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A REAPPRAISAL OF KNOWN SOURCES OF PROMPT ELECTRONS
AND PHOTONS FROM HADRON COLLISIONS

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A B S T R A C T

The prompt electron yield from known sources of pairs, including the photon continuum near $q^2 \sim 0$, is calculated for both small and large $p_\perp$. At low $p_\perp$, the prompt electron yield rises as $1/p_\perp$, while at moderate $p_\perp$ ($\lesssim 2.5$ GeV), the inclusion of a realistic low mass continuum and of additional three-body decays such as $\omega \rightarrow \pi^0 e^+ e^-$ raises the prompt electron yield by a factor of about 2 compared with previous estimates. Finally we stress the importance of measuring the $e^+ e^-$ (or $\mu^+ \mu^-$) pair mass distribution at very low invariant masses, in order to test the basic assumptions one needs to make in calculating the ℓ/π and γ/π^0 ratios.

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1. - INTRODUCTION

Much of the direct lepton yield observed ¹⁾ in hadron-hadron interactions seems to be due ^{2),3)} to the decay into lepton pairs of the well-known vector mesons ρ^0 , ω , ϕ , J/ψ (which should also include three-body decays as $\omega \rightarrow \pi^0 e^+ e^-$) and possibly higher mass vector states ^{2),4)}. Conventional estimates ²⁾⁻⁴⁾ of the single lepton yield due to vector meson decays have been based on the narrow width approximation (hadronic production cross-section times branching ratio). In the present paper, it will be pointed out that the narrow width approximation neglects an important additional contribution due to low mass ($q_{e^+e^-}^2 \ll m_V^2$) lepton pairs, which strongly influences the low transverse momentum ($p_\perp$) lepton yield, and also contributes significantly at large $p_\perp$. The vector meson photon coupling gives rise to a copious production of low invariant mass pairs, essentially due to the photon propagator $1/q^2$. Moreover, since there is good empirical evidence that the dynamics of hadronic inclusive production is determined by the transverse energy ($E_\perp = \sqrt{q^2 + q_\perp^2}$) of the produced system, and, further, the $E_\perp$ distribution is steeply falling, the production of lower invariant mass pairs should in addition be hadronically enhanced. Specifically, we will show that the vector meson dominated low mass continuum ($q^2 \ll m_V^2$) leads to a $1/p_\perp$ rise of the electron to pion ratio at low $p_\perp$ ($\lesssim 0.25$ GeV). For vector mesons with appreciable photon coupling and relatively small branching ratio into lepton pairs (i.e., for a broad object, like the ρ^0 or even the ρ'), the prompt lepton yield is enhanced also at large transverse momentum. The enhancement factor for the ρ^0 is about 1.8 in the large $p_\perp$ limit, if compared with the conventional estimate, which includes pairs with masses in the vicinity of the ρ^0 mass only.

The vector meson dominated low mass continuum yields for direct real photons a rather small value for the ratio $\gamma/\pi \sim 0.4\%$. In fact, from our considerations, taking into account mechanisms like $\omega \rightarrow \pi^0 \gamma$, $\eta' \rightarrow (\rho, \gamma) \gamma$, it is difficult to find a value of $(\gamma/\pi^0)_{\text{excess}} = (\gamma/\pi^0) - (\gamma/\pi^0)_{\pi^0} - (\gamma/\pi^0)_\eta$ much larger than 6% at large $p_\perp$ without invoking some additional photon coupling, which does not go through the known vector mesons. The existence of such an additional coupling has recently been suggested by Preparata ⁵⁾. A large direct photon yield $(\gamma/\pi^0)_{\text{excess}} \sim 20\%$ has been conjectured in some parton model calculations ⁶⁾ and in the Landau hydrodynamical model ⁷⁾. However, these models would predict a massive enhancement in the e^+e^- pair mass spectrum at small $q^2 \ll m_\rho^2$, and will make the

(ρ^0, ω) peak appear like a relatively minor effect. Hence we show that a measurement of the e^+e^- mass spectrum for small q^2 ($q^2 = 0-1 \text{ GeV}^2$) will provide a most significant test of these widely different predictions on the direct photon yield.

The decays $\omega \rightarrow \pi^0 e^+ e^-$ and $\eta' \rightarrow \rho^0 e^+ e^-$, etc., are quite analogous to π^0 and η Dalitz decays. Our estimate shows that especially $\omega \rightarrow \pi^0 e^+ e^-$ is a strong lepton source, which does not seem to have been fully accounted for in previous estimates of prompt lepton production.

In Section 2, basic formulae and basic dynamical assumptions are given. Section 3 estimates the prompt lepton production from vector meson sources including the low mass continuum at low and high transverse momenta. In Section 4, contributions from decays of the type $\omega \rightarrow \pi^0 e^+ e^- (\pi^0 \gamma)$ and $\eta' \rightarrow (\gamma, \rho) e^+ e^-$, $\eta' \rightarrow (\gamma, \rho) \gamma$, etc., are estimated. The numerical results and a comparison with the available data is presented in Section 5. The predicted low invariant mass spectrum and remarks on prompt photons are to be found in Section 6, and a few concluding remarks are given in Section 7.

2. - BASIC FORMULAE AND DYNAMICAL ASSUMPTIONS

If we neglect spin or at least polarization effects, the single lepton spectrum $ab \rightarrow \ell + x$, resulting from the production of a narrow resonance or particle $ab \rightarrow P + x$ and its subsequent decay $P \rightarrow \ell + x'$ can be expressed in the form

$$2 E_p \frac{d\sigma}{d^3 p} = B_\ell \int \frac{d^3 P}{2 E_P} \left(2 E_P \frac{d\sigma}{d^3 P} \right) \frac{1}{\Gamma_\ell} 2 E_p \frac{d\Gamma_\ell}{d^3 p} \quad (2.1)$$

where B_ℓ is the branching ratio for the parent into the channel $P \rightarrow \ell + x'$ and Γ_ℓ is the corresponding partial width. Equation (2.1) assumes P is a narrow width object. If, however, the parent is a broad resonance or a continuous mass source, then provided the production and decay mechanisms are factorizable ^{*)} (i.e., are only linked by the running mass variable), we use the following generalization

^{*)} Spin and polarization effects can readily be incorporated using the density matrix formalism.

$$2E_p \frac{d\sigma}{d^3p} = \int dm^2 \rho(m^2) B_e(m^2) \int \frac{d^3P}{2E(m^2)} \left(2E \frac{d\sigma}{d^3P} \right) \frac{1}{\Gamma_\ell(m^2)} 2E_p \frac{d\Gamma_\ell}{d^3p}, \quad (2.2)$$

where, for a broad resonance $\rho(m^2)$ is simply the Breit-Wigner excitation formula of the resonance. $\Gamma_\ell(m^2)$ and the invariant cross-section $(2E(d^3\sigma/d^3P))$ are defined in terms of Feynman amplitudes in exactly the same way as for a narrow width object.

The case of prompt leptons involving three-body decays, such as $\omega \rightarrow \pi^0 e^+ e^-$ will be discussed in Section 4, where we show that a non-negligible contribution comes from such mechanisms, at large $p_\perp$. In this section we will restrict ourselves to the production of lepton pairs from vector meson decay $V \rightarrow e^+ e^-$ and the associated low mass continuum.

For this latter case

$$\rho(q^2) 2E_q \frac{d\sigma}{d^3q} = 2E_q \frac{d\sigma}{d^3q} \frac{m_V \Gamma_V}{\pi (q^2 - m_V^2)^2 + m_V^2 \Gamma_V^2} \quad (2.3)$$

gives the mass distribution of the produced hadronic spin 1 photon source [using q^2 and q instead of m^2 and P in (2.2)], and the branching ratio is given by

$$B_e(q^2) = B_{V \rightarrow e^+ e^-} \frac{m_V^2}{q^2}, \quad (2.4)$$

and the two-body decay probability per invariant phase space element ^{*)}, by

$$\frac{1}{\Gamma_V} 2E_p \frac{d\Gamma_V}{d^3p} = \frac{2}{\pi} \delta(q^2 - 2q \cdot p). \quad (2.5)$$

*) A more refined analysis should include polarization effects, in which one discriminates between transverse and longitudinal photons.

Γ_V and m_V denote vector meson total width and mass, respectively, and the factor $1/q^2$ in (2.4) originates from the photon propagator (only one power of $1/q^2$ remains upon summing over lepton spins). The vector meson branching ratio is given by

$$B_{V \rightarrow e^+e^-} = \frac{\Gamma_{V \rightarrow e^+e^-}}{\Gamma_V}, \quad (2.6)$$

$$\Gamma_{V \rightarrow e^+e^-} = \frac{1}{3} \propto m_V / (f_V^2/4\pi) \equiv \frac{1}{12} \propto m_V / (g_V^2/4\pi),$$

where, in the case of the ρ , the vector meson photon coupling $f_\rho^2/4\pi \equiv 4(g_\rho^2/4\pi) \approx 2.4$. For simplicity, we shall assume that at high energies and small rapidities the hadronic parent spectrum (in centre-of-mass system) factorizes according to

$$2E_q \frac{d\sigma}{d^3q} = c F(E_\perp) f(y), \quad (2.7)$$

where $y = \log \left[(q_0 + q_{||}) / (q_0 - q_{||}) \right]$ is the rapidity and $E_\perp = \sqrt{q^2 + q_\perp^2}$ the transverse energy. Such a form roughly describes the known hadronic single particle spectra in the central region where one finds $F(E_\perp)$ is approximately universal⁸⁾, i.e., the production of particles with widely different masses is, to a first approximation, described by the same function $F(E_\perp)$. This has also been shown to hold for a continuum mass system by recent measurements⁹⁾ of, e.g., $\pi^+\pi^-$ pairs with masses in the range 2-3 GeV. We shall therefore assume (2.7) to be the correct form for the parent spectrum entering in (2.2). Theoretically, such an ansatz can be motivated in multiperipheral and thermodynamic models. When using (2.7) for a broad object, like the ρ^0 meson, we get the following important dynamical effect. Since $E_\perp$ is determined by the continuous mass variable q^2 and not the mass m_V^2 entering in the Breit-Wigner distribution, the effective parent mass distribution in q^2 is significantly distorted for small $q_\perp$, because $F(E_\perp)$ is a steeply falling function. [Naively this may seem to contradict a dispersion theoretic approach, in which one starts from a mass dispersion relation. This for the case of a single vector meson is approximated by the vector meson pole multiplied by the residue of this pole, i.e., at $q^2 = m_V^2$. However, the mass dispersion relation in the present case should be evaluated for fixed $E_\perp$ or more precisely for fixed Toller variable η , the latter being simply related to $E_\perp = \sqrt{q^2 + q_\perp^2}$ by

kinematics. Any other procedure would have to take into account the singularities in η_π . We shall return to the above effect on the mass distribution in Section 6, where we consider the e^+e^- pair mass spectrum.

One can look at the prompt lepton yield according to (2.2)-(2.6) in another way by approximately separating the contributions obtained from the different ranges in q^2 . First of all, for $q^2 \sim m_V^2$ the prompt lepton yield may, to a good approximation, be obtained by using the narrow width approximation, i.e.,

$$2E_q \frac{d\sigma}{d^3q} f(q^2) B \frac{m_V^2}{q^2} \simeq B_{V \rightarrow e^+e^-} 2E_q \frac{d\sigma}{d^3q} (pp \rightarrow V^0 X) \delta(q^2 - m_V^2). \quad (2.8)$$

The magnitude of this piece (i.e., the conventional estimate ²⁾⁻⁴⁾) is thus determined by the leptonic branching ratio $B_{V \rightarrow e^+e^-}$ times vector meson production. In contrast, the additional contribution from low invariant mass pairs ($q^2 \ll m_V^2$) is in a good approximation given by

$$\begin{aligned} 2E_q \frac{d\sigma}{d^3q} f(q^2) B \frac{m_V^2}{q^2} &\simeq \\ &\simeq 2E_q \frac{d\sigma}{d^3q} (pp \rightarrow V_{tr}^0 X) \frac{m_V^4}{(q^2 - m_V^2)^2} \frac{\alpha^2}{3\gamma_V^2} \frac{1}{q^2} \\ &\simeq 2E_q \frac{d\sigma}{d^3q} (pp \rightarrow V_{tr}^0 X) \frac{\alpha\pi}{\gamma_V^2} \frac{\alpha}{3\pi q^2} \end{aligned} \quad (2.9)$$

and is thus determined by $\Gamma_{e^+e^-}$ or, equivalently, the vector meson photon coupling strength $\gamma_V^2/4\pi$. The relative contributions from $q_{e^+e^-}^2 \approx m_V^2$ and $q_{e^+e^-}^2 \approx 0$ to the prompt lepton yield will thus be strongly dependent on the particular vector meson considered. The right-hand side in (2.9) may be viewed as production of virtual photons $pp \rightarrow \gamma_{virt} X$ of mass $q^2 \approx 0$ multiplied with $\alpha/3\pi q^2$ to obtain the lepton yield. Since the dynamics of the process is assumed to be determined by $E_\perp$, the cross-section for production of transversely polarized vector mesons, $pp \rightarrow V_{tr}^0 X$, is to be evaluated ^{*)} at $E_\perp = \sqrt{q^2 + q_\perp^2}$.

*) The value of the vector meson production cross-section entering (2.9) for fixed photon transverse momentum $q_\perp(\gamma)$ is the one measured for vector mesons ($q^2 = m_V^2$) at $E_\perp = \sqrt{m_V^2 + q_\perp^2(V)} = \sqrt{q^2 + q_\perp^2(\gamma)} \simeq q_\perp(\gamma)$. For $E_\perp = 0$, the vector meson production cross-section is to be extrapolated to $E_\perp = 0$. The procedure is analogous to the one used in photoproduction, where Compton scattering and vector meson photoproduction are related to each other at fixed t (and not at fixed scattering angle).

3. - LEPTON TO PION RATIO FROM REAL AND VIRTUAL VECTOR MESON DECAYS, $V \rightarrow e^+e^-$

The prompt lepton yield for, e.g., $pp \rightarrow e^-X$ from real and virtual vector meson sources, as specified by (2.2) to (2.7), is evaluated in the large $p_\perp$ and small $p_\perp$ limits in Sections 3.1 and 3.2, respectively.

3.1. - The large $p_\perp$ region

With the δ function (2.5), one angular integration over the parent spectrum may be carried out, and (2.2) thus simplifies to ^{*)}

$$2E_p \frac{d\sigma}{d^3p} = B_{V \rightarrow e^+e^-} \int \frac{dq^2}{4\mu^2} m_V^2 \rho(q^2) \frac{1}{2\pi} \int_0^{2\pi} d\varphi \int_{E_-}^{E_+} \frac{dE_q}{p} \left(2E_q \frac{d\sigma}{d^3q} \right), \quad (3.1)$$

where $p \equiv |\vec{p}|$ denotes the momentum of the observed electron or muon, μ the electron (or muon) mass, and the limits of the energy integration are given by

$$E_{\pm} = \frac{1}{2\mu^2} \left(p^0 q^2 \pm |\vec{p}| \sqrt{q^2} \sqrt{q^2 - 4\mu^2} \right). \quad (3.2)$$

The angle θ between the parent particle (virtual photon or vector meson) of mass q^2 and the observed electron is determined by

$$\cos \theta = \frac{2p^0 E_q - q^2}{2|\vec{p}||\vec{q}|}, \quad (3.3)$$

where $|\vec{q}| = \sqrt{E_q^2 - q^2}$. The azimuthal angle φ in (3.1) is defined with respect to the plane containing the beam axis and the observed electron. For $\vec{p}^2 \gg \mu^2$ and $q^2 \gg 4\mu^2$, the integration limits simplify to

$$\begin{aligned} E_- &= |\vec{p}| \left(1 + q^2/(4\vec{p}^2) \right), \\ E_+ &= |\vec{p}| q^2/\mu^2. \end{aligned} \quad (3.4)$$

^{*)} This is a widely used standard procedure. See, e.g., Ref. 10).

For electrons, (3.4) is good for almost the whole integration range in q^2 . The parent production cross-section in (3.1) depends on the parent momentum components $q_{||}$ and $q_{\perp}$ with respect to the beam direction, which are related to θ , φ and E_q via

$$\begin{aligned} q_{||} &= |\vec{q}| \cos \vartheta_z, \\ q_{\perp} &= |\vec{q}| \sin \vartheta_z, \end{aligned} \quad (3.5)$$

where

$$\cos \vartheta_z = \cos \vartheta_t \cos \vartheta - \sin \vartheta_t \sin \vartheta \cos \varphi, \quad (3.6)$$

and θ_t is the (trigger) angle of the observed electron with respect to the beam direction. In the limit of large trigger momentum $|\vec{p}|^2 \gg q^2$, from (3.3) and (3.4) we have $\cos \theta \rightarrow 1$, i.e., the parent particle has approximately the direction of the trigger particle. The φ integration becomes trivial and reduces to a factor 2π .

For the further evaluation of (3.1) let us, for simplicity, first assume a parent spectrum at large transverse momentum of the form

$$2E_q \frac{d\sigma}{d^3q} = c_V \frac{1}{E_1^n} f(x_{\perp}) \varphi(x_{||}) \quad (3.7)$$

where $X_{\perp} = 2q_{\perp}/\sqrt{s}$, $X_{||} = 2q_{||}/\sqrt{s}$ and $E_1 = \sqrt{q_{\perp}^2 + q^2}$, and let us first restrict ourselves to the narrow width approximation, by putting $\rho(q^2) = \delta(q^2 - m_V^2)$. For $\theta_t = 90^\circ$ and large trigger momentum, $p \equiv p_{\perp}$, from the above discussion, only parent particles with approximately $X_{||} \cong 0$ and $E_1 \cong E_q$ contribute. Making the variable change $E_q = yp_{\perp}$, we obtain immediately the well-known parent daughter relation ^{11), 12)}

$$2E_p \frac{d\sigma}{d^3p} = \frac{c_V \varphi(0)}{p_{\perp}^n} B \int_1^\infty dy y^{-n} f(x_{\perp} \cdot y), \quad (3.8)$$

i.e.,

$$2E_p \frac{d\sigma}{d^3p} (pp \rightarrow e^- X) = \frac{c_V f(x_{\perp}) \varphi(0)}{p_{\perp}^n} \frac{B}{(n-1)} \left(1 - \frac{x_{\perp} \cdot a}{n} + O\left(\frac{x_{\perp}^2}{n^2}\right) \right) \quad (3.9)$$

where

$$a = \frac{d}{dx_{\perp}} \ln f(x_{\perp}). \quad (3.10)$$

If $x_{\perp}$ is not too large, we can neglect the second term, and assuming (3.7) also for the pion yield, we obtain for the electron to pion ratio in the narrow width approximation

$$\frac{e}{\pi} = \frac{c_V}{c_{\pi}} \frac{B_{V \rightarrow e e^-}}{(n-1)}. \quad (3.11)$$

The parent daughter relation (3.8) may immediately be generalized to a parent spectrum, in which the power law in $E_{\perp}$ is replaced by an arbitrary steeply falling function $F(E_{\perp})$. For example, by representing $F(E_{\perp})$ locally by $F(y_{p\perp}) = F(p_{\perp})/y^{n_{\text{eff}}}$ one obtains the local parent daughter relation ($n_{\text{eff}} \gg 1$)

$$2E_p \frac{d\sigma}{d^3p} = c_V F(q_{\perp}) f(x_{\perp}) g(0) \frac{B}{n_{\text{eff}}} \left(1 - \frac{x_{\perp} a}{n_{\text{eff}}}\right), \quad (3.12)$$

where now

$$n_{\text{eff}} = -q_{\perp} \frac{\partial}{\partial q_{\perp}} \ln F(q_{\perp}). \quad (3.13)$$

For $F(E_{\perp}) = \exp(-aE_{\perp})$, we have $n_{\text{eff}} = a q_{\perp}$, so that the daughter spectrum falls off more rapidly than the parent spectrum. In our subsequent numerical estimates, we will use for $F(E_{\perp})$ the empirical formula proposed by Bourquin and Gaillard ³⁾, namely,

$$F(E_{\perp}) = \frac{1}{(1 + E_{\perp}/a)^N}, \quad (a = 2 \text{ GeV}, N = 12.3) \quad (3.14)$$

which has

$$n_{\text{eff}} = N \left(\frac{q_{\perp}}{a + q_{\perp}} \right) \quad (3.15)$$

and thus behaves as an exponential in $q_{\perp}$ for small $q_{\perp}$, but as a power law (with fixed power) for $q_{\perp} \gg a$.

With inclusion of the full photon continuum, as specified in Section 2, an analogous analysis leads to

$$2E_p \frac{d\sigma}{d^3p} = \frac{c_v f(x_L) f(0)}{p_L^n} \frac{B_{V \rightarrow e^+e^-}}{(n-1)} \int_{4\mu^2}^{\infty} \frac{dq^2}{q^2} m_V^2 \rho(q^2), \quad (3.16)$$

and the corresponding lepton to pion ratio for $p_L^2 \gg m_\rho^2$ becomes

$$\frac{e}{\pi} = \frac{c_v}{c_\pi} \frac{B_{V \rightarrow e^+e^-}}{n_{eff}} \left(1 + \frac{\Gamma_V}{\pi m_V} \ln \frac{m_V^2}{4\mu^2} \right), \quad (3.17)$$

where the second term, which has been absent in (3.11), corresponds to the contribution from low invariant mass lepton pairs. The second term, as expected, is determined by the vector meson photon coupling, and may be rewritten as

$$\left(\frac{e}{\pi} \right)_{q^2 \ll m_V^2} = \frac{c_v}{c_\pi} \frac{(\alpha^2/12\pi)(g_V^2/4\pi)}{n_{eff}} \ln \frac{m_V^2}{4\mu^2}. \quad (3.18)$$

From (3.17), the continuum contribution is rather unimportant for a relatively narrow object (such as the ω and ϕ), but yields an important additional contribution for broad resonances like the ρ and ρ' . The continuum contribution is clearly more important for electrons than for muons because of the logarithm. For electrons and for the ρ^0 , from (3.17) the contribution from the low mass continuum amounts to approximately 80% of the yield coming from pairs in the vicinity of the ρ^0 mass.

Much of the prompt electron yield, especially from the low mass continuum, originates from pairs of very low q^2 and very small opening angle Θ . Thus it is important to consider the influence of a cut-off in the opening angle (or in q^2) on the e/π ratio. An acceptance cut in the opening angle may be incorporated by restricting the energy integration in (3.1) to a value smaller than the upper limit given by (3.2) and (3.4). From elementary kinematics, in the zero lepton mass approximation, the energy of the e^+e^- pair and the opening angle Θ of the pair are connected by

$$E_q = |\vec{p}| + q^2 / (2|\vec{p}|(1 - \cos \Theta)). \quad (3.19)$$

If only leptons are counted, which belong to pairs with an opening angle larger than some specific value (given, e.g., by the dimensions of the experimental apparatus), i.e., for

$$(1 - \cos \Theta) \geq \varepsilon, \quad (3.20)$$

then the upper limit of the energy integration has to be replaced by

$$E_+ = |\vec{p}| + q^2/2|\vec{p}|\varepsilon. \quad (3.21)$$

Consistency with (3.4) requires

$$\varepsilon \geq \frac{\mu^2}{2p_{\perp}^2}. \quad (3.22)$$

Such a restriction is necessary because we have neglected the lepton mass in (3.19). With this upper limit the zero width formula (3.11) for a pair of mass squared q^2 is replaced by

$$\frac{e}{\pi} = \frac{C_V}{C_{\pi}} \frac{\mathcal{B}_{V \rightarrow e^+e^-}}{(n-1)} \frac{1}{(1+q^2/4p_{\perp}^2)^{n-1}} \left[1 - \left(\frac{1+q^2/4p_{\perp}^2}{1+2q^2/4p_{\perp}^2\varepsilon} \right)^{n-1} \right]. \quad (3.23)$$

Thus, the cut-off becomes effective for

$$\frac{2}{\varepsilon} \frac{q^2}{4p_{\perp}^2} \ll 1 \quad (3.24)$$

only. Even for a rather large value of $p_{\perp}$, with a realistic $\varepsilon \cong 0.004$ (corresponding to $\Theta = 5^\circ$) only very low mass pairs ($q^2 \lesssim 0.2 \text{ GeV}^2$ for $p_{\perp} = 5 \text{ GeV}$) from the photon continuum will be effected by the angular cut-off.

The cut-off in the case of the full photon continuum may now be handled to a good approximation as follows. For given $p_{\perp}$ and ε , the angular cut-off becomes effective only if q^2 fulfills (3.24). Thus we replace the lower limit in the q^2 integration in (3.16) by

$$q_{\min}^2 = 2 p_{\perp}^2 \epsilon, \quad (3.25)$$

which yields

$$\frac{e}{\pi} = \frac{C_V}{C_{\pi}} \frac{B_{V \rightarrow e^+e^-}}{n_{\text{eff}}} \left[1 + \frac{T_V}{\pi m_V} \ln \frac{m_V^2}{8 p_{\perp}^2 \epsilon} \right], \quad (3.26)$$

where the constant λ has been set equal to 4 so that, on substituting the value of ϵ given by (3.22) in (3.26), we recover the result (3.17). Numerically, for $\epsilon = 0.004$ and for $p_{\perp} = 2$ GeV the angular cut-off removes about 40% of the low mass continuum compared with the full yield without cut.

3.2. - The small $p_{\perp}$ region

We will show in this section that the inclusion of the low mass continuum yields an e/π ratio, which increases ^{*)} at low $p_{\perp}$ (≈ 0.1 GeV) as $1/p_{\perp}$.

For the treatment of the low $p_{\perp}$ region it is convenient to replace the integration over the parent momentum in (2.2) by an integration over the non-observed lepton momentum $k = q - p$. Equation (2.2) with (2.3) to (2.6) then becomes

$$2E_p \frac{d\sigma}{d^3p} = \frac{2}{\pi} B \int dq^2 \frac{m_V^2}{q^2} \rho(q^2) \int \frac{d^3k}{2E_k} (2E_q \frac{d\sigma}{d^3q}) \delta((p+k)^2 - q^2). \quad (3.27)$$

Upon integration over $d|\vec{k}|$ we obtain

*) A low $p_{\perp}$ increase along different lines of reasoning is also obtained by Lederman and White ¹³⁾, Ranft and Ranft ¹⁴⁾, Rückl ¹⁵⁾ and Hinchliffe and Llewellyn Smith ¹⁶⁾.

$$2E_p \frac{d\sigma}{d^3p} = \frac{2}{\pi} B \int_{\mu^2}^{\infty} dq^2 f(q^2) \int \frac{d\Omega}{8p^2(1-\cos\Theta)^2} \left(2E_q \frac{d\sigma}{d^3q} \right), \quad (3.28)$$

where in good approximation for the case of electrons the lepton mass has been neglected in the integrand. The lepton mass has to be kept, however, in the integration limits of the angular integration over the positron angles Θ, Φ , with respect to the trigger direction. The upper limit on the positron momentum, for fixed trigger momentum p , is given by

$$k_{Max} = |\vec{p}| \frac{q^2}{\mu^2}, \quad (3.29)$$

and yields for the opening angle Θ in the zero lepton mass approximation (3.28) [see Eq. (3.22)] the restriction

$$1 - \cos\Theta \geq \frac{\mu^2}{2\vec{p}^2}. \quad (3.30)$$

For electrons, this lower limit is very small, even for small trigger momentum $p \sim 0.1$ GeV. With an angular acceptance cut-off, the lower limit (3.30) is to be replaced by a larger cut-off ϵ , fulfilling

$$1 - \cos\Theta \geq \epsilon \geq \mu^2/2\vec{p}^2. \quad (3.31)$$

Substituting the parent cross-section

$$2E_q \frac{d\sigma}{d^3q} = C_V F(E_\perp^2) f(y) \quad (3.32)$$

and neglecting the rapidity dependence, $f(y) \cong f(0)$, for large trigger angle Θ_t , the leading term for $p_\perp \rightarrow 0$ of the electron to pion ratio turns out to be ^{*)}

*) Quite independently of vector dominance the $1/p_\perp$ behaviour for small $p_\perp$ holds in any model in which the production of the parent obeys (2.7) with $F(E_\perp)$ regular for $E_\perp \rightarrow 0$. For the general case, $\alpha\pi/\gamma_V^2$ in (3.33) would have to be replaced by an arbitrary constant parameter of order α .

$$\frac{e}{\pi} = \frac{1}{p_{\perp}} \frac{\alpha^2}{24 \pi^2 (\gamma_V^2 / 4\pi)} \frac{c_V}{C_{\pi}} J \cdot F, \quad (3.33)$$

where

$$J \equiv \sin \vartheta_t \int_{-1}^{1-\varepsilon} \frac{d \cos \theta}{(1 - \cos \theta)} 2 \int_0^{\pi} \frac{d\phi}{[1 - (\cos \vartheta_t \cos \theta + \sin \vartheta_t \sin \theta \cos \phi)]^2}^{1/2}$$

$$F \equiv \int_0^{\infty} F(z^2) dz. \quad (3.34)$$

Here, the pion distribution has been assumed to be also given by (3.32), with c_V replaced by c_{π} .

For a simple exponential or a power dependence on $E_{\perp}$, the integral over the transverse energy distribution can easily be evaluated to give for

$$F(E_{\perp}^2) = \begin{cases} \exp(-a E_{\perp}^2) \\ \exp(-a E_{\perp}) \\ 1 / (1 + E_{\perp}/a)^N \end{cases} \quad (3.35)$$

respectively the results

$$F \equiv \int_0^{\infty} F(z^2) dz = \begin{cases} \frac{1}{\Gamma a} \frac{\sqrt{\pi}}{2} \\ \frac{1}{a} \\ a/(N-1) \end{cases} \quad (3.36)$$

The angular integral J may be calculated for a 90° trigger angle, $\vartheta_t = 90^\circ$, using the series expansion of the elliptic integral, yielding

$$J(\vartheta_t = \pi/2) = 2\pi \left(\ln \frac{2}{\varepsilon} + \sum_{m=1}^{\infty} \frac{1}{2^{2m-1}} \frac{(2m-1)!}{(m!)^2} \right)$$

$$\approx 2\pi \left(\ln \frac{2}{\varepsilon} + 1.11 \right). \quad (3.37)$$

A numerical evaluation of $J(\theta_t)$ as a function of the trigger angle θ_t shows that it only varies by about 10% in the range $(30 < \theta_t \leq 90)$.

The $1/p_\perp$ rise in (3.33) is, of course, a characteristic feature of the low mass continuum. The magnitude of the e/π ratio at low $p_\perp$ in (3.33) is determined by the vector meson photon coupling (or $\Gamma_{e^+e^-}$) in contradistinction to the yield from pairs with masses in the vicinity of the vector meson mass, which is determined by the leptonic branching ratio. As is well known, the contribution from $q^2 \approx m_V^2$ pairs for $p_\perp \ll m_V$ becomes negligible. Explicitly, for $p_\perp \rightarrow 0$, upon integrating (3.28) in the narrow width approximation, one obtains

$$\left(\frac{e}{\pi}\right)_{q^2 \approx m_V^2, p_\perp \rightarrow 0} = \frac{1}{F(m_\pi^2)} \frac{C_V}{C_\pi} \frac{1}{2} B_{V \rightarrow e^+e^-} \int_0^\infty dz F\left(m_V^2 + \frac{m_V^2}{4} z\right), \quad (3.38)$$

and thus for $\exp(-aE_\perp^2)$,

$$\left(\frac{e}{\pi}\right)_{q^2 \approx m_V^2, p_\perp \rightarrow 0} = \frac{C_V}{C_\pi} B \frac{2}{a m_V^2} e^{-a(m_V^2 - m_\pi^2)}. \quad (3.39)$$

Numerically, for the case of the ρ^0 meson, the low $p_\perp$ prompt lepton yield (due to low mass pairs) from (3.33) turns out to be

$$\frac{e}{\pi} (\text{no acceptance cut}) = \frac{0.62 \times 10^{-5} (\text{GeV})}{p_\perp (\text{GeV})} \quad (3.40)$$

where we used $c_V = c_\pi$, $a = 2$ and $N = 12$ in (3.36), and $\gamma_p^2/4\pi = \alpha^2 m_p/12 \cdot \Gamma \cdot B$ with $B = 0.43 \times 10^{-4}$. With a 5° acceptance cut (e^+e^- opening angle $\Theta \geq 5^\circ$) one obtains

$$\frac{e}{\pi} (5^\circ \text{ acceptance cut}) = \frac{0.35 \times 10^{-5} (\text{GeV})}{p_\perp (\text{GeV})}. \quad (3.41)$$

These results are roughly two orders of magnitude higher than the (negligible) yield at $p_\perp = 0$, from pairs from the vicinity of the ρ^0 mass according to the estimate (3.39).

4. - LOW MASS SOURCES INVOLVING THREE-BODY DECAYS

Although single leptons emerging from three-body decays of the parent are much more suppressed than two-body decays, the branching ratio for the decays $\omega \rightarrow \pi^0 e^+ e^-$ and $\eta'(958) \rightarrow (\rho, \gamma) e^+ e^-$ are very large, so these contributions have to be carefully estimated at large $p_\perp$. This kind of mechanisms can be thought of as additional sources of Dalitz pairs ¹⁷⁾, and treated in much the same way, so we will only summarize the basic formulae involved in estimating their contribution. All the processes we will consider are either $1^- \rightarrow 0^- \gamma$ or $0^- \rightarrow 1^- \gamma$ transitions, so that the vertex functions have exactly the same structure. For $A(1^-) \rightarrow B(0^-) + \gamma_V$ (see Fig. 1) the amplitude is given by

$$M_{A \rightarrow B \gamma_V}^\mu = \frac{e g}{M_A} F_{AB\gamma}(Q^2) \varepsilon^{\mu\alpha\beta\gamma} \varepsilon_\alpha(P_A, \lambda_A) P_{A\beta} Q_\gamma, \quad (4.1)$$

where we assume the form factor $F_{AB\gamma}(Q^2)$ to be dominated by the nearest vector meson pole. This is essentially the Gell-Mann, Sharp, Wagner model ¹⁸⁾. However, since the normalization will in all cases be estimated by the measured branching ratio $B_{A \rightarrow B \gamma}$, the specific form of $F_{AB\gamma}(Q^2)$ will only influence the higher mass pairs emerging from this kind of mechanisms. For the case $A(0^-) \rightarrow B(1^-) + \gamma_V$, the polarization vector entering is $\varepsilon_\beta(P_B, \lambda_B)$. Using (4.1) it is a simple matter to derive the following relationship between the branching ratios of $A \rightarrow B e^+ e^-$ and $A \rightarrow B \gamma$

$$B_{A \rightarrow B e^+ e^-} = \frac{\alpha}{3\pi} \left(\ln \frac{m'^2}{4\mu^2} - \beta \right) B_{A \rightarrow B \gamma}, \quad (4.2)$$

where $m' = M_A - M_B$. B depends on the specific form factor used. In the Gell-Mann, Sharp, Wagner model for $\omega \rightarrow \pi^0 \gamma$, $\beta \simeq 1$.

In calculating the single lepton yield from (2.2), we use

$$\frac{1}{\Gamma} 2 E_p \frac{d\Gamma}{d^3 p} = \frac{2}{\pi} \int_0^1 \frac{dx}{x} g(x) \delta(M_A^2(1-x) - 2P \cdot p) \quad (4.3)$$

where $x = 2\omega/M_A$, ω being the energy of the electron in the rest frame of the parent. For the above coupling

$$g(x) = \frac{6}{\left(\ln \frac{m^2}{4M^2} - \beta\right)} \cdot \int_{4M^2}^{S_+(x)} \frac{ds}{s} |F_{AB\gamma}(s)|^2 \left[\frac{2(M_A^2 M_B^2 s)}{4} - \frac{(M_A^2 - M_B^2 + s)M_R x}{2} + \frac{M_R^4 x^2}{2} \right] \quad (4.4)$$

where

$$S_+(x) = \text{Min} \left(\frac{x(1-x)M_A^2 - M_B^2}{1-x}, m^2 \right).$$

Inserting (4.3) into (4.4), we obtain in the place of (3.1)

$$2E_p \frac{d\sigma}{d^3p} = B_{A \rightarrow B e^+ e^-} \int_0^1 dx g(x) \int_{x^{-1} + M_A^2/4p_L^2}^{\infty} dy \int_0^{2\pi} d\varphi \left(2E_p \frac{d\sigma}{d^3p} \right) \quad (4.5)$$

where again $E_p = y p$. Using the power behaved parent spectrum (3.7) and working at large p_L and small x_L , we obtain for the e/π ratio

$$\frac{e}{\pi} = \frac{C_A}{C_\pi} \frac{B_{A \rightarrow B e^+ e^-}}{(n-1)} \int_0^1 dx x^{n-2} g(x) \frac{1}{\left(1 + \frac{M_A^2 x}{4p_L^2}\right)^{n-1}} \quad (4.6)$$

For $4p_L^2 \gg M_A^2$ this leads to the asymptotic formula

$$\frac{e}{\pi} = \frac{C_A}{C_\pi} \frac{3}{2} \frac{B_{A \rightarrow B e^+ e^-}}{(n-1)^2} \left(1 - \frac{M_B^2}{M_A^2}\right)^{n-2} \quad (4.7)$$

The corresponding γ/π ratio is given by

$$\frac{\gamma}{\pi} = \frac{C_A}{C_\pi} \frac{B_{A \rightarrow B \gamma}}{(n-1)} \quad (4.8)$$

For a more general parent spectrum we can replace (n) by n_{eff} defined in (3.15). In the Table we have summarized the contributions from various mechanisms of the type discussed above. (The experimental rates for $A \rightarrow B\gamma$ have been taken from the 1974 edition of the particle data group.) The

estimates given in the Table are very approximate [since we use the asymptotic formulae (4.7) and (4.8)], however they indicate that these three-body decay contributions cannot be neglected, when one is dealing with a signal $\sim 6 \times 10^{-5}$. However, since most of the electrons produced by these mechanisms come from low mass pairs, an acceptance cut in the opening angle can eliminate a good fraction of this background.

$A \rightarrow Be^+e^-$	$B_{A \rightarrow B\gamma}$	$B_{A \rightarrow Be^+e^-}$	n_{eff}	c_A/c_π	γ/π	e/π	μ/π
$\omega \rightarrow \pi^0 e^+e^-$	0.087 ± 0.005	$8.1 \times 10^{-4}(e)$ $0.8 \times 10^{-4}(\mu)$	8	1	1.0%	2.1×10^{-5}	$< 0.3 \times 10^{-5}$
$\phi \rightarrow \eta e^+e^-$	0.03 ± 0.01	$2.6 \times 10^{-4}(e)$ $0.2 \times 10^{-4}(\mu)$	8	1/3	0.1%	0.3×10^{-6}	$< 0.05 \times 10^{-5}$
$\eta' \rightarrow \rho e^+e^-$	0.27 ± 0.02	2.0×10^{-3}	8	1	4 %	1.3×10^{-7}	0
$\eta' \rightarrow \gamma e^+e^-$	0.02 ± 0.003	$2.0 \times 10^{-4}(e)$ $3.3 \times 10^{-5}(\mu)$	8	1	0.3%	0.6×10^{-5}	0.1×10^{-5}
Total					$\sim 5\%$	$\sim 3 \times 10^{-5}$	$< 0.4 \times 10^{-5}$

5. - NUMERICAL RESULTS AND COMPARISON WITH EXPERIMENT

The results (3.17), (3.26) and (3.33) for the e/π ratio at large and small $p_\perp$ respectively, will now be supplemented with a detailed numerical calculation taking into account also the rapidity and $x_\perp$ dependence of the inclusive vector meson production cross-section. We assume it to be given by

$$2E_q \frac{d\sigma}{d^3q} = C_V F(E_\perp) f(x_\perp) \exp(-by^2), \quad (5.1)$$

where the rapidity dependence is taken from fits to the pion inclusive distributions as measured at the ISR ¹⁹⁾ for $\sqrt{s} = 53$ GeV ($b = 0.11$). The $x_{\perp}$ distribution is given by $f(x_{\perp}) = \exp(-12.5x_{\perp})$. For the transverse mass distribution we use the recent fit of Bourquin and Gaillard ³⁾ given by (3.14), which holds at small as well as at large transverse momenta. Finally, $c_{\rho} = c_{\omega} = c'_{\rho} = c_{\pi}$ and $c_{\phi} = (1/3)c_{\rho}$ and $c_{J/\psi} = (1/30)c_{\pi}$.

Figure 2 shows the full ρ^0 contribution to the single prompt electron yield including the low mass ρ^0 dominated continuum. The expected rise at low $p_{\perp}$ agrees with (3.40) for $p_{\perp} \simeq 0.1$ GeV and is dramatically different from the low $p_{\perp}$ fall-off obtained in the narrow width approximation. At large $p_{\perp}$, as expected from (3.17), we find an excess of about 80% over the narrow width approximation. Even though an acceptance cut, which cuts out pairs of opening angle larger than 5° (corresponding to 2.4×10^{-2} in solid angle), removes much of the yield at very large $p_{\perp} \sim 4$ GeV, a considerable increase over the narrow width approximation remains for $p_{\perp} \lesssim 1.5$ GeV. The yield obtained, if all pairs of mass $q^2 < 0.04 = 4m_{\mu}^2$ are removed, is also shown on Fig. 2. The signal thus obtained exceeds the narrow width approximation significantly for $p_{\perp} \simeq 0.5$ GeV only, and roughly shows what happens if muons are detected instead of electrons. Finally, let us remark that numerical evaluation based on (3.17) with n_{eff} as given by (3.15) agrees with our numerical results within less than 25% for $p_{\perp} > 2$ GeV.

Figure 3 first of all illustrates that low mass pairs have a negligible effect for narrow vector mesons with a relatively large branching ratio into lepton pairs such as the ω , ϕ , except for extremely low $p_{\perp}$. As an example for the contribution to the e/π yield due to a higher mass vector meson we also show on Fig. 3 the effect of a $\rho'(1600)$ of width $\Gamma_{\rho'} \simeq 2\Gamma_{\rho} = 300$ MeV and a photon coupling $1/\gamma_{\rho'}^2 = 0.2(1/\gamma_{\rho}^2)$ (and thus $B_{\rho} = 0.2 B_{\rho'}$) produced according to (4.1) with $c_{\rho'} = c_{\rho}$. Even though low mass pairs yield a large effect relative to the narrow width approximation, the over-all contribution is rather small, due to the small branching ratio $B_{\rho'}$. We do not enter a systematic study of higher mass vector meson contributions ^{2),4)} here. As regards the low mass e^+e^- pair yield ($q_{e^+e^-}^2 \lesssim m_{\rho}^2$) we expect ρ^0 , ω , ϕ dominance (with perhaps a ρ' contribution added) to be a relatively safe approximation just as in any other $q^2 = 0$ photon reaction.

Figure 4 finally shows the effect, on the e/π ratio, of adding up all vector meson contributions including the low mass continuum and the $\omega \rightarrow \pi^0 e^+ e^-$ and $\eta' \rightarrow \gamma e^+ e^-$ contribution as estimated in Section 4. Just as a rough guide, we have included the J/ψ contribution, following Refs. 20) and 3). The data points come from the CCRS ²¹⁾ and the CHORMN ²²⁾ measurements. It is clear that in the intermediate $p_{\perp}$ region, $p_{\perp} \lesssim 2.5$ GeV, the inclusion of low invariant mass pairs, calculated on the basis of rather conventional mechanisms, raises the prompt electron signal by a factor of about 2. The sensitivity of the prompt electron yield to acceptance cuts has to be stressed, however. This has been discussed in Section 3 and in connection with Fig. 2. Although the prompt lepton yield due to the low mass continuum rises as $1/p_{\perp}$, the rise sets in at values of $p_{\perp}$ which are too low to be compatible with the CHORMN results. Moreover, our normalization based on a smooth extrapolation from $q^2 \simeq m_p^2$ to $q^2 \simeq 0$ [i.e., $\alpha\pi/\gamma_V^2$ in (2.9)] yields a result which is too low by an order of magnitude compared with these latter data.

6. - THE e^+e^- MASS SPECTRUM AND A REMARK ON DIRECT PHOTONS

According to the ansatz (2.3), (2.4), the mass spectrum (dependence at $q_{e^+e^-}^2$) of the lepton pairs underlying our calculation of the prompt lepton yield is given by

$$\begin{aligned} 2E_q \frac{d\sigma}{d^3q dq^2} (pp \rightarrow e^+e^- + X) = \\ = 2E_q \frac{d\sigma}{d^3q} (pp \rightarrow V^0 X) \frac{m_V \Gamma_V}{\pi((q^2 - m_V^2)^2 + m_V^2 \Gamma_V^2)} B_V \frac{m_V^2}{q^2}. \end{aligned} \quad (6.1)$$

As we discussed in Section 2, the vector meson production cross-section (5.1) is to be evaluated at the transverse energy $E_{\perp}^V = \sqrt{q^2 + q_{\perp}^2}$ of the virtual photon (lepton pair) being produced. The mass spectrum at fixed $q_{\perp}$ of the produced lepton pairs thus has the shape of a distorted Breit-Wigner curve strongly enhanced at low $q^2 \approx 0$ by the photon propagator and the transverse mass dependence of the hadronic photon source. This latter effect is very pronounced at small $q_{\perp} \lesssim 1$ GeV², but becomes clearly negligible at large $q_{\perp} \sim 3$ GeV, where q^2 (for $q^2 \lesssim 1$ GeV²) becomes negligible compared with

$q_{\perp}^2$ and E_{γ} approximately independent of q^2 . The change of the low mass lepton pair spectrum with $q_{\perp}$ is seen on Fig. 5, where the mass spectrum for $pp \rightarrow e^+e^-(\mu^+\mu^-) + X$ due to the ρ^0 and ω contributions normalized at the ω mass, i.e.,

$$\begin{aligned} & 2E_{\gamma} \frac{d\sigma}{d^3q dq^2}(q_{\perp}, q^2) / 2E_{\gamma} \frac{d\sigma}{d^3q dq^2}(q_{\perp}, q^2 = m_{\omega}^2) = \\ & = \frac{F(q^2 + q_{\perp}^2)}{F(m_{\omega}^2 + q_{\perp}^2)} \left(\frac{m_f \Gamma_f B_f}{((q^2 - m_f^2)^2 + m_f^2 \Gamma_f^2)} \frac{m_f^2}{q^2} + \frac{m_{\omega} \Gamma_{\omega} B_{\omega}}{((q^2 - m_{\omega}^2)^2 + m_{\omega}^2 \Gamma_{\omega}^2)} \frac{m_{\omega}^2}{q^2} \right) \cdot \\ & \cdot 1 / \left(\frac{B_{\rho}}{m_f \Gamma_f} + \frac{B_{\omega}}{m_{\omega} \Gamma_{\omega}} \right) \end{aligned} \quad (6.2)$$

is plotted for $F(E_{\perp}) = 1/(1 + E_{\perp}/2)^{12.3}$. A measurement of the mass distribution, and specifically of its change as a function of $q_{\perp}$, thus would allow one to unambiguously verify our model assumptions for the low mass continuum (provided the π^0 and η Dalitz pair background can be subtracted).

Let us add a comment at this point on the real photon yield expected from our ansatz for the low mass continuum. From (2.9), it is simply given by the vector dominance formula

$$2E_{\gamma} \frac{d\sigma}{d^3q}(pp \rightarrow \gamma X) = \sum_{\rho^0, \omega, \phi} 2E_{\gamma} \frac{d\sigma}{d^3q}(pp \rightarrow V_{tr} X) \frac{\alpha \pi}{\delta_V^2} \quad (6.3)$$

where according to Section 2 the vector meson production cross-section is to be evaluated at $E_{\perp}^{\gamma} = q_{\perp}^{\gamma}$. The γ/π^0 ratio is thus quite independent of the photon transverse momentum and with ρ^0 yield $\cong \omega$ yield $\cong \pi$ yield approximately given by

$$\gamma/\pi^0 = \sum_{\rho^0, \omega, \phi} \frac{\alpha \pi}{\delta_V^2} \cong 0.4\%. \quad (6.4)$$

(clearly, this γ/π^0 ratio denotes the excess of photons after having subtracted all "trivial sources", as $\pi^0 \rightarrow \gamma\gamma$, $\eta \rightarrow \gamma\gamma$, $\eta' \rightarrow \rho\gamma$, etc., see the Table). The very small γ/π ratio ^{*}) (6.4) is at variance with recent conjectures ^{6),7)} on the basis of parton and quark gluon dynamics, which, for large $q_{\perp}$ ($\gtrsim 3$ GeV) for direct photons predict ratios of the order of $\gamma/\pi^0 \cong 10\%$ to 20% . The prediction (6.4) may be raised by including higher mass states along the lines of generalized vector dominance [using the usual couplings $1/\gamma_N^2 = (1/\gamma_p^2)(m_p^2/m_N^2)$ for the N^{th} vector meson of mass $m_N^2 = m_p^2(1+2N)$]. Rather drastic assumptions (e.g., maximal interference, very high mass contributions even though $q^2=0$) in strong contrast to what one has learnt from photoproduction) would have to be invoked, however, to change (6.4) appreciably.

Quite independent of the question of how to reconcile the quark gluon predictions with generalized vector dominance for the γ/π ratio, we wish to point out that the drastic differences in the low invariant mass lepton pair yield corresponding to factors of about 20 to 50 in the γ/π ratio allow for a significant experimental test. In Fig. 6, we have plotted the ratio of lepton pair to pion production

$$\frac{e^+e^-(\mu^+\mu^-)}{\pi^0} = \frac{2q^0 \frac{d\sigma}{d^3q dq^2} (pp \rightarrow e^+e^- X)}{2q^0 \frac{d\sigma}{d^3q} (pp \rightarrow \pi^0 X)} \quad (6.5)$$

as a function of the invariant mass of the lepton pair, for $q^2 \lesssim 0.9$ GeV and large $q_{\perp}$ ($\gtrsim 3$ GeV, and thus $E_{\perp} \simeq q_{\perp}$ for $q^2 \lesssim m_p^2$). We first of all show the ρ^0, ω dominance prediction including the low invariant mass vector meson dominated continuum given by ^{**)}

^{*}) There is one other source of prompt photons and possibly low mass pairs, which we have not looked into in detail, namely those arising from radiation of charged pions in the final state. A naive calculation, which ignores the problems of gauge invariance and off-shell extrapolations gives $\gamma/\pi = (8\pi\alpha \log p_{\perp}^2/m_{\pi}^2)/n(n-1)$ for a pion spectrum falling like $1/p_{\perp}^n$. This formula indicates a yield of around 2% for $p_{\perp} \sim 2$ GeV. However, in converting this into a e^+e^- pair yield requires a detailed study of gauge invariance, since the latter is problematic for photon couplings to pseudoscalar mesons.

^{**)} A more careful treatment would have to take into account ρ^0, ω interference.

$$\frac{e^+e^-(\mu^+\mu^-)}{\pi^0} = \sum_{\rho, \omega} \frac{m_\nu P_\nu B_{e^+e^-}^\nu}{\pi((q^2 - m_\nu^2)^2 + m_\nu^2 \Gamma_\nu^2)} \frac{m_\nu^2}{q^2}, \quad (6.6)$$

which for $q^2 \rightarrow 0$ ($\geq 4m_\ell^2$) reduces to

$$\frac{e^+e^-(\mu^+\mu^-)}{\pi^0} = \sum_{\rho, \omega} \frac{\alpha \pi}{g_\nu^2} \frac{\alpha}{3\pi q^2} = \frac{g}{\pi^0} \frac{\alpha}{3\pi q^2}, \quad (6.7)$$

where $\gamma/\pi^0 = 10\%$ and 20% . Differences are rather drastic : when taking into account experimental resolution, which smears out the narrow ω peak, the ρ^0, ω peak for $(\gamma/\pi^0)_{\text{excess}} = 10\%$ to 20% will show up as a minor effect on a strong lepton pair continuum, which, for $q^2 \rightarrow 0$, rises by roughly an order of magnitude above the ρ^0, ω peak. For the case of the ρ^0, ω dominated low mass continuum, the rise near $q^2 \rightarrow 0$ will in magnitude be below the ρ^0, ω peak. However, before one can draw any conclusions about direct photons from the e^+e^- pair spectrum at small q^2 , one must properly take into account not only the π^0 and η Dalitz pair background, but also contributions from $\omega \rightarrow \pi^0 e^+e^-$ and $\eta' \rightarrow (\gamma, \rho) e^+e^-$, which, in Section 4, we showed to contribute approximately 5% to 6% to the γ/π^0 ratio.

A recent ISR experiment ²³⁾ finds for $(\gamma/\pi)_{\text{excess}} = (\gamma/\pi)_{\text{measured}} - (\gamma/\pi)_{\pi^0} - (\gamma/\pi)_\eta$ about 20%. This rate should likewise yield a corresponding enhancement at small q^2 in the e^+e^- pair spectrum. (An exception would be two-body decays of the form $A \rightarrow B e^+e^-$ with $m_A \approx m_B$. For these, from Section 4, the γ/π ratio may be large, while ℓ/π is small.)

7. - CONCLUDING REMARKS

Let us briefly summarize the main points which have been made.

A) - At low $p_\perp$ ($\lesssim 300$ MeV), due to the production of low invariant mass pairs, the prompt lepton yield rises as $1/p_\perp$. It is quantitatively given by (3.33) and (3.40). Even though ρ^0 dominance has been used to

normalize this cross-section, the $1/p_{\perp}$ behaviour is not dependent on this assumption and holds quite generally (for a parent distribution which is regular for $E_{\perp} \rightarrow 0$), as it essentially follows from the presence of the photon propagator alone. With the ρ^0 dominance normalization, which should be quite reliable for $q^2 \cong 0$, the results obtained are much too low to be compatible with the CHORMN data.

B) - At moderate values of $p_{\perp}$ (~ 1 to 2.5 GeV), there is a sizeable effect due to low mass pairs originating from the low mass photon continuum and in addition from three-body decays, in particular $\omega \rightarrow \pi^0 e^+ e^-$. The over-all effect raises the prompt signal from "known sources" from about 3×10^{-5} to 6×10^{-5} , if no acceptance cuts are employed. This latter magnitude seems not inconsistent with the CCRS data.

C) - The most significant test of the considerations of this paper, and of speculations in the literature on copious real photon production at large $p_{\perp}$, consists in a careful measurement and analysis of $e^+ e^-$ pairs of small invariant mass below the ρ^0 .

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FIGURE CAPTIONS

Figure 1 The $A(1^-) \rightarrow B(0^-) + \gamma$ vertex.

Figure 2 The ρ contribution to the ℓ/π ratio including the associated low mass continuum.

Figure 3 The $\rho, \omega, \phi, \rho'(1600)$ contributions to the ℓ/π ratio including associated low mass continua.

Figure 4 Electron to pion ratio from vector meson decays, associated low mass continua and three-body electromagnetic decays compared with experimental data.

Figure 5 Mass distribution of $e^+e^-(\mu^+\mu^-)$ pairs due to ρ^0 and ω contributions showing low mass enhancement due to photon propagator and $E_{\perp}$ dependence on q^2 . Low mass enhancement is largest for small $q_{\perp}$, while only photon propagator effect remains for $q_{\perp} \gg 1$ GeV. (Experimental resolution will smear out ρ^0 - ω peak).

Figure 6 Mass distribution of $e^+e^-(\mu^+\mu^-)$ pairs for large $q_{\perp} \gg 1$ GeV. The mass distribution resulting from ρ^0 - ω dominance is compared with the one expected from the conjecture of a huge γ/π ratio.

FIG. 1

FIG. 2

FIG. 3

FIG.4

FIG.5

FIG.6