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COMMENT ON PCAC AND THE ELECTROMAGNETIC MASS SPLITTINGS
OF CHARMED MESONS

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A B S T R A C T

Strong PCAC and a generalization of Dashen's relation for the $K^+ - K^0$ mass difference as well as non-relativistic considerations are used in order to obtain insight into the mechanism which causes the mass splitting among the charmed pseudoscalar mesons D^+ and D^0 . We find a value of 7...8 MeV for $M_{D^+} - M_{D^0}$.

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Recently it was pointed out that the electromagnetic mass differences among charmed particles, especially the D^+ and D^0 mesons, may be unusually large ¹⁾. As emphasized, this could be important for the production of the various charmed mesons in the e^+e^- channel. The authors of Ref. 1) based their considerations on a crude non-relativistic ansatz for the electromagnetic splittings. In this note I would like to study the question of the D^+-D^0 mass splitting by using the conventional PCAC approach ²⁾ for the pseudoscalar mesons in combination with non-relativistic approximations. The D^+-D^0 mass difference turns out to be relatively large (about 7...8 MeV), but significantly smaller than the value of ~ 15 MeV quoted in Ref. 1).

Within strong PCAC ²⁾ the mass squared of a pseudoscalar meson is, up to photon self-energy contributions, linear in the effective quark masses. Specifically one has

$$\begin{aligned} M_{\pi^+}^2 &\approx (m_d + m_u) M_0 + \gamma_{\pi^+}, & M_{K^+}^2 &\approx (m_s + m_u) M_0 + \gamma_{K^+} \\ M_{\pi^0}^2 &\approx (m_d + m_u) M_0 + \gamma_{\pi^0}, & M_{K^0}^2 &\approx (m_s + m_d) M_0 + \gamma_{K^0} \end{aligned} \quad (1)$$

where M_0 is a common $SU(3) \times SU(3)$ invariant scale parameter, m_u , m_d , and m_s are the effective quark masses ³⁾, and $\gamma_{\pi^+} \dots$ are the various photon self-energy contributions. The u-d mass difference gives rise to the well-known u_3 term in the $SU(3)$ breaking Hamiltonian; it is generally agreed that its origin is due to the weak and associated interactions.

Dashen has emphasized ⁴⁾ that in the limit of chiral $SU(3) \times SU(3)$ one has

$$\gamma_{K^+} - \gamma_{K^0} = \gamma_{\pi^+} - \gamma_{\pi^0} \quad (2)$$

which gives

$$\frac{(M_{K^0}^2 - M_{K^+}^2) + (M_{\pi^+}^2 - M_{\pi^0}^2)}{M_{\pi^+}^2 / 2} \approx \frac{m_d - m_u}{(m_d + m_u)/2} \approx 0.56 \quad (3)$$

This value is consistent with other determinations of the u_3 term, for example by using the neutron-proton mass difference [in this case one finds a slightly smaller value : $(m_d - m_u)/((m_d + m_u)/2) \cong 0.4$, see, for example, Ref. 5) for a recent discussion].

If the masses of the charmed particles were not as heavy as observed.⁶⁾, but comparable in mass to the conventional hadron states, the notion of approximate $SU(3) \times SU(3)$ invariance of hadron dynamics could be generalized to an approximate $SU(4) \times SU(4)$ invariance. Let us study this fictitious case more in detail. Besides Eq. (1) we would have the new relations

$$M_{D^+}^2 = (m_c + m_d) M_0 + \gamma_{D^+} ; \quad M_{D^0}^2 = (m_c + m_u) M_0 + \gamma_{D^0} \quad (4)$$

where m_c is the effective charmed quark mass. Dashen's relation could be immediately generalized to

$$\gamma_{D^+} - \gamma_{D^0} \cong \gamma_{K^+} - \gamma_{K^0} \cong \gamma_{\pi^+} - \gamma_{\pi^0} \quad (5)$$

(This formula follows since the electromagnetic self-energy of the D^0 is zero, and D^+ and π^+ are related by the generalized U spin transformation $u \rightleftharpoons c$.) Suppose we set for simplicity $m_s = m_c$, implying $M_D^2 = M_K^2$. One obtains, using (5) :

$$\begin{aligned} M_{D^+}^2 - M_{D^0}^2 &\cong 2 M_D (M_{D^+} - M_{D^0}) \cong (m_d - m_u) M_0 + \gamma_{D^+} - \gamma_{D^0} \\ &\cong (m_d - m_u) M_0 + M_{\pi^+}^2 - M_{\pi^0}^2 \\ &\cong M_{K^0}^2 - M_{K^+}^2 + 2 (M_{\pi^+}^2 - M_{\pi^0}^2) \end{aligned} \quad (6)$$

which leads to

$$M_{D^+} - M_{D^0} \cong 6.5 \text{ MeV} \quad (7)$$

to be compared with 4.0 MeV in case of the K^0-K^+ mass splitting. We obtain a significantly larger mass splitting for D mesons than for K mesons : the "tadpole" contributions and the photon self-energy contributions are subtractive for K mesons, but additive for D mesons.

How can we apply these considerations to the real world, where m_c is much larger than m_s ? Since K mesons are relatively heavy, one may expect that non-relativistic approximations, which certainly are very bad for the pion system, become useful for the kaon system. In a non-relativistic picture the "tadpole" contributions to the electromagnetic mass differences correspond to the u-d mass difference $M_u - M_d$, where M_u, M_d are the non-relativistic constituent quark masses [not to be confused with the "relativistic" quark masses entering in the PCAC formula (1)], while the direct photon contributions correspond to the Coulomb contributions to the electromagnetic self-energy, which is proportional to the expectation value of $(Q_1 \cdot Q_2)/r$ (Q_i = quark charges). Thus we find, still in the fictitious case $M_D = M_K$:

$$\begin{aligned} M_{D^+} - M_{D^0} &\approx M_d - M_u + C.c. \\ &\approx \frac{M_{K^0}^2 - M_{K^+}^2 + M_{\pi^+}^2 - M_{\pi^0}^2}{2 M_D} + C.c. \\ &\approx 5.3 \text{ MeV} + C.c. \end{aligned} \quad (8)$$

(C.c : Coulomb contribution)

Extrapolating to the real case $m_c \gg m_s$, one obtains the same result as in (8) for the tadpole contributions (the charmed quark mass cancels in the mass difference).

The Coulomb contribution to the K mass difference is proportional to $(-1/3)(1/3) - (-1/3)(2/3) = 1/9$, while the contribution to the D mass difference is proportional to $(2/3)(2/3) - (-1/3)(2/3) = 2/9$ i.e., ¹⁾ $\Delta M_D^{\text{Coulomb}} = 2 \Delta M_K^{\text{Coulomb}}$. On the other hand the Coulomb contribution to ΔM_K is $(M_{\pi^+}^2 - M_{\pi^0}^2)/2M_K$, thus one obtains

$$\Delta M_D^{\text{Coulomb}} \approx 2.6 \text{ MeV} \quad (9)$$

and

$$M_{D^+} - M_{D^0} = \underbrace{5.3 \text{ MeV}}_{\text{tadpole}} + \underbrace{2.6 \text{ MeV}}_{\text{Coulomb}} = 7.9 \text{ MeV} \quad (10)$$

Note that the non-relativistic considerations violate the generalized Dashen relation (5), which is strictly valid only in the (relativistic) PCAC limit. Thus we regard the two values (7) and (10) for the D mass difference as limits for the range of the physical mass difference. Since the D meson system may well be described by non-relativistic physics because of the heaviness of the charmed quark, we expect the real D mass difference to be close to 8 MeV.

Applying analogues of the well-known $SU(6)$ mass relations, e.g.,

$$M_{D^*}^2 - M_{D^0}^2 \cong M_{D^{*+}}^2 - M_{D^{0+}}^2, \quad (11)$$

we expect the D^* mass difference also to be ~ 8 MeV.

It is interesting that our value (10) for the D mass difference is significantly lower than the value of 15 MeV as advocated in Ref. 1). The reason for it is simple : the authors of Ref. 1) used the pion mass difference in their non-relativistic calculations, while we use the "non-tadpole" part of the kaon mass difference in extrapolating to the D meson system. We believe that according to the success of chiral $SU(2) \times SU(2)$ non-relativistic considerations as the ones discussed in Ref. 1) applied to the pion system have no value.

An experimental determination of the D mass difference will enable us to distinguish between the two approaches. At the same time one will also obtain a useful insight into the interplay between the full relativistic chiral dynamics on the one side, and non-relativistic quark dynamics on the other side. If the D mass difference turns out to be much larger than 8 MeV, say 15 MeV, it would mean that the basic ideas on PCAC and the origin of the electromagnetic mass splittings have to be reinvestigated.

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