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SU(3) AS GAUGE GROUP OF THE VECTORLIKE
WEAK AND ELECTROMAGNETIC INTERACTIONS

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A B S T R A C T

A scheme of the weak and electromagnetic interactions based on the group $SU(3)$ as well as on six quark flavours and eight leptons is presented. Several aspects of the weak interactions are discussed, in particular the presence of new right-handed currents, the neutral current, the generation of the Cabibbo angle, and the dominant non-leptonic weak interaction.

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An economical description of the observed electromagnetic and weak interactions is the one based on the gauge group ¹⁾ $SU(2) \times U(1)$ and the observed four leptons as well as the four quark flavours u, d, s and c (c = charmed quark). However, within such a scheme no real unification of electromagnetism and the weak interactions is achieved (two independent coupling constants are involved, no reason for the quantization of the electric charge is provided).

In this note we should like to suggest that the observed weak and electromagnetic interactions are a subsystem of a larger set of interactions involving the simple gauge group $SU(3)$, and present an example which is vectorlike ²⁾ and is based on six quark flavours (the triplet of "light" quarks u, d, s with the charges $2/3, -1/3, -1/3$, respectively), and a triplet of "heavy" quarks c, b, h ("charm", "bottom", "heaviness", charges $2/3, -1/3, -1/3$, respectively) ^{F1)}, and eight leptons [$SU(3)$ octet]. Only one coupling constant (α) enters; the quantization of the electric charges is obtained within the system of the weak and electromagnetic interactions.

We start from the Lagrangian

$$\mathcal{L} = -\frac{1}{4} G_{\mu\nu}^i G_{\mu\nu}^i + \bar{q} [i \gamma_\mu (\partial^\mu + i g \frac{\lambda_i}{2} B^{\mu i})] q + \bar{l}^\alpha i \gamma_\mu (\partial^\mu - g f_{k\ell m} B^{\mu k}) l^m$$

$$\left(\left[\frac{\lambda_i}{2}, \frac{\lambda_j}{2} \right] = i f_{ijk} \frac{\lambda_k}{2}, \quad \text{tr}(\lambda_i \lambda_j) = 2 \delta_{ij} + \text{symmetry breaking terms} \right)$$

$$G_{\mu\nu}^i = \partial_\nu B_\mu^i - \partial_\mu B_\nu^i - g f^{ijk} B_{\nu j} B_{\mu k}; \quad i, j = 1, \dots, 8 \quad (1)$$

where q stands for the two triplets of quarks, l_i for the lepton octet, and $B_{i\mu}$ for the gauge boson octet.

The Lagrangian (1) is vectorlike: in the absence of symmetry breaking terms the gauge currents are vectorial, and parity is conserved. When the fermion mass matrix in particular is turned on, it has scalar and pseudoscalar terms, and parity is thus violated.

Our gauge group $SU(3)^{\text{weak}}$ has as subgroups the groups $SU(2)^W$, $SU(2)^V$ and $SU(2)^U$ (weak isospin, weak V spin, weak U spin), which we define analogously to the corresponding subgroups of the well-known strong interaction classification group $SU(3)$. The photon field is given by $A_\mu = \sqrt{3}/2 (B_\mu^3 + 1/\sqrt{3} B_\mu^8)$ (U spin singlet), and the generators of the weak isospin

are identified with the generators of the observed weak interactions. The bosons which couple to the raising or lowering operators of $SU(2)^W$, $SU(2)^V$ and $SU(2)^U$ are denoted by (W_+, W_-) , (V_+, V_-) , $(U_0, \bar{U}_0)$, respectively.

The mass eigenstates of the vector boson mass matrix are assumed to be as follows :

$$\begin{aligned} W_{\pm}, V_{\pm}, \quad \gamma &= \frac{\sqrt{3}}{2} (B_3 + \frac{1}{\sqrt{3}} B_8), \quad Z = -\frac{1}{2} B_3 + \frac{\sqrt{3}}{2} B_8 \\ U_1 &= \frac{1}{\sqrt{2}} (U_0 + \bar{U}_0), \quad U_2 = \frac{1}{\sqrt{2}} (U_0 - \bar{U}_0). \end{aligned} \quad (2)$$

The masses of the gauge bosons are supposed to be generated by the spontaneous symmetry breaking. By specifying the kind of representations of scalar and pseudoscalar fields used to break the symmetry one would obtain relations among the various gauge boson masses ^{F2)}. In this note we do not specify those representations but keep the gauge boson masses as parameters to be measured by experiment.

The fermions are represented by two $SU(3)$ triplets (quarks) and an $SU(3)$ octet (leptons). The representations for the left-handed (L) and right-handed (R) fermions are displayed in the Figure. The Cabibbo rotation is denoted by the definitions $d_L^1 = d_L \cos \theta' + s_L \sin \theta'$, $s_L^1 = 1$ (θ' : Cabibbo angle), an analogous rotation of the right-handed fields is denoted by $d_R^1 = d_R \cos \theta'' + s_R \sin \theta''$, $s_R^1 = 1$ (θ'' : right-handed analogue of the Cabibbo angle).

The lepton octet contains two new charged leptons belonging to an $SU(2)^W$ triplet, while the electron and muon are members of $SU(2)^W$ doublets. The electron neutrino and the muon antineutrino form together a four-component spinor field denoted by ν . The leptons N , L_3 and L_8 are neutral massive leptons. In order not to disturb the conventional picture for the K decay the mass of N has to be larger than M_K . Furthermore the U spin triplet term in the $L_3 - L_8$ mass matrix has to be larger than m_K . No such restriction exists for the U spin singlet term. [The leptons L_3, L_8 may not be eigenstates of the fermion mass matrix.]

The scheme given in the Figure is (up to trivial permutations) the only vectorlike $SU(3)$ scheme based on six quarks and one lepton octet which is not in disagreement with observed facts. We add the following remarks.

- 1) The simple gauge group $SU(3)$ fixes the basic gauge coupling constant g in Eq. (1) uniquely in terms of the electromagnetic coupling constant e . One finds $g = 2/\sqrt{3} e$. Within our normalization the Fermi constant is related

to g and M_{W+} by : $G/\sqrt{2} = g^2/8M_{W+}^2$ which gives $M_{W+} = 43.1$ GeV. If we define an angle θ by $e = g \sin\theta$ [which corresponds to the weak mixing angle in the $SU(2) \times U(1)$ theory], one obtains $\sin^2\theta = 3/4$ ($\theta = 60^\circ$).

2) The $SU(3)$ scheme contains two flavour diagonal generators. One of them is identified with the electric charge, the other is orthogonal to the photon in $SU(3)$ space. Its current is coupled to the Z boson and is identified with the observed neutral current. The hadronic neutral current is given by

$$j_\mu^{\text{neutral}} = \bar{d} \gamma_\mu \gamma_5 d - \bar{b} \gamma_\mu \gamma_5 b + \bar{s} \gamma_\mu \gamma_5 s - \bar{h} \gamma_\mu \gamma_5 h \quad (3)$$

i.e., the neutral current is the three-component of a weak U spin triplet and is an axial vector. The leptonic neutral current can easily be calculated, we denote only the terms containing electron and neutrino fields : $j_\mu^{\text{neutral}} = \bar{e} \gamma_\mu e - 2\bar{\nu}_\mu \gamma_\mu \gamma_5 \nu + \dots$ (it couples vectorially to electrons). The neutral current differential cross-sections on hadronic targets or electrons for incident neutrinos and antineutrinos are predicted to be equal and (in the scaling limit) to be proportional to $1 + (1-y)^2$. The hadronic neutral current acts effectively only on d or s quarks, and one has $\sigma^{\nu\nu}(\text{proton}) \cong \frac{1}{2} \sigma^{\nu\nu}(\text{neutron})$.

One expects parity violation effects due to the neutral current in atomic transitions, which, however, will be much smaller than the ones predicted by the $SU(2) \times U(1)$ theory : the neutral current couples vectorially to electrons, thus only the hadronic neutral current coupling causes parity violations. Those, however, are suppressed due to the smallness of the matrix elements of $\bar{d} \gamma_\mu \gamma_5 d$ in nuclei. The neutral current causes no parity violation in nuclear physics.

3) The production of charmed particles in neutrino and antineutrino scattering is analogous to the situation in the conventional $SU(2) \times U(1)$ theory. The ψ resonances are interpreted as $\bar{c}c$ bound states implying ~ 2 GeV for m_c .

Above the b threshold, b flavoured hadrons will be produced via the $(\bar{u}b)_R$ current (in antineutrino scattering predominantly off the valence u quarks in nuclei, in neutrino scattering off the $\bar{u}u$ sea), and the ratio $\sigma_{\nu\mu+}/\sigma_{\nu\mu-}$ is expected to increase. Present experimental results on the so-called $\bar{\nu}$ anomaly can be interpreted in this way, implying an effective b quark mass of ~ 5 GeV⁵).

Charmed particles decay via the left-handed W spin current by emitting hadrons or a lepton pair, or by the new V spin current by emitting hadrons. In any case they will decay predominantly into final states with strangeness ± 1 .

The b flavoured hadrons can decay via the W spin current into non-strange final states either non-leptonically or leptonically. They can also decay into charmed particles via the V spin current by emitting a non-strange hadron system.

4) The new neutral lepton N can be produced in ν_μ electron scattering.

Because of the constraint $m_N \gtrsim M_K$ the lower limit for the associated threshold is $(M_K + m_\mu)^2 / 2m_e \cong 350 \text{ GeV}$. Its main decay mode will be $N \rightarrow$ charged lepton pair + neutrino.

The new charged lepton E can be produced in ν_μ scattering off hadronic matter via the V spin current according to the reaction $\nu_\mu + u_R \rightarrow E_L^+ + d_R$. Since E can decay into a muon + neutrinos, the present experiments put rather strong constraints on the E mass ⁶⁾ ($m_E \gtrsim 6 \text{ GeV}$). No such constraint exists for the M lepton, which can be produced in ν_e scattering.

The new charged leptons E, M can be produced in pairs in e^+e^- annihilation. The decay pattern of those leptons depends on the mass matrix of the neutral leptons L_3, L_8 and N and will not be discussed in detail.

5) Interesting is the existence of combined excitations of new quarks and leptons via the V and U spin interaction. In ν_μ scattering at sufficiently high energies the u quark is transformed into an h quark via the V spin current, while the same current turns the incoming neutrino into an E^+ . Furthermore, the reaction $\nu_\mu + d \xrightarrow{U \text{ spin current}} h + \bar{L}_3(\bar{L}_8)$ exists. The h quark can decay weakly via the U spin current, the W spin current and the V spin current. In either case charged leptons will be emitted part of the time. Consequently, one obtains multi-lepton events in ν_μ scattering at very high energies.

6) Both h and b flavoured hadrons will decay part of the time via sequential weak decays which produce several strange particles (e.g., $(\bar{K}^0)$) in the final state, along with one or several charged leptons. For example, the decay chain $b \xrightarrow[V \text{ spin current}]{V \text{ spin current}} c + \bar{c} + s \rightarrow \dots$ generates three strange particles, while the decay chain $h \xrightarrow[U \text{ spin current}]{U \text{ spin current}} s + \bar{s} + b \rightarrow \dots$ can produce up to

five strange particles. Recently one has observed ⁷⁾ the production of several strange particles in high energy neutrino scattering (in association with the production of a $\mu^- - e^+$ pair). It remains to be seen if those events can find an explanation as sequential weak decays of b or h flavoured hadrons.

7) In the Figure we have displayed only the Cabibbo angle Θ' and its right-handed analogue Θ'' . Besides those angles one expects also the presence of other weak angles. We assume that all angles vanish if the weak and electromagnetic interactions are turned off, and one is left with the scheme of the Figure (with $\Theta' = \Theta'' = 0$). Furthermore it is assumed that in this limit the light leptons ν, e and the "light" quarks u, d are massless. All weak angles as well as the neutrino and electron mass and the u and d quark masses are supposed to arise through higher order effects ^{F3)}. For the electron and neutrino masses those are logarithmically divergent, and a detailed study of the spontaneous symmetry breaking, outside the scope of this paper, would be necessary to extract finite results.

The U spin weak interaction generates off - diagonal finite terms in the (d,s) mass matrix (by emission and reabsorption of the U boson), and the (d,s) mass matrix becomes

$$m_s \bar{s}_0 s_0 + \delta (m_h \bar{s}_{0R} d_{0L} + m_b \bar{s}_{0L} d_{0R}), \quad \left(\delta = \frac{\alpha}{4\pi} \log \left(\frac{M_1^2}{M_2^2} \right) \right),$$

where s_0, d_0 denote the quark fields in the limit $\alpha = 0$. The diagonalization gives as eigenstates

$$\begin{aligned} d &= (d_L, d_R) = [(\cos \Theta' d_{0L} + \sin \Theta' s_{0L}), (\cos \Theta'' d_{0R} + \sin \Theta'' s_{0R})] \\ s &= (s_L, s_R) = [(-\sin \Theta' d_{0L} + \cos \Theta' s_{0L}), (-\sin \Theta'' d_{0R} + \cos \Theta'' s_{0R})]_{(4)} \end{aligned}$$

where

$$\Theta' = \delta \cdot m_h / m_s, \quad \Theta'' = \delta \cdot m_b / m_s, \quad |m_d| = |m_s \Theta' \Theta''|.$$

No similar mechanism exists generating a mass for the u quark of order α^2 . We conclude : the Cabibbo angle as well as Θ'' are formally of order α/π , but enhanced substantially by the large ratios m_h/m_s and m_b/m_s . Solving the equation $\Theta' \cong 0.23 = \delta \cdot m_h / m_s$ by setting, e.g., $\log(M_1^2/M_2^2) = 2$, one finds $m_h/m_s \cong 200$ which gives $m_h = 20$ GeV for $m_s = 100$ MeV (PCAC estimates give ⁸⁾ m_s to be of the order of 100 MeV). This value of m_h is, of course, very uncertain due to the uncertainty in the value of m_s used, and the unknown

logarithmic factor, and we regard the estimate above only as a qualitative one : m_h must be very heavy ($m_h \gtrsim 10$ GeV). From Eq. (4) we deduce $\theta'/\theta'' = m_h/m_b$. Taking $m_b \simeq 5$ GeV, we conclude that θ'' is significantly smaller than θ' . The same mechanism which generates the Cabibbo angle generates a mass for the d quark, but not for the u quark, i.e., one finds an explanation for the presence of an isospin breaking term in the quark mass matrix which causes, e.g., the neutron to be heavier than the proton ^{F4)}.

8) The U spin weak interaction which generates θ' and θ'' generates also an effective Hamiltonian relevant for the non-leptonic weak decay of strange particles, which is pure $|\Delta I| = \frac{1}{2}$ and corresponds to the reaction $s \rightarrow d + \text{gluon}$. One finds the induced Hamiltonian

$$H = - \tau \cdot \frac{G}{\sqrt{2}} \cdot \frac{g_{st}}{4\pi^2} \bar{s} \sigma_{\mu\nu} G^{\mu\nu} (m_b d_L + m_b d_R) + h.c. \quad (5)$$

where g_{st} is the quark-gluon coupling constant, $G_{\mu\nu}$ the colour octet gluon field strength operator (we neglect colour indices), and $r = \frac{1}{2} M_W^2 (M_2^{-2} - M_1^{-2})$ (r is assumed to be of order 1). This Hamiltonian is enhanced relative to the conventional current-current Hamiltonian by the large values of m_h and m_b . Furthermore it is not multiplied by $\sin\theta'$. We assume that the Hamiltonian (5) constitutes the dominant non-leptonic Hamiltonian relevant for the weak decay of strange particles. A similar effective Hamiltonian has been discussed recently ¹⁰⁾, and all conclusions drawn there can be applied here. As emphasized, the d quark should enter the non-leptonic weak Hamiltonian predominantly left-handedly, which implies $m_h/m_b \gtrsim 3$. Thus as a possible spectrum for the heavy quark flavours emerges : $m_c \sim 2$ GeV, $m_b \sim 5$ GeV, $m_h \gtrsim 15$ GeV. The V spin weak interaction generates a new $|\Delta I| = \frac{3}{2}$ term in the non-leptonic weak interaction proportional to $\sin\theta''$ in which d_R enters. It is required that this interaction is suppressed compared to the conventional term ¹⁰⁾, implying $m_b M_W^2 / m_h M_V^2 \lesssim 1/3$. Thus the V spin weak interaction Fermi constant cannot be larger than the observed W spin Fermi constant ^{F5)}.

Finally we discuss the construction of a unified theory of the strong, electromagnetic and weak interactions, based on a simple gauge group ¹³⁾. The strong interactions are supposed to be described by QCD, based on the unbroken gauge group $SU(3)^{\text{colour}}$. A natural possibility to describe all interactions within a simple gauge group would then be one based on an exceptional group. Those groups (e.g., E_6, E_7) have typically products of $SU(3)$ groups as subgroups. Such models have been discussed recently ^{4), 14), 15)}, but they suffer from describing the weak interactions in a rather artificial way ; our

scheme of the weak interactions leaves a unified theory based on one of the exceptional groups as an attractive possibility. In particular the semi-simple group $SU(3)^{\text{colour}} \times SU(3)^{\text{weak}}$ is naturally embedded in E_7 . The fermions are supposed to transform under E_7 as the fundamental 56 representation which breaks up under $SU(3)^{\text{colour}} \times SU(3)^{\text{weak}}$ as $2 \times (3,3) + (1,8) + 2 \times (1,1) + \text{antiparticles}$, i.e., the correct fermion content under $SU(3)^{\text{weak}}$ is obtained. In addition to the lepton octet there are two neutral leptons which decouple from the normal weak interactions. Since in the E_7 scheme baryon number is not conserved, the unifying bosons must be very heavy ^{4),15)}. A possible symmetry breakdown is then given by the chain

$$E_7 \rightarrow SU_3^{\text{weak}} \times SU_6 \rightarrow SU_3^{\text{weak}} \times SU_3^{\text{colour}}$$

[here the group $SU(3)^{\text{weak}}$ is broken off first ; this type of symmetry breaking is able to generate the relatively large value of the ratio g_{st}/g (see also Ref. 12)] ^{F6)}.

Finally we emphasize the main consequence of the $SU(3)$ scheme discussed here : the structure of the weak interactions is much more involved and much richer as in the $SU(2) \times U(1)$ scheme. If $SU(3)$ is indeed the gauge group of the weak and electromagnetic interactions, many new effects are waiting to be discovered in neutrino production and e^+e^- annihilation at high energies ^{F7)}.

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FOOTNOTES

- F1) Some phenomenological aspects of three "heavy" quark flavours were discussed in Ref. 3). We follow the notation of Ref. 4).
- F2) The mass pattern [Eq. (2)] can, for example, be obtained by using an octet of scalar and pseudoscalar fields (coupled to the gauge bosons) to break the symmetry such that the 6th and 7th components, respectively, develop non-zero vacuum expectation values.
- F3) We are aware of the fact that this constraint is a constraint on the fermion mass counterterms. We have no reason besides phenomenological ones to justify it.
The weak mixing angles are typically of the form $\Theta \sim \alpha$ fermion mass ratio. Thus one can divide these angles into two classes :
a) the mass ratio is $\lesssim 1$, i.e., the angle is very small ;
b) the mass ratio is large, and the angle, although of order α/π , is much bigger than naively expected (e.g., the Cabibbo angle).
- F4) We are led to a picture of the light quark masses very similar to the one conjectured purely on numerical grounds some time ago ⁹⁾. The author of Ref. 9) assumed $m_u \simeq 0$, $m_d/m_s = (\Theta')^2$, while we find $m_u \simeq 0$, $m_d/m_s \simeq \Theta' \Theta'' = (\Theta')^2 m_d/m_h$. Typical values of this ratio are $(0.23)^2$. $m_b/m_h \sim 1/30 \dots 1/60$. Using "strong" PCAC, one obtains in case $m_u \simeq 0$: $m_d/m_s \simeq M_\pi^2/M_K^2$. $F_\pi/F_K \simeq 1/18$. We find m_d/m_s smaller than $1/18$ (typically by a factor ≥ 2) implying that there should exist rather large corrections to the naive PCAC result. Note that a naive treatment of the $K^+ - K^0$ mass difference (neglecting the electromagnetic term) gives $\Delta M_K^2/M_K^2 \simeq m_d/m_s \simeq 1/60$, i.e., the correct ratio of m_d/m_s may well be a value between $1/18$ and $1/60$.
- F5) The U spin weak interaction generates terms contributing to the $K_1 - K_2$ mass difference, which are enhanced compared to the situation in the six-quark scheme of Ref. 6) [with the $(\bar{c}s)_R$ current] by a factor $\cot^2 \Theta'$. $\cdot (m_h/m_c)^2 \cdot r^2$, which is of order $(10^2 \dots 10^3) \cdot r^2$. Using the vacuum insertion approximation in the corresponding $|\Delta S| = 2$ Hamiltonian, and $r \sim 1$, one would find a $K_1 - K_2$ mass difference which is too large ¹¹⁾. However, the vacuum insertion approximation has been criticized recently ¹²⁾. Nevertheless we prefer the ratio r to be smaller than one such that $m_h/m_c \cdot r \sim 1$, i.e., the U spin Fermi constant is smaller than G .
- F6) It is possible that the breakdown of a simple gauge group (e.g., E_7) yields the group $SU(3)_{\text{weak}} \times U(1)^N$ ($N = 1, 2, \dots$) as the gauge group of the weak and electromagnetic interactions. In this case our conclusions about the neutral current have to be modified. In particular it may acquire a vector part in the hadronic sector.
- F7) Details of this scheme, in particular of the symmetry breaking, will be discussed in a subsequent paper.

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FIGURE CAPTION

Quark-lepton scheme. The associated scheme for the antiparticles can be obtained by a CP transformation (note : $\nu_L = \nu_e$, $\nu_R = \bar{\nu}_\mu$).

FIG.1