non-leptonic transitions. Although one expects them to be stronger than the semi-leptonic ones by an order of magnitude ¹¹⁾, they are probably much more difficult to detect than the transitions discussed in this paper.

In the calculations below we have varied the masses of the baryons $C_1^{++,+,0}$ and C_0^+ in the range from 2.0 GeV to 2.8 GeV, the masses of $S^{+,0}$ and $A^{+,0}$ in the range from 2.0 GeV to 2.9 GeV and the mass of T^0 in the range from 2.0 GeV to 3 GeV. If the mass of the D or the F meson is not smaller than 1.9 GeV this choice of masses satisfies conditions (5.1). Since it may not satisfy all conditions (5.2) and because of the presence of the radiative decays as discussed above, we should keep in mind while analyzing our results that some of the charmed baryons may dominantly undergo non-weak transitions.

5.2. - Branching Ratios

The branching ratios for the semi-leptonic decays of charmed baryons into $\frac{1}{2}^+$ and $\frac{3}{2}^+$ non-charmed baryons can be estimated by means of the formulae (3.22), (3.23) and (4.11), (4.12), respectively. We first discuss the dependence of Γ on the dipole parameters m_{D^*} and m_{F^*} . These parameters control the enhancement of the Γ due to form factors. With the increase of m_{D^*} or m_{F^*} the enhancement is decreased. Clearly the effect depends on Δ . The larger Δ , the stronger the effect ^{*)}. We have found that this combined dependence of Γ on the dipole parameter m_{a^*} and Δ can be very well described for $2 \leq m_{a^*} \leq 2.3$ and $0.8 \leq \Delta \leq 1.8$ by the formula

) Notice that if conditions (5.1) are satisfied and $m_{D^}, m_{F^*} > m_D$ as we assume, then the pole of the form factor is never reached.

$$\Gamma(\Delta, \Sigma, M_1, m_{a*}) = K(\Delta, m_{a*}) L(\Delta, \Sigma, M_1) \quad (5.3)$$

with $K(\Delta, m_{a*})$ given by

$$K(\Delta, m_{a*}) = \left(1 - \frac{\Delta^2}{m_{a*}^2}\right)^{-1.8} \quad (5.4)$$

and $L()$ independent of m_{a*} . In Fig. 1 we have plotted $K(\Delta, m_{a*})$ as function of m_{a*} for various values of Δ . For illustration purposes $K(\Delta, m_{a*})$ has been renormalized so that $K(\Delta, 2.15) = 1$. All the effects discussed above are clearly seen. We observe that whereas for $\Delta \leq 1.2$ GeV Γ is practically independent of m_{a*} , for $\Delta > 1.6$ GeV it changes by a factor of 3 as m_{a*} is varied through the range considered.

We next turn to the presentation of various branching ratios. These have been plotted in Figs. 2a-j as functions of the masses of decaying baryons for two different values of the pair (m_{D*}, m_{F*}) , $(2.05 \text{ GeV}, 2.05 \text{ GeV})$ and $(2.25 \text{ GeV}, 2.3 \text{ GeV})$ ^{*)}. Only electron modes have been plotted since the muonic ones are smaller by less than 2%. This independence of the branching ratios on the lepton mass is a direct consequence of the high values of Δ ^{**)}. We recall that this is not the case of the usual decays of $\frac{1}{2}^+$ non-charmed baryons.

As expected from the discussion above the dependence on the dipole parameters is only visible for high values of M_1 , i.e., for high Δ . Here this dependence is even smaller than that given by Eq. (5.4), since the ratios of Γ 's have been plotted.

The main conclusion drawn from Figs. 2a-j is that all $\frac{1}{2}^+$ charmed baryons but T^0 should decay predominantly into $\frac{1}{2}^+$ non-charmed baryons. This feature is independent of the mass of the decaying baryon. This result is easy to understand. First the baryons $A^{0,+}$ and C_0^+ being members of the multiplet $\underline{3}^*$ cannot decay into $\frac{3}{2}^+$ non-charmed baryons which

^{*)} These values are within the range of various estimates ^{1), 13)}.

^{**) In our calculations the dependence on the lepton mass comes only through the phase space, since the induced pseudoscalar term which gives a contribution proportional to the lepton mass has been neglected.}

belong to the multiplet 10, since the decomposition of $\underline{3}^* \otimes \underline{3}$ into irreducible representations obviously does not contain 10. Thus from two possibilities, $\frac{3}{2}^+$ and $\frac{1}{2}^+$, only the latter remains for these baryons. Secondly the dominance of the decays of $C_1^{+,++,0}$ and $S^{+,0}$ into $\frac{1}{2}^+$ baryons is a result of larger phase space as compared to $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+$ transitions and consequently also due to stronger enhancement through form factors. For these reasons we have not calculated branching ratios for the decays into heavier $\frac{1}{2}^+$, $\frac{1}{2}^-$, $\frac{3}{2}^-$ and higher spin baryons, since we expect them to be much smaller than those discussed in this paper. Finally T^0 should decay predominantly into Ω^- , since there is no $\frac{1}{2}^+$ state with three s quarks, to which it could go through F transition ($\sim \cos^2 \theta$).

We next notice that the decays of T^0 , $C_1^{+,+,0}$ and C_0^+ are characterized by the dominant decays into Ω^- , $\Sigma^{+,0,-}$ and Λ^0 , respectively ^{*)}. In the case of discovery of a charmed baryon this observation could be helpful in the identification of the baryon in question. In particular, recalling our discussion on radiative decays in Section 5.1, observation of Λ^0 would indicate the inequality $m_{C_0^+} < m_{C_1^+}$, whereas appearance of Σ^0 would signal $m_{C_1^+} < m_{C_0^+}$. The situation is more complicated in the case of $S^{0,+}$ and $A^{0,+}$ since both should dominantly decay into $\Xi^{-,0}$. Fortunately as we shall see in Section 5.4, $S^{0,+}$ and $A^{0,+}$ can be distinguished by their $M(\text{Be}^+)$ distributions.

We also notice that since in the decay modes discussed D^- transitions correspond to larger phase space as compared to F^- transitions and consequently also to stronger enhancement through form factors the Cabibbo suppression is not as clearly seen as one would expect from the ratio $\sin^2 \theta / \cos^2 \theta \approx 0.06$. We have for instance for $\Delta^2/m_{a^*}^2$ not too close to 1

$$\frac{\Gamma(C_0^+ \xrightarrow{D^-} N^0 e^+ \nu)}{\Gamma(C_0^+ \xrightarrow{F^-} \Lambda^0 e^+ \nu)} \approx \frac{1}{4} ,$$

$$\frac{\Gamma(C_1^{++} \xrightarrow{D^-} N^+ e^+ \nu)}{\Gamma(C_1^{++} \xrightarrow{F^-} \Sigma^+ e^+ \nu)} \approx \frac{1}{2} ,$$

^{*)} For $\Delta^2/m_{a^*}^2$ close to 1 some changes could be expected as seen, for instance, in the case of C_0^+ or S^+ (Figs. 2a and 2g). We comment on this below.

and

$$\frac{\Gamma(s^+ \xrightarrow{\mathcal{D}^-} \Lambda^0 e^+ \nu)}{\Gamma(s^+ \xrightarrow{F^-} \Xi^0 e^+ \nu)} \approx \frac{1}{2}$$

Finally we comment on the peculiar behaviour of the branching ratios for Δ^2/m_{a*}^2 close to 1, i.e., for high values of M_1 , where the D^- transitions seem to dominate. This behaviour is due to our choice $m_{F*} > m_{D*}$. It would not take place for $m_{F*} < m_{D*}$. Certainly the value of M_1 at which the phenomenon in question starts is highly dependent on $|1 - (\Delta^2/m_{a*}^2)|$ and $m_{F*} - m_{D*}$. By requiring stability conditions (5.1) to be satisfied, and choosing $m_{D*} - m_D$ larger and $m_{F*} - m_{D*}$ smaller than the values used in our calculations, the effect under discussion can be eliminated completely. Nevertheless one should keep in mind that if the parametrization of the form factors used in the present paper is reasonable, then the effects in question may take place if, for instance, $m_{F*} - m_{D*}$ happens to be large and positive.

In such a case C_0^+ , $C_1^{++,+}$, C_1^0 , $A^{0,+}$, S^+ , S^0 and T^0 could be dominated by the decays into N^0 , $N^{+,0}$, Δ^- , $\Sigma^{0,-}$, Λ^0 , Σ^- and Ξ^- , respectively, if their masses happened to be sufficiently large.

In order to simplify our discussion, we shall assume in the following that this does not happen.

We end this section by noticing that the branching ratios do not depend strongly on the masses of the charmed baryons. This is not the case of the partial lives as is shown below. Consequently, the branching ratios are less subject to $SU(4)$ breaking than the partial lifetimes.

5.3. - The Partial Lifetimes *)

In Fig. 3a we have plotted the partial lifetimes of the baryons $C_1^{++,+0}$ and C_0^+ as functions of their masses for the intermediate values of m_{D*} and m_{F*} , $m_{D*} = 2.15$ GeV and $m_{F*} = 2.20$ GeV. The partial lifetimes

*) We recall that only semi-leptonic transitions into $\frac{1}{2}^+$ and $\frac{3}{2}^+$ non-charmed states have been taken into account.

of C_1^{++} , C_1^+ and C_1^0 are approximately the same and change from $2 \cdot 10^{-12}$ sec to $1 \cdot 10^{-14}$ sec as M_1 is varied from 2.0 GeV to 2.8 GeV. The partial lifetime of C_0^+ is larger by approximately a factor of 2 in the whole range of masses considered.

Figure 3b shows the partial lifetimes of $S^{0,+}$, $A^{+,0}$ and T^0 as functions of their masses for $m_{F*} = 2.2$ GeV and $m_{D*} = 2.15$ GeV. To a good approximation

$$\tau_{T_0} = \tau_{S,A} \cdot (6 \pm 2)$$

with τ_{T_0} changing from $1 \cdot 10^{-10}$ sec to $1 \cdot 10^{-13}$ sec as M_{T_0} is varied from 2.0 GeV to 3.0 GeV. In summary, for a given value of M_1 we have

$$\tau_{T_0} : \tau_S : \tau_A : \tau_{C_0} : \tau_{C_1} = 36 : 6 : 6 : 2 : 1.$$

Since the dependence of τ on M_1 is very strong these ratios are not true if the masses of baryons are different. Choosing for instance in accordance with conditions (5.1) and (5.2) $m_{C_0} = 2.4$ GeV, $m_{C_1} = 2.5$ GeV, $m_{A^0} = 2.7$ GeV, $m_S = 2.8$ GeV and $m_{T^0} = 3.0$ GeV we obtain $\tau_{C_0} = 0.28 \cdot 10^{-12}$ sec, $\tau_{C_1} = 0.9 \cdot 10^{-13}$ sec, $\tau_A = 0.2 \cdot 10^{-12}$ sec, $\tau_S = 0.5 \cdot 10^{-13}$ sec and $\tau_T = 0.6 \cdot 10^{-13}$ sec.

For completeness we have plotted in Fig. 4 the partial lifetimes of T^0 and A^0 as functions of their masses for two different values of the pair (m_{F*}, m_{D*}) . As expected from previous discussion the dependence on the dipole parameter is only visible for high values of M_1 .

Needless to say, the above estimates can only be at best the upper limits for the lifetimes of the baryons in question. Although we do not expect these estimates to be affected very much by the transitions into high spin states, they are probably strongly affected by the weak non-leptonic transitions and are meaningless if radiative and strong decays are present.

5.4. - The $M(\text{Be}^+)$ Distributions

The $d\Gamma/dM(\text{Be}^+)$ distributions can be estimated by means of formulae (3.26)-(3.28). Only the distributions for the dominant decay modes are presented here.

In Fig. 5a we have plotted $1/\Gamma(d\Gamma/dM)$ for the dominant decays of C_0^+ and C_1^+ for $M_1 = 2.5$ GeV and $m_{P^*} = 2.2$ GeV. Figure 5b contains corresponding distributions for S^0 and A^0 .

We recall that the baryons C_1 and S are members of 6 SU(3) representation, whereas A and C_0 belong to the multiplet 3^* . According to Eqs. (3.12) and (3.13) the form factors for these two multiplets are different. This is clearly seen in Figs. 5a and 5b. Whereas the decay spectra of 3^* multiplet are rather smooth, those of the multiplet 6 show a remarkable shoulder. This feature remains as we vary M_1 . We conclude that the $M(Be^+)$ distributions might be helpful in distinguishing between charmed baryons belonging to different SU(3) representations. In particular they may help to distinguish between the baryons A^+ and S^+ or A^0 and S^0 . As we already discussed in Section 5.2 these distinctions were very difficult on the basis of the branching ratios since both $S^{+,0}$ and $A^{+,0}$ decay dominantly into $\Xi^{0,-}$. Recalling our discussion on radiative decays we conclude that the question whether $m_{S^{+,0}} > m_{A^{+,0}}$ or $m_{S^{+,0}} < m_{A^{+,0}}$ can be in principle answered on the basis of the mass spectra. The same applies to C_1^+ and C_0^+ .

Finally we would like to point out that the value of the mass of a given charmed baryon can be determined by measuring $\langle M(Be^+) \rangle$. The relation between $\langle M(Be^+) \rangle$ and M_1 as given in our approach is presented on Fig. 6 for the dominant decay modes of C_1 , A , C_0 and S . We observe that the dependence of $\langle M(Be^+) \rangle$ on M_1 is almost linear in the range of masses discussed. To a very good approximation, we have

$$\langle M(\Lambda^0 e^+) \rangle_{C_0} = 0.33 M_{C_0} + 0.856 \quad [\text{GeV}] ,$$

$$\langle M(\Xi^- e^+) \rangle_A = 0.34 M_A + 0.98 \quad [\text{GeV}] ,$$

$$\langle M(\Sigma^0 e^+) \rangle_{C_1} = 0.42 M_{C_1} + 0.727 \quad [\text{GeV}] ,$$

$$\langle M(\Xi^- e^+) \rangle_S = 0.42 M_S + 0.8 \quad [\text{GeV}] .$$

Also here the difference between 3^* and 6 representations is clearly seen. Notice also the SU(3) property of the slopes.

We have investigated the dependence of $1/\Gamma(d\Gamma/dM)$ and of $\langle M(Be^+) \rangle$ on the mass of the dipole parameter. Since this dependence is very weak for $2.05 \text{ GeV} \leq m_{P^*} \leq 2.3 \text{ GeV}$ and $2.0 \text{ GeV} \leq M_1 \leq 2.9 \text{ GeV}$ we do not present it here.

5.5. - Energy Spectrum of the Neutrino and $\langle E_\nu \rangle$

In Fig. 7 we have plotted using Eqs. (3.30) a typical energy spectrum of the neutrino. Again the difference between the members of 3^* and 6 multiplets is clearly seen. Figure 8 contains $\langle E_\nu \rangle$ as a function of the mass of the decaying baryon. We observe that the dependence of $\langle E_\nu \rangle$ on M_1 is, to a very good approximation, linear. Again the slopes show the $SU(3)$ property.

6. - SUMMARY

In this paper we have presented estimates of the semi-leptonic decays of the charmed $\frac{1}{2}^+$ baryons into ordinary $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons. The form factors and general formulae used in our calculations are collected in Sections 3 and 4. The relevant parameters entering these formulae are to be found in Table II. Our main results, details of which occupy Section 5, are as follows.

- i) The baryons $C_1^{++,+}, C_1^+, S^{+,0}$ and $A^{+,0}$ decay dominantly into $\frac{1}{2}^+$ baryons $\Sigma^{+,0,-}, \Lambda^0, \Xi^{0,-}$ and $\Xi^{0,-}$, respectively. The dominant decay mode of the baryon T^0 has Ω^- in the final state.
- ii) The partial lifetimes of $C_1^{++,+}, C_1^+$ for $2 \text{ GeV} \leq m_{C_1} = m_{C_0} \leq 2.8 \text{ GeV}$ are $O(10^{-12} - 10^{-14})$ seconds. The partial lifetimes of $S^{+,0}$ and of $A^{+,0}$ for $2 \text{ GeV} \leq m_S = m_A \leq 3.0 \text{ GeV}$ are $O(10^{-10} - 10^{-13})$ seconds. The partial lifetime of T^0 is estimated to be six times larger than that of the baryons S for $m_{T^0} = m_S$.
- iii) The $1/\Gamma(d\Gamma/dM)$ distributions offer a possibility of distinguishing between the baryons belonging to the multiplets 3^* and 6. It is pointed out that the measurement of $\langle M(Be^+) \rangle$ may give an idea about the masses of the decaying charmed baryons.

- iv) The Cabibbo suppression of D^- transitions relatively to F^- ones is in many cases not clearly seen due to phase space effects and enhancement through form factors both being stronger for the former transitions.

We have also discussed the conditions under which the semi-leptonic decays are dominant. At best only six baryons out of nine discussed in this paper can be stable against strong and radiative decays. There will be always three baryons which will be dominated by other than weak decays. This fact as discussed in Section 5.2 may give an idea about the relative masses of some baryons.

We hope that the results of this paper will be helpful to both experimentalists and theorists in their search for charmed particles.

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Baryon	Quark content	I	I_3	Y	SU(3)
C_1^{++}	Cuu		1		6
C_1^+	$C(u d)_{\text{sym}}$	1	0	1	
C_1^0	Cdd		-1		
S^+	$C(s u)_{\text{sym}}$	$\frac{1}{2}$	$\frac{1}{2}$	0	
S^0	$C(s d)_{\text{sym}}$		$-\frac{1}{2}$		
T^0	Css	0	0	-1	
A^+	$C(s u)_{\text{anti}}$	$\frac{1}{2}$	$\frac{1}{2}$	0	<u>3*</u>
A^0	$C(s d)_{\text{anti}}$		$-\frac{1}{2}$		
C_o^+	$C(u d)_{\text{anti}}$	0	0	1	

TABLE I : Quantum numbers and quark content of the charmed $\frac{1}{2}^+$ baryons.

Decaying baryon	Baryon in the final state	A^2 or B^2	Cabibbo factor
C_0^+	Λ^0	$2/3$	$\cos \theta$
	N^0	1	$\sin \theta$
C_1^{++}	Σ^+	$2/3$	$\cos \theta$
	N^+	$2/3$	$\sin \theta$
	Σ^{*+}	$1/9$	$\cos \theta$
	Δ^+	$1/9$	$\sin \theta$
C_1^+	Σ^0	$2/3$	$\cos \theta$
	N^0	$1/3$	$\sin \theta$
	Σ^{*0}	$1/9$	$\cos \theta$
	Δ^0	$2/9$	$\sin \theta$
C_1^0	Σ^-	$2/3$	$\cos \theta$
	Σ^{*-}	$1/9$	$\cos \theta$
	Δ^-	$1/3$	$\sin \theta$
A^+	Λ^0	$1/6$	$\sin \theta$
	Σ^0	$1/2$	$\sin \theta$
	Ξ^0	1	$\cos \theta$
A^0	Σ^-	1	$\sin \theta$
	Ξ^-	1	$\cos \theta$
S^+	Ξ^0	$1/3$	$\cos \theta$
	Σ^0	$1/6$	$\sin \theta$
	Λ^0	$1/2$	$\sin \theta$
	Ξ^{*0}	$2/9$	$\cos \theta$
	Σ^{*0}	$1/9$	$\sin \theta$
S^0	Ξ^-	$1/3$	$\cos \theta$
	Σ^-	$1/3$	$\sin \theta$
	Ξ^{*-}	$2/9$	$\cos \theta$
	Σ^{*-}	$2/9$	$\sin \theta$
T^0	Ξ^-	$2/3$	$\sin \theta$
	Ω^-	$1/3$	$\cos \theta$
	Ξ^{*-}	$1/9$	$\sin \theta$

TABLE II : Parameters used in the calculation of the semi-leptonic decay modes of the charmed $\frac{1}{2}^+$ baryons into $\frac{1}{2}^+$ and $\frac{3}{2}^+$ non-charmed states. See subsection 2.2 for the definition of A and B . The minus sign in front of $\sin \theta$ has been suppressed. The values $\sin^2 \theta = 0.06$ and $\cos^2 \theta = 0.94$ have been used.

APPENDIX A - BARYON TRACE FOR $\frac{1}{2}^+ \rightarrow \frac{1}{2}^+$ TRANSITION

Let us write Eq. (3.11) as

$$\langle \frac{1}{2}^+ | J_a^\lambda | \frac{1}{2}^+, \alpha \rangle = H(\alpha) \sqrt{\frac{3}{2}} \bar{u}(p_2) \Gamma_{(\alpha)}^\lambda u(p_1) \quad (A.1)$$

then

$$B_{\mu\nu} = H^2(\alpha) \frac{3}{2} \text{Tr}((\gamma^\tau p_{2\tau} + M_2) \Gamma_\mu (\gamma^\tau p_{1\tau} + M_1) \gamma_0 \Gamma_\nu^+ \gamma_0) \quad (A.2)$$

It is convenient to introduce the variables r_i defined by ^{*})

$$\begin{aligned} r_1 &= \Delta^2 - s, & r_2 &= \Sigma^2 - s, & r_3^\pm &= s \pm \Sigma \Delta, \\ r_4^\pm &= s \pm m^2, & r_5 &= s + 2m^2, & r_6 &= 2s + m^2, \\ r_7 &= M_1^2 + M_2^2 - s \end{aligned} \quad (A.3)$$

and the tensors

$$\begin{aligned} V_{\mu\nu}^{(1)} &= p_{1\nu} p_{2\mu} + p_{2\nu} p_{1\mu}, \\ V_{\mu\nu}^{(2)} &= p_{2\nu} q_\mu + p_{2\mu} q_\nu, \\ V_{\mu\nu}^{(3)} &= p_{1\nu} q_\mu + p_{1\mu} q_\nu. \end{aligned} \quad (A.4)$$

Then $B_{\mu\nu}$ is given by

$$B_{\mu\nu} = \frac{3}{2} H^2(\alpha) \sum_{i,j=1}^3 H_i^{(\alpha)} H_j^{(\alpha)} W_{\mu\nu}^{ij} \quad (A.5)$$

^{*}) See Eqs. (3.2) and (3.6) for the definition of s , Δ and Σ .

where

$$\begin{aligned}
 W_{\mu\nu}^{11} &= 2 \left(2 V_{\mu\nu}^{(1)} - \tau_1 g_{\mu\nu} \right) , \\
 W_{\mu\nu}^{22} &= 2 \left(\tau_3^- V_{\mu\nu}^{(3)} - \tau_3^+ V_{\mu\nu}^{(2)} - 2s V_{\mu\nu}^{(1)} - \tau_2 q_\mu q_\nu - \tau_1 \Sigma^2 g_{\mu\nu} \right) / \Sigma^2 , \\
 W_{\mu\nu}^{33} &= 2 \left(2 V_{\mu\nu}^{(1)} - \tau_2 g_{\mu\nu} \right) , \\
 W_{\mu\nu}^{12} &= 4 \left(M_2 V_{\mu\nu}^{(3)} - M_1 V_{\mu\nu}^{(2)} - \tau_1 \Sigma g_{\mu\nu} \right) / \Sigma , \\
 W_{\mu\nu}^{13} &= 8i p_2^\alpha p_1^\beta \varepsilon_{\alpha\mu\beta\nu} , \\
 W_{\mu\nu}^{23} &= \frac{8i}{\Sigma} \left(M_1 p_2^\alpha + M_2 p_1^\alpha \right) q^\beta \varepsilon_{\alpha\beta\mu\nu} .
 \end{aligned}
 \tag{A.6}$$

Notice that we have kept the vector-axial-vector interference terms $W_{\mu\nu}^{13}$, $W_{\mu\nu}^{23}$, since they contribute to $d\Gamma/dM(B\ell^+)$. On the other hand, their contribution to Γ is zero.

APPENDIX B - LEPTON AND BARYON TRACE FOR $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+$ TRANSITION

As in Eqs. (3.18)

$$\sum_{\text{spins}} |M|^2 = \frac{G^2}{2} \mathcal{L}_{\mu\nu} B_{\mu\nu} \quad (\text{B.1})$$

The lepton trace is given by

$$\mathcal{L}_{\mu\nu} = 8 (k_{2\mu} k_{1\nu} + k_{1\mu} k_{2\nu} - k_1 \cdot k_2 \delta_{\mu\nu}) \quad (\text{B.2})$$

where the antisymmetric piece has been neglected since it does not contribute to Γ .

The baryon trace is given by

$$B_{\mu\nu} = [3 B(\alpha, \alpha)]^2 \text{Tr}(\Lambda_{\beta\kappa} S_{\kappa\mu} (M_1 - i\gamma \cdot P_1) \gamma_4 S_{\beta\nu}^+ \gamma_4) \quad (\text{B.3})$$

with $\Lambda_{\beta\kappa}$ being the $\frac{3}{2}$ projection operator

$$\Lambda_{\beta\kappa} = \left(\delta_{\beta\kappa} + \frac{2}{3} \frac{P_{2\beta} P_{2\kappa}}{M_2^2} - \frac{1}{3} \gamma_\beta \gamma_\kappa - \frac{i}{3M_2} (\gamma_\beta P_{2\kappa} - \gamma_\kappa P_{2\beta}) \right) \cdot (M_2 - i\gamma \cdot P_2) \quad (\text{B.4})$$

and $S_{\kappa\mu}$ given by (4.4).

Calculation of the trace in Eq. (B.3) gives

$$\begin{aligned} B_{\mu\nu} = [3 B(\alpha, \alpha)]^2 & \left(F_1^2 P_{11}^\nu + F_2^2 P_{22}^\nu + F_3^2 P_{33}^\nu + \right. \\ & + F_1 F_2 P_{12}^\nu + F_2 F_3 P_{23}^\nu + F_1 F_3 P_{13}^\nu + \\ & \left. + G_1^2 P_{11}^A + G_2^2 P_{22}^A + G_1 G_2 P_{12}^A \right) / 3M_2^2 \end{aligned} \quad (\text{B.5})$$

where

$$\begin{aligned}
 P_{11}^V &= -4\tau_1 (\delta_{\mu\nu} M_2^2 + p_{2\nu} p_{2\mu}), \\
 P_{22}^V &= -\tau_1 \tau_2 (\tau_2 \delta_{\mu\nu} + 2 V_{\mu\nu}^{(1)}), \\
 P_{33}^V &= -4\tau_2 \tau_1^2 p_{2\nu} p_{2\mu}, \\
 P_{12}^V &= 8 M_1 M_2^2 V_{\mu\nu}^{(1)} - 8 M_2^3 p_{1\mu} p_{1\nu} - 2(3\tau_1 + \tau_2) M_1 p_{2\mu} p_{2\nu} \\
 &\quad - 2\tau_1 \tau_2 M_2 \delta_{\mu\nu}, \\
 P_{13}^V &= -8\tau_1 (M_2^2 V_{\mu\nu}^{(1)} + 2(p_1 \cdot p_2) p_{2\nu} p_{2\mu}), \\
 P_{23}^V &= 4\tau_1 \tau_2 (2 M_1 p_{2\mu} p_{2\nu} - M_2 V_{\mu\nu}^{(1)}), \\
 P_{11}^A &= -4\tau_2 (\delta_{\mu\nu} M_2^2 + p_{2\nu} p_{2\mu}), \\
 P_{22}^A &= -4\tau_1 \tau_2^2 p_{2\nu} p_{2\mu}, \\
 P_{12}^A &= -8\tau_2 (M_2^2 V_{\mu\nu}^{(1)} + 2(p_1 \cdot p_2) p_{2\nu} p_{2\mu}).
 \end{aligned}
 \tag{B.6}$$

with $V_{\mu\nu}^{(1)}$ given by (A.4).

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FIGURE CAPTIONS

Figure 1 The dependence of $K(\Delta, m_{a*})$ [see, e.g., (5.4)] on the dipole parameter m_{a*} for $\Delta = 0.8, 1, 1.2, 1.4, 1.6$ and 1.8 GeV.

Figure 2 The branching ratios (%) as functions of the masses M_1 of the decaying charmed baryons for two values of (m_{F*}, m_{D*}) : $(2.05 \text{ GeV}, 2 \text{ GeV})$ solid line and $(2.3 \text{ GeV}, 2.25 \text{ GeV})$ dashed line.

- a) $C_0^+ \rightarrow Be^+\nu$; b) $C_1^0 \rightarrow Be^+\nu$; c) $C_1^+ \rightarrow Be^+\nu$;
d) $C_1^{++} \rightarrow Be^+\nu$; e) $A^0 \rightarrow Be^+\nu$; f) $A^+ \rightarrow Be^+\nu$;
g) $S^+ \rightarrow Be^+\nu$; h) $S^0 \rightarrow Be^+\nu$; h) $S^0 \rightarrow Be^+\nu$;
i) $T^0 \rightarrow Be^+\nu$.

Figure 3 The partial lifetimes τ of the charmed baryons as functions of their masses (M_1) ;

- a) the partial lifetimes of $C_1^{+,0}$ and C_0^+ ;
b) the partial lifetimes of $A^{+,0}$, S^{+0} and T^0 .

Figure 4 Typical dependence of the partial lifetimes on M_1 and on the dipole parameters m_{D*} and m_{F*} .

Figure 5 The normalized mass distributions $1/\Gamma(d\Gamma/dM(Be^+))$ at fixed $M_1 = 2.5 \text{ GeV}$ and $m_{F*} = 2.2 \text{ GeV}$ for the reactions

- a) $C_1^+ \rightarrow \Sigma^0 e^+\nu$, $C_0^+ \rightarrow \Lambda^0 e^+\nu$, and
b) $S^0 \rightarrow \Xi^- e^+\nu$, $A^0 \rightarrow \Xi^- e^+\nu$.

Figure 6 The dependence of $\langle M(Be^+) \rangle$ on the mass M_1 for the reactions $C_0^+ \rightarrow \Lambda^0 e^+\nu$, $A^0 \rightarrow \Xi^- e^+\nu$, $C_1^+ \rightarrow \Sigma^0 e^+\nu$ and $S^0 \rightarrow \Xi^- e^+\nu$.

Figure 7 A typical energy spectrum of the neutrino at fixed M_1 and m_{F*} for the member of the 6 multiplet (dashed line) and for the member of the 3* multiplet (solid line).

Figure 8 The dependence of $\langle E_\nu \rangle$ on the mass M_1 for the reactions $C_0^+ \rightarrow \Lambda^0 e^+\nu$, $A^0 \rightarrow \Xi^- e^+\nu$, $C_1^+ \rightarrow \Sigma^0 e^+\nu$ and $S^0 \rightarrow \Xi^- e^+\nu$.

FIG.1

FIG. 2a.

FIG. 2 b.

FIG. 2 c.

FIG. 2 d.

FIG. 2 e.

FIG. 2 f.

FIG. 2 g.

FIG. 2 h.

FIG. 2 i.

FIG.3 a.

FIG. 3b.

FIG. 4

FIG. 5 a.

FIG. 5 b.

FIG. 6

FIG. 7

FIG. 8