Ref.TH.2198-CERN

VECTORLIKE INTERACTIONS OF LEPTONS AND QUARKS *)

H. Fritzsch
CERN - Geneva

Invited talk given at the
International Neutrino Conference
Aachen, Germany

8 - 12 June 1976

*) To be printed in the Proceedings of the
Conference.

Ref.TH.2198-CERN
22 July 1976

1. - INTRODUCTION

During the last few years a remarkable simple picture of the world of particles and their interactions has emerged. It seems now that matter is built up from a set of basic spin $\frac{1}{2}$ objects (leptons, quarks), and that the interactions among those are mediated by vector gauge fields. General prospects exist to understand all interactions as different manifestations of an underlying grand unified theory. Such a theory must in particular provide an answer to the question how many basic fermions exist in nature. I find it remarkable that at present this question is not only addressed by theoreticians, but after the discovery of new effects, especially in e^+e^- annihilation and in neutrino scattering, also by experimentalists in their search for new leptons and new hadronic quantum numbers.

This talk is based on the following assumptions.

- (1) - The strong interactions are described by quantum chromodynamics ¹⁾ ("QCD"). The quarks come in different flavours (u,d,s,...) and in three colours ("red, green, blue") :

$$\underbrace{\begin{pmatrix} u_r & u_g & u_b \\ d_r & d_g & d_b \\ s_r & s_g & s_b \\ \vdots & \vdots & \vdots \end{pmatrix}}_{SU_3^{color}} \left. \vphantom{\begin{pmatrix} u_r & u_g & u_b \\ d_r & d_g & d_b \\ s_r & s_g & s_b \\ \vdots & \vdots & \vdots \end{pmatrix}} \right\} flavor$$

The strong interactions are generated by the exchange of eight massless vector gluons coupled gauge invariantly to the colour quantum numbers. All coloured objects are assumed to be permanently bound by the colour forces. Only colour singlet states exist as physical states, in agreement with the observed hadron spectrum ²⁾.

- (2) - The basic fermions are either colour singlets (leptons) or colour triplets (quarks). There may exist other types of fermions (colour octets, etc.), provided they are sufficiently massive in order not to leave traces in the world accessible to observation thus far.
- (3) - Until today three different quark flavours (u,d,s) have been observed in form of hadronic degrees of freedom, while the existence of at least one more flavour seems certain. We assume that the charm flavour

exists ³⁾, with $m_c \simeq 2$ GeV, and that the observed new resonances in the e^+e^- channel are $\bar{c}c$ states. The electromagnetic properties of hadrons can be understood by assigning the electric charges $2/3, -1/3, -1/3, 2/3$ to the quark flavours u, d, s, c , respectively.

(4) - The observed weak interactions are mediated by the exchange of massive gauge bosons, with masses of the order of 50 GeV. The observed universality of the weak interactions is interpreted as due to the existence of a group SU_2^W ("weak isospin"). In the simplest scheme the left-handed (L) fermions transform as doublets, and the right-handed (R) ones as singlets :

$$\begin{pmatrix} u & c \\ d' & s' \end{pmatrix}_L, u_R, c_R, d'_R, s_R; \begin{pmatrix} \nu_e & \nu_\mu \\ e^- & \mu^- \end{pmatrix}_L, e_R, \mu_R, \quad (1.1)$$

$[d' = d \cos\theta + s \sin\theta, s' = -d \sin\theta + s \cos\theta$ (θ : Cabibbo angle)]. The smallest gauge group describing the weak and electromagnetic interactions is the group ⁴⁾ $SU_2 \times U_1$.

It is well known that the $SU_2 \times U_1$ theory of the weak and electromagnetic interactions cannot be the final theory of those interactions, since no reason for the quantization of the electric charge is given within that theory. One possible way to obtain the charge quantization is to regard the $SU_2 \times U_1$ scheme as a subsystem of a larger system which is based on a simple gauge group ⁵⁾. However, realistic theories based on a simple gauge group cannot be constructed if the electromagnetic and strong interactions are vector interactions while the weak interactions are of character V-A. Thus one is invited to speculate that the weak interactions are basically a vector theory, just like the strong and electromagnetic ones, and the observed preference of the weak interactions for V-A currents in only a low energy effect, due to parity violating terms in the fermion mass matrix. Such a theory is called vectorlike. Vectorlike models have been studied recently by many people ⁶⁾⁻¹²⁾. In this talk I shall outline the general features of vectorlike theories, and study a few special models, in particular with respect to the new, interesting experimental facts reported at this Conference.

2. - VECTORLIKE WEAK INTERACTIONS

A vectorlike theory is a theory in which the parity violation is not intrinsic, but due to the presence of parity violating terms in the fermion mass matrix. In the limit where the fermion masses are set to zero the theory is a pure vector theory, and the gauge currents are vectorial. If the fermion mass matrix is turned on, it acquires both scalar and pseudo-scalar terms, and parity is violated. The unitary transformation in the fermion space which diagonalizes the fermion mass matrix and eliminates its pseudoscalar part (in the absence of CP violation) introduces apparent axial vector currents.

In the conventional gauge theory framework the fermion masses are generated by the coupling of the fermion fields to scalar fields which develop non-zero vacuum expectation values ("spontaneous symmetry breaking" ¹³⁾).

In a vectorlike theory the violation of parity is part of the spontaneous symmetry breaking, in contrast to, for example, the Salam-Weinberg model ⁴⁾ where the parity violation is an intrinsic property of the field equations.

Realistic vectorlike theories can only be constructed if there exist new leptons, and more than four quark flavours. A simple vectorlike scheme is, for example, given by the following lepton scheme :

$$\begin{pmatrix} \nu_e & N \\ e^- & E^- \end{pmatrix}_L \quad \begin{pmatrix} N & \nu_e \\ e^- & E^- \end{pmatrix}_R, \quad (2.1)$$

and an analogous scheme involving the muon. Here N is a new massive neutral lepton ($m_N > m_K$), and E^- a new charged lepton.

Introducing the fields $N_1 = (\nu_{eL}, N_R)$, $N_2 = (N_L, \nu_{eR})$, we can rewrite this scheme in a vector form :

$$\begin{pmatrix} N_1 & N_2 \\ e^- & E^- \end{pmatrix}_L \quad \begin{pmatrix} N_1 & N_2 \\ e^- & E^- \end{pmatrix}_R \quad (2.2)$$

while the mass term of the neutral fermions (neglecting a possible ν_e mass) can be rewritten as

$$\begin{aligned}
 m_N \bar{N} N &= m_N (N_L^* N_R + N_R^* N_L) \\
 &= m_N \left(\bar{N}_1 \frac{1-\gamma_5}{2} N_1 + \bar{N}_1 \frac{1+\gamma_5}{2} N_2 \right) \\
 &= \frac{1}{2} m_N (\bar{N}_1 N_1 + \bar{N}_1 N_2 + \bar{N}_2 \gamma_5 N_1 - \bar{N}_2 \gamma_5 N_2)
 \end{aligned}
 \tag{2.3}$$

The scheme (2.1) reproduces the correct V-A pattern of the observed weak interactions, provided $m_N > M_K$. It shows the following general features of vectorlike theories :

- A : Parity is violated only by pseudoscalar terms in the fermion mass matrix.
- B : The numbers of left-handed currents and right-handed currents are equal. This implies in particular the absence of anomalies ^{14),15)}.

It is worth while noting that the possibility to break parity spontaneously by pseudoscalar fields has been suggested first by Pauli in a letter to Weisskopf written shortly after the discovery of parity violation ¹⁶⁾.

One can easily see that a vectorlike theory of the hadronic weak currents based on SU_2^{weak} can only be constructed if one introduces more than four quark flavours. Let us try to do it with just four. The left-handed currents are $\begin{pmatrix} u & c \\ d' & s' \end{pmatrix}_L$. In order to construct a vectorlike scheme one has to introduce two right-handed currents. There is only one possibility to do so, namely $\begin{pmatrix} u & c \\ d'' & s'' \end{pmatrix}_R$ ($d'' = d_R \cos \theta'' + s_R \sin \theta''$, $s'' = 1$; θ'' : right-handed analogue of the Cabibbo angle). The V-A character of the weak current entering in β decay requires $\theta'' \cong 90^\circ$, which gives $\begin{pmatrix} u & c \\ s & -d \end{pmatrix}_R$. This pattern is correct for β decay, however, wrong for Λ decay (both in strength and chirality). Thus we conclude :

A vectorlike theory of the hadronic weak interactions can only be constructed if there exist more than four quark flavours and more than four leptons.

3. - VECTORLIKE MODELS BASED ON SU_2^W DOUBLETS ONLY

A radical possibility to construct vectorlike schemes is to place all leptons and quarks in SU_2^{weak} doublets. The smallest vectorlike scheme of this sort is one based on six lepton flavours and six quark flavours ⁸⁾⁻¹⁰⁾ :

$$\begin{pmatrix} u & t & c \\ d' & b & s' \end{pmatrix}_L \quad \begin{pmatrix} u & t & c \\ b & d'' & s'' \end{pmatrix}_R$$

$$\begin{pmatrix} \nu_e & N_E & \nu_\tau \\ e^- & E^- & \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_e & N_E & N_\tau \\ E^- & e^- & \mu^- \end{pmatrix}_R.$$

(3.1)

Here t and b are new flavours of quarks ("top" and "bottom") with charges $2/3$ and $-1/3$, respectively ¹⁷⁾. The left-handed quarks are rotated by the Cabibbo angle θ_c into $d' = d \cos\theta_c + s \sin\theta_c$, and $s' = -d \sin\theta_c + s \cos\theta_c$. The observed universality of the weak interactions requires that there is no (or only little) mixing of d' with b . There may, however, be substantial mixing between b and s' , i.e., the fields b and s' in the scheme above may be replaced by the rotated fields $\tilde{b} = b \cos\tilde{\theta} + s' \sin\tilde{\theta}$ and $\tilde{s}' = -b \sin\tilde{\theta} + s' \cos\tilde{\theta}$. For simplicity we have not displayed this rotation in the scheme above.

Among the right-handed quarks, the observed V-A structure of the weak currents connecting the u quark with the d' combination forbids any appreciable coupling of u_R to either d_R or s_R . Consequently its partner must be pure or nearly pure b . The linear combinations $d'' = d \cos\theta'' + s \sin\theta''$ and $s'' = -d \sin\theta'' + s \cos\theta''$ are, for the moment, left arbitrary. Below we shall argue in favour of $\theta'' \approx 0$.

The scheme contains, in addition to the conventional leptons, one new charged lepton M^- , one new massive neutral lepton N_M and the right-handed partner $(\nu_e)_R$ of the electron neutrino as well as the massive Majorana partner of the muon neutrino ν_μ . We assume that the neutral lepton N_M is a Fermi-Dirac particle (its mass term conserves lepton number). The

fields $(\nu_e)_L$, $(\nu_e)_R$ and $(\nu_\mu)_L$ are assumed to be massless in the absence of the weak interactions. The violation of lepton number conservation occurs only via the N_μ mass term. Thus neutrino-less double β decay does not occur in lowest order of the weak interaction ¹⁸⁾. However, there exists the decay $K^- \rightarrow \pi^+ + \mu^- + \mu^-$, which occurs in second order of the weak interactions [This decay was estimated in Ref. 19]. The expected branching ratio $K^- \rightarrow \pi^+ + \mu^- + \mu^- / K^- \rightarrow \text{all}$ is of the order of 10^{-14} for $M_{N_\mu} \sim$ several GeV, i.e., more than ten orders of magnitude smaller than the experimental limit. It remains to be seen, if the accuracy of the experiments can be improved such that an experimental test of the lepton number violation of the kind we are discussing becomes feasible.

In the scheme (3.1) all lepton and quark fields take part in the weak interaction. Consequently the neutral current is a vector current :

$$j_r^{\text{neutral}} = (-\frac{1}{2} + z) \bar{e} \gamma_\mu e^- + (\frac{1}{2} - \frac{2}{3} z) \bar{u} \gamma_\mu u + (-\frac{1}{2} + \frac{1}{3} z) \bar{d} \gamma_\mu d$$

$$(z = \sin^2 \theta) \quad (3.2)$$

+ terms, involving the other fermions ($\theta : SU_2 \times U_1$ mixing angle).

A direct generalization of the six-flavour scheme (3.1) is the following eight-flavour scheme ^{6),7),20)} :

$$\begin{pmatrix} u & t & c & v \\ d' & b & s' & h \end{pmatrix}_L \quad \begin{pmatrix} u & t & c & v \\ b & d & h & s \end{pmatrix}_R$$

$$\begin{pmatrix} \nu_e & N_E & \nu_\mu & N_M \\ e^- & E^- & \mu^- & M^- \end{pmatrix}_L \quad \begin{pmatrix} N_E & \nu_e & N_\mu & \nu_\mu \\ e^- & E^- & \mu^- & M^- \end{pmatrix}_R.$$

(3.3)

where we have not indicated the possibility to have weak angles between the various leptons and quarks. The eight-flavour scheme contains two new charged leptons E^- , M^- , and two new massive leptons N_E , N_M (Fermi-Dirac spinors), and five new quark flavours. The essential difference between the six-flavour and the eight-flavour scheme is that there is no need to introduce massive Majorana neutrals. The lepton mass matrix need not be lepton number violating.

4. - VECTORLIKE MODELS BASED ON SU_2^W DOUBLETS AND SINGLETS

The two models considered above are only two specific examples of a large class of vectorlike theories. In general, a vectorlike theory may include not only SU_2^W doublets, but also singlets, triplets, etc. In the following Table, I give a few examples of vectorlike theories involving SU_2^W doublets and singlets. (Note : new quarks of charge $2/3$ are denoted by $c, t, v, \dots$; new quarks of charge $-1/3$ by $b, h, \dots$. In the last column, I display explicitly the expression for the neutral current in the corresponding $SU_2 \times U_1$ theory ; $\bar{u}u$ stands for $\bar{u}_\mu u$, $(\bar{u}u)_L$ for $\bar{u}_\mu (1+\gamma_5)/2 u$, $(\bar{u}u)_R$ for $\bar{u}_\mu (1-\gamma_5)/2 u$, etc.)

Comments

Models A and B were described in the previous section. In models C, D and F, the u quark enters in the weak current both left-handedly and right-handedly. The neutral current couples vectorially to u quarks, but not to d quarks. The electron couples vectorially to the neutral current in model C, but not in D and F. In model G the right-handed currents are reserved entirely for weak transitions among the heavy leptons and quarks. As far as the light fermions are concerned, the neutral current has the same form as in the Salam-Weinberg model.

5. - VECTORLIKE LEPTONIC CURRENTS AND THEIR PHENOMENOLOGY

Any vectorlike theory implies the existence of right-handed weak currents. Typically those are relevant for the weak interactions of heavy leptons.

A) - Weak decays of heavy charged leptons

A heavy charged lepton can decay weakly via a right-handed weak current. For example, in the six-flavour scheme A the lepton E^- can decay via $E^- \rightarrow (\nu_e)_R + (\text{lepton pair} + \text{hadrons})$.

If $M_{E^-} \cong 2 \text{ GeV}$, one can approximate the emission of hadrons by the emission of essentially massless free u or d quarks, in accordance with simple scaling ideas. Thus one predicts that the new lepton decays $\sim 60\%$

Model (References)	SU ₂ doublets	SU ₂ singlets	Remarks	New quarks [charge], New leptons [charge]	Neutral current in SU ₂ × U ₁ theory z=sin ² θ (θ: SU ₂ × U ₁ mixing angle)
A (8,9,10)	$\begin{pmatrix} u & t & c \\ d' & b & s' \end{pmatrix}_L \begin{pmatrix} u & t & c \\ b & d' & s'' \end{pmatrix}_R$ $\begin{pmatrix} \nu_e & N_E & \nu_\mu \\ e^- & E^- & \mu^- \end{pmatrix}_L \begin{pmatrix} \nu_e & N_E & \nu_\mu \\ E^- & e^- & \mu^- \end{pmatrix}_R$	-	$d_R'' = \cos \theta'' d_R$ + $\sin \theta'' s_R$ $s_R'' = 1$ θ'' small (ΔI = 1/2 rule)	$c[2/3], t[2/3]$ $b[-1/3]$ $N_E[0], N_\mu[0]$ $E^-[-1]$	$\frac{1}{2} \bar{\nu} \nu - \left(\frac{1}{2} - z\right) \bar{e} e$ + $\left(\frac{1}{2} - \frac{2}{3}z\right) \bar{u} u - \left(\frac{1}{2} - \frac{1}{3}z\right) \bar{d} d + \dots$ (vector; ν stands for ν _e , ν _μ)
B (20,21)	$\begin{pmatrix} u & t & c & v \\ d' & b & s' & h \end{pmatrix}_L \begin{pmatrix} u & t & c & v \\ b & d & h & s' \end{pmatrix}_R$ $\begin{pmatrix} \nu_e & N_E & \nu_\mu & N_M \\ e^- & E^- & \mu^- & M^- \end{pmatrix}_L \begin{pmatrix} \nu_e & N_E & \nu_\mu & N_M \\ e^- & E^- & \mu^- & M^- \end{pmatrix}_R$	-	New possible weak angles neglected -	$c[2/3], t[2/3], \nu[2/3]$ $b[-1/3], h[-1/3]$ $N_E[0], N_M[0]$ $E^-[-1], M^-[-1]$	For light fermions, same as in A.
C (21)	$\begin{pmatrix} u & c \\ d' & s' \end{pmatrix}_L \begin{pmatrix} u & t \\ b & d'' \end{pmatrix}_R$ $\begin{pmatrix} \nu_e & \nu_\mu \\ e^- & \mu^- \end{pmatrix}_L \begin{pmatrix} \nu_e & N_E \\ E^- & e^- \end{pmatrix}_R$	$(t_L, c_R),$ (b_L, s_R') $(N_{E_L}^-, \nu_{\mu R})$ (E_L^-, μ_R^-)	$d_R'' = \cos \theta'' d_R$ + $\sin \theta'' s_R$ $\theta'' = 0$, or $\theta'' = \frac{\pi}{2}$	$c[2/3], t[2/3],$ $b[-1/3]$ $E^-[-1], N_E[0]$	$\frac{1}{2} \bar{\nu} \nu - \left(\frac{1}{2} - z\right) \bar{e} e$ + $\left(\frac{1}{2} - \frac{2}{3}z\right) \bar{u} u - \left(\frac{1}{2} - \frac{1}{3}z\right) \bar{d} d_L - \left(\frac{1}{2} - \frac{1}{3}z\right) (\bar{d}'' d'')_R$ + ...
D (21,22)	$\begin{pmatrix} u & c \\ d' & s' \end{pmatrix}_L \begin{pmatrix} u & c \\ b & s'' \end{pmatrix}_R$ $\begin{pmatrix} \nu_e & \nu_\mu \\ e^- & \mu^- \end{pmatrix}_L \begin{pmatrix} \nu_e & N_\mu \\ E^- & \mu^- \end{pmatrix}_R$	(b_L, d_R'') $(N_{\mu L}^-, \nu_{\mu R})$ (E_L^-, e_R^-)	$s_R'' = \cos \theta'' s_R$ + $\sin \theta'' d_R$ $\theta'' = 0$, or $\theta'' = \frac{\pi}{2}$	$c[2/3], b[-1/3]$ $N_\mu[0], E^-[-1]$	$\frac{1}{2} \bar{\nu} \nu - \left(\frac{1}{2} - z\right) \bar{e} e_L$ + $\left(\frac{1}{2} - \frac{2}{3}z\right) \bar{u} u - \left(\frac{1}{2} - \frac{1}{3}z\right) \bar{d} d_L$ - $\left(\frac{1}{2} - \frac{1}{3}z\right) \bar{s}'' s''_R + \dots$

E	$\begin{pmatrix} u & c \\ d' & s' \end{pmatrix}_L \quad \begin{pmatrix} t & c \\ d'' & s'' \end{pmatrix}_R$ $\begin{pmatrix} \nu_e & \nu_\mu \\ e^- & \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_e & \nu_\mu \\ e^- & \mu^- \end{pmatrix}_R$	(t_L, u_R) (e_L^-, e_R^-) (μ_L^-, μ_R^-)	θ'' small $(\Delta I = \frac{1}{2}$ rule, see also: A)	$c[\frac{2}{3}], t[\frac{2}{3}]$ $N_e[0], N_\mu[0]$	$\frac{1}{2} \bar{\nu}\nu - \left(\frac{1}{2} - z\right) \bar{e}e$ $+ \left(\frac{1}{2} - \frac{2}{3}z\right) \bar{u}u_L - \frac{2}{3}z \bar{u}u_R$ $- \left(\frac{1}{2} - \frac{1}{3}z\right) \bar{d}d + \dots$
F (21, 22, 11)	$\begin{pmatrix} u & c \\ d' & s' \end{pmatrix}_L \quad \begin{pmatrix} u & c \\ b'' & h'' \end{pmatrix}_R$ $\begin{pmatrix} \nu_e & \nu_\mu \\ e^- & \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_e & \nu_\mu \\ e^- & \mu^- \end{pmatrix}_R$	$(b_L, d_R), (h_L, s_R)$ $(e_L^-, e_R^-), (\mu_L^-, \mu_R^-)$	$b'' = b \cos \theta''$ $h \sin \theta''$ $h'' = 1$	$c[\frac{2}{3}], b[\frac{2}{3}], h[\frac{2}{3}]$ $E^-[-1], M^-[-1]$	$\frac{1}{2} \bar{\nu}\nu - \left(\frac{1}{2} - z\right) \bar{e}e_L + z \bar{e}e_R$ $+ \left(\frac{1}{2} - \frac{2}{3}z\right) \bar{u}u - \left(\frac{1}{2} - \frac{1}{3}z\right) \bar{d}d_L + \frac{1}{3}z \bar{d}d_R$
G (6, 7)	$\begin{pmatrix} u & c \\ d' & s' \end{pmatrix}_L \quad \begin{pmatrix} t & \nu \\ b & h \end{pmatrix}_R$ $\begin{pmatrix} \nu_e & \nu_\mu \\ e^- & \mu^- \end{pmatrix}_L \quad \begin{pmatrix} N_E & N_M \\ e^- & \mu^- \end{pmatrix}_R$	$(t_L, u_R), (\nu_L, c_R)$ $(b_L, d_R), (h_L, s_R)$ $(N_{eL}, \nu_{eR}), (N_{\mu L}, \nu_{\mu R})$ $(e_L^-, e_R^-), (\mu_L^-, \mu_R^-)$	Possible new mixing angles not displayed.	Same as in B.	$\frac{1}{2} \bar{\nu}\nu - \left(\frac{1}{2} - z\right) \bar{e}e_L + z \bar{e}e_R$ $+ \left(\frac{1}{2} - \frac{2}{3}z\right) \bar{u}u_L - \frac{2}{3}z \bar{u}u_R - \left(\frac{1}{2} - \frac{1}{3}z\right) \bar{d}d_L + \frac{1}{3}z \bar{d}d_R$ $+ \dots$ <p>(For light fermions, same as in Salam-Weinberg model).</p>

of the time into hadrons plus $(\nu_e)_R$, and 40% of the time into leptons. (In this estimate we have neglected the contributions of new flavours, which, however, will not matter, unless $M_{M^-} \gg 2 \text{ GeV}$.)

The observed dilepton events in e^+e^- annihilation²³⁾ can be interpreted as the consequence of the production of a new lepton of mass $\simeq 1.8 \text{ GeV}$. According to the estimate above, one expects 8% of the time the heavy lepton pair to disintegrate into $e^+\mu^-$ or $e^-\mu^+$ (plus neutrinos). In this scheme the new neutrino $(\nu_e)_R$ is produced in the decay of E^- . This lepton may also decay into N_E provided $M_E > M_{N_E}$, which subsequently decays into $e^- + \text{lepton pair}$ or $e^- + \text{hadrons}$. [This would lead to multi-lepton events, for example $e^+e^- \rightarrow \mu^+ + (e^- + e^- + \mu^+) + \text{neutrinos}$.] The angular distribution of the dilepton events in e^+e^- annihilation depends on the weak coupling of E^- . When the statistics of the dilepton events improves in the near future, it will be possible to check whether a right-handed current is involved or not²⁴⁾.

B) - Radiative decays of heavy charged leptons

Especially interesting is the possibility to have radiative decays of heavy charged leptons. For example in scheme A the decay $E^- \rightarrow e^- \gamma$ occurs with a branching ratio of order²⁵⁾ $(m_N/m_E)^2 \cdot \alpha/\pi$. It would be interesting to look for the radiative decays in e^+e^- annihilation, where one suspects that part of the observed total cross-section is due to the contribution of a new charged lepton²³⁾.

C) - The physics of neutral massive leptons

Many of the vectorlike schemes (A, B, C, D, E, G) contain new neutral massive leptons ("heavy neutrinos") along with new charged leptons, which are coupled by charged weak currents to the electron and muon. In general they will decay by emitting a charged lepton pair and a neutrino, e.g., in the six-flavour scheme A one has, for example, the decay $N_E \rightarrow e^- + \mu^+ + \nu_\mu$. The present experimental limits on the existence of such objects are very poor: they must only be heavier than the K meson, in order not to show up in the disintegration of the kaon. How can one see and discover the new massive neutral leptons?

One possibility is to see them in the decay of the new charged leptons. Another one is the production of a heavy neutral lepton N in ν_μ -electron scattering by the charged weak current. Because of the

constraint $m_N \gtrsim M_K$ the lowest possible threshold for such a reaction is $M_K^2/2m_e \simeq 250$ GeV. Since N decays partly via $e^- \mu^+ \nu_\mu$, one could see the production of the new neutral lepton by the appearance of trilepton events in $\nu_\mu e^-$ scattering.

It seems to me that the easiest way to discover new massive neutral leptons is to look for them in e^+e^- annihilation at very high energies where the production of the neutral leptons via the neutral current sets in. I estimated (within the six-flavour scheme A) the pair production of the massive neutral leptons to contribute about 30% of one unit in R at energies of ~ 40 GeV. Their weak decays would lead, e.g., to events like $e^+e^- \rightarrow (\mu^+ \mu^-) + (\mu^+ \mu^-)$, where both lepton pairs are produced in two jets and carry away $\sim 2/3$ of the available energy, the rest being carried away by neutrinos. The N leptons can in general also decay via $N \rightarrow \mu^- + \pi^+$, $N \rightarrow \mu^- + \rho^+$, etc., which leads to the very interesting events like $e^+e^- \rightarrow (\mu^- + \pi^+, \rho^+, \dots)_{\text{jet}} + (\mu^+ + \pi^-, \rho^-, \dots)_{\text{opposite jet}} + \text{no missing energy}$. Most interesting I find the pair production of massive Majorana neutrals, like the N_μ lepton in scheme A. Since the mass term of this lepton violates lepton number conservation, it can decay both into $\mu^- + \pi^+, \rho^+, \dots$ and into $\mu^+ + \pi^-, \rho^-, \dots$. This leads to spectacular new events, e.g., $e^+e^- \rightarrow (\mu^- \pi^+)_{\text{jet}} + (\mu^- \pi^+)_{\text{opposite jet}}$.

D) - Massive neutrinos and neutrino oscillations

In a vectorlike theory the neutrinos cannot be described by a two-component Weyl theory, as for example in the Salam-Weinberg scheme. Neutrinos are four-component spinors. Consequently the basic reason why neutrinos should be massless is lost, and it would be most naturally if neutrinos like all other fermions would have a mass. To require the neutrino to be massless becomes an artificial constraint in a vectorlike theory. For example in all models considered, except G, both the left-handed and right-handed neutrino field components enter in the neutral current. Thus the emission and absorption of Z bosons will produce a (formally logarithmically divergent) neutrino mass term. In some models (A, B, C, H) the charged weak interaction leads to a mixing between neutrinos and the massive neutral leptons, producing a neutrino mass term of the order of a few eV^{8),19)}.

Neutrino masses of the order of 1 eV, although being much below the experimental limits, would be very interesting since in this case the "neutrino sea" in the universe suspected on the basis of the observed 2.7°K "photon sea" would be a "sea" of massive, non-relativistic neutrinos. This

"sea" can provide a large, perhaps even dominant fraction of the mass of the universe, which may well be the mass necessary for a closed universe compatible with the observed red shift and deceleration parameters. Furthermore the missing mass in galactic clusters can be provided by a cloud of massive neutrinos ²⁶⁾. Estimates made on the basis of the observed astronomical parameters suggest an average neutrino mass of ~ 2 eV.

If the neutrino masses are indeed in the vicinity of 1 eV, the sea of cosmic neutrinos will consist of very non-relativistic neutrinos (temperature $\sim 10^{-4}$ °K), which will not be distributed uniformly in space (as the photons constituting the 2.7°K photon sea), but will be concentrated in the galactic clusters. Inside those clusters, e.g., also here in this room, situated in Aachen, the neutrino density will be very high (typically $\sim 10^9 \dots 10^{10}$ ν/cm^3). To prove the existence of such a dense "neutrino sea" is a great challenge for experimentalists.

Especially interesting in case of massive neutrinos is the possibility of neutrino beam oscillations ²⁷⁾. If, for example, the electron and muon neutrino are superpositions of two mass eigenstates :

$$\nu_e = \cos \theta \nu_1 + \sin \theta \nu_2$$

$$\nu_\mu = -\sin \theta \nu_1 + \cos \theta \nu_2$$

(θ : leptonic analogue of the Cabibbo angle), oscillations will occur with an oscillation length

$$l = \frac{p [\text{MeV}]}{4 |m_1^2 - m_2^2| [\text{eV}^2]} \cdot 10^9 \text{m}$$

(p : neutrino momentum). This formula is already made suitable for application by use of the relevant units. The interesting aspect of it is that the oscillation lengths for $|m_1^2 - m_2^2| \sim 1 \text{ eV}^2$ are well within the range of experiments which could be done in the laboratory, and are already being done in the case of reactor neutrinos ²⁸⁾.

Besides the $\nu_e - \nu_\mu$ oscillations there exist in general also oscillations between the electron or muon neutrino and the new neutrinos $\bar{\nu}_{eL}$ and $\bar{\nu}_{\mu R}$, e.g., $\nu_{\mu L} \rightleftharpoons \bar{\nu}_{\mu L}$. These oscillations are especially interesting since they violate lepton number conservation. One may expect that the main part of the ν_μ mass is generated by the lepton number conserving part of the weak interaction, and the lepton number violating part is small compared to it, in which case the $\nu_\mu - \bar{\nu}_\mu$ mass matrix takes the form

$$\begin{pmatrix} m & \epsilon \\ \epsilon & m \end{pmatrix}$$

where ϵ is the lepton number violating mixing term (note that in general in a vectorlike theory of leptons and quarks it is desirable to have a violation of lepton number conservation⁸⁾).

This mass matrix is very similar to the $K^0 - \bar{K}^0$ mass matrix : the eigenstates are $\nu_1 = (\nu_\mu + \bar{\nu}_\mu)/\sqrt{2}$ and $\nu_2 = (\nu_\mu - \bar{\nu}_\mu)/\sqrt{2}$. Thus in case of the $\nu_\mu - \bar{\nu}_\mu$ or $\nu_e - \bar{\nu}_e$ mixing one has a reason why the mixing angle might be 45° ; no such reason exists for the $\nu_e - \nu_\mu$ mixing.

In many vectorlike models the new helicity components of the neutrinos are coupled by weak currents to new charged leptons (see, e.g., the models A, B, C, D, E, F). Thus, at a distance where the oscillations start to become relevant, a new charged lepton (M^+) would be produced in high energy ν_μ nucleus scattering. This leads in particular to a "wrong" charge signal : $\nu_\mu + \text{hadron} \rightarrow M^+ + \text{hadrons} \rightarrow \mu^+ + \text{neutrinos} + \text{hadrons}$.

It remains to be seen if neutrino oscillations of the type discussed here take place in nature. I would like to urge that experiments be performed designed to search for the oscillations.

6. - PHENOMENOLOGY OF VECTORLIKE HADRONIC CURRENTS

A) - Weak decays of the new quark flavours

One way to see the new weak currents would be to look for them in the weak decays of the heavy quark flavours. For example, in the six-quark scheme A the charm flavour decays via a vector current. It seems feasible to measure the chirality of the charm changing weak current in the near future ²⁹⁾.

B) - New right-handed currents in neutrino scattering

In many vectorlike models (A, B, C, D, E, F) there exist new right-handed currents acting on the "valence" quarks of the nucleon u and d. These can be discovered in neutrino or antineutrino scattering. For example, the new current $(\bar{u}b)_R$ could be seen in antineutrino scattering, once the threshold for producing bottom flavoured hadrons is passed.

Neglecting the antiquark content of the nucleon, the cross-section for the reaction $\bar{\nu}_\mu + \text{nucleon} \rightarrow \mu^+ + \text{hadrons}$ can be written as $d\sigma/dy = \text{const.} \cdot E (1-y)^2$. The new current $(\bar{u}b)_R$ produces a new component in the y distribution :

$$\frac{1}{E} \frac{d\sigma}{dy} = \text{const.} [(1-y)^2 + (\text{threshold factor}) \cdot 1]. \quad (6.1)$$

The ratio $\sigma^{\bar{\nu}_\mu^+}/E$ increases by a factor 4, once all threshold factors are passed.

Recently it has been suggested ³⁰⁾ that a reliable way to take into account the relevant threshold factors is to replace the scaling variable x in the deep inelastic antineutrino nucleon scattering by the variable $\hat{x} = x + m_b^2/2\nu$ (provided $m_b^2 \gg M_{\text{nucleon}}^2$), in which case the threshold effects can be parametrized by only one parameter (m_b). The use of the variable $\hat{x}$ leads to a rather gentle onset of the new process. If we assume that the observed y anomaly ³¹⁾ is due to the new $(\bar{u}b)_R$ current, one finds ³²⁾ $m_b \simeq 5$ GeV. (Here we have set a possible mixing angle, e.g., the b-h mixing angle θ in scheme F, to zero.)

At present it is not clear if the y anomaly is due to a new weak current. If this turns out to be the case, and if $m_b \simeq 5$ GeV, one expects the first $\bar{b}b$ state to show up in e^+e^- annihilation in the region 8...10 GeV, just outside the range of the present e^+e^- machines.

A new right-handed current, acting on the d quark, e.g., $(\bar{t}d)_R$, could be seen most clearly in ν_μ scattering. However, here the effect would be less conspicuous as the corresponding effect due to the $(\bar{u}b)_R$ current in $\bar{\nu}_\mu$ scattering [a term proportional to $(1-y)^2$ is added to a constant term] :

$$1/E \cdot \frac{d\sigma}{dy} = \text{const.} (1 + (\text{threshold factor}) (1-y)^2).$$

Thus far there exists no indication in the experiments for the existence of such a current.

C) - "Wrong" sign dimuon events : are they due to sequential weak decays ?

Bottom flavoured hadrons can also be produced in ν_μ scattering (production off the sea of $\bar{u}u$ pairs) : $\nu_\mu + \bar{u} \rightarrow \mu^- + \bar{b}$. A certain, perhaps even important, fraction of the dimuon events observed at high energies may be due to this process. Especially the dominant fraction of dimuon events of the type ³³⁾ $\nu_\mu + \text{nucleon} \rightarrow \mu^- \mu^- + \text{hadrons}$ and $\bar{\nu}_\mu + \text{nucleon} \rightarrow \mu^+ \mu^+ + \text{hadrons}$ may be due to a sequential weak decay of the b quark.

Let us study this possibility within model F. The hadronic weak currents are given by

$$\begin{pmatrix} u & c \\ d' & s' \end{pmatrix}_L \quad \begin{pmatrix} u & c \\ b'' & h'' \end{pmatrix}_R \quad \begin{aligned} b'' &= \cos \theta'' b + \sin \theta'' h \\ h'' &= -\sin \theta'' b + \cos \theta'' h \end{aligned}$$

We assume the $\bar{\nu}$ anomaly to be due to b quark and h quark production via the current $\cos \theta'' (\bar{u}b)_R + \sin \theta'' (\bar{u}h)_R$. Both the b and the h quark can decay non-leptonically into the charmed quark, which in turn can decay semi-leptonically (see the Figure).

The rate for this reaction is proportional to $\cos^2\theta'' \cdot \sin^2\theta''$ (it is, of course, 0 for $\theta''=0$, the maximum is reached for $\theta''=45^\circ$). In order to see that the sequential b or h quark decay can indeed be the source of the observed equal sign dimuon events, let us take as an example $m_b \simeq 5 \text{ GeV}$, $m_h \gg m_b$. Let us suppose that in $\bar{\nu}_\mu$ scattering the b quark production contributes 30% to the total cross-section (the contributions due to h quark production are neglected), while in ν_μ scattering where the b quark can only be produced off the $\bar{u}u$ sea, it contributes 3% to the cross-section. In this case one finds

$$\frac{\sigma(\bar{\nu} \rightarrow \mu^+ \mu^-)}{\sigma(\bar{\nu} \rightarrow \mu^+)} = 0.3 \cdot \sin^2\theta'' \cdot \rho$$

$$\frac{\sigma(\nu \rightarrow \mu^- \mu^-)}{\sigma(\nu \rightarrow \mu^-)} = 0.03 \cdot \sin^2\theta'' \cdot \rho \quad (6.2)$$

ρ : muonic branching ratio for the decay of charmed particles.

The experimental data are consistent with ²³⁾

$$\frac{\sigma(\nu \rightarrow \mu^- \mu^-)}{\sigma(\nu \rightarrow \mu^-)} \simeq (0.8 \pm 0.5) \cdot 10^{-3}$$

$$\frac{\sigma(\bar{\nu} \rightarrow \mu^+ \mu^+)}{\sigma(\bar{\nu} \rightarrow \mu^+)} \simeq (0.8 \pm 0.7) \cdot 10^{-2} \quad (6.3)$$

For, e.g., $\rho = 20\%$ these numbers are reproduced for $\sin^2\theta'' \simeq 0.2$ ($\theta'' \simeq 25^\circ$).

If this picture of the origin of the wrong sign dimuon events is correct, there have to exist also trimuon events (e.g., $\nu_\mu + \text{nucleon} \rightarrow \mu^- + \mu^+ + \mu^- + \text{hadrons}$), since part of the time the decay $\bar{b} \rightarrow \bar{c}$ (or $\bar{h} \rightarrow \bar{c}$) will proceed by emitting leptons. Naive scaling estimates give a branching ratio $(\bar{b} \rightarrow \bar{c} + \mu^+ + \nu_\mu) / (\bar{b} \rightarrow \bar{c} + \text{quarks}) \sim 20\%$. Consequently the trimuon events are expected to appear on a level of 20% compared to the rate for producing "wrong" sign dimuons ³⁴⁾.

D) - New right-handed weak currents and the $\Delta I = \frac{1}{2}$ rule

New currents add new terms to the non-leptonic weak Hamiltonian which may be relevant for the non-leptonic weak decays of strange (and charmed) particles ^{8),9),10)}. However, I expect a four-quark operator like $\bar{s}h\bar{h}d$ (h : heavy quark) to contribute very little to the non-leptonic strange particle decay, due to the absence of heavy quark flavours in normal hadrons. Recently it has been emphasized that new right-handed currents involving heavy quarks can lead in QCD to the appearance of a new term in the non-leptonic Hamiltonian which may be important and even dominating, namely the quark bilinear $\bar{s}\sigma_{\mu\nu}G^{\mu\nu}d$ ($G^{\mu\nu}$: gluon field strength, summation over colour understood) ^{35),36)}. For example, in schemes A, B, D, E the new right-handed current $(\bar{c}s)_R$ is present, which generates the new term

$$H^{\text{nonlept.}} = - \frac{G}{\sqrt{2}} \sin \theta \cdot m_c \cdot \frac{g}{4\pi^2} \bar{s} \sigma_{\mu\nu} G^{\mu\nu} \frac{1+\gamma_5}{2} d + h.c. \quad (6.4)$$

(it corresponds to the decay $s \rightarrow d + \text{gluon}$, g : gluon-quark coupling constant). The important features of this new term are the following :

- (i) It contains no heavy quark fields. Thus its matrix elements between normal hadrons can be big.
- (ii) It is enhanced by m_c .
- (iii) It is pure $|\Delta I| = \frac{1}{2}$ and has all the desired properties for a non-leptonic weak Hamiltonian ³⁷⁾.

A consistent picture of all non-leptonic decays of strange particles emerges if one assumes that the term (6.4) dominates the decays ³⁵⁾. Specific wave function calculations performed recently within the MIT bag model indicate that this assumption may be consistent ³⁸⁾.

We emphasize that the term (6.2) is enhanced by the large value of m_c . A similar enhancement would not occur in the case of charmed particle decay. Thus charmed particles are supposed to have a large semi-leptonic decay mode, in agreement with recent estimates on the basis of the observed dimuon events in ν_μ hadron scattering.

Let me add : I think that the $|\Delta I| = \frac{1}{2}$ rule and other mysteries of the non-leptonic weak decays are due to a new term of type (6.4). However, it may not be due to the $(\bar{c}s)_R$ current, and not due to charmed intermediate states as in (6.4), but due to new currents and other intermediate states

[see, for example, Ref. 39]. For the moment, one may say :

The non-leptonic decays of strange particles are dominated by an effective Hamiltonian $\sim G.m_{\text{heavy quark}} \bar{s}_R \sigma_{\mu\nu} G^{\mu\nu} d_L$. Which heavy quark is really involved depends on the specific scheme of the weak interactions, and future investigations will shed more light on this question.

E) - Problems with $|\Delta S|=1$ neutral currents

In all vectorlike models involving SU_2^W doublets and singlets there is always the threatening danger to generate $\Delta S=1$ neutral currents. For example, in model C one has to impose (artificially ?) the constraint $s''=s$ or $s''=d$. Such constraints seem unnatural, and we conclude that this model can hardly be regarded as realistic. The situation is even worse for models D, E and F. Even if we set $\theta''=0$ or $\pi/2$ in these models, the higher order weak interactions will always generate a mixing between d and s (of order α) .:

$$\theta'' \approx \theta' \cdot \frac{\alpha}{\pi} \cdot \frac{m_d \cdot m_c}{m_s^2} \approx \theta' \cdot \frac{\alpha}{4} ,$$

implying a $|\Delta S|=1$ neutral current of order $\alpha/\pi \sin\theta'$, which is too large to be tolerable. Thus we conclude : the models D, E and F are unrealistic. The same is true for the related models of Gürsey and Sikivie, and Ramond ²²⁾. There are, however, no problems of this sort, if both d_R and s_R are SU_2^W singlets. This is, for example, the case in models F and G ⁴⁰⁾.

7. - PHENOMENOLOGY OF THE NEUTRAL CURRENT

A) - Hadronic neutral currents

In schemes based on SU_2^W doublets only the neutral current is a vector current, provided the gauge group is $SU_2 \times U_1$. This implies in particular $\sigma^{\nu\nu} = \sigma^{\bar{\nu}\bar{\nu}}$. The new experimental results reported at this conference are in disagreement with this prediction ⁴¹⁾. Thus $SU_2 \times U_1$ vectorlike models based on SU_2^W doublets only are ruled out ⁴²⁾.

Let us consider the various schemes described in the Table. Especially, we concentrate on the ratio $\hat{R} = \sigma_{\text{hadr.}}^{\nu\nu} / \sigma_{\text{hadr.}}^{\nu\mu}$, which is measured to be $\sim 0.5 \dots 0.7$. The effective Hamiltonian describing the weak interaction is

$$H^{\text{weak}} = \frac{4G}{\sqrt{2}} (j_r^+ j_r^- + \rho j_r^{\text{n}} j_r^{\text{n}}); \quad \rho = \frac{M_w^2}{M_z^2 \cos^2 \theta} \quad (7.1)$$

($j_\mu^+ = \bar{u}_\mu \gamma_\mu (1 + \gamma_5/2) d + \dots$, j_μ^{n} : see the Table). The parameter ρ describes the strength of the neutral current coupling.

In the simple doublet symmetry breaking scheme discussed by Weinberg ⁴⁾ one has $\rho = 1$. We assume a more general symmetry breaking mechanism and keep ρ as a free parameter ⁴³⁾. The ratio $\hat{R}$ is, of course, independent of ρ .

In the following discussion of models, we apply the naive scaling assumptions for neutrino scattering and set all antiquark densities in the nucleon to zero. We use the definitions $R^\nu = \sigma_{\text{hadr.}}^{\nu\nu} / \sigma_{\text{hadr.}}^{\nu\mu^-}$ and $R^{\bar{\nu}} = \sigma_{\text{hadr.}}^{\bar{\nu}\bar{\nu}} / \sigma_{\text{hadr.}}^{\bar{\nu}\mu^+}$; $z = \sin^2 \theta$.

Models

A, B: $\hat{R} = 1$ (excluded)

C ($\theta''=0$), D ($\theta''=\frac{\pi}{2}$): $\hat{R} = 1$ (excluded)

C ($\theta''=\frac{\pi}{2}$), D ($\theta''=0$); F:

$$R^\nu = \left(\frac{7}{12} - \frac{11}{9} z + \frac{20}{27} z^2 \right) \rho^2 \quad R^{\bar{\nu}} = \left(\frac{5}{4} - 3z + \frac{20}{9} z^2 \right) \rho^2$$

$$\hat{R} = \frac{5/4 - 3z + 20/9 z^2}{7/4 - 11/3 z + 20/9 z^2}$$

z	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$\hat{R}$	0.71	0.69	0.67	0.65	0.63	0.65	0.71	0.88	1.14

Remarks

These models are consistent with the data for $z = 0 \dots 0.6$; note the rather smooth dependence of $\hat{R}$ on z , in contrast, e.g., to the Salam-Weinberg model. For $z = 3/8 = 0.375$ the predictions of these models for inclusive neutrino scattering off hadrons coincide with the Salam-Weinberg model for the same value of z . The reason for this is simple : the neutral currents for both models in case $z = 3/8$ are equal for d quarks, while for u quarks they are either a vector (here) or axial vector (Salam-Weinberg case). If we require $\rho = 1$, consistency with the data for R^ν and $R^{\bar{\nu}}$ is reached for $z \simeq 0.3 \dots 0.4$.

$$E: \quad R^\nu = \left(\frac{7}{12} - \frac{10}{9}z + \frac{20}{27}z^2 \right) \rho^2 \quad R^{\bar{\nu}} = \left(\frac{5}{4} - 2z + \frac{20}{9}z^2 \right) \rho^2$$

$$\hat{R} = \frac{\frac{5}{12} - \frac{2}{3}z + \frac{20}{27}z^2}{\frac{7}{12} - \frac{10}{9}z + \frac{20}{27}z^2}$$

z	0	0.1	0.2	0.3	0.4	0.5
$\hat{R}$	0.71	0.75	0.8	0.83	1.04	1.26

Remarks

Consistent with data for $z < 0.1$. In this case ρ is much different from 1 ($\rho^2 \sim 0.3 \dots 0.5$), in order to give consistency with the observed values of R^ν and $R^{\bar{\nu}}$.

G:

Within our approximations, the predictions of this model coincide with the Salam-Weinberg model generalized for arbitrary ρ :

$$R^\nu = \left(\frac{1}{2} - z + \frac{20}{27}z^2 \right) \rho^2 ; \quad R^{\bar{\nu}} = \left(\frac{1}{2} - z + \frac{20}{9}z^2 \right) \rho^2$$

$$\hat{R} = \frac{\frac{1}{2} - z + \frac{20}{9} z^2}{\frac{3}{2} - 3z + \frac{20}{9} z^2}$$

z	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
$\hat{R}$	0.33	0.35	0.39	0.50	0.69	1	1.4	1.82

Remarks

Consistent with data for $z = 0.3 \dots 0.4$. The same values give $\rho \approx 1$.

Conclusions

The hadronic neutral current data are consistent with either model G (no new right-handed couplings to the valence quarks u, d), or with the case E (new right-handed coupling to the d quark), or with the cases where a new right-handed coupling to the u quark enters (but none to the d quark).

B) - Parity violation in atomic physics

In the immediate future it will be feasible to test the possibly existing parity violation due to the neutral current in atomic physics ⁴⁴⁾. Very soon one expects the first experimental results from the groups at Seattle and Oxford, investigating ²⁰⁹₈₃Bi.

The effective Hamiltonian relevant for atomic physics is in $SU_2 \times U_1$ models :

$$H_{p.v.} = \frac{G}{12} Q_W \cdot p \left\{ \frac{\vec{\sigma} \cdot \vec{p}}{2m_e} \delta^3(x) \right\} (I_3^{weak}(e_L) - I_3^{weak}(e_R)),$$

where p is the electron momentum, and the effective charge Q_W is defined by $Q_W = 4 \langle \text{nucleus} | \int j_0^n d^3x | \text{nucleus} \rangle$.

In the various models of the Table, we find (z : nuclear charge, N : neutron number) :

Model	Q_W	$I_3^W(e_L^-) - I_3^W(e_R^-)$	Q_W for $^{209}_{83}\text{Bi}$ and $z = 1/3$
A, B, C ($\theta'' = 0$)	$Z(2-4z) - 2N$	0	-197
D($\theta'' = \pi/2$)	$Z(2-4z) - 2N$	$-\frac{1}{2}$	-197
D($\theta'' = 0$), F	$Z(3-4z)$	$-\frac{1}{2}$	+183
E	$Z(-4z) - 3N$	$-\frac{1}{2}$	-489
G	$Z(1-4z) - N$	$-\frac{1}{2}$	-154

The predictions of model G (with $\rho = 1$) for the parity violation in atomic physics are identical with the prediction of the Salam-Weinberg model. We characterize all models as follows. The models A, B, C($\theta'' = 0$) give no effect. Model D($\theta'' = \pi/2$) and the models E and G give qualitatively the same results as the Salam-Weinberg model. The models D($\theta'' = 0$) and F, i.e., those models involving the $(\bar{u}b)_R$ current, give roughly the same magnitude for the parity violation as the Salam-Weinberg model but the opposite sign. We emphasize the importance of the sign ; the first, even relatively crude determination of a parity violation in atomic physics will provide us at least with the sign of Q_W , and it will be very important in discriminating among the various models.

8. - SU_2^W TRIPLETS AND SU_3

The fact that the previously considered models D($\theta'' = 0$) and F give the opposite sign for the parity violation in atomic physics as the Salam-Weinberg model depends crucially on the assignment of the right-handed electron to a SU_2^W singlet ($I_3^W(e_L^-) - I_3^W(e_R^-) = -\frac{1}{2}$). If we place the right-handed electron into a SU_2^W triplet by introducing new massive leptons :

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L ; \quad \begin{pmatrix} M^+ \\ N \\ e^- \end{pmatrix}_R$$

(8.1)

(N, M^+ : massive new leptons),

the difference $I_3^W(e_L^-) - I_3^W(e_R^-)$ becomes $+1/2$, i.e., the sign of $H_{p.v.}$ changes. In this case the sign of the parity violation in atomic physics is identical to the one obtained in the Salam-Weinberg model.

The triplet assignment for e_R^- will be forced upon us within $SU_2 \times U_1$ models, if the sign of Q_W turns out to be negative and if the $(\bar{u}b)_R$ current exists.

If e_R^- belongs to a SU_2 triplet, the neutral current interaction of the electron is changed rather profoundly. In particular the differential cross-sections for $\nu_\mu - e^-$ scattering are

$$\begin{aligned} \frac{d\sigma^\nu}{dy} &= \frac{2G^2 m_e}{\pi} E_\nu \rho^2 \left[\left(-\frac{1}{2} + z\right)^2 + (-\lambda + z)^2 (1-\gamma)^2 \right] \\ \frac{d\sigma^{\bar{\nu}}}{dy} &= \frac{2G^2 m_e}{\pi} E_{\bar{\nu}} \rho^2 \left[\left(-\frac{1}{2} + z\right)^2 (1-\gamma)^2 + (-\lambda + z)^2 \right] \end{aligned} \quad (8.2)$$

$z = \sin^2 \theta$, $\lambda = 1$ (for comparison : the corresponding expressions in the generalized S-W model are obtained for $\lambda = 0$). In order to free oneself from the parameter ρ , we consider especially the ratios

$$\bar{r}^{(\nu)} = \frac{\sigma_{\nu e^-}^{(\nu)} / E_\nu \cdot \frac{2G^2 m_e}{\pi}}{R_{hadr.}^{(\nu)}} \quad (8.3)$$

For example, one finds, using $\frac{2G^2 m_e}{\pi} E_\nu = 1.7 \cdot 10^{-11} E_\nu [GeV] cm^2$:

$$\bar{r} = \frac{\left(-\frac{1}{2} + z\right)^2 \cdot \frac{1}{3} + (-1+z)^2}{\frac{7}{12} - \frac{11}{9} z + \frac{20}{27} z^2} ;$$

z	0	0.1	0.2	0.3	0.4	0.5	0.6
$\bar{r}$	1.86	1.84	1.82	1.78	1.71	1.59	1.40

(8.4)

Experimentally, $R_{\text{hadr.}}^{\nu}$ is ~ 0.26 . For example, for $R_{\text{hadr.}}^{\nu} = 0.26$ and $z = 0.3 \dots 0.5$, we obtain $\sigma_{\nu \mu}^{\nu} e^{-} \simeq 7.10^{-42} \text{ cm}^2 E_{\nu} [\text{GeV}]$. Similarly one has $\sigma_{\nu \mu}^{\nu} e^{-} \simeq 3.10^{-42} \text{ cm}^2 E_{\nu} [\text{GeV}]$. Typically, these cross-sections are a factor 2.5 higher than the upper limits reported by the Gargamelle group ⁴⁵⁾. On the other hand they agree well with the cross-sections reported at this conference by the Aachen-Padova group ⁴⁶⁾. Thus at present the triplet possibility for e_R^{-} cannot be excluded. The experimental data to be presented at the next neutrino conference will probably be sufficiently accurate in order to exclude or verify the triplet nature of e_R^{-} within a $SU_2 \times U_1$ theory ⁴⁷⁾.

Suppose the triplet hypothesis for e_R^{-} is correct. In this case, the following model H for leptons and quarks can be constructed.

Model H:

This scheme is the smallest vectorlike extension of the scheme (8.1) incorporating the $e-\mu$ universality such that all leptons transform either as doublets or triplets. It contains two new charged leptons and two neutral ones (N, ν') . Note : $\nu_L = \nu_e$, $\nu_R = \bar{\nu}_\mu$.

The scheme H is very interesting, since it can immediately be incorporated into a SU_3 scheme, where the group $SU_2 \times U_1$ is regarded as a subgroup³⁹⁾ of SU_3 . If one adds one neutral lepton (in parenthesis), the leptons transform as an octet, while the quarks transform as triplets. The correct electric charges of the leptons and quarks follow as the consequence of the octet character of the leptons and the triplet character of the quarks.

We emphasize that in the scheme H the parity violation of the weak interaction is very simple. One notes easily that the scheme is invariant under a combined space and U spin reflection. The existence of such a symmetry may be the reason for the universality of the weak interactions [for a different model based on SU_4 this point has been emphasized recently in Ref. 48].

Within the SU_3 scheme the $SU_2 \times U_1$ mixing angle (unrenormalized) is 60° ($\sin^2 \theta = 3/4$) ; the neutral current would then be a pure axial vector. This is excluded by experiment ; thus rather large renormalization effects must exist such as to give $\sin^2 \theta < 0.6$ as required by experiment. This can only happen if the masses of the unifying bosons are superheavy implying that the interactions caused by them are negligible for phenomenology.

The SU_3 pattern of the weak interactions described above can be obtained within a unified theory of the weak, electromagnetic and strong interactions based on the exceptional group⁴⁹⁾ E_7 .

9. - OUTLOOK

In this talk I could only discuss the main aspects of vectorlike weak interaction theories. Many other problems have been discussed during the last few years, e.g., the origin of CP violation⁵⁰⁾, the origin of universality of the weak interactions, etc., which I have not mentioned.

Are the weak interactions really vectorlike ? This question can only be answered by experimentalists in their search for right-handed weak currents. For the moment, the most urgent questions are :

- A) - Is the $\bar{\nu}$ anomaly reported both by the HPWF group and the Caltech group due to the production of a new quark flavour, via a new right-handed current ?
- B) - Are neutrinos massive, and do neutrinos oscillate ?
- C) - Is there a new charged lepton with a mass of ~ 1.8 GeV, and does it decay via a right-handed current, as predicted by most vector-like theories ?
- D) - Is the right-handed electron part of a weak triplet, and is, as a consequence, the antineutrino-electron neutral current cross-section as high as discussed in Section 8 ?
- E) - Is there parity violation in atomic physics, and, if it exists, what is the sign of the effect ?

Summarizing our conclusions about the different models, in particular the ones with respect to the neutral currents, we can say that only the $SU_2 \times U_1$ models E, F, G and H have chances of being realistic. All other models suffer either from parity conserving neutral currents, or from the generation of a $|\Delta S| = 1$ neutral current of order α .

At present, neutrino physics has become one of the most active and interesting fields in particle physics. I am sure that at the next neutrino conference, like at this meeting, many new and interesting experimental facts will be reported. As it became apparent at this meeting, one has reached a stage in weak interaction physics in which it is hard to build new models of leptons and quarks which are not in contradiction with observed facts. Model building has become an exact science. Perhaps very soon theoreticians will come close to the true pattern of leptons and quarks which nature was able to hide until today.

ACKNOWLEDGEMENTS

I am indebted to P. Minkowski and my colleagues at CERN, especially J. Bernabeu and C. Jarlskog, for useful discussions.

REFERENCES AND FOOTNOTES

- 1) See, for example :
Y. Nambu - "Preludes in Theoretical Physics", North Holland (1966) ;
H. Fritzsch, M. Gell-Mann and H. Leutwyler - Phys.Letters 47B, 365 (1973) ;
S. Weinberg - Phys.Rev.Letters 31, 494 (1973).
- 2) For discussions of various aspects of colour, see :
O.W. Greenberg - Phys.Rev.Letters 13, 598 (1964) ;
M. Han and Y. Nambu - Phys.Rev. 139, 1006 (1965) ;
W. Bardeen, H. Fritzsch and M. Gell-Mann - in "Scale and Conformal Symmetry in Hadron Physics", R. Gatto, Ed., p. 139 (John Wiley and Sons, New York, 1973).
- 3) See, for example :
S.L. Glashow, J. Iliopoulos and L. Maiani - Phys.Rev. D2, 1285 (1970).
- 4) A. Salam and J.C. Ward - Phys.Letters 13, 168 (1964) ;
S. Weinberg - Phys.Rev.Letters 19, 1264 (1967).
- 5) J.C. Pati and A. Salam - Phys.Rev. D8, 1240 (1973) ;
H. Georgi and S.L. Glashow - Phys.Rev.Letters 32, 438 (1974) ;
H. Fritzsch and P. Minkowski - Ann.Phys.N.Y. 93, 193 (1974).
- 6) H. Fritzsch and P. Minkowski - Phys.Letters 56B 69 (1975).
- 7) J.C. Pati and A. Salam - Proceedings of the 1975 Palermo Conference ;
R.N. Mohapatra and J.C. Pati - Phys.Rev. D11, 2558 (1975).
- 8) H. Fritzsch, M. Gell-Mann and P. Minkowski - Phys.Letters 59B, 256 (1975).
- 9) R.L. Kingsley, S.B. Treiman, F.A. Wilczek and A. Zee - Phys.Rev. D12, 2768 (1975).
- 10) S. Pakvasa, W. Simmons and S.F. Tuan - Phys.Rev.Letters 35, 702 (1975) ;
A. De Rujulà, H. Georgi and S.L. Glashow - Phys.Rev. D12, 3589 (1975) ;
N.V. Krasnikov, V.A. Kuzmin and K.G. Chetyrkin - JETP Letters 22, 47 (1975).
- 11) R.M. Barnett - Phys.Rev., to be published.
- 12) The models discussed in the following references are also vectorlike.
H. Georgi and S.L. Glashow - Phys.Rev.Letters 28, 1494 (1972) ;
P. Fayet - Nuclear Phys. B78, 14 (1974).
- 13) P. Higgs - Phys.Letters 12, 132 (1964) ;
G. Guralnik, C.R. Hagen and T.W.B. Kibble - Phys.Rev.Letters 13, 585 (1964) ;
F. Englert and R. Brout - Phys.Rev.Letters 13, 321 (1964).

- 14) See, for example :
H. Georgi and S.L. Glashow - Phys.Rev. D6, 429 (1972) ; (these authors introduced the term "vectorlike").
- 15) D. Gross and R. Jackiw - Phys.Rev. D6, 477 (1972).
- 16) W. Pauli - Collected Scientific Papers, J. Wiley and Sons, Inc. (1964), Vol. 1. Letter to V.F. Weisskopf (in preface).
- 17) We use the notation of Harari.
H. Harari - Phys.Letters 57B, 265 (1975).
- 18) See, in this respect :
A. Halprin, P. Minkowski, H. Primakoff and S.P. Rosen - Caltech Preprint CALT-68-533 (1976).
- 19) T.P. Cheng - to be published.
- 20) H. Fritzsch and P. Minkowski - Caltech Preprint CALT-68-503 (1975).
- 21) H. Fritzsch - Copenhagen Lectures 1975, Caltech Preprint CALT-68-524 (1976).
- 22) Very similar $SU_2 \times U_1$ schemes have been discussed in :
F. Gürsey and P. Sikivie - Phys.Rev.Letters 36, 775 (1976) ;
P. Ramond - Caltech Preprint (1976).
- 23) M. Perl - See talk at this conference.
- 24) See, for example :
Y. Park and A. Yildiz - to be published ;
K. Fujikawa and N. Kawamoto - DESY Preprint 76-101 (1976).
- 25) H. Fritzsch and P. Minkowski - Caltech Preprint CALT-68-538 (1976) ;
K. Fujikawa - DESY Preprint (1976) ;
F. Wilczek and A. Zee - Princeton University Preprint (1976).
- 26) See, e.g. :
R. Cowsik and J.M. McClelland - Phys.Rev.Letters 29, 669 (1972) ;
J. Gunn - unpublished ;
G. Marx - Contribution to this conference.
- 27) B. Pontecorvo - JETP 26, 986 (1968) ;
H. Fritzsch and P. Minkowski - Phys.Letters 62B, 72 (1976) ;
M. Gell-Mann and J.B. Stephenson - unpublished ;
S. Eliezer and A.R. Swift - to be published in Nuclear Phys.
- 28) See the talks of F. Reines and R. Sobel at this conference.
- 29) A. Buras and J. Ellis - CERN Preprint TH. 2160 (1976).
- 30) R.P. Feynman - unpublished.
R.M. Barnett - to be published in Phys.Rev.

- 31) See the contribution of A. Benvenuti and B. Barish at this conference.
- 32) See also :
J. Kaplan and F. Martin - "Testing Charged Currents in Neutrino Experiments", Paris Preprint (1976).
- 33) A. Benvenuti et al. - Phys.Rev.Letters 35, 1199, 1203 and 1249 (1975).
- 34) Note, however, that the discovery of trimuon events would not necessarily imply that the "wrong" sign dimuons are due to a sequential weak decay. For example, the associated production of charm will also lead to "wrong" sign dimuons and trimuons.
- 35) H. Fritzsch and P. Minkowski - Phys.Letters 61B, 275 (1976).
- 36) R.K. Ellis - to be published.
- 37) See, in particular :
E. Golowich and B. Holstein - Phys.Rev.Letters 35, 83 (1975).
- 38) J.F. Donoghue, E. Golowich and B. Holstein - Amherst Preprint (1976).
- 39) See also :
H. Fritzsch and P. Minkowski - CERN Preprint TH. 2165 (1976).
- 40) I am indebted to P. Minkowski and S. Weinberg for discussions about these points.
- 41) See the talks of B. Barish, A. Benvenuti, T. Hansl, W.Y. Lee, L. Sulak and others at this conference.
- 42) Note, however, that new interactions, outside the $SU_2 \times U_1$ framework, may have entered the effective neutral current interaction
[see, for example, B.R. Kim - Aachen Preprint (1976)].
- 43) See, for example, the third paper in Ref. 5).
- 44) See, for example, the talk of C. Jarlskog at this conference.
- 45) See the various talks by members of the Gargamelle group at this meeting.
- 46) See, for example, the talk of F. Bobisut at this conference.
- 47) I studied the consequences of the e_R^- triplet hypotheses discussed here and in the previous section in collaboration with J. Bernabeu, C. Jarlskog and P. Minkowski.
- 48) F.J. Yndurain - CERN Preprint TH. 2158 (1976).
- 49) See the papers of F. Gürsey, P. Sikivie and P. Ramond [Ref. 22].
- 50) See, e.g. :
M. Kobayashi and K. Maskawa - Progr.Theor.Phys. 49, 652 (1973) ;
L. Maiani - to be published ;
H. Fritzsch and P. Minkowski - to be published in Phys.Letters.

"Wrong" sign dimuons due to sequential b quark decay in ν_μ scattering.

Model (References)	SU ₂ doublets	SU ₂ singlets	Remarks	New quarks [charge], New leptons [charge]	Neutral current in SU ₂ × U ₁ theory z = sin ² θ (θ: SU ₂ × U ₁ mixing angle)
A (8, 9, 10)	$\begin{pmatrix} u & t & c \\ d' & b & s' \end{pmatrix}_L \begin{pmatrix} u & t & c \\ b & d'' & s'' \end{pmatrix}_R$ $\begin{pmatrix} \nu_e & N_E & \nu_\mu \\ e^- & E^- & \mu^- \end{pmatrix}_L \begin{pmatrix} \nu_e & N_E & \nu_\mu \\ E^- & e^- & \mu^- \end{pmatrix}_R$	-	$d_R'' = \cos \theta'' d_R$ $+ \sin \theta'' s_R$ $s_R'' = 1$ θ'' small $(\Delta I = \frac{1}{2} \text{ rule})$	$c[\frac{2}{3}], t[\frac{2}{3}]$ $b[-\frac{1}{3}]$ $N_E[0], N_\mu[0]$ $E^-[-1]$	$\frac{1}{2} \bar{\nu} \nu - \left(\frac{1}{2} - z \right) \bar{e} e$ $+ \left(\frac{1}{2} - \frac{2}{3} z \right) \bar{u} u - \left(\frac{1}{2} - \frac{1}{3} z \right) \bar{d} d + \dots$ (vector; ν stands for ν_e, ν_μ)
B (20, 21)	$\begin{pmatrix} u & t & c & v \\ d' & b & s' & h' \end{pmatrix}_L \begin{pmatrix} u & t & c & v \\ b & d & h & s' \end{pmatrix}_R$ $\begin{pmatrix} \nu_e & N_E & \nu_\mu & N_M \\ e^- & E^- & \mu^- & M^- \end{pmatrix}_L \begin{pmatrix} \nu_e & N_E & \nu_\mu & N_M \\ E^- & e^- & \mu^- & M^- \end{pmatrix}_R$	-	New possible weak angles neglected	$c[\frac{2}{3}], t[\frac{2}{3}], v[\frac{2}{3}]$ $b[-\frac{1}{3}], h[-\frac{1}{3}]$ $N_E[0], N_M[0]$ $E^-[-1], M^-[-1]$	For light fermions, same as in A.
C (21)	$\begin{pmatrix} u & c \\ d' & s' \end{pmatrix}_L \begin{pmatrix} u & t \\ b & d'' \end{pmatrix}_R$ $\begin{pmatrix} \nu_e & \nu_\mu \\ e^- & \mu^- \end{pmatrix}_L \begin{pmatrix} \nu_e & N_E \\ E^- & e^- \end{pmatrix}_R$	$(t_L, c_R),$ (b_L, s_R'') $(N_{E_L}, \nu_{\mu R})$ (E_L^-, μ_R^-)	$d_R'' = \cos \theta'' d_R$ $+ \sin \theta'' s_R$ $\theta'' = 0$, or $\theta'' = \frac{\pi}{2}$	$c[\frac{2}{3}], t[\frac{2}{3}]$ $b[-\frac{1}{3}]$ $E^-[-1], N_E[0]$	$\frac{1}{2} \bar{\nu} \nu - \left(\frac{1}{2} - z \right) \bar{e} e$ $+ \left(\frac{1}{2} - \frac{2}{3} z \right) \bar{u} u - \left(\frac{1}{2} - \frac{1}{3} z \right) \bar{d} d_L - \left(\frac{1}{2} - \frac{1}{3} z \right) (\bar{d}'' d'')_R$ $+ \dots$
D (21, 22)	$\begin{pmatrix} u & c \\ d' & s' \end{pmatrix}_L \begin{pmatrix} u & c \\ b & s'' \end{pmatrix}_R$ $\begin{pmatrix} \nu_e & \nu_\mu \\ e^- & \mu^- \end{pmatrix}_L \begin{pmatrix} \nu_e & N_\mu \\ E^- & \mu^- \end{pmatrix}_R$	(b_L, d_R'') $(N_{\mu L}, \nu_{\mu R})$ (E_L^-, e_R^-)	$s_R'' = \cos \theta'' s_R$ $+ \sin \theta'' d_R$ $\theta'' = 0$, or $\theta'' = \frac{\pi}{2}$	$c[\frac{2}{3}], b[-\frac{1}{3}]$ $N_\mu[0], E^-[-1]$	$\frac{1}{2} \bar{\nu} \nu - \left(\frac{1}{2} - z \right) \bar{e} e_L$ $+ \left(\frac{1}{2} - \frac{2}{3} z \right) \bar{u} u - \left(\frac{1}{2} - \frac{1}{3} z \right) \bar{d} d_L$ $- \left(\frac{1}{2} - \frac{1}{3} z \right) \bar{s}'' s''_R + \dots$

E	$\begin{pmatrix} u & c \\ d' & s' \end{pmatrix}_L \quad \begin{pmatrix} t & c \\ d'' & s'' \end{pmatrix}_R$ $\begin{pmatrix} \nu_e & \nu_\mu \\ e^- & \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_e & \nu_\mu \\ E^- & M^- \end{pmatrix}_R$	(t_L, u_R) $(\bar{E}_L^-, \bar{e}_R^-)$ $(\bar{M}_L^-, \bar{\mu}_R^-)$	θ'' small $(\Delta I = \frac{1}{2} \text{ rule, see also: A})$	$c[\frac{2}{3}], \quad t[\frac{2}{3}]$ $N_e[0], \quad N_\mu[0]$	$\frac{1}{2} \bar{\nu}\nu - \left(\frac{1}{2} - z\right) \bar{e}e$ $+ \left(\frac{1}{2} - \frac{2}{3}z\right) \bar{u}u_L - \frac{2}{3}z \bar{u}u_R$ $- \left(\frac{1}{2} - \frac{1}{3}z\right) \bar{d}d + \dots$
F (21, 22, 11)	$\begin{pmatrix} u & c \\ d' & s' \end{pmatrix}_L \quad \begin{pmatrix} u & c \\ b'' & h'' \end{pmatrix}_R$ $\begin{pmatrix} r_e & r_\mu \\ e^- & \mu^- \end{pmatrix}_L \quad \begin{pmatrix} r_e & r_\mu \\ E^- & M^- \end{pmatrix}_R$	$(b_L, d_R), (h_L, s_R)$ $(\bar{E}_L^-, \bar{e}_R^-), (\bar{M}_L^-, \bar{\mu}_R^-)$	$b'' = b \cos \theta''$ $h \sin \theta''$ $h'' = \perp$	$c[\frac{2}{3}],$ $b[-\frac{1}{3}], \quad h[-\frac{1}{3}]$ $E^-[-1], \quad M^-[-1]$	$\frac{1}{2} \bar{\nu}\nu - \left(\frac{1}{2} - z\right) \bar{e}e_L + z \bar{e}e_R$ $+ \left(\frac{1}{2} - \frac{2}{3}z\right) \bar{u}u - \left(\frac{1}{2} - \frac{1}{3}z\right) \bar{d}d_L + \frac{1}{3}z \bar{d}d_R$
G (6, 7)	$\begin{pmatrix} u & c \\ d' & s' \end{pmatrix}_L \quad \begin{pmatrix} t & v \\ b & h \end{pmatrix}_R$ $\begin{pmatrix} \nu_e & \nu_\mu \\ e^- & \mu^- \end{pmatrix}_L \quad \begin{pmatrix} N_E & N_M \\ E^- & M^- \end{pmatrix}_R$	$(t_L, u_R), (v_L, c_R)$ $(b_L, d_R), (h_L, s_R)$ $(N_{e_L}, \nu_{e_R}), (N_{\mu_L}, \nu_{\mu_R})$ $(\bar{E}_L^-, \bar{e}_R^-), (\bar{M}_L^-, \bar{\mu}_R^-)$	Possible new mixing angles not displayed.	Same as in B.	$\frac{1}{2} \bar{\nu}\nu - \left(\frac{1}{2} - z\right) \bar{e}e_L + z (\bar{e}e)_R$ $+ \left(\frac{1}{2} - \frac{2}{3}z\right) \bar{u}u_L - \frac{2}{3}z \bar{u}u_R - \left(\frac{1}{2} - \frac{1}{3}z\right) \bar{d}d_L + \frac{1}{3}z \bar{d}d_R$ $+ \dots$ (For light fermions, same as in Salam-Weinberg model).