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A SIMPLE PHENOMENOLOGICAL DESCRIPTION OF HADRON PRODUCTION

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ABSTRACT

The inclusive production distributions of the non-leading particles in pp collisions are fitted by a simple general formula. The contributions from the ρ^0 , ω , ϕ , and J/ψ decays to the direct lepton spectra are calculated. Possible contributions from the semi-leptonic decay of the conjectured charmed meson are discussed.

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1. INTRODUCTION

The abundance of data on inclusive hadron production in nucleon-nucleon collisions can give extensive tests of phenomenological models attempting to describe that production. In a recent letter¹⁾ we have shown that a simple parametrization (BG_1) gave an adequate description of the π , ρ^0 , and J/ψ data for the energy range $\sqrt{s} = 20$ -53 GeV. In this article we will generalize the BG_1 type of description to all non-leading particles in a wider range of energies from $\sqrt{s} = 6$ GeV (PS and AGS) to $\sqrt{s} = 63$ GeV (ISR). Some properties of the leading-particle spectra will also be described. A straightforward extrapolation of the parametrization as a function of the mass will then be used to predict the production cross-sections of the hypothetical charmed particles in nucleon-nucleon collisions. Finally, we will discuss the contributions to the single-lepton spectra from the various decays.

2. NEGATIVE PION INCLUSIVE CROSS-SECTIONS

The π^- distributions were fitted in BG_1 by the expression

$$E \frac{d^3\sigma}{d^3p} = \frac{A}{(E_T + B)^C} f(y) \begin{cases} e^{-p_T} & \text{for } p_T < 1 \text{ GeV/c} \\ e^{-D(p_T-1)/\sqrt{s}} e^{-1} & \text{for } p_T > 1 \text{ GeV/c} \end{cases} \quad (1)$$

with a smooth transition around $p_T = 1$ GeV/c and $B = 2.7$ GeV, $C = 16.5$, $D = 25$ GeV, and $E_T = (m_\pi^2 + p_T^2)^{1/2}$. An explicit analytical form was not given for $f(y)$. We will include the $\sqrt{s}$ dependence at low p_T in the function $f(y)$. Since, for any value of p_T , the inclusive cross-section has to vanish for the maximum value of y , $y_{\max}(p_T)$, allowed by kinematics, $f(y)$ is *a priori* a function of three variables y , $\sqrt{s}$, and $y_{\max}(p_T)$ with

$$y_{\max}(p_T) = \ln \left(\frac{E_{\max} + p_L^{\max}}{E_T} \right)$$

$$p_L^{\max} = (p_{\max}^2 - p_T^2)^{1/2}.$$

$p_{\max}$ is the maximum allowed pion momentum. We assume that $f(y)$ is only a function of $Y = y_{\max} - y$:

$$Y = \ln \left(\frac{E_{\max} + p_L^{\max}}{E + p_L} \right). \quad (2)$$

The fits presented below will demonstrate that the variation of Y with $\sqrt{s}$ is sufficient to account for the observed $\sqrt{s}$ dependence of the invariant cross-section at low p_T . The new variable Y is invariant under Lorentz transformations along

the beam direction which makes the expression given in Eq. (1) invariant under the same transformations. A preliminary inspection of the observed $d\sigma/dy$ distributions led us to choose a form:

$$f(Y) = e^{-\alpha/Y^\beta} . \quad (3)$$

In BG₁ the values of the parameters B, C, and D were determined by a fit to the Chicago-Princeton (C-P) data²⁾ on π^- at $\sqrt{s} = 23$ GeV. The formula obtained gave a reasonable fit to the British-Scandinavian (B-S) data³⁾ at $\sqrt{s} = 23$ GeV and 53 GeV. Since we now want to describe the data in a much wider energy domain and for a great variety of particles, we must examine more critically the compatibility between the C-P data extrapolated from nuclear targets to hydrogen and the B-S data on pp collisions. As can be seen from Figs. 1-3, which include both sets of π^- data, there are indications that the extrapolated data give a somewhat steeper slope as a function of p_T than the pp data. The effect becomes very clear when one does the same comparison for K^- and $\bar{p}$. The ratio $\bar{p}/\pi^-$ obtained in the two experiments is shown as a function of p_T in Fig. 4. The C-P data give consistently a ratio about two times lower than the B-S data, and the divergence between the two sets of data seems to increase with p_T . The effect is increasingly noticeable going from π^- to K^- and finally to $\bar{p}$ and it sets in at correspondingly lower p_T . The extrapolation of the C-P data assumes a dependence of the cross-section with the atomic number A of the form $\sigma(A) = A^n \sigma_{pp}$, where n is a function of p_T and of the type of particle. The value of n is determined for each p_T from a least-squares fit to the Be, Ti, and W cross-sections. In general, $n_{\pi^-} < n_{K^-} < n_{\bar{p}}$ and the discrepancy increases with n. An obvious remedy is that n might also depend upon A. For our fits we will only use the C-P data on pions with great caution, especially at high p_T .

From Eq. (1) it is clear that at low p_T the logarithmic slope of the invariant cross-section fixes the value of B/C and it is only from the behaviour at large p_T that C, and therefore also B, are separately determined. For all the pion fits we have increased the errors on the C-P data, eliminated the highest p_T point, and fitted simultaneously the B-S data at 23 GeV and 53 GeV and the C-P data. We have also checked that, under these conditions, the values of the parameters given by the fit were only slightly modified when the C-P data were removed altogether. To determine $f(Y)$ we have used the data available²⁻⁶⁾ at $\sqrt{s} = 6, 23$, and 53 GeV covering the range $0.07 < Y < 6$.

The results of the fit are

$$\left. \begin{aligned} B &= 2 \text{ GeV}, C = 12.3, D = 23 \text{ GeV} \\ \alpha &= 5.13, \beta = 0.38 . \end{aligned} \right\} \quad (4)$$

Figure 1 shows the 90° data of the C-P and the B-S collaborations at $\sqrt{s} = 23$ GeV and at $\sqrt{s} = 53$ GeV together with the fits obtained. To test how well the factorization of the invariant cross-section $E \frac{d^3\sigma}{d^3\vec{p}} = f(Y)g(p_T)$ works, $f(Y)$ and $g(p_T)$ are displayed in Figs. 2 and 3, respectively, together with data available over most of the phase space for $\sqrt{s} = 6.15$ GeV, 23 GeV, and 53 GeV. The fits in p_T and in Y are very satisfactory except for $Y < 0.1$, where the formula gives too small a cross-section. The discrepancy reaches even a factor of five for very low values of Y , but this is not by itself an argument against the factorization. It simply shows that the function we have chosen to represent $f(Y)$ falls too rapidly for very small values of Y . Since for $Y < 0.1$, $f(Y)$ is down by a factor larger than 2×10^4 with respect to its "plateau" value measured up to $Y = 6$, we consider that the factorized form given in Eq. (1) is a good fit to the data for $6 \text{ GeV} < \sqrt{s} < 53 \text{ GeV}$.

For convenience we rewrite the formula as

$$E \frac{d^3\sigma}{d^3\vec{p}} = A_{\pi^-} \left(\frac{2}{E_T + 2} \right)^{12.3} e^{-5.13/Y^{0.38}} \times \begin{cases} e^{-p_T} & \text{for } p_T < 1 \text{ GeV/c} \\ e^{-23(p_T-1)/\sqrt{s}} e^{-1} & \text{for } p_T > 1 \text{ GeV/c} \end{cases} \quad (5)$$

where p_T is in GeV/c, E_T in GeV and

$$A_{\pi^-} = (3.78 \times 10^{-24}) \text{ cm}^2 \cdot \text{GeV}^{-2} \cdot c.$$

The formula can be applied to π^0 and π^+ , but it does not account for the increase of the π^+/π^- ratio for large values of $x = 2p_L/\sqrt{s}$, a leading-particle effect.

3. ANTIPROTON AND NEGATIVE KAON INCLUSIVE CROSS-SECTIONS

The antiproton inclusive cross-section has an intriguing behaviour as a function of p_T , as illustrated by Fig. 4, where the ratio $\bar{p}/\pi^-$ at 90° is plotted as a function of p_T for $\sqrt{s} = 23$ GeV. The two important characteristics of the antiproton cross-sections are

- i) the percentage of $\bar{p}$ amongst the particles emitted in the interaction has a maximum around $p_T = 1.5$ GeV/c as illustrated by Fig. 4 for the ratio $\bar{p}/\pi^-$;
- ii) for any p_T the antiproton cross-section increases faster with $\sqrt{s}$ than the π and K cross-sections, as can be seen in Fig. 5.

We have already discussed the discrepancy between the C-P values obtained by extrapolation of measurements on nuclei and the B-S values, and when fitting the antiproton data we will not use the C-P measurements.

To explain qualitatively the two characteristics we assume that due to a local conservation of the baryonic number an antiproton is always emitted accompanied very closely in momentum space by a nucleon. The antiproton-nucleon object R of variable mass is the "particle" which must be described by a production formula and then "decays" into antiproton and nucleon. If R is emitted with a transverse momentum p_T^R , the maximum transverse momentum of either decay products is given by:

$$p_T^{\max} = \frac{p_T^R}{2} \left(1 + \frac{1}{\beta_R} \frac{2p^*}{M_R} \right), \quad (6)$$

where $\beta_R = p_R/E_R$ and p^* is the momentum of the "decay" particle in the R c.m. system. For values of β_R close to 1, as $p^* \leq M_R/2$, $p_T^{\max} = (1/\alpha) p_T^R$ with $\alpha \geq 1$. If the p_T dependence of the R production is of the form $g(p_T^R)$, the antiproton invariant cross-section will exhibit a dependence $g(\alpha p_T^R)$ at large p_T . An extreme case corresponds to $M_R = M_{\bar{p}} + M_p$ which gives, for all values of β_R , $\vec{p}_R = 2\vec{p}_{\bar{p}}$. Equation (6) is valid for ordinary decays; in particular, for a decay into two massless particles $\alpha = 1$ and for large values of p_T^R , the daughter p_T distribution is parallel to the parent distribution. With such a model, if one assumes that the invariant cross-section of the "particle" R has the same general form as the pion cross-section, the slope of the $\bar{p}$ cross-section will be steeper than the pion slope at large p_T . We will show below that $p^* \approx 500$ MeV/c gives a good description of the $\bar{p}$ spectrum; this corresponds to $\alpha = 1.33$. For small p_T values, it is well known that the slope gets flatter for increasing masses [this is, of course, also a property of Eq. (5)], and the combination of that fact with the effect at large p_T will reproduce the first characteristic of the $\bar{p}$ spectrum. If we use the pion formula to describe the invariant cross-section, the second characteristic is qualitatively an immediate consequence of the Y and p_T dependence of Eq. (5).

To perform explicit computations, we have used Eq. (5) to describe the production of R for discrete masses. The "decay" of R into nucleon-antiproton is assumed to be isotropic. The shapes of the calculated antiproton p_T distributions are compared to the 90° data in Fig. 4, which shows that $M_R = 2.15$ GeV gives a good fit to the B-S data and that the shape changes very fast with M_R . Since no strong enhancement has been observed for an antiproton-nucleon mass of 2.15 GeV, a mass distribution must be used for M_R instead of an average mass. To introduce this distribution $F(M_R)$, the invariant cross-section of the $\bar{p}N$ object is written as:

$$\frac{\partial}{\partial M_R} \left(E \frac{d^3\sigma}{d^3\vec{p}} \right) = A_{\bar{p}N} F(M_R) \left(\frac{M_R + 2}{M_{\bar{p}} + M_p + 2} \right)^{12.3} g(p_T^R, E_T^R) f(Y^R), \quad (7)$$

where $g(p_T, E_T)$ and $f(Y)$ are, respectively, the transverse and the Y components of Eq. (5). The factor $[(M_R + 2)/(M_p + M_{\bar{p}} + 2)]^{12.3}$ has been introduced only as a matter of convenience. With this factor the variation of the invariant cross-section with M_R is directly given by the variation of $F(M_R)$.

The choice of $F(M_R)$ is somewhat arbitrary and we find that distributions of the type $e^{-8 \times \Delta M_R}$, $e^{-\Delta(M_R^2)/0.8}$ and $(\Delta M_R/M_R)^{1/2} e^{-\Delta(M_R^2)/0.7}$, where $\Delta M_R = M_R - M_p - M_{\bar{p}}$ fit the data with comparable success. The average c.m. momentum of $\bar{p}$ (or N) in the "decay" of R is 400-500 MeV/c for the three types of distributions as well as for a discrete mass $M_R = 2.15$ GeV. In Fig. 6 the invariant cross-section computed with $F(M_R) = e^{-\Delta(M_R^2)/0.8}$ is compared to the B-S data at 90° and $\sqrt{s} = 23$ GeV. The normalization imposed by the data is such that

$$A_{\bar{p}N} \int F(M_R) dM_R = 22 A_{\pi^-} . \quad (8)$$

The factor 22 between the pion and the $\bar{p}N$ normalizations may be interpreted as representing the ratio between the area of the mass distribution and the probability of emission of a single particle, in which case one would have $A_{\bar{p}N} = A_{\pi^-}$. The normalization factor is slightly different for the other types of mass distributions mentioned above. We have computed the inclusive total cross-section for the production of $\bar{p}$ in pp collision as a function of $\sqrt{s}$ using $F(M_R) = e^{-\Delta(M_R^2)/0.8}$ and the normalization given in Eq. (8). The result is shown in Fig. 7 together with the values obtained from the data by Antinucci et al.⁷⁾. These authors estimate that the accuracy of their compiled values is about 15%. The agreement between our calculation and the data points is remarkably good.

An additional comparison with the data is shown in Fig. 5. The data points represent the ratio between the cross-sections at $\sqrt{s} = 63$ GeV and $\sqrt{s} = 23$ GeV as a function of p_T for $\theta = 90^\circ$ derived by the B-S group using their own measurements. The curves are calculated with our model. The fits to the π^- , π^+ , and $\bar{p}$ data are excellent.

The last part of this section will deal with the parametrization of the K^- inclusive cross-sections. The framework is the same as for the $\bar{p}$ cross-sections, this time with the assumption of a local conservation of strangeness and baryon number. An antikaon is always emitted accompanied very closely in momentum space by a kaon. The mass distribution $F(M_R)$ of the $(K\bar{K})$ system is, however, not as simple to parametrize as the $(\bar{p}N)$ distribution. Combining the information on the invariant cross-section distributions as a function of p_T and on the inclusive total cross-section for K^- 's, we find that a distribution of the type

$$F(M_R) = 3 \times e^{-8 \times \Delta M_R} + e^{-\Delta M_R^2}$$

gives an adequate fit to the p_T distributions at $\theta = 90^\circ$ with a correctly normalized K^- total cross-section. A part of the low mass peak should be regarded as due to the ϕ meson which is produced at a rate of about 12% of the K^- inclusive cross-section. The average momentum of the K^- (or K) in the centre of mass of R is 350 MeV/c. The normalization to the data gives

$$A_{K^-K} \int F(M_R) dM_R = 3A_\pi . \quad (9)$$

We have computed the inclusive total cross-section of K^- as a function of $\sqrt{s}$ using our formula. The result is plotted in Fig. 7 together with the points obtained from a compilation of the data by Antinucci et al.⁷⁾. The estimated accuracy of the data points is about 15% as in the $\bar{p}$ case. It must be clear that the excellent agreement between the curve and the data points is not a check of the specific form used for $F(M_R)$. The shape of the curve is simply given by the $\sqrt{s}$ dependence of our formula, mostly carried by $f(Y)$, for a mass of about 1.3 GeV. The calculated ratio between the K^- cross-sections at $\sqrt{s} = 63$ GeV and $\sqrt{s} = 23$ GeV as a function of p_T for $\theta = 90^\circ$ is in good agreement with the data of the B-S group as shown by Fig. 5.

To conclude this discussion we think that the K^- data give somewhat less evidence for a composite description than the $\bar{p}$ data. In particular, one cannot use an average mass to describe the $\bar{K}K$ system, because the effective mass distribution is too wide. This is probably due to the complexity of the contributions to the K^- spectrum from K^* 's, ϕ , and other resonances. For the discussion of composite systems in the remainder of this paper we shall assume that the average value of $\Delta(M_R^2)$ is the same as for the $\bar{p}N$ system, which is equivalent for a two-particle system to an average value of the "decay" centre-of-mass momentum $\langle p^* \rangle = 400$ MeV/c.

4. INCLUSIVE CROSS-SECTIONS FOR ρ^0 , ω^0 , and ϕ

We have already shown in BG₁ that the type of production formula we use gave good fits to the observed p_T and x distributions for ρ^0 . In Fig. 8 is shown the p_T distribution of ρ^0 's obtained by the CHIF Collaboration⁸⁾ from dimuon events in neutron beryllium interactions at an average energy of 320 GeV. The dashed curve is obtained by using Eq. (5) to describe ρ^0 production. The full curve is also computed from Eq. (5), but with a smearing function $e^{-10(\Delta p_T)^2}$, which takes into account the uncertainty in p_T due mostly to the multiple scattering of the decay muons. The excellent agreement between the smeared curve and the data shows that the mass dependence of the slope introduced by the term in E_T of Eq. (5) is perfectly adequate. Of course, both ω^0 and ρ^0 contributions are included in the

experimental data, but since the masses are very close no difference is expected between the slopes for these two particles.

To obtain the normalization constant A_{ρ^0} , two types of information are available. At $\sqrt{s} = 53$ GeV Darriulat et al.⁹⁾ have measured the ratio of the invariant cross-sections $\rho^0/\pi^0 = 0.9 \pm 0.2$ at $p_T = 3.5$ GeV/c and $\theta_{cm} \approx 90^\circ$, where " π^0 " refers to the π^0 invariant cross-section corrected for the ρ contribution, with the assumption $\rho^0 = (\rho^+ + \rho^-)/2$. On the other hand, inclusive total cross-sections for ρ^0 production have been measured in two bubble-chamber experiments. Blobel et al.¹⁰⁾ get $\sigma_{\rho^0} = (1.80 \pm 0.25)$ mb with a proton incident momentum of 12 GeV/c and $\sigma_{\rho^0} = (3.49 \pm 0.42)$ mb at 24 GeV/c. At 205 GeV/c, Singer et al.¹¹⁾ find $\sigma_{\rho^0} = (10.4 \pm 3.1)$ mb. Using a form $g(p_T, E_T) f(Y)$ for the invariant cross-section, the requirements that the slope fits the CHIF data and that the behaviour at large p_T is in agreement with the Darriulat et al. measurement⁹⁾ at $p_T = 3.5$ GeV/c determine completely the inclusive cross-section. Under those conditions, even by taking $\rho^0/\pi^0 = 1.1$ at $p_T = 3.5$ GeV/c and $\sqrt{s} = 53$ GeV, the inclusive cross-sections fall short of the measured values by 20 to 30%. An increase of the low p_T slope to agree with the cross-sections would yield an unacceptable disagreement with the CHIF data. It is not unreasonable to attribute the moderate discrepancy to a failure of the factorization assumption. However, the evaluation of the inclusive ρ^0 cross-section is very delicate, because of the high combinatorial background inherent to such measurements. For instance, the value published by Singer et al.¹¹⁾ is larger by a factor 1.6 than the value announced previously with the same data, because of a different background subtraction.

Another possibility, perhaps unlikely, to reconcile the different data would be to assume that ρ 's have a distribution given by Eq. (5) multiplied by $(1 + \cos^2 \theta)$. Since the ρ/π measurement of Darriulat et al.⁹⁾ is made near $\theta = 90^\circ$, it would not be affected. In the available data no check has been made for a possible ρ/π anisotropy. Since the dimuon data and the ρ/π ratio at high p_T are not plagued by high background problems, we will choose $A_{\rho^0} = 1.3 A_{\pi^-}$, which corresponds to $\rho^0/\pi^0 = 1.1$ at $p_T = 3.5$ GeV/c and $\sqrt{s} = 53$ GeV. The value chosen for A_{ρ^0} is a factor 1.4 larger than in BG₁, where the normalization had been found in a more indirect and less reliable way. The upper curve drawn in Fig. 9, together with the data points represents the total cross-section σ_{ρ^0} as a function of $\sqrt{s}$ obtained by using Eq. (5) with $A_{\rho^0} = 1.3 A_{\pi^-}$.

There is almost no data on ω^0 production in pp collisions at high energy. Blobel et al.^{10,12)} have compared the exclusive channels $p + p \rightarrow \rho^0$ (or ω^0) + charged. They find $\rho^0/\omega^0 = 1.0 \pm 0.2$ at $\sqrt{s} = 5$ GeV and $\rho^0/\omega^0 = 1.07 \pm 0.2$ at $\sqrt{s} = 6.8$ GeV, with much less severe backgrounds than in the inclusive measurements. Although these exclusive channels represent only a fraction of the ρ^0 or ω^0 cross-sections, the result strongly suggests that ρ^0 and ω^0 are produced in equal amounts.

Since the masses of these two particles are very nearly equal, the rate equality corresponds with our formula to $A_{\omega^0} = A_{\rho^0}$, which is the normalization we will use.

The description of ϕ production introduces two new problems. The Zweig rule should inhibit the production of ϕ unaccompanied by strange particles, while $\phi K\bar{K}$ production is completely allowed. In this type of situation we will adopt the rule that the contributions of the inhibited and of the allowed processes must be computed separately and added together. We have also to deal, for the first time, with a composite system R of three particles. A distribution $F(M_R) = e^{-\Delta(M_R^2)/0.8}$ will be used as in the $\bar{p}N$ system with, in this case, $\Delta M_R = M_R - M_\phi - 2M_K$. The three-body "decay" of R will be described by phase space. The data consist of

- i) inclusive total cross-section at $\sqrt{s} = 6.84$ GeV, $\sigma_{\text{tot}} = (0.158 \pm 0.035)$ mb measured by Blobel et al.¹³⁾;
- ii) at $\sqrt{s} = 23.7$ GeV, upper limits in the range of 6% to 10% around $p_T = 2.5$ GeV/c for the ratio ϕ/π^- given by Appel et al.¹⁴⁾;
- iii) a ratio $\phi/\rho^0 = 6\%$ obtained from the $\mu^+\mu^-$ data of Antipov et al.¹⁵⁾ at $\sqrt{s} = 11.5$ GeV assuming $\sigma_{\rho^0} = \sigma_{\omega^0}$ and using our own estimate of the relative acceptance for ρ^0 (or ω^0) with respect to ϕ under the experimental conditions. We estimate the percentage error on this ratio to be less than 25%.

For large values of $p_T(\phi)$, the $\phi K\bar{K}$ contribution to the invariant cross-section is negligible, because of the three-body "decay". Therefore, the limits obtained at $\sqrt{s} = 23$ GeV correspond to upper limits for A_ϕ . The best compromise with the data corresponds to

$$A_\phi = 0.2 A_{\pi^-} \quad (10)$$

$$A_{\phi K\bar{K}} \int F(R) dM_R = 6A_{\pi^-}$$

and gives a total cross-section of 0.11 mb at $\sqrt{s} = 6.84$ GeV, a ratio ϕ/π^- of 11% at $p_T = 2.5$ GeV/c for $\sqrt{s} = 23$ GeV, and a ϕ/ρ^0 ratio of 7.3% at $\sqrt{s} = 11.5$ GeV. The cross-sections calculated as a function of $\sqrt{s}$ for the two processes are shown in Fig. 9. The ratio $A_\phi/A_{\pi^-} = 0.2$, equivalent to $A_\phi/A_{\rho^0} = 0.15$, is interpreted as a suppression effect due to the Zweig rule. We get $(\phi K\bar{K}/\phi_{\text{tot}}) = 0.24$ at $\sqrt{s} = 6.84$ GeV where the separate contributions of $\phi K\bar{K}$ and ϕ alone to the total cross-section have been studied by Blobel et al.¹³⁾. They reach the conclusion that the $\phi K\bar{K}$ contribution is negligible. In the reaction

$$p + p \rightarrow K_{\text{identified}}^\pm + h^\mp + \text{anything}$$

they have measured the percentage n_S of events with an additional identified strange particle in the "anything" as a function of the mass M_{Kh} of the (Kh) system. To compute M_{Kh} , it is assumed that $M_h = M_K$. In the ϕ mass region n_S is simply written as

$$n_S = n_S^0 [1 - p(\phi) + p(\phi\bar{K}K)] , \quad (11)$$

where n_S^0 is the percentage of the background events with additional strange particles at the ϕ mass; $p(\phi)$ and $p(\phi\bar{K}K)$ are, respectively, the percentages of ϕ and $\phi\bar{K}K$ events in the peak; $p(\phi) + p(\phi\bar{K}K) \approx 0.33$. The value found for n_S is 0.11 ± 0.02 . The conclusion on the ϕ and $\phi\bar{K}K$ relative contributions depends only upon the value of n_S^0 extrapolated from masses above 1.04 GeV. Given the data of Blobel et al.¹³⁾, we think that the value of 0.18 they obtain for n_S^0 is too high. We get $n_S^0 = 0.14 \pm 0.03$, which combined with $n_S = 0.11 \pm 0.02$ gives

$$r = \frac{\phi\bar{K}K}{\phi\bar{K}K + \phi} = (17 \pm 17)\% . \quad (12)$$

Moreover, in a recent PS experiment with incident π^- at 19 GeV/c¹⁶⁾, a ratio of ≈ 1 has been observed between the exclusive channels $(\phi K^+ K^- p \pi^-)$ and $(\phi \pi^+ \pi^- p \pi^-)$. It is not possible to deduce a value of the ratio between the inclusive cross-sections from that measurement, but $r > 0.1$ is probably a safe lower limit. The production of $\phi K^+ K^-$ has also been observed in $\bar{p}p$ interactions at 3.6 GeV/c¹⁷⁾.

Our composite model of the $\phi\bar{K}K$ production gives the important prediction that at Fermilab and ISR energies more than 50% of the ϕ 's produced in the central region ($|x| \lesssim 0.2$) should be accompanied closely in momentum space by a kaon-antikaon pair. The mean invariant mass of the $\phi\bar{K}K$ system will be about 2.3 GeV.

5. J/ψ AND ψ' PRODUCTION

Since the co-discovery of the J/ψ particle in proton beryllium collisions at $\sqrt{s} = 7.3$ GeV¹⁸⁾, its production has been measured at several energies. We assume that the J/ψ production is described by Eq. (5). The data can be compared to the predictions of our model for the y (or x) and p_T distributions and for the variation of the production cross-section as a function of $\sqrt{s}$. In Fig. 10 the shape of $d\sigma/dx$ obtained from Eq. (5) is compared to the data of Anderson et al.¹⁹⁾ at $\sqrt{s} = 16.8$ GeV. The agreement is very good; the fact that for small values of x the distribution is much flatter for J/ψ production than, for example, for pions is an immediate consequence of the $f(Y)g(p_T)$ factorization, since

$$\frac{d^2\sigma}{dx dp_T^2} \approx \frac{\sqrt{s}}{2E_{cm}} \frac{d^2\sigma}{dy dp_T^2} . \quad (13)$$

For large masses and small values of x , E_{cm} varies very slowly. The shape of $d\sigma/dx$ reproduces the flat shape of $d\sigma/dy$. On the contrary, for small masses, $d\sigma/dx \approx x^{-1} d\sigma/dy$.

In Fig. 11 the $d\sigma/dp_T^2$ distribution obtained in neutron beryllium interactions at $\sqrt{s} = 24$ GeV by the CHIF Collaboration⁸⁾ is compared with the prediction. The full curve is computed from Eq. (5) with a smearing function $e^{-3(\Delta p_T)^2}$ which corresponds to the uncertainty in p_T due to the large amount of material traversed by the decay muons. For comparison, the dashed curve is the $d\sigma/dp_T^2$ distribution obtained from Eq. (5) without smearing. The agreement of the full curve with the data shows that the p_T dependence of Eq. (5) is adequate for J/ψ production at least up to $p_T \approx 2.5$ GeV/c. In Fig. 12 the shape of the invariant cross-section as a function of p_T is compared at $\sqrt{s} = 27.4$ GeV to the data of the Columbia-Fermilab-Stony Brook experiment²⁰⁾ where the e^+e^- decay mode of the J/ψ is measured. The good agreement of the curve with the data points is once again a check of the adequacy of Eq. (5).

The above comparisons show that the model can give a good description of J/ψ production around $\sqrt{s} = 20$ GeV. We will now discuss the J/ψ inclusive total cross-sections over the complete energy range. The measured values^{15,18-22)} are plotted in Fig. 13. We have raised the original cross-sections of Aubert et al.¹⁸⁾ by a factor 1.5 to take into account the fact that the measured p_T distribution is flatter than initially assumed. The CCRS cross-section²¹⁾ has been calculated using the published value of $B d\sigma/dy|_{y=0} = (7.5 \pm 2.5) \times 10^{-33}$ cm² and the y dependence given by our model. That value of $d\sigma/dy$ corresponds to an average $p_T = 0.67$ GeV/c for J/ψ production and should be multiplied by a factor 1.6 for $\langle p_T \rangle = 1$ GeV/c. A new value of the production cross-section at $\sqrt{s} = 24$ GeV will soon be published by the CHIF Collaboration⁸⁾.

All measurements except the ISR points at $\sqrt{s} = 53$ GeV have been obtained from nucleon interactions on nuclei, mostly beryllium. A detailed study by the CHIF Collaboration⁸⁾ has shown that J/ψ production on complex nuclei is proportional to the atomic number A . This A dependence has been assumed for all the data points.

The full curve drawn in Fig. 13 corresponds to the total cross-section calculated using Eq. (5) and a normalization constant

$$A_\psi = \frac{1}{30} A_\pi. \quad (14)$$

The computed cross-section increases by a factor 100 from $\sqrt{s} = 7.3$ GeV to $\sqrt{s} = 53$ GeV. The curve is a good fit to the measurements with the possible exception of the values at large $\sqrt{s}$. The factor $1/30$ can be interpreted as a suppression due to the Zweig rule. As we have done in the ϕ case for the $\phi K \bar{K}$

Assuming that $A_{\psi'} = A_{\psi}$, the ratio between the inclusive cross-sections for ψ' and J/ψ is computed using Eq. (5). It is given in Table 1 for three values of $\sqrt{s}$. This ratio has been measured very recently at $\sqrt{s} = 27$ GeV by the Columbia-Fermilab-Stony Brook group²⁰⁾

$$\frac{\psi'}{J/\psi} = (10 \pm 3)\% ,$$

in agreement with the prediction of our model for $A_{\psi'} = A_{\psi}$.

6. TOTAL CROSS-SECTIONS AND PRODUCTION PROCESSES

We have used extensively Eq. (5) to describe the inclusive invariant cross-sections. That equation can be integrated to get the inclusive total cross-section as a function of the mass m

$$\sigma_{\text{tot}} = A \times 2.3 \times 10^{-30} \times y_{\text{max}} e^{-5.13/y_{\text{max}}^{0.38}} \frac{m}{(m+2)^{12.3}} , \quad (16)$$

where $y_{\text{max}} = \ln [(E_{\text{cm}}^{\text{max}} + p_{\text{cm}}^{\text{max}})/m]$; $p_{\text{cm}}^{\text{max}}$ and $E_{\text{cm}}^{\text{max}}$ are, respectively, the maximum momentum and energy of the "particle" of mass m in the centre-of-mass system.

The rules of the game are the same as for the invariant cross-section:

- for single particles, $A \approx 1$, or the suppression factor when the production is inhibited by the Zweig rule;
- for a composite system, m is the sum of the masses and $A \approx 5-20$;
- the production obeys a local conservation of certain quantum numbers like baryon number and strangeness;
- when a process is suppressed by the Zweig rule, the inclusive total cross-section is the sum of the contributions of Eq. (16) for that process and for the simplest Zweig-allowed system which includes the same particle.

With this simple set of rules we have been able to describe over a wide range of energy the inclusive distributions and cross-sections of the non-leading particles from the pion mass up to the J/ψ and ψ' masses. That mass range corresponds to five orders of magnitude for Eq. (16). This formula gives also a reasonable estimate for what could be the cross-section of the hypothetical $J/\psi D\bar{D}$ production, another three orders of magnitude down.

A detailed study of the leading-particle distributions goes beyond the scope of this article, but we want to make a few observations which are direct consequences of our model.

- i) Since in the non-leading mode the proton-antinucleon and the antiproton-nucleon distributions are the same, the difference between the proton and antiproton distributions represents the leading protons. In Fig. 14 we have plotted as a function of p_T the data of the B-S Collaboration³⁾ at 90° for the quantity $\Delta\sigma = (\sigma_{\text{inv}}^p - \sigma_{\text{inv}}^{\bar{p}}) \times e^{4p_T}$ at $\sqrt{s} = 23, 31, 45$, and 53 GeV. For each energy a horizontal line has been drawn at a level of 4 mb. The $\sqrt{s}$ dependence of $\Delta\sigma$ is weak in sharp contrast with the behaviour of the antiproton cross-section. The dashed line shows how the 4 mb level of $\sqrt{s} = 23$ GeV would look at $\sqrt{s} = 53$ GeV if $\Delta\sigma$ had the same $\sqrt{s}$ dependence as the antiproton cross-section.
- ii) A similar argument can be applied to the difference between K^+ and K^- distributions, which corresponds to the excitation of the leading proton into a YK^+ system. The 90° distributions have only a weak dependence upon $\sqrt{s}$ as in the leading-proton case. The difference $[\sigma_{\text{tot}}^{\text{incl}}(p + p \rightarrow K^+ + \dots) - \sigma_{\text{tot}}^{\text{incl}}(p + p \rightarrow K^- + \dots)]$ rises from 2.5 mb at $\sqrt{s} = 6.15$ GeV to a "plateau" value of about 3.5 mb in the ISR energy range.

7. DIRECT LEPTON SPECTRA

In this section we will use the parametrization and the normalizations of the vector meson and the ψ productions discussed in Sections 3 and 4. We will not study in detail the contribution from kaon and hyperon decays to the lepton spectra. The sum of those contributions corresponds always to lepton/pion ratios smaller than 10^{-5} for values of the lepton transverse momentum larger than 1 GeV/c. At low p_T the $K_{\ell 3}$ contribution may become important and it is subtracted in the published data. As the hypothetical charmed meson D ($\bar{D}$) may give an important contribution to the lepton spectra through its semi-leptonic decays, we will now discuss the prediction of our model for the D ($\bar{D}$) production in proton-nucleon collisions. Our rule of local conservation of quantum number applies to charm and the production of the composite system $R = D + \bar{D}$ is described by the equivalent of Eq. (7) for D and $\bar{D}$ with $F(M_R) = e^{-\Delta(M_R^2)/0.8}$. Figure 15 represents the inclusive total cross-section for $D\bar{D}$ production as a function of $\sqrt{s}$ with a similar choice of normalization as for $\bar{p}N$ production

$$A_{D\bar{D}} \int F(M_R) dM_R = 20 A_{\pi^-} \quad (17)$$

for $M_D = 2.2$ GeV and $M_{\bar{D}} = 1.8$ GeV.

The contributions of the various decays to the ratio ℓ/π between the lepton and pion invariant cross-sections at 90° are plotted as a function of p_T in Fig. 16 for $\sqrt{s} = 23$ GeV and in Fig. 17 for $\sqrt{s} = 53$ GeV²³⁾. For the charmed-meson

contribution we have assumed a $D \rightarrow K\ell\nu$ decay with a V-A matrix element. The curves labelled $D\bar{D}$ in Eqs. (16) and (17) correspond to $M_D = 2.2$ GeV, with $\sigma_B = 1.8 \times 10^{-30}$ cm² at $\sqrt{s} = 53$ GeV and $\sigma_B = 0.4 \times 10^{-30}$ cm² at $\sqrt{s} = 23$ GeV. With the normalization of Eq. (17), these values correspond to $B = 15\%$. The D contribution depends only weakly upon the exact form taken for the matrix element.

Since the p_T distribution of J/ψ production is only measured up to about 2 GeV/c (see Figs. 11 and 12), the J/ψ contribution to the ℓ/π ratio beyond $p_T \approx 2.5$ GeV/c is entirely determined by the behaviour of our production formula at large p_T .

In Fig. 16 the unlabelled dashed curve represents a subtotal, which does not include the D contribution. In the top part of the figure that curve and the total full curve are compared to the 90° data of the C-P group²⁴⁾ for μ/π and that of the Columbia-Fermilab group²⁵⁾ for e/π . A similar comparison is done in Fig. 17 at $\sqrt{s} = 53$ GeV with the final data of the CCRS group²⁶⁾ at $\theta_{cm} = 90^\circ$ and to the CHORMN data²⁷⁾ at $\theta_{cm} = 30^\circ$. At both energies, the curve with the D contribution fits well the data with the exception of the CHORMN measurement at low p_T . Since the latter data are taken at a different angle, Fig. 18 shows a comparison between the D contributions for $\sqrt{s} = 53$ GeV at $\theta_{cm} = 30^\circ$ and at $\theta_{cm} = 90^\circ$. The curves are computed with our model for two values of M_D . In both cases, the same normalization, given by Eq. (17), has been used for the production cross-sections. Figure 18 illustrates three important facts:

- i) For a given mass the D contribution to the ℓ/π ratio is about 20% lower at $\theta_{cm} = 30^\circ$ than at $\theta_{cm} = 90^\circ$. This effect applies also to the contributions from other decays.
- ii) When M_D decreases from 2.2 GeV to 1.8 GeV, the maximum of the ℓ/π contribution is shifted downwards in p_T by about 150 MeV/c, due to the decrease of the Q value of the decay.
- iii) Except for that small shift the $M_D = 1.8$ GeV and $M_D = 2.2$ GeV curves have similar shapes. For $M_D = 1.8$ GeV values of $\sigma_B = 3.6 \times 10^{-30}$ cm² at $\sqrt{s} = 53$ GeV and $\sigma_B = 0.8 \times 10^{-30}$ cm² would yield total curves for the ℓ/π ratio very similar to those drawn in Figs. 16 and 17. With $B = 15\%$, the corresponding normalization for $M_D = 1.8$ GeV would be 8 instead of 20 in Eq. (17).

Several experimental searches are aiming to observe $e\mu$ coincidences in pp collisions which could be a signature for the production and subsequent leptonic decays of $D\bar{D}$. We have calculated with our model for $M_D = 2$ GeV and $\sqrt{s} = 53$ GeV the predicted distributions of the $e\mu$ mass, the lepton momentum, the lepton angle, and the lepton angular correlation which are shown in Fig. 19. The $e\mu$ mass

spectrum and the lepton momentum distribution reach their maximum values at about 700 MeV/c² and 500 MeV/c, respectively. As an indication of the background shape, we have also drawn in Fig. 19 the negative pion angular distribution, and a comparison between curves (e) and (c) shows that the ratio signal/background is about twenty times better at 90° than in the forward (backward) direction, whereas the signal is only five times lower. The average cosine of the angle between the two leptons is about +0.5.

Is it possible to fit the CHORMN data with the semi-leptonic D decays by lowering the Q value of the decay? In the charm framework it seems difficult for M_D to be much lower than $M_\psi/2$. However, we have only considered the $D \rightarrow K\ell\nu$ decays. A K^* or a $(K + n\pi)$ system in the final state would certainly decrease the Q value. Hinchliffe and Llewellyn Smith²⁸⁾ have studied in some detail the $K^*(890)$ case using our model, but for producing D singly instead of producing a $D\bar{D}$ system. Their conclusion that for $M_D = 1.8$ GeV and $\sigma_B = 30 \times 10^{-30}$ cm² for the decay $D \rightarrow K^*\ell\nu$ a maximum value of $\ell/\pi = 3 \times 10^{-4}$ can be obtained at $p_T = 450$ MeV/c remains valid with our type of production. But this is still a long way from accounting for the CHORMN data. A simple integration shows that a $\sigma_B = 150-200 \times 10^{-30}$ cm² would be necessary to explain fully the data. In addition, because of the experimental level of the ratio ℓ/π around $p_T = 1$ GeV/c, such a large value of σ_B would imply

$$\frac{D \rightarrow K\ell\nu}{\Sigma(D \rightarrow K\ell\nu + n\pi)} \lesssim (2 - 3) \times 10^{-2} ,$$

a rather unlikely prospect.

The Pennsylvania-Stony Brook group²⁹⁾ has recently observed a rise of the e/π ratio at $\theta_{cm} = 90^\circ$ for $p_T < 0.8$ GeV/c in pp collisions for incident momenta 10, 15, and 24 GeV/c. The rise is similar to the CHORMN results, whereas for 0.8 GeV/c $< p_T < 1.5$ GeV/c the data are compatible with the e/π value expected from vector mesons. If one is willing to accept both experimental results and a common source for the low p_T effect, the source cannot be charmed mesons because of threshold effects.

The measurements done at Serpukhov³⁰⁾ at $\theta_{cm} = 90^\circ$ for $p_T > 1.8$ GeV/c and 7 GeV $< \sqrt{s} < 11$ GeV had already shown that under these conditions the μ/π ratio was at the level expected from vector meson contributions. The Pennsylvania-Stony Brook data give a similar result for the e/π ratio in a somewhat lower p_T range. These results indicate that for $p_T > 1$ GeV/c the ℓ/π ratio at $\theta_{cm} \approx 90^\circ$ increases from the vector meson level $\sim 3 \times 10^{-5}$ at low values of $\sqrt{s}$ to 1×10^{-4} at the ISR and Fermilab energies. We have shown that the J/ψ could contribute most of the excess for $p_T > 3$ GeV/c and that if charmed mesons were produced with

cross-sections of 10-25 μb at $\sqrt{s} = 53 \text{ GeV}$, and 2.5-6 μb at $\sqrt{s} = 23 \text{ GeV}$, they would fill in the remaining valley at lower p_T . These values of the production cross-sections are in good agreement with a simple extrapolation in mass of our description for lower mass systems, for instance, the $\bar{p}N$ system.

In Table 2 the various contributions to the ℓ/π ratio are given for six values of p_T and for five values of x . This table gives clear examples of how the kinematics of the parent-daughter relationship coupled in some cases with the production and "decay" of composite systems influence the behaviour of the ℓ/π ratio as a function of x . The centralism hierarchy is as follows:

- i) Single-particle production followed by two-body decay. Example: ρ at $p_T = 1.5 \text{ GeV/c}$; the ℓ/π ratio decreases slowly by about a factor 2.5 between $x = 0$ and $x = 0.7$.
- ii) Single-particle production followed by three-body decay.
- iii) Production of a two-particle composite system followed by a two-body decay, or more central, a three-body decay of the particles. Example of the latter case: $D\bar{D}$ at $p_T = 1.5 \text{ GeV/c}$; the ℓ/π ratio decreases by a factor of about 30 between $x = 0$ and $x = 0.3$.
- iv) Production of a three-particle composite system followed by a two- or three-body decay. Example with two-body decay: $\psi D\bar{D}$ at $p_T = 1.5 \text{ GeV/c}$; the ℓ/π ratio decreases by a factor of about 30 between $x = 0$ and $x = 0.3$.

The numbers given above depend upon the masses of the particles and the Q values of the decays; they are only meant as an illustration of a general behaviour.

In addition to the e/π measurements at low p_T of the CHORMN and the Pennsylvania-Stony Brook groups, there are two low p_T measurements of direct muon production which do not fit into our description:

- a) For $p_T < 400 \text{ MeV/c}$ and $\sqrt{s} = 23 \text{ GeV}$, Buchholz et al.³¹⁾ find $\mu^-/\pi^- = (1.83 \pm 0.42) \times 10^{-4}$ and $\mu^+/\pi^+ = (6.4 \pm 1.3) \times 10^{-5}$ at $x = 0.3$ and $\mu^-/\pi^- = (4.2 \pm 2.3) \times 10^{-5}$ at $x = 0.5$. The ratio μ^+/μ^- is 1.05 ± 0.1 at $x = 0.3$.
- b) At $\sqrt{s} = 27 \text{ GeV}$, Leipuner et al.³²⁾ have measured the μ^- and μ^+ intensities at three average values of x : 0.57, 0.26, and 0.135. Using a parametrization of the pion and kaon production, they compute the decay corrections and the μ^-/π^- and μ^+/π^+ ratios. Their results for negative particles can be parametrized by $\mu^+/\pi^- = (x + 0.1)^{-1} \times (1.2 \pm 0.2) \times 10^{-5}$ and the ratio μ^+/μ^- is about 1.2. Due to multiple scattering, the mean transverse momentum of the direct muons produced in that experiment is uncertain, but is probably not larger than 500-600 MeV/c.

It is clear from Table 2 that the data for $p_T \lesssim 500$ MeV/c cannot be fitted for any value of x with the contributions we have described. It seems premature, in view of the state of flux of some of the data and of the speculative nature of the charm contribution for $1 \text{ GeV/c} < p_T < 2 \text{ GeV/c}$, to elaborate further on the origin of the low p_T leptons.

8. CONCLUSION

We have shown that the inclusive production of non-leading particles in pp collisions can be parametrized with a single and simple expression for the invariant cross-section as a function of $E_T = (m^2 + p_T^2)^{1/2}$, p_T , and y , over a wide range of energies $6 \text{ GeV} < \sqrt{s} < 63 \text{ GeV}$. The description assumes a factorization of the y and (p_T, E_T) dependences which is well verified by the π^- data.

Using the measurements supplemented by our production formula mostly for extrapolation purposes, we have computed the contributions from ρ^0 , ω , ϕ , and J/ψ to the single-lepton spectra. For $p_T \gtrsim 3 \text{ GeV/c}$ these contributions are sufficient to explain the lepton measurements, but they are too low by a factor 2-3 in the range $1 \text{ GeV/c} \lesssim p_T \lesssim 2 \text{ GeV/c}$.

From a simple extrapolation in mass of our production formula, we have deduced production cross-sections for the conjectured charmed meson and we find that the contribution of the $D \rightarrow K\ell\nu$ decay could eliminate the lepton deficit for $p_T \gtrsim 1 \text{ GeV/c}$. Typical production cross-sections for the charmed meson would be $4 \times 10^{-30} \text{ cm}^2$ at $\sqrt{s} = 23 \text{ GeV}$ and $15 \times 10^{-30} \text{ cm}^2$ at $\sqrt{s} = 53 \text{ GeV}$.

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Table 1

Ratio of ψ' and J/ψ production cross-sections

$\sqrt{s}$	7.3 GeV	23 GeV	53 GeV
ψ'/ψ	0.06	0.12	0.20

Table 2

Contributions to lepton/pion ratio in units of 10^{-6} at $\sqrt{s} = 23$ GeV
 $(x = 2p_L/\sqrt{s}, p_T \text{ in GeV/c}). a = \rho + \omega, b = \phi, c = \phi(K\bar{K}),$
 $d = \psi, e = \psi(DD), f = D\bar{D}.$

$x \backslash p_T$	0.1	0.3	0.7	1.5	3.5	5.5	
0	2	13	18	18	13	9	a
		1	4	5	4	3	b
		1	5	2			c
				9	71	90	d
				3			e
	1	5	30	29	2		f
	3	20	57	66	90	102	total
0.1	3	9	17	16	12	8	a
		1	4	5	4	3	b
		1	4	1			c
				9	71	89	d
				2			e
	1	3	12	20	1		f
	4	14	37	53	88	100	total
0.3	2	4	12	13	11	7	a
			2	4	4	3	b
				6	64	78	c
				1			d
							e
							f
	2	4	14	24	79	88	total
0.5	1	2	7	10	8	6	a
			1	3	3	2	b
				3	36	42	c
							d
							e
							f
	1	2	8	16	47	50	total
0.7		2	3	7	5	3	a
			1	2	3	2	b
				1	13	17	c
							d
							e
							f
	0	2	4	10	21	22	total

Figure captions

- Fig. 1 : Invariant cross-sections for π^- inclusive production in proton-nucleon collisions as a function of p_T at 90° in the c.m. system. The curves are calculated from Eq. (5).
- Fig. 2 : The invariant cross-section for π^- inclusive production divided by the function $g(p_T)$ versus $Y = y_{\max} - y$. The data points cover a wide range of c.m. angles and transverse momenta. The curves represent $f(y)$, which is independent of $\sqrt{s}$. To get a separation between the three energies, multiplicative factors of 10 for $\sqrt{s} = 23$ GeV and of 10^2 for $\sqrt{s} = 53$ GeV have been used.
- Fig. 3 : The invariant cross-section for π^- inclusive production divided by the function $f(y_{\max} - y)$ versus p_T . The curves are calculated from Eq. (5). For $\sqrt{s} = 6.15$ GeV and for $\sqrt{s} = 53$ GeV the vertical scale has been shifted for a clearer separation.
- Fig. 4 : The $\bar{p}/\pi^-$ ratio as a function of p_T at $\sqrt{s} = 23$ GeV and $\theta_{\text{cm}} = 90^\circ$. Black dots are obtained from the B-S data and crosses from the C-P measurements. The shapes of the curves are calculated from Eq. (5) for the production of an object of mass M_R which "decays" into anti-proton and nucleon.
- Fig. 5 : Ratios between cross-sections at 63 GeV and 23 GeV as a function of p_T at $\theta_{\text{cm}} = 90^\circ$. The data for $\pi^\pm$, $\bar{p}$, and K^- are compared to the curves from our model.
- Fig. 6 : The invariant cross-section for $\bar{p}$ inclusive production as a function of p_T at $\sqrt{s} = 23$ GeV and $\theta_{\text{cm}} = 90^\circ$. The curve is calculated from Eq. (7) with $F(M_R) = \exp [-\Delta(M_R^2)/0.8]$.
- Fig. 7 : The inclusive total cross-sections for $\bar{p}$ and K^- production as a function of $\sqrt{s}$. The data points are obtained from a compilation by Antinucci et al.⁷⁾. The curves are calculated from Eq. (7), where $F(M_R)$ and the normalizations are given in the text.
- Fig. 8 : dN/dp_T^2 distribution for ρ^0 and ω production. The data of the CHIF Collaboration⁸⁾ at $\sqrt{s} = 24$ GeV are compared to the prediction of Eq. (5) without smearing (dashed curve) and with a smearing function $\exp [-10(\Delta p_T)^2]$ (full curve).
- Fig. 9 : The inclusive total cross-sections for ρ and ϕ production as a function of $\sqrt{s}$. The curves are calculated with our model. At $\sqrt{s} = 23$ GeV the experimental upper limit of the ϕ cross-section is computed by taking $\phi/\pi^- \leq 10\%$ at large p_T ¹⁴⁾.

- Fig. 10 : $d\sigma/dx$ distribution for J/ψ production. The shape obtained from Eq. (5) is compared to the data of Anderson et al.¹⁹⁾ at $\sqrt{s} = 16.8$ GeV.
- Fig. 11 : dN/dp_T^2 distribution for J/ψ production. The data of the CHIF Collaboration⁸⁾ at $\sqrt{s} = 24$ GeV are compared to the prediction of Eq. (5) without smearing (dashed curve) and with a smearing function $\exp[-3(\Delta p_T)^2]$ (full curve).
- Fig. 12 : Production cross-section times branching ratio for $J/\psi \rightarrow e^+e^-$ versus p_T at $\sqrt{s} = 27.4$ GeV. The Columbia-Fermilab-Stony Brook data²⁰⁾ are compared to the shape predicted by our model.
- Fig. 13 : The inclusive total cross-section for the production of J/ψ as a function of $\sqrt{s}$. Some data points have been calculated from the published values as indicated in the text. The full curve has been obtained from Eq. (5) with a normalization constant $A_\psi = A_\pi/30$. The $\psi D\bar{D}$ curve has been computed for $M_D = 2.2$ GeV. The curves labelled D (1.8 GeV) and D (2.2 GeV) represent the sum of the ψ and $\psi D\bar{D}$ contributions for $M_D = 1.8$ GeV and $M_D = 2.2$ GeV, respectively. The normalization of Eq. (17) has been used for the $\psi D\bar{D}$ contributions.
- Fig. 14 : The difference between the proton and antiproton cross-sections multiplied by $\exp(4p_T)$ versus p_T . The data points are from the B-S Collaboration³⁾ at $\theta_{cm} = 90^\circ$.
- Fig. 15 : The inclusive total cross-section for D ($\bar{D}$) production in proton-nucleon collisions as a function of $\sqrt{s}$.
- Fig. 16 : The contributions of the various decays to the ℓ/π ratio as a function of p_T for $\sqrt{s} = 23$ GeV and $\theta = 90^\circ$. In the top part of the figure the data points are compared to the sum of the contributions with $D\bar{D}$ (full curve) and without (dashed curve).
- Fig. 17 : The contributions of the various decays to the ℓ/π ratio as a function of p_T for $\sqrt{s} = 53$ GeV and $\theta = 90^\circ$. In the top part of the figure the data points are compared to the sum of the contributions with $D\bar{D}$ (full curve) and without (dashed curve). For $p_T < 1.3$ GeV/c the CCRS points correspond effectively to an average between the 53 GeV and 45 GeV data.
- Fig. 18 : Contributions to the e/π ratio at $\sqrt{s} = 53$ GeV from $D \rightarrow K e \nu$ for $M_D = 1.8$ GeV and $M_D = 2.2$ GeV at two angles $\theta_{cm} = 30^\circ$ and $\theta_{cm} = 90^\circ$. For both masses the normalization factor of Eq. (17) has been used.

Fig. 19 : Distributions for the leptons from $D\bar{D}$ semi-leptonic decays for $M_D = 2$ GeV and $\sqrt{s} = 53$ GeV in the pp c.m. system. The curves are:

- a) $dN/dM_{e\mu}$ versus the electron-muon mass $M_{e\mu}$ (bottom horizontal scale);
- b) dN/dp_ℓ versus the lepton momentum p_ℓ (bottom horizontal scale);
- c) $dN/d \cos \theta_\ell$ versus the cosine of the lepton angle with the beam direction $\cos \theta_\ell$ (top horizontal scale);
- d) $dN/d \cos \theta_{e\mu}$ versus the cosine of the angle between the two leptons $\cos \theta_{e\mu}$ (top horizontal scale);
- e) $dN/d \cos \theta_{\pi^-}$ versus $\cos \theta_{\pi^-}$ for inclusive negative pion production. The shape should be compared to the curve (c).

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Fig. 19