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REGGEON FIELD WITH NON-VANISHING VACUUM EXPECTATION VALUE

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A B S T R A C T

It is argued that if the vacuum expectation value of the Reggeon field does not vanish the solution of the Reggeon calculus corresponds to the leading singularity of intercept one. The physical interpretation of the Reggeon field theory with unstable vacuum is discussed.

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I. INTRODUCTION

In recent investigations of the Reggeon calculus much attention is given to models which have a single pole with intercept larger than one as the input (bare) singularity. Structure of the exact singularity is then determined examining the large distance (or infra-red) behaviour. This has been attempted by Abarbanel and Bronzan ¹⁾ and Migdal, Polyakov and Ter Martirosyan ²⁾ with use of the renormalization group technique. However, the "strong coupling" solution which they find exists only in the case when the intercept α_0 of the bare trajectory equals exactly the "critical" value $\bar{\alpha}_0$ which is uniquely determined by the remaining parameters ^{2),3)}.

Somewhat earlier, the Reggeon calculus with bare intercept larger than one has been studied by Bronzan ⁴⁾ and Cardy ⁵⁾, with results which differ qualitatively from those of the renormalization group approach. The most important feature of their results is that there is no critical constraint: instead, there is a lower bound for the bare intercept. The elastic amplitude they have obtained corresponds to a uniformly absorbing disc of constant opacity and a radius increasing like $\log s$, which leads to a $(\log s)^2$ behaviour of total cross-section. Consequently the s channel unitarity is not violated.

It is natural to assume ⁶⁾ that the results of Cardy and Bronzan are true for $\alpha_0 > \bar{\alpha}_0$. When $\alpha_0 < \bar{\alpha}_0$ the output singularity is below one, so that the total cross-section falls as a power of s . The strong coupling solution, valid for $\alpha_0 = \bar{\alpha}_0$, corresponds then to a point of transition between two possible patterns of asymptotic behaviour. Any change from this critical value leads to a result which is qualitatively different from the strong coupling solution. Consequently, the strong coupling solution should be considered unstable, and its relevance to physics is very much in doubt.

There is a close analogy between this picture and the phenomena of phase transition. When $\alpha_0 < \bar{\alpha}_0$ the exact propagator of the Reggeon field falls exponentially at large distances. This is also true for the spin-spin correlation function in a ferromagnet above the critical temperature. At $\alpha_0 = \bar{\alpha}_0$ a phase transition occurs. Finally, $\alpha_0 > \bar{\alpha}_0$ is a case of a temperature below the critical point. In analogy with the spontaneous magnetization in spin systems, one expects that the Reggeon field would then have a non-vanishing vacuum expectation value.

In this paper we discuss the Reggeon calculus for $\alpha_0 > \bar{\alpha}_0$ from this point of view. We find that if the vacuum expectation value of the Reggeon field does not vanish, the intercept of the leading singularity is exactly one. The results of Cardy and Bronzan obtain a simple interpretation within this picture. We also comment on the recent paper of Abarbanel et al.⁷⁾ and give a brief discussion of the inclusive distributions in the Reggeon calculus with $\alpha_0 > \bar{\alpha}_0$.

Some of the ideas discussed below were included in an earlier paper⁶⁾. After most of the work reported here had been completed, we learnt about related results of Amati, Le Bellac, Ciafaloni and Marchesini⁸⁾.

II. VACUUM EXPECTATION VALUE OF THE REGGEON FIELD

The existence of a non-vanishing vacuum expectation value of the Reggeon field may be established as follows. We introduce the sources : $J_1\phi$, $J_2\phi^\dagger$ uniformly distributed within a finite region of the impact parameter and the rapidity : $|x| < L$, $|y| < T$. The expectation value of the Reggeon field at $(x,y)=(0,0)$ may then be evaluated in a diagrammatical expansion. We then take the limit $L, T \rightarrow \infty$ and next $J_1 \rightarrow 0$. The result is the vacuum expectation value.

The sources should act in the same way as the external particles. In the case of the Pomeron field theory, this may be achieved with purely imaginary sources : $iJ_1\phi + iJ_2\phi^\dagger$. Expanding the weight factor under the functional integral, we obtain

$$e^{-i(J_1\phi + J_2\phi^\dagger)} = (1 - iJ_1\phi + \frac{1}{2}i^2\phi^2 + \dots)(1 - iJ_2\phi^\dagger + \frac{1}{2}i^2\phi^{\dagger 2} + \dots) \quad (\text{II.1})$$

so that the multi-Pomeron cuts appear with correctly alternating signs.

Looking at the diagrams in the (x,y) representation, it is now easy to see that the expectation value of the Pomeron field, if non-zero, must be purely imaginary : $\langle \phi \rangle = i\nu$. The same holds for the vacuum expectation value of the $\phi^\dagger$ field. As the boundary conditions (and, in general, the Lagrangian) are not Hermitean, there is no reason to require a Hermitean vacuum. Consequently, in general $\langle \phi \rangle$ and $\langle \phi^\dagger \rangle$ are not complex conjugate to each other and the vacuum expectation value of the real part of the Reggeon field : $\phi_R = \frac{1}{2}(\phi + \phi^\dagger)$ is not real. Though ϕ_R is constrained to be real in the functional integration, this is not a contradiction because the average is calculated with a complex weight.

We want to stress that the non-Hermitean boundary conditions should be used even if the Lagrangian of the Pomeron field is Hermitean, like in the $\lambda(\phi^\dagger\phi)^2$ model. This has not been recognized in a recent paper ⁹⁾, the conclusion of which (that the spontaneous symmetry breaking cannot produce an imaginary triple-Pomeron coupling) is consequently incorrect.

We start from a most simple example : the Regge eikonal model. In this case we are interested in a vacuum expectation value not of the Reggeon field itself, but of the function $f[\phi] = 1 - e^{i\beta\phi}$ where β is the coupling of the Pomeron to the external particles. Summing the diagrams for finite L, T we obtain

$$\langle 1 - e^{i\beta\phi} \rangle_{J,L,T} = 1 - e^{-\beta J \int_{|x|<L} d^2x \int_0^T dy D_0(x,y)} \quad (II.2)$$

where

$$D_0(x,y) = \frac{e^{\Delta y - \frac{|x|^2}{4\alpha'y}}}{4\alpha'y}, \quad \Delta = \alpha_0 - 1 \quad (II.3)$$

is a bare Pomeron propagator. If the intercept α_0 is larger than one, the integral in (II.2) diverges in the limit $L, T \rightarrow \infty$ and we obtain the non-vanishing vacuum expectation value

$$\langle 1 - e^{i\beta\phi} \rangle = \lim_{J \rightarrow 0} \lim_{L,T \rightarrow \infty} \langle 1 - e^{i\beta\phi} \rangle_{J,L,T} = 1 \quad (II.4)$$

The impact parameter representation of the scattering amplitude in the Regge eikonal model

$$T(y,b) = 1 - e^{-\beta^2 D_0(b,y)} \quad (II.5)$$

is given by the expectation value of $1 - e^{i\beta\phi(Q,0)}$ in the presence of sources localized at (b,y) . If $y \rightarrow \infty$, $|b|^2 < 4\alpha'y^2$, there is a complete absorption : $T(y,b) \rightarrow 1$. That is, for y large enough, the effect of a localized perturbation is again the vacuum expectation value.

Consider now the theory of Pomerons interacting through the triple-Pomeron coupling. The Lagrangian is

$$\mathcal{L}[\phi, \phi^+] = \frac{1}{2} \phi^+ \overleftrightarrow{\partial}_y \phi - \alpha' \nabla \phi^+ \cdot \nabla \phi + \Delta \phi^+ \phi - \frac{i g}{2} (\phi^+ \phi^2 + \phi^{+2} \phi) \quad (\text{II.6})$$

($\Delta = \alpha_0 - 1$). The expectation values of the fields ϕ and ϕ^+ are estimated in a tree diagram approximation. Note that in this approximation, the critical intercept $\bar{\alpha}_0$ is equal to one. However, we expect that it will be larger than one when the loop corrections are included.

In this paper we do not attempt to calculate the sum of the tree diagrams explicitly. Instead we assume that the (finite and non-vanishing) vacuum expectation value exists and we examine the consequences. As summing tree diagrams is equivalent to solving the classical field equations

$$\frac{\partial \phi^+}{\partial y} - \alpha' \nabla^2 \phi^+ - \Delta \phi^+ + \frac{i g}{2} (\phi^{+2} + 2 \phi^+ \phi) = 0 \quad (\text{II.7})$$

$$-\frac{\partial \phi}{\partial y} - \alpha' \nabla^2 \phi - \Delta \phi + \frac{i g}{2} (\phi^2 + 2 \phi^+ \phi) = 0$$

the vacuum expectation value, if it exists, is determined by the (x, y) independent solution of (II.7). Apart from the trivial solution: $\langle \phi \rangle = \langle \phi^+ \rangle = 0$ there are three other solutions. The first one corresponds to the anti-Hermitean perturbation $iJ(\phi + \phi^+)$:

$$\langle \phi \rangle = \langle \phi^+ \rangle = -i \frac{2\Delta}{3g} \quad (\text{II.8})$$

According to Ref. 7) it is, however, unstable. The two remaining solutions may be obtained by summing the diagrams which include only emitting or only absorbing sources, respectively:

$$\langle \phi \rangle = -i \frac{2\Delta}{g} \quad \langle \phi^+ \rangle = 0 \quad (\text{II.9a})$$

$$\langle \phi \rangle = 0 \quad \langle \phi^+ \rangle = -i \frac{2\Delta}{g} \quad (\text{II.9b})$$

Rewriting the Lagrangian (II.6) in terms of the $\chi = \phi - \langle \phi \rangle$, $\chi^\dagger = \phi^\dagger - \langle \phi^\dagger \rangle$ fields which have vanishing vacuum expectation values, we obtain in the first case (II.9a) :

$$\mathcal{L} = \frac{1}{2} \chi^\dagger \overleftrightarrow{\partial}_y \chi - \alpha' \nabla \chi^\dagger \cdot \nabla \chi - \Delta \chi^\dagger \chi - \Delta \chi^{+2} - \frac{ig}{2} (\chi^\dagger \chi^2 + \chi^{+2} \chi) \quad (\text{II.10})$$

and symmetrically for (II.9b). The resulting theory is then a Reggeon field theory with intercept lower than one ⁷⁾ ($1-\Delta$ in the no-loop approximation).

III. ASYMPTOTIC BEHAVIOUR OF SCATTERING AMPLITUDE AND INCLUSIVE DISTRIBUTIONS

If the shifted fields $\chi^\dagger, \chi$ describe the physical Pomeron [as assumed in Ref. 7)], one is led to the conclusion that in the Reggeon calculus with the bare intercept larger than the critical value, the intercept of the output Pomeron is below one. The Pomeron with intercept one is then obtained only in the critical case corresponding to the strong coupling solution (here we disregard the possibility of the spontaneous symmetry breaking). This is in disagreement with the results of Cardy ⁵⁾ and Bronzan ⁴⁾, suggesting that for the bare intercept larger than the critical value, the output Pomeron has again the intercept one.

This apparent contradiction is, however, easy to resolve. The point is that the two-point function corresponding to the sum of the Reggeon diagrams considered by Cardy and Bronzan for $\alpha_0 > \bar{\alpha}_0$ is not a propagator of the Reggeon field theory. To see this, consider the Reggeon propagator expressed in terms of diagrams corresponding to the perturbative expansion around the unstable vacuum state with $\langle \phi^\dagger \rangle = \langle \phi \rangle = 0$. The internal lines correspond then to the bare (intercept α_0) Pomeron propagators defined in (II.3) and the vertices are the usual Gribov vertices so that no couplings like $g\phi^n$ or $g\phi^{\dagger m}$ should appear. We shall write $\langle 0 | \dots | 0 \rangle$ to indicate a sum of these diagrams. The propagator of the Reggeon field with boundary conditions appropriate to the stable ground state $|v\rangle$ [e.g., (II.9a)] is then defined as follows :

$$\langle v | T \phi(y, x) \phi^\dagger(0, g) | v \rangle = \lim_{J \rightarrow 0} \lim_{L, T \rightarrow \infty} \langle 0 | T \phi(y, x) \phi^\dagger(0, g) e^{-iJ \int d\gamma' \int d^2x' \phi^\dagger(\gamma', x')} | 0 \rangle \quad (\text{III.1})$$

Similar formulae hold for other Green's functions of the Reggeon field (one example is the field expectation value discussed in the preceding Section). In the case of the (II.9b) ground state in the exponential factor ϕ^+ should be replaced by ϕ .

Note that as either $\langle \phi \rangle$ or $\langle \phi^+ \rangle$ vanishes in the ground state, the propagator of the shifted field is equal to (III.1). Consequently (III.1) is the two-point function calculated in the SLAC group paper ⁷⁾.

The exponential factor in (III.1) corresponds to the external (emitting) sources distributed uniformly within the region $|y| < T$, $|x| < L$. These are additional sources with no connection to the external particles. In other words, the Reggeon diagrams summed in (III.1) include the tadpole contributions. Notice that if J would be set to zero before the limit $L, T \rightarrow \infty$ is taken, the tadpole terms would vanish. The result would then be different from (III.1): this is just a statement of an instability of the $|0\rangle$ vacuum. The two-point function calculated by Bronzan ⁴⁾ includes no tadpole contributions. It is, in our notation

$$G^{(1,1)}(y, x; 0, 0) = \langle 0 | T \phi(y, x) \phi^+(0, 0) | 0 \rangle \quad (\text{III.2})$$

As explained above, this is not equal to (III.1) if the $|0\rangle$ vacuum is unstable [so that the limits in (III.1) do not commute]. In this case (III.2) is not a propagator of the Reggeon field theory. One can say that the two-point function (III.2) and the usual propagator of the Pomeron field theory (III.1) are evaluated with different boundary conditions. The propagator is obtained if the Pomeron is emitted into the stable ground state of the field. This is the case considered in the SLAC group paper ⁷⁾. But in the case of the two-point function of Cardy and Bronzan, the initial state is the one with $\langle \phi \rangle = \langle \phi^+ \rangle = 0$: it is unstable when $\alpha_0 > \bar{\alpha}_0$.

Consequently the results of Ref. 7) and of Refs. 4) and 5) contradict each other because they correspond to different physical interpretations of the Reggeon field theory. The assumption of Abarbanel et al. ⁷⁾ is evidently that the physical Pomeron should be identified with the leading j plane singularity of the exact propagator of the Reggeon field theory. As explained below, we think that this interpretation is not likely to be correct for $\alpha_0 > \bar{\alpha}_0$.

The physical Pomeron of Cardy and Bronzan is defined in terms of Reggeon diagrams with bare Pomeron propagators corresponding to the perturbative expansion around the state with $\langle \phi \rangle = \langle \phi^\dagger \rangle = 0$. The diagrams with bare Pomerons emitted or absorbed by the vacuum (the tadpole terms) are not included. That is, there is no perturbation due to the distributed external sources: there are only localized sources corresponding to the external particles. When $\alpha_0 < \bar{\alpha}_0$ this sum of diagrams gives correctly the propagator of the Reggeon field theory. However, when $\alpha_0 > \bar{\alpha}_0$ it is not so because the original vacuum becomes unstable.

To see which approach is correct, let us look at the derivation of the Reggeon calculus from the underlying field theory of strong interactions. It has been recently shown by Ciafaloni, Marchesini and Veneziano¹⁰⁾ that the topological expansion in the high energy limit may be expressed in terms of Reggeon diagrams. Moreover, in this approach it is possible to define the bare quantities of the resulting Reggeon calculus (like the bare Pomeron, Reggeon and their coupling constants) in terms of the Feynman diagrams of the underlying field theory. Consequently, it is rather the Reggeon diagrammatic expansion (with bare propagators and couplings) and not the Reggeon field theory itself which has direct physical interpretation.

The Reggeon diagrams with tadpoles are not obtained from the topological expansion. The same is true for the earlier derivation of the Reggeon calculus from the so-called hybrid diagrams. The boundary conditions corresponding to the stable ground state of the Reggeon field theory are consequently unphysical when the bare Pomeron intercept exceeds the critical value. (This has been also noted in the second paper of the SLAC group.) The Reggeon calculus derived from the underlying field theory corresponds to the perturbative expansion around the state with $\langle \phi \rangle = \langle \phi^\dagger \rangle = 0$ which, from the point of view of the Reggeon field theory, is unstable for $\alpha_0 > \bar{\alpha}_0$.

Our conclusion is that in the case of the bare Pomeron intercept larger than the critical value, the correct approach is that of Cardy and Bronzan^{4),5)}. The Reggeon field theory with stable vacuum state in this case has no direct physical interpretation because it cannot be based on the underlying field theory. Consequently, the results of Abarbanel et al.⁷⁾ and the "Goldstone boson" theories of the Pomeron as well^{9),11),12)}, from this point of view, should be rejected [see, however, the recent work of Dyatlov¹³⁾]. [To avoid misunderstanding, we note that the tadpoles we speak about have nothing to do with tadpoles discussed in Ref. 14). The latter correspond to the Pomerons emitted by the scattering particles and are included in the Gribov perturbative expansion: in our language they are not "tadpoles" at all.]

We conclude that the physical Pomeron is described by the two-point function with boundary conditions corresponding to the unstable initial state : $\langle \phi \rangle = \langle \phi^\dagger \rangle = 0$. It is of interest that the "grey disc" behaviour of the scattering amplitude may be easily understood from this point of view. If the localized perturbation (the external particle emitting a Pomeron) is introduced, then in its immediate neighbourhood the system will switch to the stable ground state with non-vanishing expectation value of the Pomeron field. That is, a region of space is created inside which the average value of the field does not vanish. This region is expanding with constant velocity, so that in the case of a pointlike source it has the form of a disc in the impact parameter plane with radius increasing like $\log s$ because $y = \log s$ is a time variable of the Reggeon field theory.

This behaviour is most easily understood in the classical approximation. Consider the case of a localized (emitting) external source acting at the point $(y, b) = (0, 0)$ [it corresponds to the term $-iJ\delta(y)\delta^2(b)\phi^\dagger(y, b)$ in the Lagrangian density function]. The classical field ϕ is then obtained solving the equation

$$\frac{\partial \phi}{\partial y} - \Delta \phi - \alpha' \nabla^2 \phi + \frac{ig}{2} \phi^2 = 0$$

with the initial condition $\phi(0, b) = -iJ\delta^2(b)$ so that the field is zero almost everywhere. The term $ig\phi^\dagger\phi$ does not appear in the equation because there are no absorbing external sources.

As long as the field ϕ is small ($\phi \ll \langle \phi \rangle = 2\Delta/ig$) we can neglect the self-interaction term and use the free field approximation

$$\phi(y, b) \approx \phi(0, 0) \frac{e^{\Delta y - \frac{|b|^2}{4\alpha'y}}}{4\alpha'y}$$

When y increases, this approximation is restricted to $|b| > V \cdot y$ where $V \approx 2\sqrt{\alpha'\Delta}$. That is, $\phi \approx 0$ outside the disc. Inside the disc the field moves towards the fixed point value $\phi = \langle \phi \rangle$. The vicinity of the fixed point is reached after a finite time $\sim 1/\Delta$ so that the width of the "edge" is finite (it is of order $\sim V/\Delta$). Once the field is near the fixed point its further evolution is best described in terms of the shifted field $\chi = \phi - \langle \phi \rangle$ with intercept below one and consequently finite correlation length. That is, within the disc, the field is equal to $\langle \phi \rangle$ except for short-range corrections.

In the Appendix of this paper we argue that similar behaviour may be expected also in the case of the two-point function of the physical Pomeron : $\langle 0 | T \phi(y, b) \phi^\dagger(0, 0) | 0 \rangle$. More exactly, we expect that, for $|b|$ and/or y large enough

$$\langle 0 | T \phi(y, b) \phi^\dagger(0, 0) | 0 \rangle \rightarrow - \langle \phi^\dagger \rangle \langle \phi \rangle \Theta(Vy - |b|) \quad (\text{III.3})$$

where $V = \text{const } (\sim 2\sqrt{\alpha' \Delta})$ and the vacuum expectation values $\langle \phi^\dagger \rangle$, $\langle \phi \rangle$ are calculated with only absorbing and only emitting sources, respectively. In the Appendix this result is derived with some assumptions about the behaviour of the field expectation values in the presence of external sources.

The enhanced diagram contribution to the imaginary part of the amplitude equals then $\beta^2 \langle 0 | T \phi(y, b) \phi^\dagger(0, 0) | 0 \rangle$ where β is a coupling of the bare Pomeron to external particles. Provided that $\beta^2 |\langle \phi^\dagger \rangle \langle \phi \rangle| \leq 1$, this is in agreement with elastic unitarity. The amplitude behaves like the uniformly absorbing grey disc expanding like $\log s$ in the impact parameter plane. This agrees with the results of Cardy ⁵⁾ and Bronzan ⁴⁾.

The opacity of the absorbing disc is determined by the expectation value of the Pomeron field. Note that as the average value of the ϕ field should be calculated with only emitting sources and the average of the $\phi^\dagger$ with only absorbing ones, these values do not correspond to one vacuum state of the Reggeon field theory. In particular, in the case of the model with triple-Pomeron coupling [the Lagrangian is given by (II.6)], the average values $\langle \phi \rangle$, $\langle \phi^\dagger \rangle$ are those given by (II.9a) and (II.9b), respectively. For a justification of this conjecture, see the Appendix. It is also in agreement with Bronzan's explicit calculation ⁴⁾.

We expect the corrections to the leading term (III.3) to have short range and the edge of the disc should be of constant width. As explained before, this width is of order $V(1/\Delta) \sim 2\sqrt{\alpha' \Delta}$, so that it is essentially determined by the inverse "mass" of the shifted field. This is again in agreement with Ref. 5).

We see that our conjecture (III.3), based on simple quasi-classical arguments, agrees quantitatively with the results of calculating explicitly the sum of diagrams to all orders of the perturbation theory expansion ^{4), 5)}. It is then of interest to consider the generalization of (III.3) to the three-point function of the Pomeron field :

$$\begin{aligned} \langle 0 | T \phi(y_1, b_1) \phi(y_2, b_2) \phi^+(0, 0) | 0 \rangle &\rightarrow \\ &\rightarrow - \langle \phi \rangle^2 \langle \phi^+ \rangle \Theta(V_{y_1} - |b_1|) \Theta(V_{y_2} - |b_2|) \end{aligned} \quad (\text{III.4})$$

This simple form of the three-point function indeed can be obtained in the super-Reggeon calculus of Cardy ⁵⁾. It has not been noticed by Cardy because in his discussion of the three-point function one diagram linear in the triple-Pomeron coupling has been neglected. The only contribution to the three-point function comes from the diagram which is zeroth order in the superpropagator interactions and it is just of the form (III.4).

We are now able to estimate the inclusive distribution in the triple-Regge limit. With $\langle \phi \rangle$ and $\langle \phi^+ \rangle$ equal to (II.9a) and (II.9b), respectively, we obtain a correct positive sign of the inclusive distribution provided that the triple-Pomeron coupling constant is positive. The contribution from the triple-Regge region to total cross-section behaves like ($Y = \log s$)

$$|\beta^2 \langle \phi \rangle^2 \langle \phi^+ \rangle| \int d^2x \Theta(VY - |x|) = |\beta \langle \phi \rangle| \sigma_{\text{tot}} \quad (\text{III.5})$$

so that the triple-Regge region contributes asymptotically a constant fraction of the total cross-section.

The inclusive distribution in the central region may also be estimated. On this point we disagree with Cardy, who has suggested ⁵⁾ that the leading term is cancelled. In fact, the diagrams which Cardy supposed to cancel the leading term of the inclusive distribution cancel among themselves, according to the cutting rules of Abramovskii, Gribov and Kancheli ¹⁵⁾. We obtain (y is the rapidity in the laboratory frame)

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma}{dy} \sim \frac{(Y-y)^2 y^2}{Y^2} \quad (\text{III.6})$$

There is no Feynman scaling and the multiplicity is increasing like $(\log s)^3$.

The inclusive distributions in the central region have been recently calculated by Ciafaloni and Marchesini ¹⁴⁾ with somewhat different approximations. Their results are, however, in agreement with ours.

Finally, we now give a brief discussion of the relation of this work to the recent paper of Amati, Le Bellac, Ciafaloni and Marchesini ⁸⁾. Their results are qualitatively similar to ours and are based on the same physical picture. However, the asymptotic formula for the S matrix given in Ref. 8) cannot be used to estimate the two-point function (enhanced diagram contribution) which is the one calculated in this paper. Note that this is also true in the classical approximation in zero dimension ¹⁶⁾, where the asymptotic S matrix is finite, while all many-point functions are obviously divergent in the $y \rightarrow \infty$ limit. Consequently, we think it unlikely that our results [as well as those of Bronzan ⁴⁾] are compatible with the prescription for the asymptotic S matrix given in Ref. 8), which makes use of the stable vacuum states. The possible explanation is that the prescription of Ref. 8) may not correspond to summing only usual Gribov diagrams.

The other important result of Ref. 8) is the proof that the (finite and non-vanishing) vacuum expectation value of the Reggeon field does indeed exist for $\alpha_0 > \bar{\alpha}_0$. This has been assumed, but not proved, in the present paper.

IV. CONCLUSIONS

The main conclusion of this paper is that if the Reggeon field has a finite and non-vanishing vacuum expectation value, then the exact solution of the Reggeon calculus corresponds to the leading singularity of intercept one. The most important advantage of our solution over the renormalization group "strong coupling" one is that the bare intercept α_0 is not constrained to be equal to some critical value. We expect that our solution is valid in the case when $\alpha_0 > \bar{\alpha}_0$ where $\alpha_0 = \bar{\alpha}_0$ is the constraint under which the "strong coupling" solution is obtained. If this interpretation is correct, the widely discussed "strong coupling" solution is unphysical because it corresponds to the point of instability : a transition between two stable patterns of asymptotic behaviour.

Our results indicate that the Reggeon calculus with $\alpha_0 > \bar{\alpha}_0$ may be in agreement with s channel unitarity. The mechanism which ensures unitarity appears to be connected to the existence of the finite and non-vanishing vacuum expectation value of the Pomeron field. The resulting behaviour of the scattering amplitude (the grey absorbing disc with a radius expanding like $\log s$ in the impact parameter plane) has been frequently obtained before in explicitly unitary models like that of Auerbach, Aviv, Sugar and Blankenbecler ¹⁷⁾.

The Reggeon calculus we speak about is not another name for the Reggeon field theory. According to our argument, the Reggeon field theory is unphysical for $\alpha_0 > \bar{\alpha}_0$ because it does include the class of Reggeon diagrams not obtained from the underlying field theory of strong interactions. This is the reason why our results are in disagreement with those of Abarbanel et al. 7).

As higher order couplings and unenhanced graphs are important in our solution, it does not seem strange that the renormalization group method has not been able so far to give similar results. On the other hand, our results are in agreement with calculations of Bronzan 4) and Cardy 5) and provide them with a simple physical interpretation.

The asymptotic behaviour we discussed is valid at energies corresponding to the radius of the absorbing disc much larger than the range of the χ field or the width of the edge : $\log s \gg 1/V\sqrt{\alpha'}/\Delta$. As $V = 2\sqrt{\alpha'\Delta}$ 5) this means $\log s \gg 1/2\Delta$. Assuming that $\Delta \approx 0.1$ we obtain $\log s \gg 5$, that is the region of accessible energies. This is particularly interesting if we note that the "strong coupling" solution is supposed to be valid for $\log s \gg 400$! 2)

But even in the case if the present energies are too small for our solution, there is one result which is important for phenomenology at present energy, namely that the intercept of the bare Pomeron is not constrained to be equal to the critical value. It is a free parameter and may be varied independently from the Reggeon coupling constants.

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A P P E N D I X

In the following, our conjecture (III.3) for the two-point function of the Pomeron field is derived with some assumptions about the expectation value of the Pomeron field in the presence of external sources localized in time (the rapidity variable).

As before, the symbol $\langle 0 | \dots | 0 \rangle$ denotes the (formal) sum of Gribov diagrams with no tadpoles. The field expectation values under discussion are

$$\Phi_{(\gamma, \beta; \bar{\gamma})}^+ [G] \equiv \langle 0 | T \phi^+(\gamma, \beta) e^{-i \int G(b) \phi(\bar{\gamma}, b) d^2 b} | 0 \rangle \quad (A.1)$$

$$\Phi_{(\gamma, \beta; \bar{\gamma})} [G] \equiv \langle 0 | T \phi(\gamma, \beta) e^{-i \int G(b) \phi^+(\bar{\gamma}, b) d^2 b} | 0 \rangle$$

They correspond respectively to the source terms $-iG(b)\delta(y-\bar{y})\phi(y, b)$ and $-iG(b)\delta(y-\bar{y})\phi^+(y, b)$ in the Lagrangian density function. We write $\Phi^+[G]$, $\Phi[G]$ to indicate the functional dependence on the source distribution function $G(b)$. Note that for $Y > \bar{Y}$ ($Y < \bar{Y}$) the expectation value Φ^+ (Φ) is identically zero.

Replacing the function $G(b)$ in Φ^+ by the field operator $i\phi^+(\bar{y}, b)$ we obtain the operator

$$\begin{aligned} \Phi_{(\gamma, \beta; \bar{\gamma})}^+ [i\phi^+(\bar{y}, b)] &\equiv \int d^2 b \left. \frac{\delta \Phi^+}{\delta G(b)} \right|_{G=0} i\phi^+(\bar{y}, b) + \\ &+ \frac{1}{2} \int d^2 b \int d^2 b' \left. \frac{\delta^2 \Phi^+}{\delta G(b) \delta G(b')} \right|_{G=0} i\phi^+(\bar{y}, b) i\phi^+(\bar{y}, b') + \\ &+ \dots \end{aligned} \quad (A.2)$$

Here $\delta/\delta G(b)$ is the functional derivative with respect to $G(b)$, $\Phi^+[0] \equiv 0$.

The two-point function of the Pomeron field may then be written as follows :

$$\langle 0 | T \phi^+(\gamma, \beta) \phi(\gamma', \beta') | 0 \rangle = \langle 0 | T \Phi_{(\gamma, \beta; \bar{\gamma})}^+ [i\phi^+(\bar{y}, b)] \phi(\gamma', \beta') | 0 \rangle \quad (A.3)$$

where $Y < \bar{y} < Y'$. In terms of diagrams this corresponds to dividing the rapidity integrations into two regions: $(Y, \bar{y})$ and $(\bar{y}, Y')$, respectively, and summing first over all interactions within the rapidity interval $(Y, \bar{y})$. In doing this, we use the identity

$$D_0(y_1 - y_2, b_1 - b_2) \equiv \int d^2 b \, D_0(y_1 - y, b_1 - b) D_0(y - y_2, b - b_2)$$

for any internal line joining the vertices belonging to different regions of integration. (A.3) may also be derived straight from the generating functional for Gribov diagrams.

We shall discuss only the case of small slope and large intercept of the bare Pomeron trajectory, so that in (A.3) it is possible to have $\Delta(\bar{y} - Y) \gg 1$ and $\alpha'(\bar{y} - Y) \ll 1$ simultaneously [we do not require, however, that $\alpha'(Y' - Y)$ is small]. We also assume that quantum corrections are unimportant for $y \lesssim \bar{y} - Y$; this is possible if the Pomeron interaction constants are small enough. We can then estimate $\Phi_{(Y, B; \bar{y})}^+$ in the zero slope classical approximation. The functional $\Phi_{(Y, B; \bar{y})}^+$ becomes then a function depending on the field at the point $b = B$ alone (when $\alpha'y \rightarrow 0$ the free propagators approach δ functions in the impact parameter plane). We write this function as a Fourier transform with respect to $\phi^+(\bar{y}, B)$:

$$\Phi_{(Y, B; \bar{y})}^+[\bar{\phi}^+(\bar{y}, b)] \rightarrow \int_{-\infty}^{\infty} dz \, \widetilde{F}_{\bar{y}-Y}(z) e^{-iz\phi^+(\bar{y}, B)} \quad (A.4)$$

Re-placing Φ^+ in (A.3) by this approximation, we get

$$\begin{aligned} \langle 0 | T \phi^+(Y, B) \phi(Y', B') | 0 \rangle &\approx \\ &\approx \int_{-\infty}^{\infty} dz \, \widetilde{F}_{\bar{y}-Y}(z) \langle 0 | T \phi(Y', B') e^{-iz\phi^+(\bar{y}, B)} | 0 \rangle = \\ &= \int_{-\infty}^{\infty} dz \, \widetilde{F}_{\bar{y}-Y}(z) \Phi_{(Y', B'; \bar{y})} [z\delta^2(b-B)] \end{aligned} \quad (A.5)$$

where $\Phi[z\delta^2(b-B)]$ is the expectation value of ϕ corresponding to the source distribution function $G(b) = z\delta^2(b-B)$, as defined in (A.1).

In the case of the triple-Pomeron interaction, the classical solution in zero slope limit is known ¹⁶⁾ and we can obtain $\tilde{F}_y(z)$ explicitly. For large Δy , it is

$$\tilde{F}_y(z) \rightarrow \langle \phi^+ \rangle \left\{ \delta(z) - \bar{\tau} \langle \phi^+ \rangle e^{-\Delta y} e^{-\bar{\tau} z \langle \phi^+ \rangle e^{-\Delta y}} \Theta(z) \right\}$$

$[\langle \phi^+ \rangle = 2\Delta/ig]$ is the vacuum expectation value given in (II.9a) .:

We can now rewrite (A.5) as follows

$$\langle 0 | T \phi^+(Y, \underline{B}) \phi(Y', \underline{B}') | 0 \rangle \approx - \langle \phi^+ \rangle \int_0^\infty dz e^{-z} \Phi_{(Y', \underline{B}'; \bar{y})} \left[\frac{z e^{\Delta y}}{\bar{\tau} \langle \phi^+ \rangle} \delta^2(\underline{b} - \underline{B}) \right] \quad (A.6)$$

The first term in $\tilde{F}_y(z)$ does not contribute, because $\Phi[0] \equiv 0$.

According to the arguments given in Section III of this paper we expect that the expectation value of the field ϕ at the point $(Y', \underline{B}')$ with external emitting sources localized at $(Y, \underline{B})$ for large $Y' - Y$ approaches the vacuum expectation value $\langle \phi \rangle$ everywhere within the disc $|\underline{B}' - \underline{B}| < V |Y' - Y|$ where V is a constant. Outside the disc (with the exception of the finite width "edge") we expect that Φ is small and that it vanishes for large $|\underline{B}' - \underline{B}|$. This behaviour has indeed been obtained in the tree approximation ^{8), 14)}. We assume that it is not significantly changed by quantum corrections.

Neglecting the edge of the disc (which gives non-leading $\sim \log s$ contribution to the amplitude) we obtain

$$\Phi_{(Y', \underline{B}'; \bar{y})} [z \delta^2(\underline{b} - \underline{B})] \approx \langle \phi \rangle \Theta(V(Y' - Y) - |\underline{B}' - \underline{B}|)$$

and

$$\langle 0 | T \phi^+(Y, \underline{B}) \phi(Y', \underline{B}') | 0 \rangle \approx - \langle \phi^+ \rangle \langle \phi \rangle \Theta(V(Y' - Y) - |\underline{B}' - \underline{B}|) \quad (A.7)$$

in agreement with (III.3). In the tree approximation $V = 2\sqrt{\alpha'} \Delta$.

The minus sign in (A.7) is obviously necessary to have the s channel unitarity satisfied. Notice that it is not a result of our convention of using imaginary coupling constants. In fact, in the formalism of ^{8), 14), 16)},

with real coupling constant and the vacuum expectation values the Pomeron propagator must be negative.

Instead of assuming the zero slope approximation for Φ^+ it is also possible to derive (A.7) assuming that $\Phi_{(Y,B;\bar{Y})}^+[G(b)]$ is essentially independent of the source distribution function outside the disc. (A.4) is then replaced by the functional integration and the argument becomes more complicated. It is not necessary to require that $\Delta \gg \alpha'$ and that $\bar{Y} \rightarrow Y$ in this case.

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