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INVERSE BREMSSTRAHLUNG PROCESS TRANSITION PROBABILITY

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A B S T R A C T

We present the calculation in quantum electrodynamics of the transition probability for the inverse bremsstrahlung process. The expected number of events is also given for an experiment which was recently proposed to study the inverse bremsstrahlung reaction.

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In a previous note, one of the authors ¹⁾ called attention to an elementary electromagnetic process, the inverse bremsstrahlung, where an electron and a photon beam scatter in the Coulomb field of a target nucleus and electrons of higher energy than the incoming ones emerge from the matter target. It is the purpose of this paper to present the calculation in quantum electrodynamics of the transition probability for the process in question, together with an evaluation of the number of counts per unit time for the proposed experimental conditions.

From the relevant Feynman diagrams, shown in the Figure, the differential transition probability per unit electron, photon and nucleus densities, and unit interaction volume, averaging and summing over polarizations, is calculated ^{*)} :

$$\frac{dW}{d\Omega} = \frac{4\pi Z^2 \alpha^3}{\omega^3 m^4} \frac{p_2}{q^4 \epsilon_1} \times \\ \times \left\{ -q^2 \left[\vec{k} \times \left(\frac{\vec{p}_2}{X_1} + \frac{\vec{p}_1}{X_2} \right) \right]^2 + 4 \left[\vec{k} \times \left(\frac{\epsilon_1 \vec{p}_2}{X_1} + \frac{\epsilon_2 \vec{p}_1}{X_2} \right) \right]^2 - \frac{2\omega^2}{X_1 X_2} \left[\vec{k} \times (\vec{p}_2 - \vec{p}_1) \right]^2 \right\} \quad (1)$$

Here

$$p_{1\mu} = (\vec{p}_1, \epsilon_1), \epsilon_1 = \sqrt{p_1^2 + m^2}; p_{2\mu} = (\vec{p}_2, \epsilon_2), \epsilon_2 = \sqrt{p_2^2 + m^2}; k_\mu = \omega(\hat{k}, 1), \omega = \epsilon_2 - \epsilon_1$$

are the momenta of the incoming and outgoing electrons and of the incoming photon in the system of the target, $q_\mu = (\vec{p}_1 + \vec{k} - \vec{p}_2, 0) \equiv (\vec{q}, 0)$ is the momentum transmitted to the static nucleus of charge Ze , $X_1 = 2p_2 \cdot k/m^2$, $X_2 = -2p_1 \cdot k/m^2$, and m is the electron mass.

One immediately sees from Eq. (1) that $dW/d\Omega$ vanishes when $\vec{p}_1$, $\vec{p}_2$ and $\vec{k}$ are all in the same direction.

In the extreme relativistic limit ($\epsilon_1/m \gg 1$, $\omega/m \gg 1$), $dW/d\Omega$ has a sharp maximum in the neighbourhood of the direction $\hat{p}_1$ of the incoming electron, i.e., when both $\hat{k}$ and $\hat{p}_2$ lie in a cone around $\hat{p}_1$ of angular aperture of order $\Theta \sim m/\epsilon_2$. In particular, for $\hat{k} = \hat{p}_1$, Eq. (1) reduces to

*) We follow the conventions, as well as the notations, of the textbook by Akhiezer and Berestetskii ²⁾. In particular, the square of the amplitude for the inverse bremsstrahlung is easily obtained from the square of the bremsstrahlung amplitude (see their Section 29.2), by simply reversing the sign of the real photon momentum.

$$\frac{dW_p}{d\Omega} = \frac{4\pi Z^2 \alpha^3}{\omega^3 m^4} \frac{p_2}{\epsilon_1 q^4} \left(-\frac{q^2}{\chi_1^2} + \frac{4\epsilon_1^2}{\chi_1^2} - \frac{2\omega^2}{\chi_1 \chi_2} \right) (\vec{k} \times \vec{p}_2)^2 \quad (2)$$

which in the extreme relativistic limit can be written as $(\hat{p}_1 \cdot \hat{p}_2 = \cos\theta)$

$$\begin{aligned} \frac{dW_p^{RL}}{d\Omega}(\epsilon_1, \omega, \theta) &\simeq \\ &\simeq \frac{8\pi Z^2 \alpha^3}{\omega m^2 \epsilon_2^2} \frac{\theta^2}{\left(\theta^2 + \frac{\omega^2 m^4}{4\epsilon_1^2 \epsilon_2^4}\right)^2 \left(\theta^2 + \frac{m^2}{\epsilon_2^2}\right)} \left[1 + \frac{2m^2 \epsilon_1}{\omega^2 \epsilon_2} \frac{1}{\theta^2 + \frac{m^2}{\epsilon_2^2}} \right] \end{aligned} \quad (3)$$

Starting from Eq. (3), the angular integration of the differential probability is easily performed for parallel direction of the photon with the incident electron, getting the result

$$W_p^{RL}(\epsilon_1, \omega) \simeq 16\pi^2 Z^2 \alpha^3 \frac{\epsilon_1^2 + \epsilon_2^2}{m^4 \omega^3} \left[\lg \frac{2\epsilon_1 \epsilon_2}{m\omega} - \frac{(\epsilon_1 + \epsilon_2)^2}{2(\epsilon_1^2 + \epsilon_2^2)} \right] \quad (4)$$

in which terms that are less divergent when m goes to zero were dropped. A lengthier calculation, retaining only the logarithmic term, gives for the general case $(\hat{k} \cdot \hat{p}_1 = \cos\theta_1)$

$$W_p^{RL}(\epsilon_1, \omega, \theta_1) \simeq \frac{16\pi^2 Z^2 \alpha^3}{\epsilon_1^4 \omega^3} \frac{\epsilon_1^2 + \epsilon_2^2}{\left(\theta_1^2 + \frac{m^2}{\epsilon_1^2}\right)^2} \left[1 - \frac{4m^2 \epsilon_2}{\epsilon_1(\epsilon_1^2 + \epsilon_2^2)} \frac{\theta_1^2}{\left(\theta_1^2 + \frac{m^2}{\epsilon_1^2}\right)^2} \right] \lg \frac{2\epsilon_1 \epsilon_2}{m\omega} \quad (5)$$

As it must be, the right-hand side of Eq. (5) reduces to the right-hand side of Eq. (4) for $\theta_1 = 0$ (in leading logarithmic approximation).

So much for the transition probability of the inverse bremsstrahlung process.

In the actual experiment proposed by Diambrini ¹⁾, the photon beam is made up by the bremsstrahlung photons produced in the target by the incoming electron beam itself. The density of photons at the end of the target of length ℓ , per unit frequency and unit solid angle, will be given by

$$\frac{d^2 g}{d\omega d\Omega} = \ell \rho_e \rho_N \frac{d^2 \delta_B}{d\omega d\Omega}(\epsilon_1, \omega, \theta) \quad (6)$$

where ρ_e and ρ_N are the number of electrons and nuclei per unit volume. $d^2\sigma_B/d\omega d\Omega$ is the differential bremsstrahlung cross-section in the material of the target, for incoming electrons of energy ϵ_1 , integrated over the angles of the outgoing electron. To first order in l/l_R (the length of the target in unit of its radiation length), we find that the number of counts per unit time in the high energy inverse bremsstrahlung reaction is ^{*})

$$\dot{N} = \frac{Vl}{2} \rho_e^2 \rho_N^2 \int_{\bar{\omega}}^{\epsilon_1} d\omega \int d\Omega \frac{d^2\sigma_B}{d\omega d\Omega}(\epsilon_1, \omega, \theta) W(\epsilon_1, \omega, \theta) \quad (7)$$

where $(\epsilon_1 + \bar{\omega})$ is the minimum energy of the outgoing electrons, and V is the interaction volume. The factor $\frac{1}{2}$ comes from averaging over the target length of the photon density. The maximum energy reached by the outgoing electrons is $2\epsilon_1$.

Both $d^2\sigma_B/d\omega d\Omega$ and W_1 are strongly peaked in the forward direction in a cone of angular aperture of order m/ϵ_1 . We then write

$$\dot{N} \simeq A \frac{Vl}{2} \rho_e^2 \rho_N^2 \int_{\bar{\omega}}^{\epsilon_1} d\omega W(\epsilon_1, \omega, 0) \frac{d\sigma_B}{d\omega}(\epsilon_1, \omega) \quad (8)$$

where A is the appropriate constant reduction factor, of order 1, and ²⁾

$$\begin{aligned} \frac{d\sigma_B}{d\omega}(\epsilon_1, \omega) &= \int d\Omega \frac{d^2\sigma_B}{d\omega d\Omega} \simeq \\ &\simeq 4 \left(\frac{\alpha}{m}\right)^2 Z^2 \alpha \frac{\epsilon_2}{\omega \epsilon_1} \left(\frac{\epsilon_1^2 + \epsilon_2^2}{\epsilon_1 \epsilon_2} - \frac{2}{3} \right) \left(\lg \frac{2\epsilon_1 \epsilon_2}{m\omega} - \frac{1}{2} \right) \end{aligned} \quad (9)$$

Since in the extreme relativistic limit the angular dependence of $d^2\sigma_B/d\omega d\Omega$ has the following behaviour ²⁾

$$\frac{d^2\sigma_B}{d\omega d\Omega} \sim \left(\theta^2 + \frac{m^2}{\epsilon_1^2} \right)^{-2}$$

from Eqs. (5) and (8) we can approximately take $A \sim 1/3$. Inserting Eqs. (4) and (9) into Eq. (8), the final integration over the photon energy ω is easily performed. To leading order in $\bar{\omega}/\epsilon_1$, one gets

^{*}) We are neglecting here any effect due to the polarization of the bremsstrahlung photons.

$$\dot{N} \approx \frac{2^8 \pi^2}{27} \nu g_e^2 g_N^2 e \lambda_c^3 \alpha^2 Z^4 r_0^4 \left(\frac{\epsilon_1}{\bar{\omega}} \right)^3 \frac{1}{\gamma} \times$$

$$\times \left(\lg^2 \frac{2\epsilon_1^2}{m \bar{\omega}} - \frac{13}{6} \lg \frac{2\epsilon_1^2}{m \bar{\omega}} + \frac{11}{9} \right) \quad (10)$$

where $\nu = e_1/m$, $\lambda_c = 1/m$ is the Compton wavelength of the electron, and r_0 is the classical electron radius $r_0 = \alpha/m$.

The formulae that we have worked out do not take into account the screening effect. As a matter of fact, in extreme relativistic situations like where we want to apply them, there is essentially complete screening. For $\epsilon_1 \gg 137 Z^{1/3} m$, upon introducing the atomic form factor for the Thomas-Fermi model, the screening is taken into account (in leading logarithmic approximation) by simply replacing the factor in parentheses in the right-hand side of Eq. (10) by $\log^2(183 Z^{-1/3})$.

Introducing the radiation length of the target

$$l_R = (4Z^2 g_N \alpha r_0^2 \lg(183 Z^{-1/3}))^{-1}$$

and the number of incoming electrons per unit time $\dot{n}_e = \rho_e (V/l)$, our final result is

$$\dot{N} \approx \frac{16\pi^2}{27} \dot{n}_e g_e \left(\frac{l}{l_R} \right)^2 \lambda_c^3 \left(\frac{\epsilon_1}{\bar{\omega}} \right)^3 \frac{1}{\gamma} \quad (11)$$

$\dot{N} \approx 1.3/h$ is the number of counts per hour that are obtained if the following numerical values are inserted in Eq. (11):

$$g_e = 10^{12} \text{ el/cm}^3, \quad \gamma = 2.2 \times 10^4, \quad n_e = 7.2 \times 10^{14} \text{ el/mc}$$

$$\bar{\omega}/\epsilon_1 = 2 \times 10^{-2}, \quad l/l_R = 1/2.$$

These values for ρ_e , γ , $\dot{n}_e$ are not far from the SLAC electron beam working conditions.

Before concluding, we want to show how Eq. (11) compares with the number of events $\dot{N}_{CF}$ per unit time that one would get in the same experimental situation from the Compton scattering in flight of the bremsstrahlung photons by the electron beam. $\dot{N}_{CF}$ can be estimated as

$$\dot{N}_{CF} \approx \frac{1}{2} \beta_e \dot{n}_e \frac{\ell^2}{\ell_R} \sigma_T (1 - \beta_1/\epsilon_1) \quad (12)$$

where $\sigma_T = 8\pi/3 r_0^2$ is the Thomson limit of the Compton cross-section, which is relevant here because in the rest frame of the electron the photon energies are always less than m .

From Eqs. (11) and (12), inserting the previously given numerical values, one has then

$$\frac{\dot{N}_{CF}}{\dot{N}} \approx \frac{9}{4\pi} \frac{\ell_R}{\lambda_c} \alpha^2 \gamma \left(\frac{\bar{\omega}}{\epsilon_1} \right)^3 \left(1 - \frac{\beta_1}{\epsilon_1} \right) = 10^{-3} \div 10^{-4} \ell_R (cm) \quad (13)$$

The smallness of this ratio comes mainly from the factor $(1 - (\beta_1/\epsilon_1))$ which is present in $\dot{N}_{CF}$ and is proportional to the flux density of the photons relative to the incoming electrons. The inverse bremsstrahlung process gives a number of events several orders of magnitude larger than the Compton scattering in flight because what matters there is not the flux density of the photons relative to the incoming electrons, which is small as they are essentially travelling together, but rather the electron or the photon fluxes relative to the target nucleus.

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FIGURE CAPTION

Lowest order Feynman diagrams for the inverse bremsstrahlung reaction.

FIG. 1

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