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TRIGGER BIAS IN LARGE p_T REACTIONS

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ABSTRACT

We explore how particle multiplicities in large transverse momentum reactions are affected by the way in which the detection apparatus is triggered. For example, we predict that the multiplicity of particles accompanying a large p_T hadron is increased when it is required that there be a second large p_T hadron in the opposite hemisphere. The mechanism responsible for this effect is shown also to make correlations between pairs of large p_T pions produced on opposite sides depend only weakly on their charges.

We include a list of further experimental tests, feasible with available apparatus, that are suggested by the two-jet picture of large p_T reactions.

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1. INTRODUCTION

It has been shown in a number of papers ¹⁾⁻⁵⁾ that the existing experimental data ⁶⁾⁻¹¹⁾ on correlations in large transverse momentum reactions are well described by a two-jet picture. In this picture, two jets of approximately equal and opposite large transverse momenta are formed by some as yet unknown dynamical mechanism, and each jet then fragments into a system of hadrons in a manner that is supposed to be independent of the formation of the jet.

In all the experiments performed so far, the detection apparatus is triggered by a hadron having large transverse momentum. The need for one of the jets to provide this hadron severely distorts the way in which this jet fragments. In this paper we continue the discussion that we began in I on the effects of this trigger bias. We also list several consequences of the jet picture that can be tested in forthcoming experiments.

We begin in Section 2 by recapitulating what we learnt in I about jet fragmentation. Throughout this paper we continue to assume that the jet fragmentation obeys a scaling law. This means that the distribution of momentum components of the jet fragments parallel to the jet axis depends only on the fractions y of the total jet momentum that are taken by the various fragments. Further verification of this is an important task for future experiments, as also is the investigation of the distribution of momentum components perpendicular to the jet axis. At present it seems ^{1),9)} that the latter is sharply limited, with an average value of about 300 MeV/c. However, all the quantities that we discuss in this paper are integrated over angles, so that for our present purposes this information matters only in that it determines the experimental acceptance needed to measure them.

In Section 3 we consider the associated multiplicities, both on the trigger side and on the opposite side. A consequence of the trigger bias is that they are very different on the two sides. In Section 4 we investigate how these associated multiplicities change when it is required that, in addition to the large p_T trigger particle, there be a large p_T particle on the opposite side. We find that they are increased.

In Section 5 we discuss the corresponding problem when the additional large p_T particle is on the trigger side. We estimated in I that the probability of obtaining on the same side a pair of particles whose combined transverse momentum is p_T should be comparable with that of obtaining a single particle of transverse momentum p_T . Nevertheless, the associated multiplicities turn out to be rather greater when the p_T is shared between two triggering hadrons, and so also is the probability of obtaining a further large transverse momentum hadron on the opposite side.

The study of correlations in large p_T reactions has so far been mainly on the kinematic level, so that little is known about the quantum numbers of the jets. One interesting piece of information has, however, been obtained by the CCRS collaboration ⁸⁾: correlations between a pair of large p_T pions on opposite sides are, within experimental error, independent of their charges. In particular, the correlations $\pi^+\pi^+$ and $\pi^+\pi^-$ seem to be equal.

At first sight this is a rather severe constraint on dynamical models that might be constructed. However, we shall argue in Section 6 that the approximate charge independence of the correlations is largely a consequence of trigger bias. This means that if, in spite of the trigger bias, there are small differences between opposite side correlations for different charge combinations, so small that they are at present masked by experimental error, this can have important significance.

We conclude in Section 7 with a list of further experimental tests of the two-jet picture of large p_T reactions. These are chosen to be mostly feasible with available apparatus, though of course there are many vital questions about large p_T reactions that can only be answered with new equipment.

2. JET FRAGMENTATION

In I we parametrized the cross-section for the production of a pair of jets of almost equal and opposite transverse momenta P_x in the form

$$\frac{d\sigma^{\text{jet}}}{dP_x} \equiv \Phi(P_x, \sqrt{s}) = \frac{A}{P_x^{n-1}} \quad (2.1)$$

Both A and n are functions of P_x and $\sqrt{s}$, but for given $\sqrt{s}$ they are almost constant for presently relevant values of P_x , namely between 2 and 6 GeV/c. At the upper ISR energies, $n \approx 9$ in this range of P_x . The cross-section (2.1) is integrated over the angles at which the two jets are produced, and also over their invariant masses. It is also summed over their quantum numbers; what these are is, at present, altogether unknown ¹²⁾.

We suppose that, in the absence of trigger bias, fragments of type h from a jet are distributed in fractional momentum y parallel to the jet axis according to a function $F_1^h(y)$. The assumption that, for a given h , the function F_1^h depends only on y is the scaling hypothesis, which occurs naturally in models ¹³⁾. The function is averaged over the jet mass and the possible different quantum numbers of the jet. An assumption made in I was

that the fact that the relative weightings of the different components in this average are surely variable should not have too important an effect when one considers the relatively numerous pion fragments h . The information that is so far available does not allow a more refined analysis.

We found in I that the correlation data suggest that $F_1^h(y)$ should not vanish at $y=1$. We obtained a good fit to all the data by including a term $K^h\delta(y-1)$, corresponding to the possibility that the jet consists of a single particle ¹²⁾. There are, of course, other ways of achieving $F_1^h(1) \neq 0$. For example, if a jet has large mass, then when its total transverse momentum is not too large some of its decay fragments can move backwards and the forward-moving fragments can have $y \geq 1$. In such a case, which is used in the model of Preparata and Rossi ⁵⁾, the function F_1^h does not satisfy the scaling hypothesis. We shall continue to assume the scaling, and take $F_1^h(y)$ to vanish outside the range $0 \leq y \leq 1$.

A particular example of a jet is a resonance produced with large p_T . The mean number of decay mesons grows with the resonance mass M , with typical energies of the order of 0.5 GeV in the parent resonance rest frame. This effectively enforces a transverse momentum cut-off perpendicular to the resonance line of flight. The scaling of the function $F(y)$ for $p_T \gg M$ is the result of Lorentz invariance. In particular, it is known ⁹⁾ that, at large p_T , the ρ is produced at approximately the same rate as is a π with the same p_T . The fragmentation of an unpolarized ρ contributes a constant L_ρ to $F_1^\pi(y)$. We explained in the Appendix of I that this contribution is hard to detect in several of the present experiments, for reasons of acceptance. The remaining part of $F_1^h(y)$ is assumed ^{1),2),13)} to diverge as $y \rightarrow 0$, so as to give a multiplicity of particles in the jet that rises indefinitely with p_T . This part will contain contributions from multibody resonances, such as the ω . An interesting question is whether in fact it is built up entirely from an infinite sequence of resonances ¹²⁾.

The well-known low-mass resonances apparently do not have a dominant role. Indeed we found in I that a good fit to the available data on correlations is provided by

$$F_1^\pi(y) = B^\pi \frac{(1-y)^2}{y} + L_c + K^\pi \delta(y-1) \quad (0 < y \leq 1) \quad (2.2)$$

with coefficients

$$\begin{aligned} B^\pi &\approx 3/5 \\ L_e &\approx K^\pi \approx B^\pi/60 \end{aligned} \quad (2.3)$$

which are approximately independent of the pion charge. Even though K^π and L_ρ are so small relative to B^π , the first term in (2.2) does not by itself adequately describe the data: the relative importance of the other terms is magnified by the trigger bias. It remains that an uncontrived jet has only a small ρ (and probably ω) component, namely $B \gg L$.

The ρ production contributes a term $L_\rho \delta(1-y_1-y_2)$ to the fragmentation function $F_2^{\pi\pi}(y_1, y_2)$ that describes the inclusive production of two unequal-charge pions. Including the continuum contribution that we found to work well in I,

$$\begin{aligned} F_2^{\pi\pi}(y_1, y_2) &= C^{\pi\pi} \frac{(1-y_1-y_2)^2}{y_1 y_2} + L_e \delta(1-y_1-y_2) \\ &\quad (y_1, y_2 > 0, \quad y_1 + y_2 \leq 1) \end{aligned} \quad (2.4)$$

with

$$C^{\pi\pi} \approx \frac{3}{5} B^{\pi\pi} \gg L_e. \quad (2.5)$$

We guess that $F_2^{\pi^0\pi^0}$ is similar to the unequal-charge fragmentation functions, though of course without the contribution from the ρ . We shall not use explicit forms for $F_2^{\pi^+\pi^+}$ and $F_2^{\pi^-\pi^-}$; we expect them to be comparable in magnitude with $F_2^{\pi^+\pi^-}$ when (y_1+y_2) is small, but rather smaller for large (y_1+y_2) -- this assumes that jets do not usually have total charge greater than one unit, and that when the particles y_1 and y_2 take most of the momentum of the jet, they also, on average, take most of its charge.

Finally, we shall make use of the three-pion inclusive fragmentation function, for which we shall assume the form

$$F_3^{\pi^0\pi^0\pi^+}(y_1, y_2, y_3) = D^{\pi\pi\pi} \frac{(1-y_1-y_2-y_3)^2}{y_1 y_2 y_3} \quad (2.6)$$

with

$$D^{\pi\pi\pi} \approx \frac{3}{5} C^{\pi\pi}. \quad (2.7)$$

Such a form enforces the same global p_T distribution, whether one considers one or several particles. This is to some extent supported by present data ⁶⁾ comparing π^0 and $2\pi^0$ distributions, and is worth a thorough experimental analysis. Our assumptions of a global p_T balance between the two jets and scaling in jet decay also leads to the same power dependence on p_T whether considering one particle or two particles on opposite sides.

We shall use the distributions (2.2), (2.4) and (2.6) to illustrate the consequences of trigger bias. We found in I that (2.2) and (2.4) provide good fits to the available data, but they may well have to be modified substantially when more is known. At best, they are approximations to the true functions. For example, the ω contribution will add a constant to (2.4), though probably a small one. Because (2.4) and (2.2) are related by a momentum sum rule, this will entail also a small modification to (2.2). For our purposes, the key features of the distributions are that they allow a finite probability for all the jet momentum to be taken by one particle, and they give a multiplicity that rises eventually logarithmically with p_T . These are the two key facts in our analysis.

3. SINGLE-HADRON TRIGGER

Our fragmentation functions $F_1^h(y)$ are normalized so as to satisfy the momentum sum rule

$$\sum_{\substack{\text{types of} \\ \text{hadron } h}} \int F_1^h(y) y dy = 1. \quad (3.1)$$

The contribution of the single-pion components to this integral is $3K^\pi$. That is, in an experiment where there is a calorimeter trigger that measures the total jet momentum, this is the probability that the triggering jet consists of just a single pion. Because of (2.3), this is only about 5%. Similarly, the probability that the triggering jet consists only of a ρ is about 5%.

If, instead, the trigger is a single large p_T pion, the situation is rather different. Then one does not know the total momentum of the trigger-side jet, and it turns out that there is now a high probability that the trigger-side jet consists of the triggering pion only. From (2.1) and (2.2), the cross-section for the inclusive production of a large p_T pion is ¹⁾

$$\begin{aligned} \frac{d\sigma^\pi}{dp_\pi} &= 2 \int \frac{dP_\pi}{P_\pi} \Phi(P_\pi, \sqrt{s}) F_1^\pi(p_\pi/P_\pi) \\ &= \frac{2A}{p_\pi^{n-1}} \left[\frac{2B^\pi}{n(n-1)(n-2)} + \frac{L_p}{n-1} + K^\pi \right]. \end{aligned}$$

(3.2)

Putting $n=9$ and using (2.3), we see that the last term contributes some 75% to this expression, and this is therefore the probability that the trigger-side jet consists of the triggering pion alone. The probability that the triggering pion is produced from a ρ is about 10%, according to (3.2).

This means, of course, that the average multiplicity of associated hadrons in the trigger jet, accompanying the single-pion trigger, should not be very large. In 75% of the events, the accompanying multiplicity in the jet is zero. We have explained in I that, both in practice and in principle, there are difficulties in measuring the total multiplicity of particles in a jet. This is because of the problem of separating out the low p_T particles that belong not to the jet, but to the background. However, for a sufficiently large p_0 , one can consider the multiplicity of particles having transverse momentum greater than p_0 . To be clear of the background, p_0 must be at least $\frac{1}{2}$ GeV/c, and preferably nearer to 1 GeV/c or more.

Consider first the case of a calorimeter trigger where the total transverse momentum P_x of the jet is supposed to be known. [Again there are problems of measuring P_x , because of the background. However, for large enough P_x these problems are not acute -- because of the factor y in the integrand of (3.1), hadrons with transverse momentum less than any fixed p_0 contribute a negligible fraction to P_x at sufficiently large P_x .] The number of hadrons of type h having transverse momentum greater than p_0 is

$$n^h(P_x; p_0) = \int_{p_0/P_x}^1 dy F_1^h(y) \quad (3.3)$$

Inserting (2.2), we find

$$n^\pi(p_x; p_0) = B^\pi \left[\log \frac{p_x}{p_0} - \frac{3}{2} + 2 \frac{p_0}{p_x} - \frac{1}{2} \left(\frac{p_0}{p_x} \right)^2 \right] + L_e \left(1 - \frac{p_0}{p_x} \right) + K^\pi \quad (3.4)$$

for each of the three pion charge states. Somewhat arbitrarily, we set $p_0 = 500$ MeV/c and plot $3n^\pi$, the total number of pions, in Fig. 1. We estimate that this is also approximately equal to the total number of charged hadrons having transverse momentum greater than 500 MeV/c. Notice that n^π scales: it is a function of the ratio p_0/p_x only. Unless p_0/p_x is close to 1, the p and single-pion terms contribute a negligible fraction of the multiplicity.

Suppose now that the trigger is a π^0 of large transverse momentum p_x . Then we no longer know the total jet momentum P_x , which for given p_x will vary from event to event. Its average value is ¹⁾

$$\begin{aligned} \langle P_x \rangle &= \frac{2 \int dP_x \Phi(P_x, \sqrt{s}) F_1^{\pi^0}(p_x/P_x)}{d\sigma^{\pi^0}/dp_x} \\ &= p_x \frac{\frac{2 B^\pi}{(n-1)(n-2)(n-3)} + \frac{L_e}{n-2} + K^\pi}{\frac{2 B^\pi}{n(n-1)(n-2)} + \frac{L_e}{n-1} + K^\pi} \end{aligned} \quad (3.5)$$

With (2.3) this works out to be $1.1 p_x$ when $n=9$. The plot of Fig. 1 is locally sufficiently linear for it to be valid to use this average value to calculate the average charged multiplicity (of hadrons with transverse momentum greater than 500 MeV/c) in the opposite-side jet. Because $\langle P_x \rangle$ is so close to p_x , the resulting curve plotted against p_x looks little different from the plot in Fig. 1. Various experiments ^{6),9)-11)} have measured the opposite-side charged multiplicity. Because of their different experimental cuts and different ways of dealing with the background,

their results cannot be exactly compared with this plot. However, the general agreement is good.

As we have said, the trigger bias results in the average multiplicity in the trigger-side jet being rather smaller. For instance, the average number of π^+ associated with the π^0 trigger, and having transverse momentum greater than p_0 , is

$$n_{\text{same}}^{\pi^0; \pi^+}(p_x; p_0) = \frac{2 \int_{p_0+p_x}^{\infty} \frac{dP_x}{P_x} \int_{p_0/P_x}^{1-p_x/P_x} dy \Phi(P_x, \sqrt{s}) F_2^{\pi^0 \pi^+}(\frac{p_x}{P_x}, y)}{d\sigma^{\pi^0}/dp_x}$$

(3.6)

With (2.1) and (2.4), this gives

$$\frac{d\sigma^{\pi^0}}{dp_x} \cdot n_{\text{same}}^{\pi^0; \pi^+} = \frac{2A}{(p_0+p_x)^{n-1}} \left[\frac{L_e}{n-1} + \frac{2C^{\pi\pi}}{n(n-1)(n-2)} Q_n(u) \right]$$

(3.7a)

where

$$u = \frac{p_x}{p_0 + p_x}$$

$$Q_n(u) = u^{1-n} \int_0^u \frac{y^n dy}{1-y} + \frac{1}{(n-2)u} + \frac{1}{n-1} + \frac{u}{n}$$

(3.7b)

With $n=9$, and (2.3), (2.5), (3.2), we find that $n_{\text{same}}^{\pi^0; \pi^+}$ takes the values 0.006 at $p_x/p_0=2$, 0.015 at $p_x/p_0=3$, 0.03 at $p_x/p_0=5$ and 0.2 at $p_x/p_0=20$. Even if one multiplies these numbers by 3, say, to include all charged particles, these values are much less than in the opposite-side jet. The contribution of the ρ to these average multiplicities falls from about 60% at $p_x/p_0=2$ to about 35% at $p_x/p_0=20$.

The relative importance of the ρ is here magnified by the trigger bias. If, on the other hand, one were to require that there be an additional large transverse momentum particle p'_x on the opposite side, its relative importance would fall sharply with increasing p'_x (the reasons for this are similar to those discussed in the next Section).

The CERN SFM experiment 412 measures ⁹⁾ same-side charged multiplicity distributions for different ranges of p_T , with a trigger π^0 having $p_x \geq 2$ GeV/c. If one makes a subtraction for the background, their results are in fair agreement with our calculations, though we have perhaps a little too much contribution from the ρ . Our parametrization for the jet fragmentation and our assumption of equal ρ and π yields are, of course, more working hypotheses than definite statements.

Notice that the expression on the right-hand side of (3.7a) does not explicitly contain K^π . However, if we put $K^\pi = 0$ the normalization of $d\sigma^\pi/dp_x$, given by (3.2), would be reduced by a factor of about 4, and so the value of $n_{\text{same}}^{\pi^0; \pi^+}$ given by (3.7a) would be some four times larger. The data certainly do not favour this.

4. TWO-HADRON TRIGGER (OPPOSITE SIDES)

Suppose there is a hadron trigger of large transverse momentum p_x , which we take to be a π^0 for definiteness. Suppose that there is required also to be a second large transverse momentum particle p'_x on the opposite side; for definiteness, let this also be a π^0 . Then we show that the multiplicity $n^{\pi^0, \pi^0; \pi^+}(p_x, p'_x; p_0)$ of charged pions accompanying the first π^0 is greater than is calculated in (3.6), and, further, that this multiplicity increases with p'_x . This has been, to some extent, verified by the SFM experiment ⁹⁾.

It is easy to obtain a qualitative understanding of the origin of this prediction. We showed in (3.5) that with a single triggering π^0 the transverse momentum P_x has an average value that is just a little greater than the transverse momentum p_x of the π^0 . However ¹⁾, for given p_x the value of P_x varies about its mean $\langle P_x \rangle$ from event to event. Because the opposite-side jet does not readily give a large fraction of its momentum to a single particle, requiring that there be a large p_T particle on the opposite side results in the values of the jet momentum greater than $\langle P_x \rangle$ being favoured. Because multiplicities rise with P_x , the result follows.

For simplicity, we neglect ρ production, that is set $L_\rho = 0$ as in I. Then, for $z = p'_x/p_x < 1$,

$$\begin{aligned} \left. \frac{d^2 \sigma^{\pi^0 \pi^0}}{dp_x dp'_x} \right|_{\text{opp}} &= 2 \int_{p_x}^{\infty} \frac{dP_x}{P_x^2} \Phi(P_x, \sqrt{s}) F_1^{\pi^0}\left(\frac{p_x}{P_x}\right) F_1^{\pi^0}\left(\frac{p'_x}{P_x}\right) \\ &= \frac{4 A (B^\pi)^2}{z p_x^n n(n-1)(n-2)} E_n(\alpha^\pi; z) \end{aligned} \quad (4.1)$$

where

$$\alpha^n = \frac{K^n}{2 B^n} n(n-1)(n-2) \quad (4.2)$$

and

$$E_n(\alpha; z) = 1 - \frac{2(n-2)}{n+1} z + \frac{(n-2)(n-1)}{(n+1)(n+2)} z^2 + \alpha (1-z)^2 \quad (4.3)$$

The average transverse momentum of the jet is now

$$\begin{aligned} \langle p_x \rangle^* &= \frac{2 \int_{p_x}^{\infty} \frac{dP_x}{P_x} \Phi(P_x, \sqrt{s}) F_1^\pi\left(\frac{p_x}{P_x}\right) F_1^\pi\left(\frac{p'_x}{P_x}\right)}{\left. \frac{d^2 \sigma^{\pi^0 \pi^0}}{dp_x dp'_x} \right|_{\text{opp}}} \\ &= \frac{n p_x}{n-3} \frac{E_{n-1}(\alpha^\pi; z)}{E_n(\alpha^\pi; z)} \end{aligned} \quad (4.4)$$

With $n=9$ and (2.3), this gives a $\langle p_x \rangle^*$ that rises from $1.2 p_x$ at $z=\frac{1}{2}$ to $1.8 p_x$ as $z \rightarrow 1$. These values are to be compared with $1.1 p_x$ obtained from (3.5), which is the same as (4.4) at $z=0$.

Instead of calculating the resulting charged multiplicity, we shall consider the simpler, related quantity

$$R^{\pi^0, \pi^0; \pi^+}(p_x, p'_x; p_0) = \frac{\frac{d^3 \sigma^{\pi^0 \pi^0 \pi^+}}{dp_x dp'_x dp_0}}{\left. \frac{d^2 \sigma^{\pi^0 \pi^0}}{dp_x dp'_x} \right|_{\text{opp}}} \quad (4.5)$$

and compare it with the corresponding quantity associated with a single large p_T π^0 trigger:

$$R^{\pi^0, \pi^+}(p_x; p_0) = \frac{\left. \frac{d^2 \sigma^{\pi^0 \pi^+}}{dp_x dp_0} \right|_{\text{same}}}{\frac{d\sigma^{\pi^0}}{dp_x}} \quad (4.6)$$

In the numerator of (4.5) is the three-particle inclusive cross-section, where the $\pi^0(p_x)$ and the $\pi^+(p_0)$ go off on one side, and the $\pi^0(p'_x)$ goes off on the other. We find, by calculations similar to those above and in I, that

$$\frac{R^{\pi^0, \pi^0; \pi^+}(p_x, p'_x; p_0)}{R^{\pi^0; \pi^+}(p_x; p_0)} = (1 + \alpha^\pi) \frac{E_n(0; \frac{p'_x}{p_x + p_0})}{E_n(\alpha^\pi; p'_x/p_x)} \quad (4.7)$$

For $p'_x \ll p_x$, this ratio is 1, as of course it should be. From (2.3) and (4.2), $\alpha^\pi \approx 4$, and so from (4.3) with $n=9$ we calculate the values of the ratio (4.7) given in the table. Evidently, quite large effects are to be expected. Large effects have indeed been found by the SFM experiment ⁹⁾, though their numerical results quoted are not in a form that may readily be compared with our calculations.

5. TWO-HADRON TRIGGER (SAME SIDE)

We showed in I that the probability of obtaining two large transverse momentum pions p_{1x}, p_{2x} emerging more or less together with combined transverse momentum $p_{1x} + p_{2x} = p_x$, is comparable with that of obtaining a single pion of transverse momentum p_x . We shall suppose that p_{1x} and p_{2x} are both π^0 , so that we know that they are not in a ρ configuration. Similar results to those we shall obtain apply to a $(\pi^+\pi^-)$ trigger if the ρ configuration is excluded, either by suitable choice of the acceptance ¹⁾ or by measuring the mass of the pair and discarding events in the ρ region ¹⁴⁾.

The basic point is that, as we have seen in (3.2), with the single-pion trigger there is a 75% chance that the trigger-side jet consists of the trigger particle only, but when two-body resonances are excluded there

is a much smaller probability that the pair of triggering pions form the whole jet. This means that the average multiplicity of charged particles emerging together with the $(\pi^0\pi^0)$ is greater than that emerging with the π^0 . Further, the average jet momentum selected by the $(\pi^0\pi^0)$ trigger is greater, so that the average multiplicity in the opposite-side jet is greater and the momentum distribution of the particles in the opposite-side jet is changed. The effects may be even greater for a $(\pi^+\pi^+)$ or $(\pi^-\pi^-)$ trigger because, as we indicated in Section 2, we expect such a combination to take an even smaller fraction of the total jet momentum. However, we have no means of estimating in this case.

The average jet momentum for the $(\pi^0\pi^0)$ trigger is

$$\langle P_x \rangle = \frac{2 \int_{p_x}^{\infty} \frac{dP_x}{P_x} \Phi(P_x, \sqrt{s}) F_2^{\pi\pi}(\frac{p_{1x}}{P_x}, \frac{p_{2x}}{P_x})}{2 \int_{p_x}^{\infty} \frac{dP_x}{P_x^2} \Phi(P_x, \sqrt{s}) F_2^{\pi\pi}(\frac{p_{1x}}{P_x}, \frac{p_{2x}}{P_x})} \quad (5.1)$$

where $F_2^{\pi\pi}$ is given by (2.4) with L_p set equal to 0. This gives

$$\langle P_x \rangle = \frac{n}{n-3} p_x \quad (5.2)$$

which is $1.5 p_x$ when $n=9$. This is to be compared with the value $1.1 p_x$ that we obtained in (3.5) for the single π^0 trigger. From Fig. 1, we can estimate that the resulting increase in the average number of opposite-side charged particles with transverse momentum greater than 500 MeV/c is roughly constant at about 0.6 for $3 < p_x < 10$.

The momentum distribution of these opposite-side particles is, of course, also changed. Let their transverse momentum be p'_x . In I we calculated the charged particle distribution dN^{π^0}/dz , where $z = p'_x/p_x$, for the case of a single π^0 trigger p_x . We obtained (neglecting the ρ)

$$\begin{aligned}
 \frac{dN^{\pi^0}}{dz} &= \sum_{\text{charged } h'} \frac{B^{h'}}{1+\alpha^{\pi}} z^{-1} E_n(\alpha^{\pi}; z) & z < 1 \\
 &= \sum_{\text{charged } h'} \frac{B^{h'}}{1+\alpha^{\pi}} z^{1-n} E_n(\alpha^{h'}; z^{-1}) & z > 1
 \end{aligned} \tag{5.3}$$

where $\alpha^{h'}$ is defined in (4.2) and E_n in (4.3). This distribution is plotted in Fig. 2; the bump around $z=1$ is the additional contribution, not shown in (5.3), corresponding to the possibility that both jets consist of a single particle (see I). The plot assumes the value $\alpha^{\pi} \approx 4$, corresponding to the relation between K^{π} and B^{π} in (2.3). We explained in I that this is necessary if the normalization of the calculated curve is to agree with the data from the SFM 412 experiment ⁹⁾. We assumed $\sum_{\text{charged } h'} B^{h'} = 9/5$; compare (2.3).

The corresponding distribution for the $(\pi^0 \pi^0)$ trigger is

$$\begin{aligned}
 \frac{dN^{(\pi^0 \pi^0)}}{dz} &= \sum_{\text{charged } h'} B^{h'} z^{-1} E_n(0; z) & z < 1 \\
 &= \sum_{\text{charged } h'} B^{h'} z^{1-n} E_n(0; z^{-1}) & z > 1
 \end{aligned} \tag{5.4}$$

and there is no additional bump at $z=1$. This distribution is also shown in Fig. 2: the difference is substantial!

Consider now the associated multiplicity of charged particles in the trigger-side jet. For a single π^0 trigger, we have calculated this in (3.7). For the double π^0 trigger, a similar calculation gives

$$\left. \frac{d^2 \sigma^{\pi^0 \pi^0}}{dp_{1x} dp_{2x}} \right|_{\text{same}} \cdot n_{\text{same}}^{(\pi^0 \pi^0); \pi^+} = \frac{4 A D^{\pi \pi \pi}}{n(n-1)(n-2)} \frac{p_x}{p_{1x} p_{2x}} \frac{Q_n(u)}{(p_0 + p_x)^{n-1}} \tag{5.5}$$

where $D^{\pi^0\pi^0\pi^+}$ is the coefficient of $F_3^{\pi^0\pi^0\pi^+}(y_1, y_2, y_3)$: see (2.6) and (2.7). Inserting the expression for the same-side two-particle inclusive cross-section that we calculated in I, we obtain

$$n_{\text{same}}^{(\pi^0\pi^0);\pi^+} = \frac{D^{\pi^0\pi^0}}{C^{\pi^0\pi^0}} u^{n-1} Q_n(u) \quad (5.6)$$

This gives $n_{\text{same}}^{(\pi^0\pi^0);\pi^+}$ rising from about $2n_{\text{same}}^{\pi^0;\pi^+}$ at $p_x/p_0 = 2$ to about $3n_{\text{same}}^{\pi^0;\pi^+}$ at $p_x/p_0 = 20$.

6. CHARGE CORRELATIONS

The result (4.4) has an important consequence for the study of differences between correlations for different charge combinations of large p_T particles emerging on opposite sides. One would hope that such a study would give information about the quantum numbers of the two jets. One would expect that, say, a high $p_T \pi^+$ is more likely to come from a positive-charge jet than a negative-charge jet, and the reverse for a π^- . Hence the result of the CCRS collaboration⁸⁾, that the opposite-side high p_T correlation $\pi^+\pi^+$ is approximately equal to $\pi^+\pi^-$, at first sight seems to imply a surprising lack of correlation between the charges of the two jets. However, we now argue that in fact the charges of the high p_T pions are correlated only weakly with the charges of their parent jets, so that the approximate equality of the $\pi^+\pi^+$ and $\pi^+\pi^-$ correlations gives, in practice, no information about the correlation between the charges of the two jets. More accurate measurements will be needed to obtain such information: a strong correlation between the charges of the jets is to be expected to give only a small difference between the $\pi^+\pi^+$ and $\pi^+\pi^-$ correlations. Conversely, a small difference between the latter correlations gives significant information about the charges of the jets.

According to (4.4), for $z = p'_x/p_x < 1$

$$\frac{p'_x}{\langle p_x \rangle^*} = \frac{n-3}{n} z \frac{E_n(\alpha^\pi; z)}{E_{n-1}(\alpha^\pi; z)} \quad (6.1)$$

For $n=9$, this increases from 0 at $z=0$ to $6/11$ as $z \rightarrow 1$. (These extremum values are independent of how much single-particle component there is in the jets, that is they are independent of the value of α^π .) That is, the particle that has the lesser transverse momentum on average does

not take more than about half the momentum of its parent jet -- for z close to 1, both particles take about half on average, while for smaller z the slower particle takes rather less than half.

For a particle that takes even half the momentum of its parent jet, one should not expect a very large correlation between the charge of the particle and that of the jet. We do not have a reliable way of estimating this correlation, but it may be that low p_T inclusive production in the pion fragmentation region is a rough guide. When the produced pion takes half the momentum of the incoming pion, the ratio $(\pi^+p \rightarrow \pi^+X)/(\pi^+p \rightarrow \pi^-X)$ is known ¹⁵⁾ to be about 3. Allowing also for the production of neutrals, this means that only about half the pions produced with half the momentum of the π^+ are themselves π^+ . Likewise, if we had a π^0 beam we can expect that half the pions produced with half the π^0 momentum would be charged.

Assuming that we can take these values to apply also to the fragmentation of large p_T jets, we can calculate the maximum difference between the correlations $\pi^+\pi^+$ and $\pi^+\pi^-$ for two pions of equal and opposite transverse momenta, that is $z=1$. The difference will be maximum if it is assumed that one of the two jets always has positive charge and the other always has negative charge: then $(\pi^+\pi^+)/(\pi^+\pi^-) \approx 60\%$. If, on the other hand, the charge combinations (0+) and (0-) for the jets are also equally likely, this ratio increases to about 85%.

The CCRS collaboration ⁸⁾ also have some results on charge correlations for two high p_T pions emerging on the same side. According to our result (5.2), they take on average a total of $2/3$ of the momentum of their parent jet. This is a large enough fraction for us to expect a sizeable correlation between their combined charge and the total charge of their parent jet. But it will surely not be anything near 100%, and so the experimental result ⁸⁾ that $(\pi^+\pi^+)/(\pi^+\pi^-) \gtrsim \frac{1}{2}$ cannot be used to infer the presence of jets of total charge 2.

7. EXPERIMENTAL SHOPPING LIST

In I we used existing experimental data to construct a quantitative two-jet picture of large p_T reactions. In this paper the emphasis has been on further experimental predictions and we now gather these together. We also include in our list some further questions that we have not discussed in this paper, but the answers to which are experimentally accessible, given enough statistics, and interesting. The list should be read in conjunction with one we have given previously ¹⁶⁾, which still remains valid.

(i) At the time that we compiled the previous list, there was a lack of any direct evidence of a jet structure. Since then, such evidence has begun to accumulate ⁹⁾: in events with more than one large p_T particle ($p_T > 1$ GeV/c, say), their momenta are coplanar, and when they emerge on the same side they are strongly correlated in rapidity. This still needs further experimental investigation, to determine just to what extent a two-jet structure completely describes all large p_T events.

(ii) In this connection we should mention a point that was briefly touched on in I, and was first emphasized by Combridge ¹⁷⁾. If, as is essentially always the case in practice, the triggering system is such that in the triggering devices there is a net flow of transverse momentum to one side, the background will be biased towards the opposite side. This is important because, although the background contains mostly low p_T particles, it will also have a tail of high p_T particles. There is some sign in the results of the CERN SFM experiment ⁹⁾ that this background distortion can be important. To study this trigger bias of the background, one should trigger on a pair of high p_T hadrons p_{1x}, p_{2x} emerging on opposite sides, and examine the associated distributions on both sides far away in rapidity from the trigger particles as $z = p_{2x}/p_{1x}$ varies.

(iii) A key assumption in our analysis has been the scaling hypothesis for jet fragmentation. There are a number of indirect ways in which one might try to verify this:

- (a) is the p_T dependence for the production of a pair of large transverse momentum particles emerging more or less together with combined transverse momentum p_T similar to that for the production of a single particle of transverse momentum p_T ? There are already indications that this is true, from the ACHM collaboration ⁶⁾. See Eq. (5.6) of I;
- (b) is the average number of charged particles having transverse momentum greater than p_0 (where p_0 is chosen so as to be largely clear of the background) opposite to a trigger particle of large transverse momentum p_x , a function only of the ratio p_0/p_x ? See (3.4) and the remarks following (3.5). In more detail, is the distribution in transverse momentum p'_x of the opposite-side charged particles a function only of $z = p'_x/p_x$? See (5.3);
- (c) repeat (b) with the trigger replaced by a pair of high transverse momentum particles of combined transverse momentum p_x . See (5.4);
- (d) is the multiplicity of particles having transverse momentum greater than p_0 (chosen so as to be largely clear of the background)

accompanying a large transverse momentum trigger particle p_x a function only of the ratio p_0/p_x ? See (3.7) together with (3.2).

(iv) If, in addition to a trigger particle of transverse momentum p_x , it is required that there be on the opposite side a particle of large transverse momentum p'_x , is the multiplicity of particles accompanying p_x increased? Does it increase with increasing $z = p'_x/p_x$? See Section 4. There is evidence for this from the SFM experiment 9).

(v) When a single trigger particle of large transverse momentum p_x is replaced by a pair of particles whose combined transverse momentum is p_x (not in a p configuration), are there substantial increases in the associated multiplicities on both sides? See Section 5. Does the momentum distribution dN/dz of opposite-side particles change in the way shown in Fig. 2? These are tests of whether indeed the trigger-side jet can be just a single particle.

(vi) For a single-particle trigger, is there a bump around $z=1$ in the distribution dN/dz of opposite-side particles? This would demonstrate that it is possible for both the jets simultaneously to consist of single particles only: see I. Notice that the width of the bump depends on how much it is possible for the total transverse momenta of the two jets to differ, the difference being compensated by an asymmetric distribution of the background. The width in z may be expected to shrink as the p_T of the trigger is increased.

(vii) What is the role of resonances? Because of the problem of the background, to examine the contribution from a given resonance one must look at the rare events where all its decay products have large momentum. For example, to study the ω one must make three-pion mass plots in events where all three pions have large momentum. For production at wide angle this requires large aperture.

(viii) From the available evidence it seems that in a jet the momentum distribution of the jet fragments perpendicular to the jet axis is

- (a) axisymmetric, and
- (b) sharply cut off with an average value of about 300 MeV/c, independent of the longitudinal momentum of the fragment.

These points need confirming.

(ix) Assuming that they are true, and combining this information with measured multiplicities 6), 9)-11), one can guess what a typical jet should look like: Fig. 3. These drawings are for jets free from trigger bias,

for example the opposite-side jet when there is a single-particle trigger (at least, approximately so). They underline the need to have as large a total transverse momentum as possible: only then does a typical jet have several charged particles than can reasonably clearly be distinguished from the background, and only then is it jet-like in the sense of being narrowly confined in angle. Guesses such as those drawn in Fig. 3 are important for designing experiments triggered with calorimeters. Such experiments will be a crucial test of the jet picture: we calculated in I that the cross-section for producing a pair of jets both free from trigger bias should be some two orders of magnitude greater than that for producing a single particle with the same transverse momentum.

(x) The study of the correlation between the angles θ_1, θ_2 with which the two jets emerge is important for gaining an understanding of the dynamics underlying jet production. We intend to study this in a future paper ¹⁸⁾. When θ_2 is comparatively small, is the corresponding jet often just one of the initial nucleons, scattered through a wide angle?

(xi) While we expect that most large p_T events have two jets, are there events with more than two jets? In such events, is one of the jets one of the initial nucleons? Multi-jet events need not be coplanar: only the vector sum of their transverse momenta need approximately vanish ¹⁹⁾. If the total quantum numbers of a jet can be meson-like, there can be an odd number of them; if they are quark-like ¹²⁾ their total number must be even, as is also the case in the model of Preparata and Rossi ⁵⁾.

(xii) A mechanism that can produce a zero charge meson-like jet can instead directly produce a large p_T photon. We have said that the cross-section for producing a jet of given transverse momentum is two orders of magnitude times that of producing a single pion of the same transverse momentum. Perhaps one jet in four has zero charge and zero strangeness. Multiplying by the fine structure constant, we obtain a crude estimate of 20% for the ratio γ/π at a given p_T . This is in agreement with the results of a recent experimental analysis ²⁰⁾; a more detailed calculation of Escobar ²¹⁾, based on a particular model, gives predictions that are slightly smaller.

(xiii) We emphasize that if we are to learn what is the nature of the jets, it is essential to obtain information about quantum number correlations between high p_T particles. For example, if one triggers on a high p_T kaon, where is the opposite-strangeness particle? We have not considered these matters here, partly because predictions about these matters require detailed models and we have no very great confidence in any particular model (this

is why the experiments are so important: they will clearly distinguish between models), and partly because the experiments will need extensive new apparatus.

We have written this paper with the explicit context of the ISR in mind. However, most of what we have said obviously applies also to SPS or FNAL energies, where the disadvantage of lower energy is to some extent outweighed by the importance of comparing results from different beams. In the case of a proton beam, at these lower energies the value of n is greater, $n \approx 11$ or 12 , so that there are quantitative differences. For example, from (3.2) the probability that the trigger-side jet consists of the triggering pion alone now rises to 85%, and the average associated multiplicity on the trigger side is correspondingly reduced. The average jet momentum (4.4) for the case of two opposite-side high p_T particles is decreased, so that the effects we describe in Section 4 are less pronounced and the charge dependence of the opposite-side correlations is a little less weak (see Section 6). In Fig. 2 the lower curve, corresponding to a single pion trigger, will change a little; the upper curve, corresponding to a double-pion trigger, will now be rather closer to it.

Table

Values of the ratio (4.7).

$p_0 = \frac{1}{2}$

	$p'_x = \frac{1}{2}$	1	2	4	8
$p_x = 2$	1.3	1.8	9.4		
5	1.1	1.2	1.4	3.4	
10	1.0	1.1	1.1	1.3	3.1

$p_0 = 1$

	$p'_x = \frac{1}{2}$	1	2	4	8
$p_x = 2$	1.3	2.1	13		
5	1.1	1.2	1.5	4.0	
10	1.0	1.1	1.2	1.4	3.4

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* * *

FIGURE CAPTIONS

- Fig. 1 : Calculated average number of charged hadrons with transverse momentum greater than 500 MeV/c in a jet of total transverse momentum P_x .
- Fig. 2 : Momentum distributions for opposite-side charged particles, for a single π^0 and a double π^0 trigger. The variable z is the ratio of the transverse momentum of the observed charged particle to the total transverse momentum of the trigger.
- Fig. 3 : Guesses for plan views of typical jets emerging at 90° in the centre of mass, drawn to scale. The broken lines denote neutral particles.

FIG. 1

FIG.2

FIG.3