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ON SOME QUESTIONS CONCERNING
A QUARK THEORY OF HADRONIC MATTER

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A B S T R A C T

This paper analyzes the consequences of a recent quark theory in several different directions of hadron dynamics. A tight, consistent and very predictive framework is seen to emerge.

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INTRODUCTION

The aim of this paper is to contribute to the discussion and hopefully the clarification of some important questions concerning a recently proposed quark theory of hadronic matter ^{1),2),3)}.

While the introduction and the discussion of quark spin effects, internal quantum numbers and three-quark systems is necessary to turn our theoretical proposal into a realistic theory of hadrons, it is nevertheless crucial at this stage to ask ourselves whether it is possible to make our general framework agree with some necessary general requirements. And to accomplish this we do not need to specify the otherwise important properties like spin, and internal quantum numbers.

The two main questions which I shall try to answer in this paper are the proper description of off-shell effects and an adequate definition of current operators on the space of physical states' wave functions.

In previous work ¹⁾⁻³⁾, the importance of these problems was adequately stressed, but the answer to them was left somewhat open, needing a detailed analysis which had not yet been carried out.

I am quite aware that some of the conclusions reached in this paper may have to be reassessed in the future, in the face of a deeper and more accurate analysis. However, I have not refrained from bringing up these points now, just to stimulate discussion on very important issues which are not peculiar to this approach, but rather generally invest theories where the quark degree of freedom is unobservable (i.e., "confined"). We can convince ourselves that the analyticity, the off-shell behaviour and the currents' structure of a theory where the fundamental fields are confined, must drastically differ from conventional quantum field theory, upon which much of our intuition is based. It is precisely the unavailability of well-established reference points which makes an investigation of such questions so difficult. In order to voyage as safely as possible in such quicksands, it is necessary that we check our answers by means of a number of calculations of processes of fundamental importance.

For this reason extensive calculations will be reported on such processes as high energy scattering, deep inelastic phenomena and production of hadrons at large p_T . The very satisfactory picture one thus obtains can be taken as an indication that we may be moving along a sensible track.

The plan of the paper is as follows. In Section 1 the basics of our approach shall be briefly reviewed ; Section 2 contains a more detailed discussion of the calculational rules, especially of the extension of our rules in the case of complex loop momenta. Section 3 takes up the question of how to normalize the physical states, and therefore of how to introduce currents in a physically meaningful way. High energy hadron scattering and the problem of Reggeization will be discussed in Sections 4 and 5. Deep inelastic phenomena are treated in Section 6, while hadron production at high p_T is the object of Section 7. The final part will be devoted to the conclusions.

1. - THE PHYSICAL HADRON STATES

A. - The meson ($\bar{q}q$) wave function and its equations of motion

The simplest description of a meson state is given by the wave function (wf)

$$\psi(p; x_1, x_2) = e^{i p \frac{x_1 + x_2}{2}} \psi(p; x) \quad (1.1)$$

where p is the total four-momentum of the $q\bar{q}$ system and $x_{1,2}$ are the four-co-ordinates of the $q\bar{q}$ pair. Note that (1.1) is a trivial consequence of translational invariance.

Quark confinement, i.e., the impossibility of observing quarks as real physical states, is introduced through the requirement :

$$\psi(p; x) = 0 \quad \text{for } x \notin R^4(p), \quad (1.2)$$

where $R^4(p)$ is a certain bounded space time domain (bag) ^{*)}, depending on the momentum p of our state, whose structure will be specified shortly.

*) The reader may notice some superficial similarities between this approach and the so-called MIT bag ⁴⁾.

To determine the quark "motion" inside the bag $R^4(p)$ we need some differential equations for the wf (1.1). The simplest choice appears to be

$$\begin{aligned} -\square_1 \psi(p; x_1, x_2) &= m^2 \psi(p; x_1, x_2) \\ -\square_2 \psi(p; x_1, x_2) &= m^2 \psi(p; x_1, x_2) \end{aligned} \quad (1.3)$$

for $x \in R^4(p)$. However, as discussed at length elsewhere ^{1),3)}, when one converts (1.3) into equations for $\psi(p, x)$; i.e.,

$$\begin{aligned} \left(\frac{p}{2} - i \frac{\partial}{\partial x}\right)^2 \psi(p; x) &= m^2 \psi(p; x) \\ \left(\frac{p}{2} + i \frac{\partial}{\partial x}\right)^2 \psi(p; x) &= m^2 \psi(p; x) \end{aligned} \quad (1.4)$$

or, equivalently

$$\left(\frac{p^2}{4} - \square\right) \psi(p; x) = m^2 \psi(p; x) \quad (1.5a)$$

$$p \cdot \frac{\partial}{\partial x} \psi(p; x) = 0 \quad (1.5b)$$

one encounters the difficulty that (1.5b) cannot be matched with the solution (1.2) outside the bag, due to its first order character. The simplest way out of this difficulty is to modify (1.5b) to

$$\left(p \cdot \frac{\partial}{\partial x}\right)^2 \psi(p; x) = -p^2 \omega_0^2 \psi(p; x), \quad (1.5c)$$

with the further constraint that only that solution of (1.5c) should be kept which corresponds to the minimum eigenvalue ω_0 . Our equations (1.5a) and (1.5c) will only be compatible with those bags $R^4(p)$ which in the rest frame, $p \equiv (M, \vec{0})$, factorize into a space bag (R_s) and a time bag (R_t) , i.e.,

$$R^4(M, \vec{0}) \equiv \left\{ \begin{array}{l} |\vec{x}| \leq R_s(M) \\ |t| \leq R_t(M) \end{array} \right. \quad (1.6)$$

In the rest frame (1.5a) and (1.5c) become respectively

$$\left[\frac{M^2}{4} - \frac{\partial^2}{\partial t^2} + \vec{\nabla}^2 \right] \psi_M(\vec{x}, t) = m^2 \psi_M(\vec{x}, t) \quad (1.7)$$

and

$$\frac{\partial^2}{\partial t^2} \psi_M(\vec{x}, t) = -\omega_0^2(M) \psi_M(\vec{x}, t). \quad (1.8)$$

The solutions of these equations which vanish at the boundary of the region $R^4(M, \vec{0})$ given in (2.6) are easily found to be

$$\psi_{nlm}(M; \vec{x}, t) = N_{nl}(M) \cos \omega_0 t \ j_\ell \left[\sqrt{\frac{M^2 + \omega_0^2}{4} - m^2} |\vec{x}| \right] Y_\ell^m(\theta, \varphi) \quad (1.9)$$

with the eigenvalue conditions :

$$\omega_0(M) = \frac{\pi}{2R_t(M)} \quad (1.10)$$

and

$$\left[M^2 + 4\omega_0^2(M) - 4m^2 \right]^{1/2} = \frac{2\beta_{nl}}{R_s(M)}. \quad (1.11)$$

$N_{nl}(M)$ is a normalization factor to be determined later, $j_\ell(z)$ is the spherical Bessel function of order ℓ , β_{nl} its n th positive zero, and $Y_\ell^m(\theta, \varphi)$ is the normalized spherical harmonic. Note that (1.10) and (1.11) will give us a spectrum labelled by ℓ , the angular momentum, and n , the radial quantum number, only after we have specified the dependence of R_s and R_t on the mass M of the meson state. The important massive quark model (MQM) criterion of universal quark mass behaviour of quark Green's functions ⁵⁾ yields, as discussed elsewhere ¹⁾⁻³⁾,

$$R_s, R_t \xrightarrow{M^2 \text{ large}} R^2 M \quad (1.12)$$

which when inserted into (1.10) and (1.11) provides a set of exchange degenerate Regge trajectories which are almost linear and parallel. The magnitude of R^2 in (1.12) is fixed by the slope of our Regge trajectories : for $\alpha' \cong 1 \text{ GeV}^{-2}$ one obtains $R^2 \cong \pi \text{ GeV}^{-2}$. We end this subsection by recording the general structure of the high mass mesons wf in momentum space (see Fig. 1)

Figure 1 The momentum space wave function

$$\psi_{n\ell m}(p, k) \simeq (2\pi)^2 \frac{2\pi}{R^2} \delta_{R^2}\left(\frac{p^2+k^2-m^2}{4}\right) \delta_{R^2}(pk) Y_\ell^m(\Omega_k) \quad (1.13)$$

where Ω_k is the direction of the three vector

$$\hat{k}_i = [\Lambda^{-1}(\vec{p}) k]_i \quad (1.14)$$

$\Lambda^{-1}(\vec{p})$ being the Lorentz transformation bringing our state to rest. The function $\delta_{R^2}(z)$ is a "fat δ function", characterized by the following two properties :

$$\int_{-\infty}^{+\infty} dz \delta_{R^2}(z) = 1 \quad (1.15)$$

$$\delta_{R^2}(0) = \frac{R^2}{\pi} \quad , \quad (1.16)$$

in the limit $R^2 \rightarrow \infty$ $\delta_{R^2}(z)$ tends to a Dirac δ function. A very simple $\delta_{R^2}(z)$, particularly useful in the calculations, is given by

$$\delta_{R^2}(z) = \frac{1}{\pi} \frac{\sin R^2 z}{z} . \quad (1.17)$$

Finally, in (1.13) the factor $N_{n\ell}$ has been chosen in such a way that

$$\int \frac{d^4k}{(2\pi)^4} |\psi_{nem}(p,k)|^2 = 1 ; \quad (1.18)$$

one should keep in mind that this particular choice is only suggested by convenience and has no physical motivation. A physically meaningful normalization can only be given by introducing the electric current ; this shall be discussed in Section 3.

B. - Decoupling the high ℓ states. The fire sausage

We have just derived a meson spectrum consisting of approximately linear and parallel Regge trajectories. On one hand this looks very promising, but on the other it may lead us into serious trouble if quarks do not somehow decouple the meson states whose angular momentum ℓ is higher than a value ℓ_0 given by

$$\ell_0 = \frac{M}{2} R_1(M) \quad (1.19)$$

where $R_1(M)$ can increase at most logarithmically with the mass of the state.

The reason for this is quite simple ; it stems from the fact that if the strong interactions are to have a range which increases at most logarithmically with energy, the maximum impact parameter of a $q\bar{q}$ collision must also be so limited. Calling $R_1(M)$ such maximum value, (1.19) follows immediately.

Now, the quark coupling to the meson states is uniquely determined by the wf, and we can easily check that if the normalization is given by (1.13) at a given mass M all angular momentum states (up to $\ell_{\max} = (R^2 M^2)/\pi$) are coupled with comparable strength. In our previous work ^{1),3)} an inadequate definition of the electromagnetic current lead us to a normalized wf simply proportional to (1.13), and to the consequent necessity to decree away the

troublesome states. We shall see later that from a physically reasonable electromagnetic current we can derive this decoupling and we shall also be able to give a very physical characterization of $R_{\perp}(M)$ appearing in (1.19).

Given that at a large mass M only those states are coupled for which $l \leq l_0$, we can get a nice intuitive picture of the space time structure of the typical meson state of mass M . We can in fact construct a set of coherent states characterized by the direction $\Omega_0 \equiv (\theta_0, \varphi_0)$, by taking the following linear combination of the wave functions (2.13)

$$|\Omega_0, M\rangle = \frac{1}{l_0^2} \sum_{l=0}^{l_0} \sum_{m=-l}^{+l} Y_l^{-m}(\Omega_0) \psi_{nlm}(p, k). \quad (1.20)$$

Notice that from the asymptotic relation

$$M^2 \simeq \frac{\pi}{R^2} (2n+l) \quad (1.21)$$

in (1.20) n is not an independent index. Going back to configuration space these states correspond to cylindrical space domain (see Fig. 2), which we shall call fire sausage (FS), of height $R_{\parallel} \simeq R^2 M$ and width $R_{\perp}(M)$, containing stationary quark waves moving with very small divergences $[\delta \simeq (R_{\perp}(M))/R^2 M]$ along the direction Ω_0 .

Figure 2 The space structure of a fire sausage

On introducing an interaction among three bags along the lines we shall discuss in the next section, we can study the decay properties of such a peculiar hadronic structure. This is a short summary of the main results of such a study ²⁾ :

- i) the strongly preferred decay configuration of the FS of mass M is a linear decay chain (Fig. 3) in which the initial FS cascades down emitting at each step a low mass state (π) and a lower mass FS oriented around the initial direction ;

Figure 3 The linear decay chain of an excited $q\bar{q}$ system

- ii) calling $\Gamma(M)$ the width of FS of mass M , we get asymptotically

$$M\Gamma(M) \longrightarrow \lambda \frac{2\pi}{R^2} \quad (1.22)$$

where λ depends on the coupling among bags ;

- iii) the one-particle yield exhibits strong interaction scaling, i.e.,

$$2E \frac{d^3N}{dp^3} \simeq f(x, p_T) \quad (1.23)$$

where $x = 2p_\ell/M$ and p_ℓ and $p_\perp$ are the particle momenta longitudinal and transverse with respect to the FS direction Ω_0 ;

- iv) the distribution in transverse momentum in (1.23) is strongly cut-off with an average value given roughly by

$$\langle p_\perp^2 \rangle \simeq \frac{\pi}{R_1^2(M)} \quad ; \quad (1.24)$$

v) when $x \rightarrow 0$ $f(x, p_\perp)$ in (1.23) tends to a non-zero limit, as a consequence the decay multiplicity of the FS increases logarithmically with its mass M :

$$\langle n \rangle = c \log M^2 + \text{const} \quad (1.25)$$

where c is given by

$$c = \int d^2 p_\perp f(0, p_\perp) .$$

These results suggest that what is experimentally observed, from hadronic jets in e^+e^- annihilation to usual peripheral high energy hadron interactions, originate from the decay of a small number of FS's which get produced in a variety of ways, depending, as we shall see later on, on the particular process one considers.

C. - The $q\bar{q}$ Green's function. The Regge states

From the bag states considered so far we can construct the $q\bar{q}$ Green's function following the usual procedure (see Fig. 4)

Figure 4 The diagrams making up the $q\bar{q}$ Green's function

In order to have crossing invariance G must be the sum of two pieces,

$$G = G_s + G_t \quad (1.26)$$

representing the contributions of bag states in the s and in the t channel. Note that the u channel, carrying non-zero triality, must be free from singularities. Let us study the G_s contribution first ; we write

$$G_s = \sum_{n\ell m} \frac{\psi_{n\ell m}(p, k) \psi_{n\ell m}^*(p, k')}{s - m_{ne}^2} \quad (1.27)$$

where ψ is a physically normalized wf (see Section 3), and $s = p^2$ is the square of the centre-of-mass energy. We are interested in the high energy behaviour of G_s . Making use of (1.13), we can write

$$G_s = (2\pi)^4 \left(\frac{2\pi}{R^2}\right)^2 \delta_{R^2}\left(\frac{m_1^2 + m_2^2}{2} - m^2\right) \delta_{R^2}\left(\frac{m_1^2 - m_2^2}{2}\right) \delta_{R^2}\left(\frac{m_1'^2 + m_2'^2}{2} - m^2\right) \delta_{R^2}\left(\frac{m_1'^2 - m_2'^2}{2}\right) G_s(s, t) \quad (1.28)$$

where

$$G_s(s, t) = \sum_{n\ell} \frac{1}{Z_{n\ell}} \frac{(2\ell+1) P_\ell(\cos\theta_s)}{s - m_{n\ell}^2 + i(m_{n\ell}')^2} \quad (1.29)$$

$m_{1,2}^2 = ((p/2) \pm k)^2$, $m_{1,2}'^2 = ((p/2) \pm k')^2$, $Z_{n\ell}$ is the factor needed to correctly normalize the wf and $\cos\theta_s \approx (1 + (2t/s))$. Note that in (1.29) we have added to the dispersive denominator an imaginary part which, according to the discussion of the previous section [see (1.22)] is asymptotically constant. We shall now calculate the imaginary part $\text{Im } G_s(s, t)$ with the approximation $(1/Z_{n\ell}) = (2/Z_0) \theta(\ell_0 - \ell)$ *) where ℓ_0 is given by (1.19). From (1.29) we easily get 1)-3)

$$\text{Im } G_s(s, t) \rightarrow \frac{R^2}{2Z_0} \sum_{\ell=0}^{\frac{\sqrt{s}}{2} R_1(s)} (2\ell+1) P_\ell(\cos\theta_s) \approx \frac{R^2 R_1^2}{2Z_0} \frac{J_1(R_1 \sqrt{t})}{R_1 \sqrt{t}} \quad (1.30)$$

*) This approximation is adequate only for small t . At large t one must be more careful as will be shown in Section 7.

exhibiting the "primeval pomeron" behaviour of the massive quark model ⁵⁾.
As for the G_t contribution, we have for large s :

$$G_t \simeq (2\pi)^4 \left(\frac{2E}{R^2}\right)^2 \delta_{R^2}\left(\frac{m_1^2+m_1'^2}{2}-m^2\right) \delta_{R^2}\left(\frac{m_1^2-m_1'^2}{2}\right) \\ \delta_{R^2}\left(\frac{m_2^2+m_2'^2}{2}-m^2\right) \delta_{R^2}\left(\frac{m_2^2-m_2'^2}{2}\right) G_t(s,t) \quad (1.31)$$

where

$$G_t(s,t) = \sum_{n\ell} \frac{(2\ell+1) P_\ell(-\cos\theta_+)}{Z_{n\ell}(t-m_{n\ell}^2)}, \quad (1.32)$$

and $\cos\theta_+ = (1 + (2s/t - 4m^2))$. By a Sommerfeld-Watson transform we convert (1.32) into

$$G_t(s,t) = \sum_{n=1}^{\infty} \frac{\pi \alpha_n'(t) [2\alpha_n(t)+1]}{\sin \pi \alpha_n(t)} \frac{1}{Z_n(\alpha_n)} P_{\alpha_n(t)}\left(1 + \frac{2s}{t-4m^2}\right) \quad (1.33)$$

which is asymptotically Regge behaved :

$$G_t(s,t) \xrightarrow{s \rightarrow \infty} \frac{\pi \alpha_1(t) (2\alpha_1(t)+1)}{\sin \pi \alpha_1(t)} \frac{1}{Z_1(\alpha_1)} \frac{\Gamma(1+2\alpha_1)}{\Gamma(1+\alpha_1)^2} \left(\frac{-s}{4m^2}\right)^{\alpha_1(t)} \quad (1.34)$$

Thus the Regge behaviour of $G_t(s,t)$ is controlled by a set of trajectories $\alpha_0(t)$ corresponding precisely to the bag states. These trajectories are exchange degenerate, a crucial property in order to ensure the absence of physical states (i.e., a discontinuity for $s < 0$) corresponding to unobserved non-zero triality states. This means that at sufficiently high $s > 0$, when we look at the discontinuities in the s channel we do not find only bag states but also a continuum associated with the Regge pole. These states must be looked upon as genuine physical states of a character different from simple bag states ; we shall call them "Regge states". They will fill up the final states with π 's in a way similar to the bag states. The only important difference between bag and Regge states at high energy is that the former builds up diffractive scattering while the latter generates normal Regge behaviour.

2. - HADRON SCATTERING AMPLITUDES

A. - Implementing unitarity in a perturbative way

In the preceding section we have explored the consequences of some simple general ideas on the structure and the spectrum of meson systems. We shall now take up the problem of how can a precise framework for hadronic interactions be introduced. As discussed elsewhere ¹⁾⁻³⁾ this can be done along the lines of the massive quark model ⁵⁾, and it can be shown that it amounts to a diagrammatical expansion of the S matrix which parallels that of a $\lambda\phi^3$ perturbation theory. Actually further development of these ideas has shown the incompleteness of such a scheme and the necessity for a piece of the type $\mu\phi^4$ (see Section 4). Enlarging accordingly our scheme, the parallelism between the S matrix expansion and the fictitious $\lambda\phi^3 + \mu\phi^4$ perturbation theory is spelled out by the following transcription (see Fig. 5)

- i) ϕ propagator $\rightleftharpoons$ $q\bar{q}$ Green's function
- ii) ϕ^3 vertex $\rightleftharpoons$ irreducible six-point kernel V_6
- iii) ϕ^4 vertex $\rightleftharpoons$ irreducible eight-point kernel V_8

Figure 5 Dictionary leading from $\lambda\phi^3 + \mu\phi^4$ to our perturbative expansion

To any given order in λ and μ , which should then have the rôle of relevant expansion parameters, we can draw the diagrams of $(\lambda\varphi^3 + \mu\varphi^4)$ and then proceed to substitute the φ propagators with $q\bar{q}$ Green's functions, the φ^3 with the irreducible six-point kernel V_6 , and the φ^4 with the kernel V_8 .

Thus the whole construction of unitary hadron scattering amplitudes boils down to determining the structure of G , V_6 , V_8 and the rules governing their juxtaposition. The structure of G has been discussed in the preceding section, the forms of V_6 and V_8 will be discussed in the next paragraphs.

B. - The irreducible kernel V_6 , the coupling of three bags and its extension to complex momenta

Let us look at the coupling of three-bag states (Fig. 6)

Figure 6 The coupling of three-bag states

V_6 determines the probability amplitude for the quarks of one bag, say p_1 , to disappear from it and reappear as the constituents of a two-bag system (p_2, p_3) . According to a geometrical picture of the interaction this amplitude should be zero whenever the space-time regions spanned by the three bags do not overlap. A way of achieving this is to write

$$V_6(x_1, \dots, y_3) = \mu^6 \delta^4(x_1 - x_2) \delta^4(y_2 - y_3) \delta^4(x_3 - y_1) + \text{perm.} \quad (2.1)$$

where μ^2 is a parameter, with dimension of a $(\text{mass})^2$, playing the rôle of a coupling constant. It can be interpreted as the amplitude for a quark to tunnel from one bag to another. According to (2.1) the three-bag coupling is given in momentum space by the diagrams in Fig. 7, i.e., by the expression

$$A(p_1 \rightarrow p_2 + p_3) = \frac{\mu^6}{Z_1^{1/2} Z_2^{1/2} Z_3^{1/2}} \int \frac{d^4 k}{(2\pi)^4} \psi_{n_1 l_1 m_1}(p_1, k) \psi_{n_2 l_2 m_2}^*(p_2, k + \frac{p_2}{2}) \psi_{n_3 l_3 m_3}^*(p_3, k - \frac{p_2}{2}) + (p_2 \leftrightarrow p_3) \quad (2.2)$$

Figure 7 The diagrams describing the three-bag coupling

Substituting the approximate expression (1.13) correctly normalized one gets (we set for convenience $m^2 = 0$)

$$A(p_1 \rightarrow p_2 + p_3) \approx \mu^6 \frac{(2\pi)^2}{(Z_1 Z_2 Z_3)^{1/2}} \left(\frac{2\pi}{R^2} \right)^3 \int d^4 k \delta_{R^2} \left(\frac{p_1^2}{4} + k^2 \right) \times \delta_{R^2} (p_1 k) \delta_{R^2} \left[\frac{p_2^2}{4} + \left(k + \frac{p_2}{2} \right)^2 \right] \delta_{R^2} [p_2 \cdot (k + \frac{p_2}{2})] \delta_{R^2} \left[\frac{p_3^2}{4} + \left(k - \frac{p_2}{2} \right)^2 \right] \times \delta_{R^2} [p_3 \cdot (k - \frac{p_2}{2})] Y_{l_1}^{m_1}(\Omega_k) \bar{Y}_{l_2}^{m_2}(\Omega_{k + \frac{p_2}{2}}) \bar{Y}_{l_3}^{m_3}(\Omega_{k - \frac{p_2}{2}}) + (p_2 \leftrightarrow p_3). \quad (2.3)$$

This quite formidable expression can be evaluated, to lowest order in $1/R^2$, by making use of the δ like properties of the "fat δ function" δ_{R^2} .

We get quite easily :

$$\begin{aligned}
 a(p_1 \rightarrow p_2 + p_3) \simeq & \frac{\mu^6 (2\pi)^2}{4(Z_1 Z_2 Z_3)^{1/2}} \left(\frac{2\pi}{R^2}\right)^3 \int_{-1}^{+1} dz \int_0^{2\pi} d\varphi Y_{\ell_1}^{m_1}(\theta, \varphi) \\
 & \times Y_{\ell_2}^{-m_2}(\Omega_{\vec{k}+\vec{p}_2}) Y_{\ell_3}^{-m_3}(\Omega_{\vec{k}-\vec{p}_2}) \delta_{R^2}^4 \left[\frac{1}{4} (m_1^2 - m_2^2 - m_3^2) + \frac{1}{4} \lambda^{1/2} (m_1^2, m_2^2, m_3^2) Z_{\vec{k}\vec{p}} \right] \quad (2.4)
 \end{aligned}$$

$z_{\vec{k}\vec{p}}$ is the cosine of the angle between the vector $\vec{k}$ and the decay momentum $\vec{p} = 1/2m \lambda(m_1^2, m_2^2, m_3^2)$, where $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$ is the well-known triangular function, which is positive provided we are in the physical decay region $[m_1 \geq (m_2 + m_3)]$. Notice that (2.4) is well behaved for $\lambda \geq 0$, but for $\lambda < 0$ $\delta_{R^2}^4$ develops exponentially exploding tails and we should try and define such a coupling in a way which does not show this disease. This will be done shortly, but before doing it we would like to analyze the structure of (2.4).

The presence of the term $\delta_{R^2}^4$ makes the integrand strongly peaked for $z_{\vec{k}\vec{p}} = 1$, and we get immediately

$$\begin{aligned}
 a(p_1 \rightarrow p_2 + p_3) \simeq & \frac{\mu^6}{(Z_1 Z_2 Z_3)^{1/2}} \left(\frac{2\pi}{R^2}\right)^3 [(2p_2 + 1)(2p_3 + 1)]^{1/2} \delta_{n_2 0} \delta_{n_3 0} Y_{\ell_1}^{m_1}(\theta, \varphi) \\
 & \times \int_{-1}^{+1} dz \delta_{R^2}^4 (a - bz) \quad (2.5)
 \end{aligned}$$

where θ, φ are the polar co-ordinates of the decay momentum $\vec{p}$, and

$$a = \frac{1}{4} (m_1^2 - m_2^2 - m_3^2) \quad (2.6)$$

$$b = \frac{1}{4} [(m_1^2 - m_2^2 - m_3^2)^2 - 4m_2^2 m_3^2]^{1/2}.$$

Thus (2.5) and (2.6) tell us that the matrix element strongly favours the decay of the highly excited state p_1 into a light meson (say a π) and another high mass state. This is the reason why the decay of a FS proceeds through a linear chain, a fact which has been stressed in the previous section.

Let us now turn to the case when $\lambda < 0$. Its interest lies in the fact that, in order to compute virtual effects coming from bag exchanges we shall need to know how the three-bag vertex behaves in the unphysical region. To see what happens in Appendix 1 we report a calculation of a simplified form of the vertex by means of Gaussian integration over the loop momentum. The disease for $\lambda < 0$ manifests itself by the presence in the final expression of a logarithmically divergent piece. We cure this by a subtraction procedure, and then determine the subtraction by the requirement of continuity at $\lambda = 0$, the boundary of the physical region. As a result [see (A1.8) and (A1.9)], we get the following prescription : whenever a complex momentum flows through a quark line the related "fat δ function" is to be defined as (a, b real)

$$\delta_{R^2}(a+ib) = \frac{1}{2\pi} \int_0^{R^2} d\alpha \cos(\alpha a) e^{-|b|\alpha} \quad (2.7)$$

Thus for $\lambda < 0$, the vertex is given by (2.4), where the expression

$$\delta_{R^2} \left[\frac{m_1^2 - m_2^2 - m_3^2}{4} + i \frac{|\lambda|^{1/2}}{4} \epsilon_{\mu\nu\rho\sigma} \vec{k}_\mu \vec{k}_\nu \vec{k}_\rho \vec{k}_\sigma \right]$$

is to be interpreted according to (2.7).

C. - The kernel V_8 and the four-bag coupling

The ideas of the preceding paragraph can be immediately carried over to the description of the kernel V_8 and the four-bag coupling (Fig. 8). Thus we write

Figure 8 The four-bag coupling

$$V_8(x_1, \dots, y_4) = \rho^8 \delta^+(x_1 - x_3) \delta^+(y_3 - x_4) \delta^+(y_4 - y_2) \delta^+(x_2 - y_1) \quad (2.8)$$

+ permutations

ρ^2 is a mass parameter which should be equal to μ^2 , if our interpretation of μ^2 as a quark tunnelling amplitude is correct. From (2.8) the four-bag coupling in momentum space is given by the diagrams in Fig. 9.

Figure 9 The four-bag coupling diagrams

D. - Graphical rules. Off-shell effects

The hadrons' S matrix is therefore given by a collection of diagrams which can be reduced to a network of quark lines joining quark Green's functions and hadron wave functions. The rules to evaluate any such diagram are the following ^{*)} :

- i) each incoming bag of momentum p_i and relative quark momentum k_i is introduced through the properly normalized wf $\psi(p_i, k_i)$;
- ii) each outgoing bag state of momentum p_f and relative quark momentum k_f is given by $\psi^*(p_f, k_f)$;
- iii) each "exchanged" bag state (see Fig. 10) of momentum p_ℓ and relative momentum k_ℓ is given by $(i^\ell) \psi(p_\ell, k_\ell)$, where ℓ is the angular momentum of the state ;

^{*)} For the reasons previously mentioned these rules turn out to be slightly different from the ones given in Refs. 1)-3).

Figure 10 An exchanged bag state

- iv) for each quark line we must multiply by μ^2 ;
- v) four-momentum at each two-quark hadron vertex must be conserved ;
- vi) for each loop we have an integration $d^4\ell/(2\pi)^4$;
- vii) $\bar{q}q$ Green's functions are totally connected and their discontinuities correspond to physical hadron states.

These rules, however, are not unambiguous. In order to lift the ambiguity we must specify the procedure by which off-shell effects are to be calculated. To be concrete, let us take the simple Born diagram in " π - π scattering" (Fig. 11). According to our rules we are led to a

Figure 11 The lowest order diagram in $\pi\pi$ scattering and its quark counterpart.

two-loop integral of four wf's and one $\bar{q}q$ Green's function. The question now is how to calculate the Green's function to be inserted in the two-loop calculation. There are two procedures which give in principle different results ^{*)}.

- 1) Calculate first the Green's function as a function of s and t at fixed quark masses, i.e., put the imaginary part (1.30) into an s dispersion relation at fixed t . Insert then such a Green's function into the two-loop integral.
- 2) Calculate the s discontinuity of the whole diagram first and then insert it in a fixed t dispersion relation.

While, obviously, the two procedures give the same imaginary part (on-shell), the real parts in principle differ drastically. It is clear that in general only procedure 2 is in agreement with usual dispersion relations, and therefore should be followed throughout. In more complicated situations we shall have to perform a thorough analysis of the correct analyticity properties of the amplitudes under consideration before we can decide about the correct calculational procedure.

^{*)} In the MQM framework ⁵⁾ procedure 2 was implicitly assumed.

3. - CURRENTS AND NORMALIZATIONS

A. - A simple way to introduce currents

Let us now suppose that our quarks, still Lorentz scalars, form an isospin doublet. The states one can build out of a $q\bar{q}$ pair will then carry isospin 1 and 0, and their wave functions are given by ($\alpha = 0, \dots, 3$; $i, j = 1, 2$)

$$(\psi_{n\ell m}^\alpha)_i^j = (\tau^\alpha)_i^j \psi_{n\ell m}(p, k) \quad (3.1)$$

where $\psi_{n\ell m}(p, k)$ is the wave function (1.9), and for simplicity the isotriplet and the isosinglet have been assumed degenerate. The simplest and most natural way ^{*)} to introduce an isospin current is to write for the current quark vertex operator (see Fig. 12) :

$$\text{disc}_{q^2} [\Gamma_\mu^\beta(q, k)]_i^j = \left(\frac{\tau^\beta}{2}\right)_i^j \sum_n (2\pi) \delta(q^2 - m_n^2) \mu^\mu \int \frac{dk'}{(2\pi)^4} 2k'_\mu \psi_{n1}(q, k') \psi_{n1}^*(q, k) \quad (3.2)$$

Figure 12 The vector meson dominated current-quark vertex operator

^{*)} This was done in Refs. 1)-3).

Any current matrix element will be calculated by first folding (3.2) with the relevant wave functions and then getting the real part by dispersing in q^2 . This definition coincides in an obvious manner with generalized vector meson dominance ⁶⁾, whose phenomenological success is well known. Using (1.5c) we can check that the current is conserved, i.e.,

$$q^\mu \text{disc}_{q^2} [\Gamma_\mu^\beta(q, k)]_i^j = 0 \quad (3.3)$$

In Appendix 2 a calculation of the form factor discontinuity is reported for the generic state of angular momentum ℓ and mass M . Upon inserting the discontinuity in the q^2 dispersion relation we obtain for the charge form factor :

$$\langle p\ell\beta | J_\mu^\alpha(0) | p'\ell\gamma \rangle = (p+p')_\mu \text{Tr} \left(\frac{\tau^\alpha}{2} [\tau^\beta, \tau^\gamma] \right) F_{M\ell}^{\text{ch}}(q^2) \quad (3.4)$$

where for $M \gg m_1 = m_\rho$

$$F_{M\ell}^{\text{ch}}(q^2) \simeq \frac{2\mu^6}{R^2 Z_\rho Z_\ell(M)} \frac{1}{m_\rho M} \frac{1}{m_\rho^2 - q^2} P_\ell \left[\sqrt{1 + \frac{m_\rho^2}{M^2}} \right] \quad (3.5)$$

The normalization condition $F_{M\ell}^{\text{ch}}(0) = 1$ leads to :

$$Z_\ell(M) \simeq \frac{2\mu^6}{R^2 Z_\rho} \frac{1}{m_\rho^3 M} P_\ell \left[\sqrt{1 + \frac{m_\rho^2}{M^2}} \right] . \quad (3.6)$$

For low partial waves $[\ell \lesssim (m_\rho/M)]$ asymptotically one has :

$$Z_\ell(M) \xrightarrow{M \rightarrow \infty} \frac{c}{M} . \quad (3.7)$$

If we go back to Section 1, we see that (3.7) wrecks the nice concept of the "primeval Pomeron" [see Eq. (1.3)], and with it the successful phenomenology of the MQM. This result is therefore unacceptable, and the only way for us to salvage the situation is to admit that (3.2) does not represent the whole isospin current.

The vanishing of $Z_\ell(M)$ when M becomes very large can be understood if we think that the currents can measure the isospin content of our hadrons only through their hadronic fluctuations, the vector mesons. Any massive vector meson can sample the isospin charge only in space region of the order of its Compton wavelength $1/m_V$.

On the other hand the space volumes of our states increase linearly with their mass, and massive vector mesons can only "see" a fraction of the charge of high mass states which decrease with the mass M in agreement in (3.7).

B. - Adding a long range piece to the currents

How can we cure the pathological result (3.7) ? The only way out of this difficulty seems to admit that the vector dominated piece is only one contribution to the isospin currents, and that the full current contains a direct coupling to the quarks comprising a hadronic state.

The idea that currents can couple directly to quarks is certainly not new and arises quite naturally ^{*)}. However, it is not widely appreciated that it is far from an innocent and harmless hypothesis. In fact if we try to implement it, we can see right away that all our (unproven) ideas about form factors and their analyticity properties are incorrect.

Suppose, in fact, we know how to introduce this additional direct coupling of the current to the quarks, we will end up with calculating diagrams of the type in Fig. 13. Such diagrams are quite familiar to nuclear physicists, it suffices to recall the calculation of the deuteron form factor. There the q^2 dependence arises from the well-known anomalous thresholds which are sensitive to the structure of the wave function. Also there is no trouble with dispersion relations due to the fact that anomalous singularities do arise from the configuration where all lines are on the "mass shell" ^{**)}. When we carry this picture

^{*)} For instance in the parton approach of Feynman and Bjorken ⁷⁾.

^{**)} See for example the discussion in Ref. 8).

Figure 13 The direct current quark coupling contribution to the hadron form factor

over to quarks, this does not make sense any more ; and we must face the serious possibility that our ideas about form factors and their analyticity have to be revised.

I am aware that this step is a momentous one and may lead the whole theory into disaster, but I have found no other way out. On the other hand this new piece of the currents might be just what we need to account for the seemingly abundant production of photons in high energy collision ⁹⁾.

I propose that in addition to (3.2) the isospin current has a "direct" quark contribution, $\hat{J}_\mu^a$, and the matrix element between any two bag states is given by

$$\langle p_1, \alpha | \hat{J}_\mu^a(x) | p_2, \gamma \rangle = Z_0 \text{Tr} \left(\frac{\tau^a}{2} [\tau^\alpha, \tau^\gamma] \right) \int dy \psi^*(p_2; xy) \frac{\overleftrightarrow{\partial}}{\partial x_\mu} \psi(p_1; xy) \quad (3.8)$$

where Z_0 is a constant carrying the dimension of a mass squared. In order to ascertain that the new current piece is indeed conserved we must utilize (1.5a) which is equivalent to

$$-\frac{1}{2}(\square_x + \square_y) \psi(p; xy) = m^2 \psi(p; xy) \quad ; \quad (3.9)$$

and the vanishing of ψ at the boundary.

From (3.8) we can readily evaluate the charge form factor for any state of mass M and angular momentum ℓ , in particular we get

$$\hat{F}_{M\ell}^{ch}(0) = \frac{Z_0}{2Z_\ell(M)} . \quad (3.10)$$

Putting (3.6) and (3.10) together one gets the normalization condition :

$$Z_\ell(M) \simeq \frac{Z_0}{2} + \frac{2\mu^6}{Z_p M m_p^3} P_\ell \left[\sqrt{1 + \frac{m_p^2}{M^2}} \right] . \quad (3.11)$$

This result not only reinstates the "primeval Pomeron" behaviour, in fact for $\ell \lesssim (M/m_p)$ at large M

$$Z_\ell(M) \simeq \frac{Z_0}{2} ;$$

but for $\ell \gg M/m_p$ $Z_\ell(M)$ increases like $(1/M)e^{+\ell(m_p/M)}$, and this implies that the high angular momenta decouple from the two-quark Green's function exactly in the way which was discussed in Section 1. One can estimate that (3.11) corresponds to the approximation made in Section 1 :

$$\frac{1}{Z_\ell(M)} = \frac{2}{Z_0} \Theta(\ell_0 - \ell) \quad (3.12)$$

with

$$\begin{aligned} \ell_0 &= \frac{M}{2} R_1(M) \\ R_1 &= \frac{1}{m_p} \log \left(\frac{M^2}{M_0^2} \right) , \end{aligned} \quad (3.13)$$

where M_0 is a mass whose explicit expression depends on the constants appearing in (3.11).

It is worth noticing that the "primeval Pomeron" acquires [see (3.13) and (1.30)] a logarithmic behaviour as well as a shrinkage. A discussion of how this behaviour affects scaling and its possible breaking will be presented in Section 6.

Thus we see that while, on one hand, the original MQM picture ⁵⁾ is basically confirmed, on the other, one sees emerging new interesting structures which get the whole approach closer to Nature. We end this section by adding an item to our list of calculational rules in Section 2 :

- viii) currents are given by the sum of two contributions which in the fictitious φ theory are given by the diagram in Fig. 14. The "vector dominated" piece is given by the quark diagram in Fig. 15, while the "direct" piece is described by the diagram in Fig. 13, multiplied by the normalization constant Z_0 .

Figure 14 The current coupling in the fictitious φ theory

Figure 15 The "vector dominated" piece of the current

4. - HADRON-HADRON SCATTERING TO LOWEST ORDER

We begin in this section a series of calculations of several interesting hadronic processes. As emphasized in the Introduction the importance of these calculations lies in the fact that from them we should get a clear idea about the real meaning of the rules of Section 2. Even though the phenomenologically crucial quark spin has no place in all this, the reader should appreciate that many of these results, when properly transposed, are indeed relevant for the actual hadronic world.

A. - The production of resonances and of a single FS

The φ diagram and its quark correspondent is reported in Fig. 16, where the external hadron shall be always taken as the ground $q\bar{q}$ state.

Figure 16 The hadron-hadron scattering amplitude to lowest order

The scattering amplitude at low energy will have poles in s corresponding to the positions of low mass resonances. At high s we calculate (see Appendix 3)

$$\text{Im } Q_s(s, t) \rightarrow \frac{c}{s} \quad , \quad (4.1)$$

which leads to a resonance cross-section decreasing like $1/s^2$. Thus single FS production in high energy hadron-hadron collisions is strongly suppressed. If, instead of FS production, we are interested in the production of Regge states, a calculation similar to the previous one shows that such production cross-section goes as $s^{-5/2}$.

B. - Bag exchange. The question of Reggeization

The amplitude we want to compute now is the one shown in Fig. 17.

Figure 17 The bag exchange contribution to lowest order

Proceeding in a standard way we write

$$a_t = \sum_{nl} g_{nl}^2 \frac{(2l+1) (-1)^l P_l(\cos\theta_t)}{t - m_{nl}^2} \quad (4.2)$$

where g_{nl} is the coupling constant of the excited state nl to the ground states, and

$$\cos\theta_t = 1 + \frac{2s}{t} . \quad (4.3)$$

Equation (4.2) is nothing but the model of Van Hove ¹⁰⁾, and it is well known to lead to Regge behaviour. Furthermore the amplitude a_t exhibits for s positive and bigger than a certain s_0 a cut coming from the divergence of the series defined by (4.2). But what is the meaning of this

cut ? From Fig. 17, it is immediately clear that it cannot correspond to a physical state, but rather to a state consisting of a Regge state and a pair of non-interacting (spectators) quarks. Thus (4.2) and (4.3) lead us into the fatal difficulty of yielding a discontinuity which cannot correspond to an observable physical state. Is there any way out ? Well, let us go back to (4.3), $\cos\theta_t$ is unambiguously defined only for $t = m_{n\ell}^2$ (i.e., on-shell) ; when we go off-shell ($t \neq m_{n\ell}^2$) (4.3) is a possible extrapolation, but far from unique. We shall show that if rather than via (4.3) we extrapolate according to

$$\cos\theta_t = 1 + \frac{2s}{m_{n\ell}^2} \quad (4.4)$$

our trouble will disappear and the amplitude defined by (4.2) and (4.4) is actually analytic in s . To see this we need only recall that our particles lie on Regge trajectories $m_{n\ell}^2 \sim a_n + \alpha' \ell$, and that when ℓ increases so does $m_{n\ell}^2$, which makes the convergence of (4.2) for s fixed a trivial matter to prove.

In Appendix 4 the behaviour of α_t for $s \rightarrow \pm\infty$ and t fixed and negative is estimated to be :

$$Q_t \xrightarrow{s \rightarrow +\infty} \frac{a(t)}{s^{3/2}} \quad (4.5)$$

$$\xrightarrow{s \rightarrow -\infty} \frac{b(t)}{s^2} , \quad (4.6)$$

which makes it legitimate to neglect this contribution at high energy, except in some special cases for $t \approx 0$ where non-Reggeized π exchange may play an important rôle.

To conclude, the main point of this section is that, in defining the rules to compute "off-shell" behaviours, we should be guided only by the criterion of physical interpretability of the amplitude discontinuities arising from our rules. This criterion excludes the interpretation of the diagram in Fig. 17 as a Van Hove model, and forces us to find an alternative solution. It is utterly non-trivial that such solution exists in the definition (4.4) of the meaning of $\cos\theta_t$, and that the amplitude so defined does not appreciably contribute to the high energy behaviour of hadron-hadron

scattering [see (4.5) and (4.6)]. In the language of dispersion relations (4.4) corresponds to writing a t dispersion relation keeping s and not $\cos\theta_t$ fixed ; this strongly suggests that in calculating "off-shell" behaviours the correct procedure to follow will always be to disperse in the variable we want to continue, while keeping all other independent Lorentz scalar fixed.

5. - HIGH ENERGY HADRON-HADRON SCATTERING : THE POMERON AND REGGE BEHAVIOUR

We have seen in the preceding section that the lowest order diagram contribution to high energy hadron-hadron scattering is negligible. Here we want to discuss how Pomeron and Regge behaviour may emerge from our approach. Once we have disposed of single FS production, as an unlikely high energy process [see Eq. (4.1)], the perturbative approach described in Section 2 leads us to consider the production of two FS's as the dominant high energy process. This is actually what will be shown in this section ; but before entering in the midst of calculations I would like to recall that in previous work ³⁾ the belief was held that a diagram like the one reported in Fig. 18 would lead to the experimentally observed Pomeron and Regge behaviours.

Figure 18 The two FS production diagram generated by the iteration of the φ^3 coupling

However, carrying out a calculation similar to the one reported in Appendix 4, this belief turns out to be unfounded, and one must conclude that one cannot get an acceptable high energy behaviour for the two FS's production processes through the "bag exchange" of Fig. 18.

As anticipated in Section 2, we must have a bag coupling scheme which in addition to the three-bag vertex allows also a direct four-bag coupling (see Fig. 19).

A. - The four-bag coupling and the Pomeron

The s discontinuity of the two-body scattering amplitude is given by the diagram in Fig. 19.

$$\begin{aligned} k_1 - k_2 &= k'_1 - k'_2 \\ s &= (p_1 + p_2)^2 = (p'_1 + p'_2)^2 \\ k &= (p_1 - p_2)^2 = (p'_2 - p'_1)^2 \end{aligned}$$

Figure 19 The Pomeron diagram

The calculation is completely standard : we write down the wf's and the Green's functions in the approximate forms (1.13) and (1.30), respectively. We parametrize all momenta à la Sudakov :

$$q = \alpha p_1 + \beta p_2 + q_1$$

and carry out all the integrals in the longitudinal variables $(\alpha, \beta, \dots)$ by means of the "fat" δ functions and obtain :

$$\begin{aligned}
 \text{disc}_s A \simeq & 2^5 u^{16} \left(\frac{2}{Z_0} \right)^4 \left(\frac{2r}{R^2} \right)^2 R_1^2 s \int_{-\frac{1}{2}}^{+\frac{1}{2}} d\alpha \int_{-\frac{1}{2}}^{+\frac{1}{2}} d\beta \frac{(\frac{1}{2}-\alpha)(\frac{1}{2}-\beta)}{(\frac{1}{2}+\alpha)(\frac{1}{2}+\beta)} \\
 & \times \int d^2 \vec{k}_1 \int d^2 \vec{k}_2 \int d^2 \vec{k}'_1 \delta_{R^2}^2 \left(\frac{\vec{k}_1^2}{\frac{1}{2}+\alpha} \right) \delta_{R^2}^2 \left(\frac{\vec{k}_2^2}{\frac{1}{2}+\beta} \right) \\
 & \times \delta_{R^2}^2 \left[\frac{\vec{p}_4^2 + \vec{k}_1'^2}{\frac{1}{2}+\alpha} + \frac{\vec{p} \cdot \vec{k}_1'}{\frac{1}{2}+\alpha} (2\alpha) \right] \delta_{R^2}^2 \left[\frac{\vec{p}^2 + (\vec{k}_1' - \vec{k}_1 + \vec{k}_2)^2}{\frac{1}{2}+\beta} + \frac{\vec{p} \cdot (\vec{k}_1 - \vec{k}_2 - \vec{k}_1')}{\frac{1}{2}+\beta} 2\beta \right] \\
 & \times \frac{J_1(R_1 |\vec{k}_1 - \vec{k}_1' - \vec{p}_2|)}{R_1 |\vec{k}_1 - \vec{k}_1' - \vec{p}_2|} \frac{J_1(R_1 |\vec{k}_1 - \vec{k}_1' + \vec{p}_2|)}{R_1 |\vec{k}_1 - \vec{k}_1' + \vec{p}_2|} \quad (5.1)
 \end{aligned}$$

where $\vec{k}_1, \dots$ are two-dimensional vectors and $t = -\vec{p}^2$. By using the approximations (A3.5) and (A3.6) we can perform the three two-dimensional integrals, and get :

$$\text{disc}_s A \simeq \frac{9}{2} \left(\frac{R_1^2}{\pi} \right)^4 \left(\frac{2u^2}{Z_0} \right)^4 \pi R_1^2 s \exp \left(\frac{R_1^2}{16} t \right) \quad (5.2)$$

We have indeed obtained a diffractive amplitude. From (3.13) we see that the total hadron-hadron cross-section goes asymptotically like $(\log s)^2$ and that the diffraction peak shrinks with energy. Our calculation, therefore, confirms that the origin of diffraction in hadron collisions is the production of two fire sausages through the four-bag coupling mechanism. Such a diffractive behaviour lends itself to a geometrical interpretation in terms of a grey disc with a radius expanding logarithmically with energy.

To get the full amplitude we follow our general strategy and write a dispersion relation in s with t fixed (5.2) being twice the imaginary part. The fact that the amplitude thus obtained is already a

quite good approximation to the real high energy amplitude, does nothing but confirm that the expansion of the S matrix envisaged in Section 2 is a meaningful one.

B. - The Regge behaviour

To obtain the observed non-leading Regge behaviour, we need simply identify one of the blobs in Fig. 19 with the discontinuity of the part of the Green's function G_t , which according to (1.34) gives rise asymptotically to Regge behaviour. Setting

$$\text{disc}_s G_t(s,t) \rightarrow c \left(\frac{s}{s_0} \right)^{\alpha(t)}$$

and performing a calculation analogous to the one reported above, we easily obtain the non-leading piece of $\text{disc}_s A$ as :

$$\text{disc}_s A \simeq k \left| \frac{s}{s_0} \right|^{\alpha(t)}. \quad (5.3)$$

This result may appear surprising ; we have in fact obtained unadulterated Regge behaviour in spite of the complicated structure of the diagram leading to (5.3). We would have rather expected a Regge cut type behaviour. The reason for this is that at very high energy the t distribution of $\text{disc} G_s$ shrinks faster ($\sim \log^2 s$) than that of $\text{disc} G_t$, thus dominating the convolution between the two Green's functions. Putting (5.3) into a fixed t s dispersion relation reproduces the correct Regge phase in the full amplitude. We can therefore conclude that in our approach Regge behaviour does not arise from the diagram in Fig. 17 but from the one in Fig. 19. However, amusingly enough, due to the peculiar structure of the Green's function G_s , everything goes as if we were calculating precisely the incriminated diagram of Fig. 17.

6. - THE SCALING PHENOMENON

After having successfully passed the (qualitative) test of hadronic scattering at high energy and recovered all essential experimental features, we must ask ourselves whether such a nice picture survives another crucial test : the deep inelastic phenomena. In this section our theory will demonstrate its healthy status in this respect also. In order not to encumber the notation the case of scalar currents and scalar targets shall be considered throughout. A full account of the spin structure will be possible only after we have introduced the quark spin degree of freedom.

A. - " e^+e^- annihilation" into hadrons

From the discussion of the currents' structure carried out in Section 3 the coupling of the current to a generic hadronic system is made up of two distinct contributions : the "vector dominated" one and the "direct" contribution. The scalar current, whose effects will be calculated here, will accordingly be given by :

$$J = J^{VD} + J^{Direct} \quad (6.1)$$

In calculating the e^+e^- annihilation process into hadrons we must therefore consider the two diagrams in Fig. 20 ; notice that we are neglecting interference terms between the two current pieces. That this is legitimate shall become apparent in the following

Figure 20 The diagrams contributing to " e^+e^- annihilation"

(a) - The vector dominated contribution

The high energy behaviour of the imaginary part ρ_a of the vacuum polarization function is given by

$$\rho_a(q^2) \rightarrow \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} \int \frac{d^4 k'}{(2\pi)^4} \text{disc}_S G_S . \quad (6.2)$$

According to (1.28) and (1.30), (6.2) becomes :

$$\begin{aligned} \rho_a(q^2) \rightarrow & \frac{1}{2} \frac{1}{(2\pi)^4} \int d^4 k \int d^4 k' \frac{R^2 R_1^2}{8\pi Z_0} \cdot q^2 \frac{J_1(R_1 \sqrt{(k-k')^2})}{R_1 \sqrt{(k-k')^2}} \\ & \times \left(\frac{2\pi}{R^2}\right)^2 \delta_{R^2}(qk) \delta_{R^2}(qk') \delta_{R^2}\left(\frac{q^2}{4} + k^2\right) \delta_{R^2}\left(\frac{q^2}{4} + k'^2\right) . \end{aligned} \quad (6.3)$$

The k' integration is easily made by use of the identity (in the sense of distribution theory) ⁵⁾ :

$$\delta_{R^2}(a^2) \delta_{R^2}(b^2) \frac{J_1(R_1 \sqrt{(a-b)^2})}{R_1 \sqrt{(a-b)^2}} \rightarrow \frac{\pi}{R_1^2} \delta_{R^2}(a) \int da \delta^4(b-a) \quad (6.4)$$

Thus we get

$$\begin{aligned} \rho_a(q^2) \rightarrow & \frac{1}{16\pi^2} \frac{1}{R^2 Z_0} \int d^4 k \delta_{R^2}(qk) \delta_{R^2}\left(\frac{q^2}{4} + k^2\right) \\ & = \frac{1}{16\pi} \frac{1}{R^2 Z_0} + O\left(\frac{1}{\sqrt{q^2}}\right) , \end{aligned} \quad (6.5)$$

where the non-leading term originates from the Regge behaviour of the G_t piece of the Green's function. In order to appreciate the result (6.5) it is necessary to calculate the "canonical contribution" reported in Fig. 21 ; a trivial calculation yields immediately :

$$\rho^{\text{can}}(q^2) \rightarrow \frac{1}{16\pi} , \quad (6.6)$$

Figure 21 The canonical contribution to e^+e^- annihilation

i.e., the imaginary part of the vacuum polarization function for large q^2 goes to a constant limit given numerically by (6.6). Introducing a familiar notation we can define R_a , the ratio between the hadronic and the leptonic annihilation, and obtain

$$R_a = \frac{P_a(q^2)}{\rho^{\text{can}}(q^2)} \rightarrow \frac{1}{R^2 Z_0} + O\left(\frac{1}{\sqrt{q^2}}\right). \quad (6.7)$$

Thus we see that the hadronic production cross-section has a pointlike, scaling behaviour and the ratio R_a tends to a constant which is in principle different from one, being determined by two crucial hadronic parameters like R^2 and Z_0 . This is at variance with the general belief that this ratio should be one.

(b) - The direct contribution

We now must turn to the contribution of the direct current coupling. Recalling the discussion of Section 2B, we need consider only the configuration where a low mass hadron recoils against a high mass FS.

Proceeding in a, by now, standard fashion we find that also this term scales and for R_b we obtain the unexpectedly large numerical result :

$$R_b \approx 2. \quad (6.8)$$

The fact that R_b does not depend on the hadronic parameters can be understood by the cancellation between the Z_0^2 terms arising from the current coupling and the Z_0^{-2} term appearing in the denominator coming from the normalization. No particular weight, however, should be given to the particular numerical results (6.7) and (6.8), what is quite important is that both contributions (a) and (b) give rise to a scaling behaviour as observed at high energy in the SPEAR experiments.

What about the one-particle distributions ? If you recall the discussion in Section 1B, you can convince yourself that Bjorken scaling holds for the "vector dominated" contribution due to the scaling properties of the decay mechanism of the FS. As for the "direct" contribution we also get a Bjorken scaling as can be checked without any particular trouble.

B. - "Deep inelastic scattering"

To lowest order we have the two contributions reported in Fig. 22.

Figure 22 The two lowest order contributions to "deep inelastic scattering"

The evaluation of (a) presents the problem of defining the off-shell extrapolation of the vector mesons m_1 and m_2 . The procedure we shall follow, as previously discussed, is to evaluate first the discontinuity $W_a(s; m_1^2, m_2^2)$ of our diagram by keeping m_1 and m_2 fixed. To get then the physical discontinuity $W_a(s; q^2)$ we shall disperse in m_1^2 and m_2^2 by keeping s for fixed, i.e.,

$$W_a(s; q^2) = \sum_{m_1, m_2} g^{Jm_1} g^{Jm_2} \frac{W(s; m_1^2, m_2^2)}{(m_1^2 - q^2)(m_2^2 - q^2)} \quad (6.9)$$

where

$$\begin{aligned} g^{Jm} &= \left(\frac{2}{Z_0}\right)^{1/2} \int \frac{d^4k}{(2\pi)^2} \left(\frac{2\pi}{R^2}\right) \frac{1}{\sqrt{4\pi}} \delta_{R^2}(qk) \delta_{R^2}\left(\frac{m^2}{4} + k^2\right) \\ &\simeq \left(\frac{2}{\pi Z_0}\right)^{1/2} \frac{1}{4R^2} \end{aligned} \quad (6.10)$$

is the current-hadron coupling constant. From the calculation reported in Appendix 5, we get [see (A5.6)]

$$\begin{aligned} W_a(s; m_1^2, m_2^2) &\simeq \left(\frac{2\mu^4}{Z_0}\right)^3 (2\pi)^4 \left(\frac{4\pi}{R^2}\right)^3 \frac{3\pi R^2}{|s - m_1^2|} \\ &\times \frac{\delta^4}{R^2} \left[\frac{\mu_0^2}{4} \frac{(m_1^2 - m_2^2)^2}{(s - m_1^2)(s - m_2^2)} \right] \frac{R_1^2}{R_1^2 + \frac{8R^2}{\pi} \left(1 - \frac{m_1^2 + m_2^2}{2s}\right)}. \end{aligned} \quad (6.11)$$

The very interesting feature of this result consists in the peaking of $W_a(s; m_1^2, m_2^2)$ for $m_1^2 = m_2^2$, exhibited by the "fat δ ". Thus in this approach the so-called "diagonal" approximation of generalized vector dominance ¹¹⁾ arises in a very natural way. Moreover, this same mechanism will be always operative in suppressing the interferences between the "direct" and the "hadron dominated" pieces of the currents; a fact already used in the preceding paragraph.

Inserting (6.10) and (6.11) into (6.9) we have ($x = Q^2/2v$)

$$W_a(v, Q^2) \simeq \frac{1}{2v} F_a(x), \quad (6.12)$$

where

$$F_a(x) = \frac{\mu^{1/2}}{Z_0^4} (2\pi)^2 3R^2 (1-x) \int_0^\infty dx_1 \int_0^\infty dx_2 \frac{1}{|1-x_1|} e^{-\frac{2R^2 \mu^2 (x_1-x_2)^2}{(x_1+1)^2 (x_2+1)}} \times \frac{1}{(x+x_1(1-x))(x+x_2(1-x))} \frac{R_1^2}{R_1^2 + \frac{8}{\pi} R^2 \left(1 - \frac{x_1+x_2}{2}\right)}. \quad (6.13)$$

The features of $F_a(x)$ worth noticing are :

$$F_a(x) \xrightarrow{x \rightarrow 0} \text{const} \quad (6.14)$$

and

$$F_a(x) \xrightarrow{x \rightarrow 1} (1-x). \quad (6.15)$$

What does all this mean ? Let us look at the free field diagram (Fig. 23). We immediately calculate

$$W^{\text{can}}(\nu, Q^2) = \frac{1}{2\nu} F^{\text{can}}(x) \quad (6.16)$$

where

$$F^{\text{can}}(x) = (2\pi) \delta(1-x). \quad (6.17)$$

Figure 23 The free field contribution to "deep inelastic scattering"

Thus our results (6.12) and (6.13) show scaling with a "structure function" $F_a(x)$ which is highly non-trivial. Equation (6.12) is the leading behaviour ; if we add the Regge behaved piece $\text{disc}_s G_t$ we obtain a non-scaling contribution of the form

$$W_a'(v, Q^2) = \frac{1}{(2v)^{3/2}} F_a'(x) \quad (6.18)$$

with

$$F_a'(x) \xrightarrow{x \rightarrow 1} (1-x)^{1/2}. \quad (6.19)$$

This piece could play an important rôle ⁵⁾ in the observed pattern of breaking of scaling behaviour near $x=1$.

We now turn to the contribution (b) ; the relevant calculation is reported in the Appendix 6, which shows that we have a behaviour which simulates scaling

$$W_b(v, Q^2) = \frac{1}{2v} F_b(x, v) \quad (6.20)$$

with

$$F_b(x, v) = \frac{12 R_1^2 (1-x)}{R_1^2 (1-x) + \frac{5}{\pi} R^2}. \quad (6.21)$$

In fact, due to the logarithmic dependence of R_1^2 on s , (6.21) exhibits a peculiar scaling breaking behaviour near $x=1$. For x small $F_b(x, v)$ tends like $F_a(x)$ to a constant.

We want to ask the question whether W_a and W_b are enough to take care of the scaling phenomenon in deep inelastic scattering. Phenomenologically we know that the data are compatible with a "Regge behaviour" of scaling functions near $x=0$. In our scalar case such a behaviour would mean for the scaling function $F(x)$:

$$F(x) \xrightarrow{x \rightarrow 0} x^{-\alpha(0)-1}, \quad (6.22)$$

and setting $\alpha(0) = 1$ for the leading Pomeron behaviour we get $F(x) \sim x^{-2}$. The constant limit reached by both $F_a(x)$ and $F_b(x)$ obviously shows that such terms must be unimportant for $x \rightarrow 0$, and it behoves upon us to show that we do have a contribution which exhibits a behaviour such as (6.22). The lowest order candidate for this is the Pomeron diagram in Fig. 24, which

Figure 24 The Pomeron contribution to "deep inelastic scattering"

we shall compute by following a procedure analogous to the one employed for $W_a(s, Q^2)$. The calculation of $W_P(v, Q^2)$ is reported in Appendix 7. The result exhibits again scaling behaviour but this time

$$F_P(x) \xrightarrow{x \rightarrow 0} \frac{1}{x^2} (\log x)^2, \quad (6.23)$$

in complete agreement with our expectations and (5.2). On the other hand, when $x \rightarrow 1$ $F_P(x)$ clearly vanishes much faster than either F_a and F_b .

This concludes our analysis of deep inelastic phenomena. It is almost superfluous to stress that in this approach we could in principle discuss several other topics like form factors, structure of final states and massive μ pairs production. However, carrying out such an extensive programme will be really meaningful only after we have introduced in a realistic way the quark internal degrees of freedom : $SU(3)$, $SU(4)$?, ..., and spin.

7. - LARGE p_T

It is by turning our attention to this very interesting piece of physics, which has been unravelled for the first time at the CERN ISR that we conclude our analysis. Again no extensive discussion will be carried out of these processes, and we shall simply concentrate on the leading contribution to large p_T particle production in the kinematical region for which $x_T = (2p_T/\sqrt{s}) \ll 1$.

As already discussed in the framework of the MQM we can isolate two distinct contributions : A) the totally inclusive FS decay, and B) the semi-inclusive decay at large angles. We shall see that the results previously obtained in the MQM framework will be naturally reproduced.

A. - The totally inclusive FS decay at large angle

At very high energy production of large p_T hadrons can occur according to the following mechanism : a $q\bar{q}$ pair produced along the collision direction according to the usual mechanism (see Fig. 19) can propagate and emerge at large angle and yield along such direction a corresponding FS which by its canonical decay (see Section 1B) produces hadrons at large angle and consequently large p_T .

To evaluate the amplitude for the pair to emerge at large angle we must sort out the large t behaviour of the Green's function G_s given in (1.29). Going to the impact parameter representation and making use of the results of Section 3 [see (3.11)] we write :

$$\text{Im } G_s(s,t) = \frac{1}{2} R_s^2 \int_0^\infty b db \frac{J_0(b\sqrt{-t})}{(1 + \frac{a}{\sqrt{s}} e^{\frac{b m_p}{2}})} \quad (7.1)$$

and defining the new variable $x = b\sqrt{-t}$ we have

$$\text{Im } G_s(s,t) = \frac{R_s^2}{2(-t)} \int_0^\infty x dx \frac{J_0(x)}{1 + \frac{a}{\sqrt{s}} e^{\frac{m_p}{2} \frac{x}{(-t)^{1/2}}}} \quad (7.2)$$

Expanding the exponential for large t we obtain :

$$\text{Im } G_s(s,t) \rightarrow \frac{R^2 s}{2(-t)} \left[\frac{-\text{amp}}{\sqrt{-st}} + O\left(\frac{1}{\sqrt{t}}\right) \right], \quad (7.3)$$

and feeding $\text{Im } G_s(s,t)$ in a dispersion relation we get asymptotically a purely imaginary amplitude ^{*}) :

$$G_s(s,t) \rightarrow i \frac{R^2 s^{1/2}}{(-t)^{3/2}} \left(\frac{-\text{amp}}{2} \right) + O\left(\frac{1}{\sqrt{s}}, \frac{1}{\sqrt{t}}\right). \quad (7.4)$$

To compute the production of large p_T hadrons through this mechanism, use will be made of the following formula :

$$2E \frac{d^3\sigma}{dp^3} \simeq \int d^4k \frac{d^4\sigma}{dk^4} \delta^4(k - \frac{p}{x}) f(x), \quad (7.5)$$

where $d^4\sigma/dk^4$ is the production cross-section of the FS as a function of the vector k which can be taken as the four-momentum of one of its quarks, $x = 2|p|/M$ is the usual Feynman variable, and M is the FS mass.

From the results of Section 5 and (7.4) we can write the following expression for the FS production cross-section :

$$\begin{aligned} \frac{d^4\sigma}{dk^4} \sim & 18 \sigma_{TOT} \int_{\frac{4p_T^2}{s}}^1 \frac{dM^2}{s} \int_0^{1-\frac{M^2}{s}} \frac{d\xi}{\sqrt{\frac{M^2}{s} + \frac{\xi^2}{4}}} \frac{M^2}{s} (1 - |\xi| - \frac{M^2}{s}) \\ & \times \left(\frac{\text{amp} R^2}{2} \right)^2 \frac{1}{M(k_0 - k_3)^2} \frac{1}{\left(\frac{\xi}{2} - \sqrt{\frac{M^2}{s} + \frac{\xi^2}{4}} \right)^3}, \end{aligned} \quad (7.6)$$

and inserting it into (7.5), we get :

$$\begin{aligned} 2E \frac{d^3\sigma}{dp^3} \simeq & \frac{18}{s^2} \sigma_{TOT} \int_{\frac{4p_T^2}{s}}^1 \frac{dM^2}{s} \int_0^{1-\frac{M^2}{s}} \frac{d\xi}{\sqrt{\frac{M^2}{s} + \frac{\xi^2}{4}}} \\ & \frac{M^2}{s} (1 - |\xi| - \frac{M^2}{s}) \left(\frac{R^2 \text{amp}}{2} \right)^2 \frac{8s}{M^2(1-\omega\theta)^3} f\left(\frac{2p}{\sqrt{s}} \left(\frac{s}{M^2}\right)^{1/2}\right). \end{aligned} \quad (7.7)$$

^{*}) Actually a more careful evaluation yields in addition a real term going like $1/s$; but this will not change any of the following conclusions.

Thus asymptotically we get the "scaling" $1/p_T^4$ behaviour. The meaning of this result and its experimental evidence are fully discussed elsewhere ⁵⁾⁻¹³⁾. However here we must recall that such a behaviour should be valid only at very high energies and very small x_T ; in an intermediate situation like the one at ISR the leading rôle should be played by the semi-inclusive decay of the FS, which will be now calculated.

B. - The semi-inclusive FS decay at large angle

We must now compute the diagram in Fig. 25, which describes a mechanism for producing large p_T hadrons which at not too high energies is much more efficient than the canonical fire sausage decay.

Figure 25 The semi-inclusive FS decay

Fixing $t = (k_1 - p)^2$ and K^2 we calculate our diagram by dispersing in $(k_1 + k_2)^2 = M^2$. Using $\text{Im } G_S$ as given by (7.3), we have

$$\hat{A}(K^2, M^2, t) \simeq \text{const} \int_0^\infty \frac{dm^2}{m^2 - M^2 + i\epsilon} \frac{m}{(-t)^{3/2}} \frac{1}{|m^2 - K^2|} \quad (7.8)$$

where the last factor comes from the loop integration [see, for instance, (A5.6)]. Scaling the variables by M^2 , we have :

$$\hat{A}(K^2, M^2, t) \rightarrow \text{const} \frac{1}{(-t)^{3/2} s^{1/2}} \frac{1}{1 - \frac{K^2}{M^2}} \quad (7.9)$$

To get the final answer we must take (7.9), square it, sum over all states $\mathcal{M}$ [which gives a factor $R_{\perp}^2(\mathcal{M}^2/4)$] and finally multiply everything by the q pair distribution in a way analogous to the one employed in the preceding paragraph. Thus :

$$2E \frac{d^3\sigma}{dp^3} \sim \frac{1}{s^4} \frac{1}{(1-\cos\theta)^3} \left(\frac{\sqrt{s}}{2p}\right)^5 \int_{\frac{4p_{\perp}^2}{s}}^1 \frac{dH^2}{s} \int_0^{1-\frac{H^2}{s}} \frac{dF}{\sqrt{\frac{H^2}{s} + \frac{F^2}{4}}} \frac{H^2}{s} \left(1 - |F| - \frac{H^2}{s}\right) \left(\frac{H^2}{s}\right)^{-3/2} \left(1 - \frac{2p}{\sqrt{s}} \left(\frac{s}{H^2}\right)^{1/2}\right). \quad (7.10)$$

which reproduces the result of the MQM, and the experimentally observed p_T^{-8} behaviour. We do not expect that the introduction of spin will change these results.

CONCLUSIONS

It seems appropriate to conclude by trying to assess where does the proposed approach stand. Let me emphasize once again that any conclusion one can extract from this work can only be qualitative due to the neglect of a realistic set of internal quantum numbers. What has then been achieved ?

Starting from a minimally simple description of meson states in terms of the quark co-ordinates we have obtained an infinite number of meson states having well-defined space-time properties. The spectrum is quite closely described by a set of parallel linear and exchange degenerate Regge trajectories.

Having constructed our meson states we have attacked the problem of introducing an interaction among them following the ideas of the massive quark model. The picture emerging is very simple and quite appealing, however, it does not give us yet the possibility to calculate physical processes. In order to do this we must introduce current operators which will give a physical normalization to our states, i.e., through the correct specification of the electric charge. At this point, something very distressing happens ; if we want to define the current operators through vector dominance the normalization, which requires an extension of the computational rules to deal with complex momenta (see Appendix 1), is found to be totally unacceptable. This prompts us to reconsider the definition of current operators, and to propose a new definition which takes also into account the possibility for current operators to couple directly to quark lines. This proposal is a very momentous one, it entails a drastic change of our views about the analytic structure of form factors, which may help us in explaining some puzzling photon behaviours in high energy collisions.

The previous distress now turns itself into gratification, because not only do we recover the nice properties of the massive quark model, but also we obtain gratis the damping of the states with angular momentum exceeding $\ell_0 = (M/2)R_1$, where R_1 increases logarithmically with M .

At this point the framework is precise enough so that we can carry out calculations of several high energy processes. The question of Reggeization is taken up in Section 4 and it is shown that the diagram usually associated with Regge behaviour (Fig. 17) does not Reggeize but

rather falls quite fast at high energy, thus presenting no problem with unitarity. The high energy behaviour of two-body scattering, discussed in Section 5, shows the relation of the Pomeron with the production of two fire sausages, as well as its peculiar geometrical nature. A similar behaviour is also shown by the normal Regge term.

Deep inelastic physics is amenable to a similar kind of analysis, which yields Bjorken scaling behaviour, and a physically appealing distinction between "valence" quarks and "sea" quarks.

Finally, large p_T physics is analyzed. The massive quark model picture led to the expectation that the "scaling" p_T^{-4} behaviour is delayed by kinematical effects, so that even at ISR energies the dominant behaviour is still p_T^{-8} . This is completely confirmed by the results of Section 7.

I believe that the quark theory of hadronic phenomena recently proposed is showing a remarkable fitness for attacking the basic problems of hadrodynamics. It will not take long before we shall be able to answer the question whether these ideas can lead us to the long sought mastering of strong interactions.

APPENDIX 1

In this Appendix we shall calculate the simplified overlap integral $(p_1 = p_2 + p_3 ; m_1^2 = p_1^2)$

$$I(m_1^2, m_2^2, m_3^2) = \int d^4k \delta_{R^2}(k^2) \delta_{R^2}((k-p_2)^2) \delta_{R^2}((k-p_1)^2) \quad (A1.1)$$

where

$$\delta_{R^2}(k^2) = \frac{1}{\pi} \frac{\sin R^2 k^2}{k^2} = \frac{1}{\pi} \int_0^{R^2} d\alpha \cos \alpha k^2$$

In the physical region $(\lambda = \lambda(m_1^2, m_2^2, m_3^2) \geq 0)$ we can immediately calculate to lowest order in R^2 :

$$I \simeq \pi \int_{-1}^{+1} dZ \delta_{R^2} \left[\frac{m_1^2 - m_2^2 - m_3^2}{2} - \frac{1}{2} \lambda^{\frac{1}{2}} Z \right] \quad (A1.2)$$

We are now going to calculate (A1.1) in a different way which will give us the possibility to extend our result to the complex momentum case $\lambda < 0$.

We write

$$\begin{aligned} I &= \frac{1}{\pi^3} \int_{-R^2}^{+R^2} d\alpha_1 \int_{-R^2}^{+R^2} d\alpha_2 \int_{-R^2}^{+R^2} d\alpha_3 \int d^4k e^{i[\alpha_1 k^2 + \alpha_2 (k-p_2)^2 + \alpha_3 (k-p_1)^2]} \\ &= \frac{1}{i\pi} \int_{-R^2}^{+R^2} d\alpha_1 \int_{-R^2}^{+R^2} d\alpha_2 \int_{-R^2}^{+R^2} d\alpha_3 \frac{e^{i(\alpha_1 + \alpha_2 + \alpha_3)}}{(\alpha_1 + \alpha_2 + \alpha_3)^2} \exp \left\{ i \frac{\alpha_2 \alpha_3 m_2^2 + \alpha_1 \alpha_2 m_3^2 + \alpha_1 \alpha_3 m_1^2}{\alpha_1 + \alpha_2 + \alpha_3} \right\} , \end{aligned} \quad (A1.3)$$

where the second line has been obtained through Gaussian integration. On changing variables $\alpha_1 \rightarrow \alpha = \alpha_1 + \alpha_2 + \alpha_3$, and neglecting higher order terms in R^2 , we can perform a Gaussian integration over α_3 to get

$$I \simeq \frac{1}{2i \pi^{1/2}} \int_{-R^2}^{+R^2} d\alpha_2 e^{-i\alpha_2 \frac{m_1^2 - m_2^2 - m_3^2}{2}} \int_{-R^2}^{+R^2} \frac{d\alpha \mathcal{E}(\alpha)}{|\alpha|^{3/2}} e^{-i\frac{\pi}{4} \mathcal{E}(\alpha)} \exp \left[\frac{i}{4} \left(\alpha m_1^2 + \frac{\alpha^2 \lambda}{\alpha} \right) \right]. \quad (A1.4)$$

To leading order in R^2 , by integrating over α we obtain :

$$I \simeq \frac{1}{2i} \int_{-R^2}^{R^2} d\alpha_2 e^{-i\alpha_2 \frac{m_1^2 - m_2^2 - m_3^2}{2}} \left\{ \frac{e^{i(\alpha_2^2 \lambda)^{1/2}}}{(\alpha_2^2 \lambda)^{1/2}} - \text{c.c.} \right\} \quad (A1.5)$$

For $\lambda \geq 0$,

$$I(\lambda \geq 0) \simeq \pi \int_{-1}^{+1} dz \delta_{R^2} \left[\frac{m_1^2 - m_2^2 - m_3^2}{2} - \frac{\lambda^{1/2}}{2} z \right] \quad (A1.6)$$

in agreement with (A1.2) ; while for $\lambda < 0$ we have :

$$I(\lambda < 0) \simeq - \int_0^{R^2} d\alpha_2 \cos \left(\alpha_2 \frac{m_1^2 - m_2^2 - m_3^2}{2} \right) \frac{e^{-\alpha_2 |\lambda|^{1/2}}}{\alpha_2 |\lambda|^{1/2}}, \quad (A1.7)$$

which diverges logarithmically. We cure this disease by subtracting the singular part of the integrand at $\alpha_2 = 0$. The subtraction term can then be determined by imposing the natural requirement that at the boundary of the physical region ($\lambda = 0$) the two functions coincide. Thus, instead of (A1.2) we get uniquely :

$$I(\lambda < 0) \simeq + \int_0^{R^2} d\alpha \cos \left(\alpha \frac{m_1^2 - m_2^2 - m_3^2}{2} \right) \left[\frac{1 - e^{-\alpha |\lambda|^{1/2}}}{\alpha |\lambda|^{1/2}} \right] \quad (A1.8)$$

One can interpret this result by saying that whenever a complex momentum flows through a quark line the correct extension of the fat δ function in this situation is given by (a, b real) :

$$\delta_{R^2}(a+ib) = \frac{1}{2\pi} \int_0^{R^2} d\alpha e^{-|b|\alpha} \cos(\alpha a). \quad (A1.9)$$

APPENDIX 2

In this Appendix we shall carry out an approximate calculation of the charge form factor of an arbitrary bag of mass M and angular momentum ℓ .

We begin with the contribution to the discontinuity $\text{disc}_q^2 \langle M\ell | J_\mu(0) | M\ell \rangle_i$ of the i th vector meson. Using the wave function given by (1.13) and the rules of Section 3

$$\begin{aligned} \text{disc}_q^2 \frac{1}{2\ell+1} \sum_m \langle p, \ell m | J_\mu(0) | p', \ell m \rangle_i &= \text{Tr} \left(\frac{\tau^a}{2} [\tau^a, \tau^b] \right) (2\pi) \delta(q^2 - m_i^2) \\ &\mu^b \left(\frac{2\pi}{R^2} \right)^4 \frac{1}{(4\pi)^2} \frac{1}{Z_i Z_\ell(M)} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_i^2} \right) \int dk (p' + 2k)_\mu \\ &\delta_{R^2}(pk) \delta_{R^2}\left(\frac{p^2}{4} + k^2\right) \delta_{R^2}\left(p'(k - \frac{q}{2})\right) \delta_{R^2}\left(\frac{p'^2}{4} + (k - \frac{q}{2})^2\right) \\ &\delta_{R^2}\left(q(k + \frac{p'}{2})\right) \delta_{R^2}\left(\frac{q^2}{4} + (k + \frac{p'}{2})^2\right) P_\ell(\cos \hat{\theta}) , \end{aligned} \quad (\text{A2.1})$$

where $\cos \hat{\theta}$ is the angle between the vector $\vec{k}$ and the vector defined by (1.14) relative to the wf $\psi_\ell(p', k - (q/2))$. Notice that we have chosen to evaluate (A2.1) in the rest frame of the state $|p, \ell m\rangle$. Proceeding in the usual way we obtain :

$$\begin{aligned} \text{disc}_q^2 \frac{1}{2\ell+1} \sum_m \langle p, \ell m | J_\mu(0) | p', \ell m \rangle_i &\simeq (2\pi) \delta(q^2 - m_i^2) \frac{\mu^b}{(4\pi)^2} \left(\frac{2\pi}{R^2} \right)^4 \frac{1}{Z_i Z_\ell(M)} \\ &(p+p')_\mu \int \frac{d\Omega_k}{4} \delta_{R^2}^4 \left(\frac{m_i^2}{4} + \frac{\lambda^2}{4} \cos \theta_{pk} \right) P_\ell(\cos(\theta - \theta')) \end{aligned} \quad (\text{A2.2})$$

where $\lambda(M^2, M^2, m_i^2) = m_i^2(m_i^2 - 4M^2)$ is the triangular function determining the magnitude of the centre-of-mass momentum $\vec{p}$ through the relation

$$|\vec{p}| = \frac{1}{2M} \lambda^{1/2}(M^2, M^2, m_i^2) , \quad (\text{A2.3})$$

and $\cos\theta_{pk}$ is the angle between $\vec{p}$ and the vector $\vec{k}$. By using (1.14) we calculate

$$\cos\theta' = \frac{2M^2 - m_i^2}{2M^2} \frac{\frac{|\vec{p}|}{2} + \frac{M}{2} \cos\theta_{pk} - \frac{|\vec{p}| m_i^2}{4M^2}}{\left[\frac{M^2 m_i^2}{4} + \frac{|\vec{p}| M}{2} \cos\theta_{pk} + \frac{1}{M^2} \left(\frac{m_i^2}{4} - \frac{|\vec{p}| M}{2} \cos\theta_{pk} \right)^2 \right]^{1/2}} \quad (A2.4)$$

In view of the fast decrease of the integrand of (A2.2) with the mass of the vector meson m_i , we need only consider the contribution of the first vector meson (i.e., $i=1$), which we call ρ with mass m_ρ . For $M^2 > m_\rho^2/4$, λ is negative and according to (A2.3), $|\vec{p}|$ becomes purely imaginary. Upon substituting (2.7) for the δ_{R^2} function appearing in (A2.2), we notice that the integrand is strongly peaked for $\cos\theta_{pk}=0$, and inserting this value in (A2.4) we obtain approximately :

$$\frac{1}{2\ell+1} \sum_m \text{disc}_q = \langle p\ell m \beta | J_\mu^d(0) | p'\ell m \gamma \rangle \simeq (2\pi) \delta(q^2 - m_\rho^2) \quad (A2.5)$$

$$(p+p')_\mu \text{Tr} \left(\frac{\epsilon^\mu}{2} [\tau^\beta, \tau^\gamma] \right) C_\ell^{(p)}(M)$$

where

$$C_\ell^{(p)}(M) \simeq \frac{\mu^6}{Z_p Z_\ell(M)} \left(\frac{2\pi}{R^2} \right)^4 \frac{1}{8\pi} \int_{-1}^{+1} dz \delta_{R^2} \left(-\frac{m_\rho^2}{4} + i|\lambda|^{1/2} z \right) P_\ell(\sin\theta'(z_0)) \quad (A2.6)$$

which for M large becomes

$$C_\ell^{(p)}(M) \xrightarrow{M \text{ large}} \frac{4\mu^6}{R^2 Z_p Z_\ell(M)} \frac{1}{m_\rho M} P_\ell \left(\sqrt{1 + \frac{m_\rho^2}{M^2}} \right) \quad (A2.7)$$

APPENDIX 3

Here we shall carry out the calculation of the diagram appearing in Fig. 16. Following the steps which lead to (2.5), the imaginary part of the scattering amplitude is given by :

$$\begin{aligned} \text{Im } A \simeq & \pi^4 \mu^2 \left(\frac{z}{Z_0}\right)^3 \left(\frac{2\pi}{R^2}\right)^6 \int_{-1}^{+1} dz \int_0^{2\pi} d\varphi \int_{-1}^{+1} dz' \int_0^{2\pi} d\varphi' \times \\ & \times \sum_{lmn}^{l_0} \delta_{R^2}^4\left(\frac{m_{ne}^2}{4}(1-z)\right) \delta_{R^2}^4\left(\frac{m_{ne}^2}{4}(1-z')\right) \frac{Y_l^m(\theta, \varphi) Y_l^{-m}(\theta', \varphi') (mP)_{ne}}{(\varepsilon - m_{ne}^2)^2 + (mP)_{ne}^2} \end{aligned} \quad (\text{A3.1})$$

We use the well-known addition theorem to perform the sum over m :

$$\sum_{m=-l}^l Y_l^m(\theta, \varphi) Y_l^{-m}(\theta', \varphi') = \frac{2l+1}{4\pi} P_l(\cos \hat{\theta}) \quad (\text{A3.2})$$

where

$$\cos \hat{\theta} = \cos \theta \cos \theta' + \sin \theta \sin \theta' \cos(\varphi - \varphi').$$

For large s (A3.1) becomes

$$\begin{aligned} \text{Im } A \simeq & \pi^4 \mu^2 \left(\frac{z}{Z_0}\right)^3 \left(\frac{2\pi}{R^2}\right)^6 \frac{R^2}{2} \int_{-1}^{+1} dz \int_0^{2\pi} d\varphi \int_{-1}^{+1} dz' \int_0^{2\pi} d\varphi' \\ & \times \frac{s R_\perp^2}{2} \frac{J_1(R_\perp \sqrt{-t})}{R_\perp \sqrt{-t}} \delta_{R^2}^4\left(\frac{s}{4}(1-z)\right) \delta_{R^2}^4\left(\frac{s}{4}(1-z')\right), \end{aligned} \quad (\text{A3.3})$$

where we have used, for small t , the asymptotic formula :

$$\sum_{l=0}^{l_0} (2l+1) P_l(\cos \theta) = 2l_0^2 \frac{J_1(R_\perp \sqrt{-t})}{R_\perp \sqrt{-t}}. \quad (\text{A3.4})$$

The steep peaking of the $\delta_{R^2}^4$ function allows us to integrate quite easily over z' , φ' and φ , and obtain :

$$\text{Im} Q \simeq \pi^4 \left(\frac{4\mu^4}{Z_0} \right)^3 R^2 R_1^2 \left(\frac{2\pi}{R^2} \right)^3 (2\pi)^2 \int_{-1}^{+1} dz \frac{J_1 \left(R_1 \sqrt{\frac{z}{2}(1-z)} \right)}{R_1 \sqrt{\frac{z}{2}(1-z)}} \delta_{R^2}^4 \left(\frac{z}{2}(1-z) \right) \quad (\text{A3.5})$$

The integral in (3.5) can be easily carried out by use of the following approximations (valid for small z)

$$J_1 \left(\frac{z}{2} \right) \simeq \frac{1}{2} e^{-\frac{z}{2}} \quad (\text{A3.6})$$

and

$$\delta_{R^2}^4(z) \simeq \left(\frac{R^2}{\pi} \right)^4 e^{-\frac{2R^2}{\pi}|z|} \quad (\text{A3.7})$$

We obtain finally :

$$\text{Im} Q \simeq (2\pi)^5 \left(\frac{4\mu^2}{Z_0} \right)^3 (4\pi^2 R^2)^2 \frac{4\mu^2}{s} \frac{8\pi^2 R_1^2}{\pi R_1^2 + 8R^2} \quad (\text{A3.8})$$

APPENDIX 4

We want to calculate the high energy behaviour of α_t given by (4.2) according to the prescription (4.4) :

$$\alpha_t = \sum_{nl} g_{nl}^2 \frac{(2l+1) (-1)^l}{t - m_{nl}^2} P_l \left(1 + \frac{2s}{m_{nl}^2} \right), \quad (A4.1)$$

where the coupling g_{nl} is easily calculated from (2.5) ; one gets :

$$\begin{aligned} g_{nl}^2 &\simeq \frac{c}{m_{nl}^2} \quad \text{for } l \leq l_0 = \frac{m_{nl}}{2} R_1 \\ g_{nl}^2 &\simeq 0 \quad \text{for } l > l_0. \end{aligned} \quad (A4.2)$$

Keeping l fixed, the sum over n can be converted approximately into an integral over m^2 , according to

$$\sum_n \rightarrow \frac{R^2}{2\pi} \int_{4l^2/R_1^2}^{\infty} \frac{dm^2}{4l^2/R_1^2} \quad (A4.3)$$

where the approximate spectrum equation

$$m_{nl}^2 \simeq \frac{\pi}{R^2} (2n+l)$$

has been used.

Thus we write

$$\alpha_t \simeq c^2 \sum_{l=0}^{\infty} (2l+1) \frac{R^2}{2\pi} (-1)^l \int_{\frac{4l^2}{R_1^2}}^{\infty} \frac{dm^2}{m^4} \frac{1}{m^2 - t} P_l \left(1 + \frac{2s}{m^2} \right) \quad (A4.4)$$

which, defining $x = \sqrt{|s|/m}$, becomes

$$\alpha_t \simeq \frac{R^2 c^2}{2\pi} \frac{1}{|s|^2} \sum_{l=0}^{l_0} (2l+1) (-1)^l \int_0^{\frac{\sqrt{|s|} R_1}{2l}} \frac{dx}{1 - \frac{t}{|s|} x^2} \frac{x^3}{x^4} P_l \left(1 \pm 2x^2 \right) \quad (A4.5)$$

where in the P_l argument the upper sign stands for positive s , and the lower for negative s .

To perform the ℓ sum more easily we introduce in the m^2 integration the smooth cut-off $e^{-2\ell x(sR_\perp^2)^{-2}}$ rather than the sharp $\theta((\sqrt{s}R_\perp/2\ell)-x)$; and we write

$$a_t \approx \frac{R_c^2}{(2\pi)|s|^2} \sum_{\ell=0}^{\infty} (2\ell+1) (-1)^\ell \int_0^{\frac{\mu}{1-\frac{t}{s}x^2}} dx \frac{x^3}{1-\frac{t}{s}x^2} e^{-x \frac{2\ell}{\sqrt{s}R_\perp}} P_\ell(1 \pm 2x^2), \quad (A4.6)$$

where μ is the lowest mass in the hadron spectrum. To perform the ℓ sum we would like to use the identity

$$\sum_{\ell=0}^{\infty} (2\ell+1) (-1)^\ell \zeta^\ell P_\ell(\xi) = \frac{1-\xi^2}{[1+\xi^2+2\xi\zeta]^{\frac{3}{2}}} \quad (A4.7)$$

which is valid, however, only for $|\xi| \leq 1$. In view of the fact that a_t is real and analytic in s due to the convergence of the series, for $|\xi| > 1$ we extrapolate (A4.9) by taking the average

$$\bar{\Sigma}(\zeta, \xi) = \frac{1}{2} \left(\Sigma(\zeta, \xi+i\epsilon) + \Sigma(\zeta, \xi-i\epsilon) \right) \quad (A4.8)$$

On setting

$$\zeta = e^{-\frac{2x}{R_\perp \sqrt{|s|}}}, \quad \xi = (1 \pm 2x^2),$$

and inserting (A4.8) into (A4.6), we get

$$a_t \xrightarrow{s \rightarrow +\infty} \frac{1}{|s|^{\frac{3}{2}}} \quad (A4.9)$$

$$a_t \xrightarrow{s \rightarrow -\infty} \frac{1}{|s|^2} \quad (A4.10)$$

APPENDIX 5

By means of the identity (6.4) we can dispose of one of the two loops and reduce our diagram to a simple four-bag overlap :

$$W_a(s, m_1^2, m_2^2) \simeq \mu^{12} \left(\frac{2}{Z_0}\right)^3 \left(\frac{2\pi}{R^2}\right)^6 \pi R^2 (2\pi)^4 \frac{R_1^2}{R_1^2 + \frac{g}{\pi} R^2 \left(1 - \frac{m_1^2 + m_2^2}{s}\right)} \int d^4k \delta_{R^2}^4 \left(q_1 k + \frac{s - m_1^2 - \mu_0^2}{4}\right) \delta_{R^2}^4 \left(q_2 k + \frac{s - m_2^2 - \mu_0^2}{4}\right) \quad (A5.1)$$

where the factor $R_1^2/\dots$ has the same origin as in (A3.7), and $\mu_0^2 = p_1^2 = p_2^2$ is the mass of the target. In the centre-of-mass frame the momenta of the targets p_1 and p_2 such that $(p_1 - p_2)^2 = t = 0$ are given by :

$$p_1 = \left(\frac{s + \mu_0^2 - m_1^2}{2\sqrt{s}}, 0, 0, |\vec{p}_1| \right) \quad (A5.2)$$

$$|\vec{p}_1| \simeq \frac{1}{2\sqrt{s}} \left[(s - m_1^2 - \mu_0^2)^2 - 4m_1^2 \mu_0^2 \right]^{1/2}$$

$$p_2 = \left(\frac{s + \mu_0^2 - m_2^2}{2\sqrt{s}}, |\vec{p}_2| \sin\theta, 0, |\vec{p}_2| \cos\theta \right) \quad (A5.3)$$

$$|\vec{p}_2| = \frac{1}{2\sqrt{s}} \left[(s - m_2^2 - \mu_0^2)^2 - 4\mu_0^2 m_2^2 \right]^{1/2}$$

where

$$\cos\theta \simeq 1 + \mu_0^2 s \frac{(m_1^2 - m_2^2)^2}{(s - m_1^2)^2 (s - m_2^2)^2} ; \quad (A5.4)$$

notice that for $m_1 \neq m_2$ $\cos\theta > 1$; i.e., $t=0$ is in the unphysical region.

Integrating over k_0 and $|\vec{k}|$ by means of the δ_{R^2} functions in (A5.1), we obtain

$$W_a(s; m_1^2, m_2^2) \simeq \mu^{12} \left(\frac{2}{Z_0} \right)^3 \left(\frac{2\pi}{R^2} \right)^6 (2\pi)^4 \frac{\pi R^2}{4} \frac{R_\perp^2}{R_\perp^2 + \frac{8}{\pi} R^2 \left(1 - \frac{m_1^2 + m_2^2}{2s} \right)}$$

$$\int_{-1}^{+1} dz \int_0^{2\pi} d\varphi \delta_{R^2}^4 \left[\frac{s - m_1^2 - \mu_0^2}{4} + \frac{\sqrt{s}}{2} |\vec{p}_1| z \right] \delta_{R^2}^4 \left[\frac{s - m_2^2 - \mu_0^2}{4} + \frac{\sqrt{s}}{2} |\vec{p}_2| (\cos\theta + \sin\theta \sqrt{1-z^2} \cos\varphi) \right] \quad (A5.5)$$

For $|s - m_1^2| \gtrsim 1/R^2$ the integral can be approximately evaluated as :

$$W_a(s; m_1^2, m_2^2) \simeq \mu^{12} \left(\frac{2}{Z_0} \right)^3 \left(\frac{2\pi}{R^2} \right)^3 (2\pi)^4 \frac{\pi^2 R^2}{|s - m_1^2|}$$

$$\frac{R_\perp^2}{R_\perp^2 + \frac{8R^2}{\pi} \left(1 - \frac{m_1^2 + m_2^2}{s} \right)} \delta_{R^2}^4 \left(\frac{\mu_0^2 s}{4} \frac{(m_1^2 - m_2^2)}{(s - m_1^2)^2 (s - m_2^2)^2} \right) \quad (A5.6)$$

APPENDIX 6

The diagram in Fig. 22b is given by the expression :

$$W_b(s, Q^2) = Z_0^2 \left(\frac{2}{Z_0}\right)^2 \left(\frac{\pi}{R^2}\right)^4 \frac{R^2}{2} \frac{1}{(4\pi)^2} \sum_{l,m}^{R_1 \sqrt{s}} \int_{-1}^{+1} dz \int_0^{2\pi} d\varphi Y_l^m(\theta, \varphi) \\ \times \delta_{R^2}^4\left(\frac{s+Q^2}{4}(1-z)\right) \int_{-1}^{+1} dz' \int_0^{2\pi} d\varphi' Y_l^{-m}(\theta', \varphi') \delta_{R^2}^4\left(\frac{s+Q^2}{4}(1-z')\right). \quad (A6.1)$$

Using the addition theorem for spherical harmonics we have :

$$W_b(s, Q^2) = \frac{1}{(4\pi)^2} \frac{R^2}{8} \left(\frac{2\pi}{R^2}\right)^4 \sum_{l=0}^{R_1 \sqrt{s}} \int_0^{2\pi} d\varphi \int_0^{2\pi} d\varphi' \int_{-1}^{+1} dz \int_{-1}^{+1} dz' (2l+1) \\ P_l(\cos \hat{\theta}) \delta_{R^2}^4\left(\frac{s+Q^2}{4}(1-z)\right) \delta_{R^2}^4\left(\frac{s+Q^2}{4}(1-z')\right), \quad (A6.2)$$

where

$$\cos \hat{\theta} = zz' + (1-z^2)^{1/2} (1-z'^2)^{1/2} \cos(\varphi - \varphi').$$

We now sum over all partial waves up to $R_1(\sqrt{s}/2)$ and using (1.30), we get :

$$W_b(s, Q^2) = \frac{R^2}{8} \frac{1}{(4\pi)^2} \left(\frac{2\pi}{R^2}\right)^4 \frac{R_1^2 s}{4} \int_0^{2\pi} d\varphi \int_0^{2\pi} d\varphi' \int_{-1}^{+1} dz \int_{-1}^{+1} dz' \\ \times \frac{2 J_1(R_1 \sqrt{\frac{s}{2}(1-\cos \hat{\theta})})}{R_1 \sqrt{\frac{s}{2}(1-\cos \hat{\theta})}} \delta_{R^2}^4\left(\frac{s+Q^2}{4}(1-z)\right) \delta_{R^2}^4\left(\frac{s+Q^2}{4}(1-z')\right). \quad (A6.3)$$

By using the approximations (A3.5) and (A3.6), and after integrating over z' , we obtain

$$W_b(s, Q^2) = \frac{\pi}{4} \frac{R_1^2 s}{(s+Q^2)} \int_{-1}^{+1} dz e^{\frac{-R_1^2}{16} s (1-z)} e^{-\frac{R^2}{2\pi} (s+Q^2) (1-z)} \quad (A6.4)$$

and making the last trivial integration we get :

$$W_b(s, Q^2) \simeq \frac{12}{s+Q^2} \frac{R_1^2 s}{R_1^2 s + \frac{8}{\pi} R^2 (s+Q^2)} \quad (A6.5)$$

This fills the gap left in (A5.1), by showing the necessity of introducing there the factor

$$\frac{R_1^2 s}{R_1^2 s + \frac{8}{\pi} R^2 (s+Q^2)}$$

APPENDIX 7

Using the identity (6.4) we can reduce the calculation of $W_p(s, Q^2)$ to the evaluation of the diagram in Fig. 26.

Figure 26 The diagram equivalent to Fig. 24

We shall proceed as follows : first compute $(d^4 W_p / dq^4)(s; m_1^2, m_2^2, q)$, then disperse in m_1^2, m_2^2 , by keeping $s, p_1 q = v_2, q_1 q = v_1$ and q^2 fixed and finally integrate over $d^4 q$ to get $W_p(s, Q^2)$.

i) we write :

$$\begin{aligned}
 \frac{dW_p(s, m_1^2, m_2^2, q)}{dq} &\simeq \frac{\pi^3 \mu^{16}}{4R^2 Z_0} \left(\frac{Z}{Z_0} \right)^3 \left(\frac{2T}{R^2} \right)^6 \frac{R^2 R_1^2}{8} q^2 \\
 &\times \int d^4 k \delta_{R^2}(k^2) \delta_{R^2}(s + q^2 - 2v_1 - 2v_2 + 2qk) \delta_{R^2}^2(-2p_1 k) \\
 &\times \delta_{R^2}^2(-2p_2 k) \delta_{R^2}^2(q^2 - 2v_2 + 2qk) \delta_{R^2}^2(q^2 - 2q(p_2 - k)) .
 \end{aligned} \tag{A7.1}$$

Defining p_1 and p_2 as in (A5.2) and (A5.3), we notice the kinematical relations :

$$q^2 + q_\perp^2 = \frac{1}{s} \left(2v_1 v_2 \frac{\sqrt{s}}{|\vec{p}_1|} - v_2^2 \frac{m_1^2}{|\vec{p}_1|^2} \right) \quad (A7.2)$$

$$q_0 = \frac{1}{\sqrt{s}} (v_1 + v_2) \quad (A7.3)$$

$$q_3 = \frac{1}{\sqrt{s}} \left(\frac{s m_1^2}{2 p_1 \sqrt{s}} v_2 - v_1 \right). \quad (A7.4)$$

We integrate over $k_\perp$, and k_3 by using $\delta_{R^2}(k^2)$ and $\delta_{R^2}^2(-2p_1 k)$ and obtain

$$\begin{aligned} \frac{dW_P}{dq} &\simeq \frac{\mu^6 \pi^3}{4R^2 Z_0} \left(\frac{2}{Z_0} \right)^3 \left(\frac{2\pi}{R^2} \right)^6 \frac{R_\perp^2 R^2}{8} q^2 \frac{R^2}{\pi} \frac{1}{2|\vec{p}_1|} \cdot \\ &\times \int dk_0 \delta_{R^2}^2 \left(\frac{k_0}{\sqrt{s}} \mu_0^2 \frac{s(m_1^2 - m_2^2)^2}{(s - m_1^2)^2 (s - m_2^2)} \right) \delta_{R^2}^2 \left(\frac{(m_1^2 - m_2^2) 2v_2}{(s - m_1^2)} \right) \\ &\times \delta_{R^2}^2 (sq^2 - 2v_1 - 2v_2 - 2k_0(\sqrt{s} - q_0 - q_3)) \delta_{R^2}^2 (2v_1 - s + 2k_0 \sqrt{s}), \end{aligned} \quad (A7.5)$$

and integrating over k_0 by means of $\delta_{R^2}[sq^2 - 2v_1 - 2v_2 - 2k_0(\sqrt{s} - q_0 - q_3)]$ we obtain :

$$\begin{aligned} \frac{dW_P}{dq} &\simeq \frac{\pi^3 \mu^6}{4R^2 Z_0} \left(\frac{2}{Z_0} \right)^3 \left(\frac{2\pi}{R^2} \right)^6 \frac{R_\perp^2 R^2}{16} q^2 \frac{R^2}{s - m_1^2 - 2v_2} \\ &\times \delta_{R^2}^2 \left(\frac{s + q^2 - 2v_1 - 2v_2}{s - m_1^2 - 2v_2} \frac{\mu_0^2 (m_1^2 - m_2^2)^2}{2(s - m_1^2)(s - m_2^2)} \right) \delta_{R^2}^2 \left(\frac{2v_2}{s - m_1^2} (m_1^2 - m_2^2) \right) \\ &\times \delta_{R^2}^2 \left(\frac{(s - m_1^2) q^2 + 2v_2 m_1^2 - 4v_1 v_2}{s - m_1^2 - 2v_2} \right). \end{aligned} \quad (A7.6)$$

ii) We now disperse over m_1^2 and m_2^2 and multiply by the current-hadron couplings (6.10), and obtain :

$$\begin{aligned} \frac{dW}{dq} P(q; s, Q^2) &= \frac{\mu^{16}}{32 Z_0} \left(\frac{2}{Z_0} \right)^3 \left(\frac{2\pi}{R^2} \right)^6 \frac{R^4 R_1^2}{16} q^2 \\ &\times \frac{1}{s} \int_0^1 dx_1 \int_0^1 dx_2 \frac{1}{(1-x_1 - \frac{2v_2}{s})} \frac{1}{(Q_s^2 + x_1)(Q_s^2 + x_2)} \delta_{R^2}^2 \left(2v_2 \frac{x_1 - x_2}{1-x_1} \right) \\ &\times \delta_{R^2}^2 \left[\frac{1+q^2/s - \frac{2v_2}{s} - \frac{2v_2}{s}}{1-x_1 - \frac{2v_2}{s}} \frac{\mu^2 (x_1 - x_2)^2}{2(1-x_1)(1-x_2)} \right] \delta_{R^2}^2 \left(\frac{q^2 (Q_s^2 + x_1) + q^2 (1+Q_s^2/s) + 2v_2 \frac{2v_2/s}{s+Q^2}}{1-x_1 - \frac{2v_2}{s}} \right). \end{aligned} \quad (A7.7)$$

iii) We finally integrate over $d^4 q$ to obtain $W_P(s, Q^2)$. We have, neglecting Q^2/s

$$\begin{aligned} W_P(s, Q^2) &\rightarrow \frac{\mu^{16}}{Z_0^4} \frac{\pi}{2} \frac{R^4 R_1^2}{s} \int \frac{dq_3}{2q_0} \int d^2 q \frac{q^2}{2v_2} \left(\frac{s}{Q^2} \right) e^{-\frac{R^2}{\pi} (q^2 \frac{Q^2}{s})} \\ &= \frac{\mu^{16}}{4 Z_0^4} \frac{\pi^2}{s} \left(\frac{s}{Q^2} \right)^2 R_1^2 R^4 \end{aligned} \quad (A7.8)$$

Thus we have obtained for $W_P(s, Q^2)$ a scaling behaviour

$$W_P(s, Q^2) = \frac{1}{2\nu} F_P(x) \quad (A7.9)$$

with

$$F_P(x) \xrightarrow{x \rightarrow 0} \frac{1}{x^2} \log^2 x . \quad (A7.10)$$

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