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NEUTRAL AND WEAK CURRENTS IN e^+e^- ANNIHILATION TO HADRONS

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ABSTRACT

In e^+e^- annihilation processes the interference term between the electromagnetic current and any other neutral currents present provides important information about the nature of the neutral currents. We compute the interference term for the semileptonic inclusive processes $e^+e^- \rightarrow h(p, s)X$ and $e^+e^- \rightarrow h_1(p_1)h_2(p_2)X$ and for the semileptonic exclusive processes $e^+e^- \rightarrow \pi(p_1)\pi(p_2)$ and $e^+e^- \rightarrow \pi(p_1)\pi(p_2)\pi(p_3)$ where the symbols h , p , s , and X (with or without subscripts) denote hadron, momentum, spin and "anything". We consider what information may be obtained from each of these channels with particular reference to the possibilities for determining the space-time nature of the neutral current. The observable strength of the weak interactions and the optimum beam energy are estimated.

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1. - INTRODUCTION

Although neutral currents have been observed ¹⁾ at a level comparable with charged weak currents, very little can at present be said about their space-time and internal symmetry properties. This is because the observation of neutral currents has so far been possible only in neutrino induced reactions, for example $\nu + N \rightarrow \nu + X$ and $\bar{\nu}_\mu + e \rightarrow \bar{\nu}_\mu + e$. In contrast to the charged current case no decay process appears capable of providing the necessary information.

Several authors ^{*)} have discussed the problem of deducing the space-time properties of neutral currents in neutrino reactions. Of course, if a two-component neutrino picture is true no scalar (S) or pseudoscalar (P) type couplings are possible at the neutrino vertex. Even allowing S, P type couplings at the neutrino vertex an ambiguity arises since the final state neutrino is not observed. If this should be different from the initial neutrino then vector (V) and axial vector (A) type couplings alone can simulate ⁵⁾ some effects which would otherwise require S, P or tensor (T) couplings. Thus the search for neutral current effects in processes not involving neutrinos is very important ⁶⁾, since in this case there is no doubt about the nature of the interacting leptons and S or P type couplings are, in principle, allowed.

With this in mind it seems useful to consider other reactions in which neutral current effects may manifest themselves and to determine what we can hope to learn of the properties of the neutral current from each reaction. Much has been done ⁷⁾ in discussing neutral current effects in e^+e^- annihilation reactions ⁸⁾⁻¹⁷⁾ but they have concentrated on V and A forms of coupling ^{**)}. However, no firm experimental evidence exists for or against this form of neutral current coupling and in this paper we include the other possible forms S, P and T.

^{*)} For example, Refs. 2), 3), 4). Our list of references is meant to be only illustrative but not complete.

^{**)} The types S and P were considered in Ref. 14) for some exclusive final states in e^+e^- annihilation. Also see our remarks (Section 5) about some recent work in this direction.

In a short note ⁶⁾ we have already discussed the analysis of the interference term between the electromagnetic and weak currents in the process $e^+e^- \rightarrow h(p)X$ where the electron and/or the positron are polarized but the polarization of the hadron h is not observed. There are several advantages in looking at this process. Firstly since the leptons interacting are electron and positron there is no ambiguity of the type mentioned above for processes involving neutrinos. Secondly it turns out that S, P and T type currents contribute only to order m_μ/E in the process $e^+e^- \rightarrow \mu^+\mu^-$ (m_μ is the mass of the muon and E is the beam energy), when the muon polarization is not observed. This follows (in the limit $n=0$) because S, P and T couplings produce μ^+ and μ^- with the same helicity while the electromagnetic V coupling creates them with opposite helicities. The final muon state from the S, P or T coupling is therefore orthogonal so that formed by the electromagnetic coupling and the interference between them vanishes. To detect S, P and T currents in annihilation processes one must either measure the muon polarization or one must consider other final states. The reaction $e^+e^- \rightarrow h(p)X$ is the simplest alternative. It also gives information on the hadronic coupling of the neutral current.

A general advantage ⁶⁾ of looking for neutral current effects in e^+e^- annihilation processes arises since the electromagnetic current is always present, and has a known structure. Since for presently accessible energies we expect the neutral current electromagnetic interference term to dominate over the pure neutral current term (which we subsequently drop) we are looking for effects due to interference of S, P, V, A, T type couplings with the pure V type electromagnetic term. This allows us to derive simple and definite tests for the space-time nature of the neutral current.

Finally it should be remembered ¹³⁾ that the neutral currents observed in neutrino induced reactions may not be equally coupled to other reactions. For instance suppose its strength be changed by a factor D for e^+e^- and for hadrons, but changed by a factor $1/D$ for $\bar{\nu}\nu$. Then if in neutrino induced hadronic reactions the neutral current effect appears at order 1, in e^+e^- annihilation to hadrons it appears at order D^2 . D large then gives large neutral current effects in annihilation to hadrons. There may also be neutral currents in addition to those already seen which do not couple appreciably to the neutrino channel.

This rather prolonged discussion should serve, we hope, to stress the importance of looking at all possible channels for neutral current effects.

The organisation of the paper is as follows. Section 2 is devoted to a semi-quantitative study of the optimum conditions necessary for observing the effects of weak neutral currents in e^+e^- annihilation, and to some general properties⁶⁾ of the effective Lagrangian which is assumed to respect lepton locality. We also consider the problem of separating neutral current effects from effects generated by the possible weak decays of (charged) hadrons or heavy leptons. We give in Section 3.1 more details than Ref. 6) of the interference term in $e^+e^- \rightarrow h(p)X$. In Section 3.2 we extend the analysis to the case $e^+e^- \rightarrow h(p,s)X$ when the hadron polarization is also measured (an interesting example would be $h = \Lambda$ or Σ baryons). Then in Section 3.3 we consider the inclusive channel $e^+e^- \rightarrow h_1(p_1)h_2(p_2)X$ where the momenta of the hadrons h_1 and h_2 are measured. In each case we discuss what may be learnt in addition by going from $e^+e^- \rightarrow hX$ to the more detailed final state. Of particular interest here is the following circumstance. Because of the limited kinematics of $e^+e^- \rightarrow h(p)X$, the electromagnetic weak interference term can reveal⁶⁾ only some of the possible components of the neutral weak current. Components of this weak current having complementary transformation properties under discrete symmetries (our emphasis is on space reflection) can be studied through the electromagnetic weak interference term in $e^+e^- \rightarrow h(p,s)X$ and $e^+e^- \rightarrow h_1(p_1)h_2(p_2)X$. The components which could be already studied in $e^+e^- \rightarrow h(p)X$ can of course also be studied in the other reactions, and often with greater experimental ease because of the extended kinematics of the more detailed final states. These questions are discussed further in Section 3. We apply in Section 4 a similar analysis to the simplest exclusive channels $e^+e^- \rightarrow \pi\pi, \pi\pi\pi$. A summary of the main points is given in Section 5, together with some comments on some recent literature on the subject. The construction and details of the hadronic structure functions contributing in the various processes, and the computation of the cross-section in terms of these structure functions is given in the Appendix and Tables 1 and 2. The Appendix applies directly to $e^+e^- \rightarrow h_1(p_1)h_2(p_2)X$, but modifications needed in going over to the other inclusive and exclusive final states are indicated in the text. Our emphasis is on the electromagnetic weak interference term, but the pure photon term can be easily obtained from it.

2. - NEUTRAL CURRENT EFFECTS IN e^+e^- ANNIHILATION

2.1. - Optimum conditions for observing the effects

We are interested in neutral currents which couple to the e^+e^- vertex. Such neutral currents will produce effects in Bhabha scattering $g-2$, etc., and to be compatible with experimental data from these processes, the possible magnitude of the neutral current coupling to leptons will be bounded. For V or A type coupling of the neutral current, bounds have been obtained ¹³⁾ which (roughly) say that the effective V (or A) coupling to the e^+e^- vertex should be less than $15\sqrt{G_F}$, where G_F is the Fermi constant.

In neutrino induced reactions the observed neutral weak current, in a V, A framework, is of order G_F in the amplitude. For equal coupling to all reactions ($D=1$ above) this implies a coupling at the e^+e^- vertex of $\sqrt{G_F}$. For other values of D this may be arbitrarily enhanced or suppressed.

With a coupling of order $\lesssim G_F$ the weak electromagnetic interference term will dominate the purely weak term except at very high energies, or near the mass of the new intermediate boson (if it exists) mediating the neutral current. We therefore ignore the pure weak neutral current term in the following analysis.

Which final states are interesting in e^+e^- annihilation ? Clearly the difficulty in observing an exclusive channel lies in the expected counting rate. In order to enhance the ratio of the weak electromagnetic interference term to the purely electromagnetic background we need to go to high energies to suppress the photon term via its propagator. (It is assumed the mass in the neutral current propagator is large compared to the centre-of-mass energy.) However, we expect the exclusive channels to have a form factor suppression at high energies both for the electromagnetic and neutral current amplitudes. Therefore for a high absolute rate in the interference term we would like as low an energy as possible. Can these two aims be reconciled to yield useful results ?

Let us try to make a quantitative estimate. Let g_l be the dimensionless coupling of the neutral current to e^+e^- and g_h the dimensionless coupling to a particular hadronic channel (h.c.). Then for the interference and pure electromagnetic terms we have

$$\sigma_{e^+e^- \rightarrow h.c.}^{\text{Interference}} \approx \frac{g_l g_h}{(-q^2 + M_Z^2)} \frac{e^2 m_p^4}{q^2 (q^2 + m_p^2)^2} q^2 \quad (1a)$$

$$\sigma_{e^+e^- \rightarrow h.c.}^{\text{E.M.}} \approx \frac{e^4}{(q^2)^2} \frac{m_p^4}{(q^2 + m_p^2)^2} q^2 \quad (1b)$$

where e is the electron charge, $q^2 = 4E^2$ and we have chosen the form factors to be proportional to $(m_p^2/(q^2 + m_p^2))$; M_Z and m_p are the masses of the neutral intermediate boson, Z , and the proton, respectively. We have assumed here that the only energy dependence arises due to the propagators and the chosen form factors.

The ratio of these terms, r_1 , is

$$r_1 = \frac{\sigma_{e^+e^- \rightarrow h.c.}^{\text{Interference}}}{\sigma_{e^+e^- \rightarrow h.c.}^{\text{E.M.}}} \approx \frac{g_l g_h}{M_Z^2} \frac{q^2}{e^2} \quad (2)$$

The number of events observed from the interference term, n_1 , depends on the luminosity $\mathcal{L}_1$ and the time $\mathcal{T}$ of the experiment

$$n_1 = \sigma_{e^+e^- \rightarrow h.c.}^{\text{Interference}} \times \mathcal{L}_1 \times \mathcal{T} \quad (3)$$

To estimate the observable coupling strength, write $(g_l g_h / M_Z^2) = a_1 G_F$ and solve these equations :

$$q^2 = \left[\frac{n_1}{\mathcal{L}_1 \mathcal{T}} e^4 m_p^4 \right]^{\frac{1}{3}} \quad (4)$$

and

$$a_1 = \frac{e^2}{q^2} \frac{r_1}{G_F} \quad (5)$$

We choose $\mathcal{L}_1$ optimistically as $10^{32} \text{ cm}^{-2} \text{ sec}^{-1}$ and $\mathcal{T} = 100$ days, for all numerical estimates in this subsection.

Trying not to be too demanding on experimental accuracy, we take $r_1 = 0.1$ and $n_1 = 100$. Then the above equations suggest an optimum value of $q^2 = 140 \text{ GeV}^2$ to observe an $a_1 = 6.5$. Clearly the parameters we have chosen are rough and our form factors have a somewhat arbitrary form. However, these considerations suggest it is difficult to see effects of order G_F in exclusive channels, even at high energies.

We might also ask what can we hope to see at low energies. Taking $q^2 = 1 \text{ GeV}^2$ in Eq. (3) we have

$$a_1 \approx \frac{n_1}{12 \times 10^5} \quad (6a)$$

$$r_1 \approx \frac{n_1}{12 \times 10^9} \quad (6b)$$

For $r_1 = 0.001$, corresponding to a signal to background ratio of only 0.1%, we find $n_1 = 1.2 \times 10^7$ and $a_1 = 10$, so it would appear barely possible, even in view of our crude approximations, to see effects of order G_F .

It is possible, however, that neutral current effects in annihilation are enhanced by a factor D^2 relative to neutrino induced reactions and for D^2 large ($\gtrsim 10$) then exclusive channels would be capable of yielding useful information about neutral currents. Exclusive channels do have the advantage that they can give information about the internal symmetry properties of the neutral weak current. In Section 4 we discuss the exclusive channels $e^+e^- \rightarrow \pi\pi$, and $\pi\pi\pi$ in detail.

If we wish to avoid the small cross-section due to the presence of a form factor we must go to inclusive channels. There we believe form factors are not present. Indeed at presently available energies the ratio of hadron cross-section to muon pair production in e^+e^- annihilation is ~ 5 , so inclusive cross-sections are of the order of the point-like $\mu^+\mu^-$ cross-section at these energies.

To estimate the magnitude of neutral currents that may be observed in the case of inclusive channels we may use Eqs. (1) without the form factors. This assumes no energy dependence in the amplitude for weak or electromagnetic terms apart from propagator effects. (For space-like q^2 in $\nu p \rightarrow \nu X$ this assumption appears to be compatible with the data.) Then the ratio of the interference term to the electromagnetic term is unchanged but the absolute number of events is enhanced by a factor $(q^2 + m_p^2)^2 / m_p^4$ relative to Eq. (1a). Then n_1 is independent of q^2 and we have

$$a_1 = \frac{n_1}{L_1 J G_F e^2} \quad (7a)$$

$$r_1 = a_1 G_F \frac{q^2}{e^2} \quad (7b)$$

With $q^2 = 900 \text{ GeV}^2$, corresponding to the next generation of e^+e^- machines, and for $a_1 = 1$, then $r_1 = 0.1$. Thus we may expect 3×10^5 events coming from the interference term. We conclude that inclusive channels seem capable of giving useful information about $O(G_F)$ neutral currents at q^2 around 900 GeV^2 .

2.2. - The effective Lagrangian

In order to discuss semi-leptonic processes we work with an effective Lagrangian for the weak neutral current coupling the electron field ψ to hadrons given by

$$\begin{aligned} \mathcal{L} = & g_S \bar{\psi} \psi J^1 + i g_P \bar{\psi} \gamma_5 \psi J^2 + g_V \bar{\psi} \gamma^\mu \psi J_\mu^3 \\ & + g_A \bar{\psi} \gamma^\mu \gamma_5 \psi J_\mu^4 + g_T \bar{\psi} \sigma^{\mu\nu} \psi J_{\mu\nu}^5 \end{aligned} \quad (8)$$

where J^1 and J^2 may contain scalar and/or pseudoscalar parts, J_μ^3 and J_μ^4 may contain vector and/or axial vector parts, and $J_{\mu\nu}^5$ transforms as a tensor and/or a pseudotensor. The currents J^i are Hermitian and the couplings g_i real.

In computing the cross-section arising from the interference between the neutral current and the photon we have the structure

$$\begin{aligned} & \text{Tr} [\not{p}_+ (1 + \gamma_5 \not{B}_+) \gamma_\mu \not{p}_- (1 + \gamma_5 \not{B}_-) \Gamma_i] (\text{Hadronic vertex})^{\mu, i} \\ & = \text{Tr} [(1 - \gamma_5 \not{h}_+ + \gamma_5 \not{s}_+) \not{p}_+ \gamma_\mu (1 + \gamma_5 \not{h}_- + \gamma_5 \not{s}_-) \not{p}_- \Gamma_i] (\text{Hadronic vertex})^{\mu, i} \end{aligned} \quad (9)$$

where the polarization four vectors $B_\pm$ are written as

$$B_\pm^\mu = \frac{1}{m_e} (h_\pm p_\pm^\mu) + s_\pm^\mu + O(m_e) \quad (10)$$

and the momentum, longitudinal polarization and transverse polarization of the electron and positron are p_- , h_- , s_- and p_+ , h_+ , s_+ , respectively, and Γ_i can be the leptonic S, P, V, A, T couplings. Here, and throughout, we have dropped terms of order (m_e/E) . The decomposition (10) will be used for $\vec{p}_+ = -\vec{p}_- = -E\hat{Z}$ where $\hat{Z}$ is a unit vector along the Z axis. $\vec{s}_\pm$ lie in the XY plane. Note $-p_\pm \cdot s_\pm \equiv \vec{p}_\pm \cdot \vec{s}_\pm = 0$. The electron mass was denoted by m_e .

As noted in Ref. 6), the form of the leptonic tensor as given in Eq. (9) means that, irrespective of any details of the hadron coupling, for S, P and T only terms proportional to the transverse polarization (with or without a multiplicative longitudinal polarization) contribute. For V and A, only terms independent of (with or without multiplicative longitudinal polarizations) or quadratically dependent on transverse $e^\pm$ polarization contribute. Thus observation, in any hadronic channel, of effects linearly proportional to a transverse positron or electron polarization would be unambiguous evidence for the presence of S, P or T type coupling. This test becomes very valuable⁶⁾ because no observation of some other characteristic weak effect such as parity violation is necessary. In contrast characteristic weak interaction signatures are needed for the coupling types V and A in order to associate an observed effect with a neutral weak current.

The situation is further confused in the V, A case by the possibility that some of the states (charm?) produced via one photon exchange in e^+e^- annihilation may decay weakly giving rise to parity violating

effects. Thus the electromagnetic production and semi-leptonic weak decay of a heavy lepton could contribute to our inclusive cross-sections. Similarly the electromagnetic production and weak decay of a charmed hadron could also give rise to parity violating effects in our inclusive cross-sections. How can we distinguish such effects from neutral current effects ?

One immediate difference arises due to the different propagators involved in the two cases. The neutral current electromagnetic interference term involves only one photon propagator and one Z propagator. For $q^2 \ll M_Z^2$ the interference cross-section $\propto$ constant. In contrast the electromagnetic production of charmed states or heavy leptons involves two photon propagators and the cross-section $\propto 1/q^2$, cf. Eq. (7b). Observation of one or other of these q^2 dependences in a parity violating term will clearly signal whether it comes from a (parity violating) neutral current or from the weak decay of an electromagnetically produced state.

A second difference arises since the parity violation occurring in the weak decay of an electromagnetically produced state occurs at the hadronic end. Thus only those parity violating kinematic terms which arise from a parity violating hadronic "vertex" can occur. In contrast, neutral currents may produce parity violating effects at either the leptonic vertex or the hadronic "vertex". Thus in the latter case there will be more possible kinematic forms which signal parity violation and observation of one of these, which can only arise due to parity violation at the leptonic end, will clearly signal a neutral current effect, even without observation of the propagator effect. We will construct the hadronic "vertex" in the Appendix and the subsequent sections, and it will be clear which parity violating terms correspond to parity violation at the leptonic vertex or at the hadronic "vertex". For example, in Table 2, we give the contributions to the cross-section as defined in the Appendix for the process $e^+e^- \rightarrow h(p)X$. The entry for $i = V, A$ gives the neutral current electromagnetic interference term for V or A coupling of the neutral current. Any terms with g_A correspond to axial coupling at the leptonic vertex and cannot occur through the electromagnetic production and weak decay of some state. Thus the only parity violating term that can occur for a non-neutral current effect is the term associated with $\text{Im}[g_V W_3]$. It corresponds to the hadronic "vertex" $\epsilon_{\mu\nu\alpha\beta} q^\alpha p^\beta$ (here q is the total centre-of-mass four momentum) which is indeed odd under space reflection. The three other kinematic forms which are odd under space reflection can only arise through a neutral current electromagnetic interference with axial coupling at the leptonic vertex.

We will not consider the possible effects of weak decays of electromagnetically produced states further except to note that their possible contribution to the cross-section may be read off from our Tables by, as above, looking at the entries $i = V, A$ and setting $g_A = 0$. Whether specific produced states can produce these terms requires further analysis ; see Ref. 18).

In order to compute the interference term we need to know the hadronic "vertex". The general form for this is given in the Appendix for the case that two four vectors in addition to the centre-of-mass momentum are observed in the final hadron state (two final state momenta or one momentum and a polarization vector). In Table 1 the cross-section is given for the general case. In the next sections we consider the form of this cross-section for the specific channels mentioned above.

3. - INCLUSIVE HADRON PROCESSES

3.1. - $e^+e^- \rightarrow h(p)X$

In this section we are interested in the case where only one hadron is observed and only its momentum p is measured. We will consider all possible polarizations of the electron positron system.

Since there are only two independent vectors at the hadron vertex, namely the centre-of-mass momentum q_μ and the hadron momentum p_μ , not all the tensors defined in the Appendix are needed. The ones that remain are :

(i) for $i = S, P$

$$H_\mu^i(p, q) = f(q^2, p \cdot q) p_\mu \quad (11)$$

(ii) for $i = V, A$

$$H_{\mu\nu}^i(p, q) = -g_{\mu\nu} W_1(q^2, p \cdot q) + p_\mu p_\nu W_2(q^2, p \cdot q) + \epsilon_{\mu\nu\alpha\beta} q^\alpha p^\beta W_3(q^2, p \cdot q) \quad (12)$$

(iii) for $i = T$

$$\begin{aligned}
 H_{\mu\rho\tau}^i(b,q) = & (q_\rho b_\tau - q_\tau b_\rho) b_\mu F_1(q^2, b \cdot q) \\
 & + (g_{\rho\mu} b_\tau - g_{\tau\mu} b_\rho) F_2(q^2, b \cdot q) \\
 & + \epsilon_{\rho\tau\alpha\beta} b^\alpha q^\beta b_\mu P F_1(q^2, b \cdot q) \\
 & + \epsilon_{\rho\tau\mu\alpha} b^\alpha P F_2(q^2, b \cdot q)
 \end{aligned} \tag{13}$$

where terms that contribute only to order (m_e/E) have been dropped. In Eq. (13) note that terms of the form $\epsilon_{\mu\rho\tau\alpha} q^\alpha$ and $g_{\mu\rho} q_\tau - g_{\mu\tau} q_\rho$ do not contribute as may be seen by applying the Dirac equation to the leptonic vertex after contracting it on the hadronic tensor. Note also that, as explained further in the Appendix, the electromagnetic index μ is conserved in Eq. (11), (12) and (13), up to order (m_e/E) .

Thus there are only three structure functions necessary to describe the V, A coupling and four to describe T coupling. The cross-section, using Eqs. (11), (12) and (13) may now be written in space terms of these structure functions and this is done in Table 2. The various terms in Table 2 may be distinguished by their different angular dependences and their dependence on the initial polarization. This we have discussed in detail in Ref. 6) and we merely stress some points here.

(A) For S, P or T types no observation of a characteristic weak effect ⁶⁾ (e.g., parity violation) is necessary to distinguish it from electromagnetic terms. For V, A types one must isolate non-electromagnetic effects by a characteristic weak effect. See, however, the remarks at the end of Section 2 concerning possible ambiguities in this characteristic signal due to the electromagnetic production of high mass leptonic or hadronic states which predominantly decay weakly.

(B) The behaviour of the various terms under parity inversion is shown in Table 2. For S or P coupling only scalars, but not pseudoscalars can be formed out of the available hadronic momenta. In this case some of the possible components of neutral weak currents cannot be observed, namely those with pseudoscalar couplings at the hadronic vertex. To look for these currents we must go to a more detailed final state (see the next subsection). For V, A and T couplings both scalar and pseudoscalar observables arise, allowing a study of both parity violating and parity conserving neutral currents for these coupling types. However, to look for all these terms it is necessary to have many polarization configurations at the leptonic vertex. It may be more convenient experimentally to look at a more detailed hadronic channel with a more restricted lepton polarization range (see next section).

(C) The charge conjugation (C) behaviour of the various terms depends on the final state. If h is its own antiparticle $\bar{h}$ then behaviour under C is given by the substitution $+$ $\rightleftharpoons$ $-$ in the subscripts of the leptonic variables. Since the hadronic form factors depend only on C even quantities, the C properties are those of the kinematics given in Table 2.

If $h \neq \bar{h}$ the terms given in Table 2 do not correspond to definite C. Under C the form factor F_h for h becomes the form factor $F_{\bar{h}}$ for $\bar{h}$. In order to project out contributions with definite C, we consider the cross-section combinations

$$\begin{aligned} \sigma_{\pm} &= \sigma(e^-(c_1) + e^+(c_2) \rightarrow h(p)x) \\ &\quad \pm \sigma(e^-(c_1) + e^+(c_2) \rightarrow \bar{h}(p)x) \end{aligned} \quad (14)$$

where c_1 and c_2 denote all characteristics of the e^- and e^+ , respectively. Then we form $\bar{\sigma}_{\pm}$ by

$$\begin{aligned} \bar{\sigma}_{\pm} &= \sigma(e^+(c_1) + e^-(c_2) \rightarrow \bar{h}(p)x) \\ &\quad \pm \sigma(e^+(c_1) + e^-(c_2) \rightarrow h(p)x) \end{aligned} \quad (15)$$

If we denote by A_1 and A_2 the kinematic terms called K^i in Eq. (A.7) and Tables 1 and 2 which are antisymmetric and symmetric, respectively, under the replacement $+$ $\rightleftharpoons$ $-$ in the subscripts of the leptonic variables, then these cross-sections may be symbolically written as the product of the form factors and the kinematic parts

$$\sigma_{\pm} = (F_L \pm F_R)(A_1 + A_2) \quad (16)$$

$$\bar{\sigma}_{\pm} = (F_R \pm F_L)(-A_1 + A_2) \quad (17)$$

C invariance requires $\sigma_{\pm} = \bar{\sigma}_{\pm}$ so we see that the A_1 terms in $\sigma_{\pm}$ are C odd, the A_2 terms in $\sigma_{\pm}$ are C even. For σ_{-} the situation is reversed. In Table 2 we list the C properties of the various kinematical forms for $\sigma_{\pm}$.

(D) The behaviour of the various terms in Table 2 under time reversal cannot be predicted directly. This is because final state interactions introduce unknown phases. However, if we consider the cross-sections $\sigma_{\pm}$ defined above, since the various terms contributing to them have definite properties under space reflection and C, we may, assuming CTP invariance, deduce their behaviour under time reversal. Without making assumptions we have no reason to suppose that time reversal non-invariant terms in the neutral current are smaller than the invariant terms and we throughout keep both types.

(E) A further classification of hadronic neutral currents of types other than V or A has been proposed by Kingsley et al.³⁾ For the purpose of this classification they restrict the Lagrangian to be purely non-derivative and also time reversal invariant, and they call "first rank" those currents which may be constructed from a single quark multiplet. The others are called "second rank" currents. We note that first rank S, P or T currents would contribute only to a parity even part of the Lagrangian in Eq. (8). This follows since restriction to a single quark multiplet implies that the S and P (similarly the tensor and pseudotensor) hadronic currents have the same charge conjugation properties as their leptonic counterparts. Then from CPT invariance, time reversal invariance of the interaction Lagrangian implies parity invariance; see also Table 3.

Similarly we may show that second rank S, P or T currents correspond to net parity violation in the Lagrangian. Thus isolating the first and second rank currents in annihilation processes corresponds just to looking for parity even and odd terms, respectively, when the coupling is S, P

or T. Then the points noted in (A) and (B) above apply. Also, of course, contributions which arise from the derivatives at the hadron vertex in our analysis are not covered by the rank classification.

However, since there is no a priori reason to assume that time reversal non-invariant currents are absent this classification scheme is somewhat speculative. In this connection we note that if Wolfenstein's suggestion¹⁹⁾ of non-diagonal neutral currents is applied to a non-derivative current-current Lagrangian (as is presupposed in the above classification of Kingsley et al.³⁾), one obtains time reversal non-invariant currents in neutrino induced processes at the same level as the invariant ones, even if one restricts to a single quark multiplet and to V, A currents.

It is important to note that Kingsley et al.'s classification³⁾ into first and second rank interactions is made on the basis of quark operators used in a purely non-derivative Lagrangian. Such a Lagrangian could be applied to deep inelastic neutral current scattering of neutrinos by hadrons in constituent models like the quark parton model if neutral current interactions were to scale. Present data are too meagre to provide conclusive evidence for or against such a scaling. Our analysis is not restricted to purely non-derivative hadronic couplings nor indeed to a quark model ; we allow the most general possible hadronic couplings.

Though, as mentioned in point (D) above, independent tests of time reversal invariance are difficult to perform in practice, it is useful to distinguish Kingsley et al.'s classification into first and second rank interactions from (a) their classification into diagonal and non-diagonal quark operators, and (b) their classification into time-reversal invariant and non-invariant interactions ; all three classifications are different. For the types S, P and T in their non-derivative neutrino quark Lagrangian, Table 3 shows the behaviour of the four possible contributions to the Lagrangian under space reflection, charge conjugation and time reversal, for a given neutrino coupling (for example $\nu\bar{\nu}$). Wolfenstein's "non-diagonality" refers to the neutrino end^{5),19)}, but the "non-diagonality" in Table 3 refers to only the quark end³⁾ and only a single neutrino field is involved as in Kingsley et al.'s discussion. The diagonality column is seen to be only a statement of the charge conjugation behaviour ; the rank becomes second due to non-diagonality or due to time reversal non-invariance. For the types V and A the diagonal and non-diagonal structures get interchanged for the last two rows in this Table. Then the non-diagonal quark structures correspond to time reversal non-invariant terms in the Lagrangian and are usually ignored.

It is now seen that the classifications corresponding to the last two columns of Table 3 make strong use of the non-derivative nature of the lepton quark Lagrangian and do not apply to the most general hadronic vertex.

3.2. - $e^+e^- \rightarrow h(p,s)X$

In this section we extend the analysis to include the case that the observed hadron's polarization is measured. This could be done if $h(p,s)$ were a spin $\frac{1}{2}$ hadron such as Λ , Σ , Ξ whose polarization may be measured via its decay products. In this case not all the form factors found in the Appendix contribute since the polarization vector s_μ can, at most, appear linearly. a_μ and b_μ in the Appendix are now p_μ and s_μ . Moreover the dependence of the form factors on the only invariant, $q \cdot s$ involving s_μ can also be at most linear. This does not necessarily mean that the total number of contributing structure functions is less than that for the channel $h_1 h_2 X$ considered in the Appendix. For example, for a V, A theory the number is eleven for both the channels $h_1 h_2 X$ and $h(p,s)X$. While the structure functions W_2^{ss} , W_2^{nn} and W_2^{sn} of Eq. (A.3) do not contribute for the channel $h(p,s)X$ because these require more than linear dependence on s_μ , there are effectively three new structure functions because

$$W_i \rightarrow W_i + (q \cdot s) W_i'$$

The new structure functions W_i' are needed for the choices $W_1 = W_1$, W_2^{pp} and W_3^{pq} of Eq. (A.3) of the Appendix. Similarly, the total number of contributing structure functions in Eq. (18) below is even more (four) than that for $h_1 h_2 X$ (three) for an S, P theory. But in all cases the kinematical structure of the interference term can be deduced from Table 1.

If we display the dependence on s explicitly then the form factors for example for the case $i = S, P$ are

$$\begin{aligned} H_\mu^i(p, q, s) = & \left[f^i(q^2, p \cdot q) + q \cdot s f'^i(q^2, p \cdot q) \right] p_\mu \\ & + f^s(q^2, p \cdot q) s_\mu + f^n(q^2, p \cdot q) n_\mu \end{aligned} \quad (18)$$

where $n_\mu = \epsilon_{\mu\alpha\beta\gamma} p^\alpha s^\beta q^\gamma$.

It may be seen that the cross-section resulting from these new terms also may be deduced from Table 2. Firstly $f(q^2, p, q)$ of Table 2 must be replaced by $f^p(q^2, p, q) + q \cdot s f'^p(q^2, p, q)$ and secondly new terms arise with $f(q^2, p, q)$ replaced by $f^s(q^2, p, q)$ [or $f^n(q^2, p, q)$] and p replaced by s (or n). Both these replacements are needed for all the types S, P, V, A and T of the weak current. Note there is a difference with respect to the channel $h_1(p_1)h_2(p_2)X$ treated in the Appendix and discussed in Section 3.3 below. The pseudovector in the present case is $\vec{s}$ and the vectors are $\vec{p}, \vec{q}$ and $\vec{n}$; for $h_1(p_1)h_2(p_2)X$ the pseudovector is $\vec{n}$ and the vectors are $\vec{p}_1, \vec{p}_2$ and $\vec{q}$.

The channel $h(p, s)X$ provides valuable information on the discrete symmetry properties of the weak neutral currents. As mentioned in point (D) of the preceding subsection, it is not in general useful to consider time reversal and we do not consider it further. For charge conjugation, the channel $h(p, s)X$ can be analyzed in the same way as $h(p)X$ as in point (C) of the preceding subsection: one just replaces p by the variables (p, s) . For space reflection, the channel $h(p, s)X$ provides more opportunities as compared to $h(p)X$ because a pseudovector ($\vec{s}$) is now available. We now consider examples of this.

For leptonic coupling types S and P , the only contributions to the weak electromagnetic interference term in $e^+e^- \rightarrow h(p)X$ are respectively parity conserving and parity violating, corresponding to the term f^p of Eq. (18). The contributions associated with the form factor f^s have the opposite parity properties. The complete set can thus be investigated in $e^+e^- \rightarrow h(p, s)X$. Similarly the parity properties of the effects associated with the form factor f'^p are opposite to those associated with f^p and f^n because the extra factor $q \cdot s$ is a pseudoscalar. Structure functions like f'^p which differ from the corresponding form factor f^p [which itself already appears in $e^+e^- \rightarrow h(p)X$] only through the new factor $q \cdot s$ of course appear for all possible leptonic coupling types; the possibility of studying weak currents having parity properties opposite to those which already couple in $e^+e^- \rightarrow h(p)X$ through these new structure functions therefore exist for all coupling types S, P, V, A and T .

Because of their different angular dependences, effects associated with f^n provide new opportunities to study weak neutral currents having the parity properties associated with f^p which could already be studied in $e^+e^- \rightarrow h(p)X$.

For a V leptonic coupling, a net parity violating effect requires e^+ and/or e^- longitudinal polarization in $e^+e^- \rightarrow h(p)X$ (see Table 2). In $e^+e^- \rightarrow h(p,s)X$, the leptonic V coupling gives a parity violating effect even with unpolarized $e^\pm$, providing an experimentally more convenient probe for an effective axial vector coupling at the hadron vertex. The structure function in question is W_2 of Eq. (A.3) with $r_\mu = s_\mu$, $t_\nu = p_\nu$. Similarly for a leptonic A coupling, observation of a net parity violation (i.e., an effective vector coupling at the hadron end) requires polarized e^+ and/or e^- beams in $e^+e^- \rightarrow h(p)X$ but can be studied with unpolarized $e^\pm$ in $e^+e^- \rightarrow h(p,s)X$: this time the relevant structure function is W_3 with, for example, $u^\alpha = s^\alpha$ and $r^\beta = p^\beta$ or q^β in Eq. (A.3). Of course, effects with unpolarized $e^\pm$ are not ⁶⁾ possible for S, P, T couplings so that observation of these effects could establish the presence of the appropriate V, A component of the neutral weak interaction (modulo the earlier remarks concerning parity odd effects arising from electromagnetic production and weak decay of "new" states), but not shed light on possible S, P, T couplings. Similar examples involving n_μ, s_μ and $q.s$ can be given in the T case to study weak currents with various parity properties. In this case, however, both parity odd and parity even terms already appear in $e^+e^- \rightarrow h(p)X$ (see Table 2). Of course, the components of neutral currents having parity properties which could be investigated in $e^+e^- \rightarrow h(p)X$ can also be studied in $e^+e^- \rightarrow h(p,s)X$ through an enlarged set of kinematical forms which can be deduced from Table 1 and the Appendix. We have only illustrated some advantages of $e^+e^- \rightarrow h(p,s)X$ in revealing new components of the weak currents and in studying parity violating terms (which are a good signal of weak interaction effects) more easily.

3.3. - $e^+e^- \rightarrow h_1(p_1)h_2(p_2)X$

For the case that the momenta p_1 and p_2 of the two final state hadrons are measured, all the form factors given in the Appendix may contribute. The cross-section in terms of these form factors is given in Table 1.

The addition of an extra vector at the hadronic vertex means we can define an axial vector $n_\mu = \epsilon_{\mu\alpha\beta\gamma} p_1^\alpha p_2^\beta q^\gamma$. However, unlike the case involving spin s in Section 3.2, this axial vector cannot form a purely hadronic parity odd invariant since

$$n \cdot p_1 = n \cdot p_2 = n \cdot q = 0 \quad (19)$$

and the generalization $W_1 \rightarrow W_1 + q.sW_1$ of the preceding subsection does not apply now. However, we may generate terms with opposite parity by substituting p by n in the entries of Table 2. Thus, for example, in the S, P_1 case

$$H_\mu(p_1, p_2, q) = f^{p_1} p_{1\mu} + f^{p_2} p_{2\mu} + f^n n_\mu \quad (20)$$

The terms corresponding to f^n will have parity properties opposite to those corresponding to f^{p_1} and f^{p_2} . Thus again we may look for components of the weak neutral current that do not contribute in the simpler process $e^+e^- \rightarrow h(p)X$. What terms these are has already been indicated in Section 3.2. The correspondence between the two cases $h(p,s)X$ and $h_1(p_1)h_2(p_2)X$ is that: (i) the pseudovectors in the two cases are $\vec{s}$ and $\vec{n}$, respectively; (ii) the vectors are $\vec{p}, \vec{q}, \vec{n}$ and $\vec{p}_1, \vec{q}, \vec{p}_2$, respectively; (iii) a_μ, b_μ of the Appendix are p_μ, s_μ and $p_{1\mu}, p_{2\mu}$, respectively. Of course, the resulting kinematic and angular structure of the corresponding terms in the two channels can be quite different for a neutral current contribution of a given parity behaviour; this could be an advantage.

For charge conjugation, a simple analysis like that of Eqs. (14) to (17) of subsection 3.1 can be given by considering $e^+e^- \rightarrow \bar{h}_1\bar{h}_2X$ along with $e^+e^- \rightarrow h_1h_2X$. The dependence of the form factors on $(p_1-p_2) \cdot q$ can then be seen to be restricted for special cases like $h_1 = h_2$ and $h_1 = \bar{h}_2$. In contrast to $e^+e^- \rightarrow h(p)X$ or $h(p,s)X$, the C properties are not determined in general by the kinematical forms because of the possible presence of odd powers of $q \cdot (p_1-p_2)$ in the form factors.

For V, A neutral currents, interesting comments have been made by Pais and Treiman¹⁶⁾. Their results all follow from our expressions in Table 1. However, our results are more complete and give the dependence of the experimental spectrum on all the variables including lepton polarization. The parity odd correlation $(\vec{p}_1 \times \vec{p}_2) \cdot \hat{K}$ where $2\vec{K} = \vec{p}_- - \vec{p}_+$ is interesting. Firstly it involves parity violation, a characteristic weak interaction effect. Secondly it involves no spin measurement, neither leptonic nor hadronic, an advantage over $e^+e^- \rightarrow h(p)X$, which needed polarized $e^\pm$ beams to produce a parity odd effect, or over $e^+e^- \rightarrow h(p,s)X$. The corresponding simplest exclusive final state is $\pi^+\pi^-\pi^0$ [see comment (2) following Eq. (22) below]. Thirdly this component only arises if the weak current has a V or A component.

purely S, P or T types cannot contribute ⁶⁾ for unpolarized beams. This particular term arises from either of the terms $\text{Re}(g_V^{rt})$ or $\text{Im}(g_A^{uv})$ or $\text{Im}(g_A^{w_4})$ in Table 1 for $r(t)=n$, $t(r)=p_1$ or p_2 ; $u(v)=n$, $v(u)=q$ or p_1 or p_2 ; $a=p_1$, $b=p_2$. These three terms give the form

$$(C - D_1 \hat{p}_1 \cdot \hat{K} - D_2 \hat{p}_2 \cdot \hat{K}) (\vec{p}_1 \times \vec{p}_2) \cdot \hat{K}$$

as found by Pais and Treiman ¹⁶⁾. The coefficient C is more interesting because it is associated with an A leptonic coupling, while the coefficients $D_{1,2}$ go with a V coupling. $D_{1,2}$ (but not C) can arise from the electromagnetic production and subsequent weak decay of "new" states, and so C is a characteristic weak neutral current effect.

4. - EXCLUSIVE HADRON PROCESSES

As mentioned above, it may be possible to observe the weak electromagnetic interference effects in exclusive hadronic channels if the effective coupling is sufficiently larger than G_F . We consider the simplest two exclusive channels.

4.1. - $e^+e^- \rightarrow \pi\pi$

The cross-section for this process is simply obtained from the cross-section for $e^+e^- \rightarrow hX$. Some simplifications occur relative to the inclusive case. The electromagnetic and neutral current amplitudes may be formed separately and the hadronic vertex expanded in terms of the momenta available. The full tensor determining the effective structure functions may then be formed by taking the product of the electromagnetic and neutral current amplitudes. However, since no spin or momenta have to be summed over there can be no mixing between the electromagnetic and neutral current Lorentz indices. Thus, for example, in the V, A case no $g_{\nu\mu}$ term is present for a $\pi\pi$ final state. Another possible simplification arises due to the constraints of PCT invariance. In the absence of hadronic final state interactions this means that certain of the form factors arising in Table 2 may be zero. When final state interactions are allowed these form

factors may become non-zero but they will be determined in terms of the other form factors via $\pi\pi$ phase shifts, provided inelastic channels are neglected. Above the inelastic threshold PCT alone imposes no constraints on the $\pi\pi$ channel. The C properties of the $\pi\pi$ state also simplify the results obtained for the inclusive channel hX . This is because the antiparticle of a pion is also a pion and therefore the C properties are easily stated. The C even channel $\pi^0\pi^0$ does not couple to the photon in lowest order ; thereby relatively enhancing the weak neutral current contribution. However, the interference term is also absent and we do not consider this channel further ; it is experimentally more difficult to detect than the $\pi^+\pi^-$ channel. For the $\pi^+\pi^-$ channel the C properties can be simply read from the cross-section expressions as we shall see.

A fourth simplification refers to the leptonic coupling type T, the J^5 term of Eq. (8), which reduces to the types S and P, up to order m_e/E , and apart from multiplicative factors $\vec{k}\cdot\vec{p}$ where $2\vec{k}$ and $2\vec{p}$ are the relative momenta in initial and final states. This happens because Lorentz indices for the electromagnetic and weak indices do not mix and is shown using the Dirac equation on the fermion states. Accordingly we drop T coupling for this channel.

In order to make these points more explicit, and to provide a contrast with the channel $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ discussed in the next section, we explicitly give the structure of the interference term for $e^+e^- \rightarrow \pi^+\pi^-$.

$$\begin{aligned}
 & -\vec{p}_- \cdot (\vec{p}_- \times (\vec{s}_+ + \vec{s}_-)) \Im \alpha_1 + E \vec{p}_- \cdot (\vec{s}_+ - \vec{s}_-) \Im \alpha_2 \\
 & + E \vec{p}_- \cdot (h_+ \vec{s}_- - h_- \vec{s}_+) \Re \alpha_1 + \vec{p}_- \cdot ((h_- \vec{s}_+ + h_+ \vec{s}_-) \times \vec{p}_-) \Re \alpha_2 \\
 & + \{ \vec{p}_- \cdot (\vec{s}_+ \times \vec{p}_-) \vec{p}_- \cdot \vec{s}_- + \vec{p}_- \cdot \vec{s}_+ \vec{p}_- \cdot (\vec{s}_- \times \vec{p}_-) \} E \Im \alpha_3 \\
 & + \{ (1 - h_- h_+ + \vec{s}_+ \cdot \vec{s}_-) (\vec{p}_- \cdot \vec{p}_- \vec{p}_- \cdot \vec{p}_- - \vec{p}_- \cdot \vec{p}_- \vec{p}_- \cdot \vec{p}_-) + 2 \vec{p}_- \cdot \vec{s}_- \vec{p}_- \cdot \vec{s}_+ E^2 \} \Re \alpha_4 \\
 & + (h_- - h_+) (\vec{p}_- \cdot \vec{p}_- \vec{p}_- \cdot \vec{p}_- - \vec{p}_- \cdot \vec{p}_- \vec{p}_- \cdot \vec{p}_-) \Re \alpha_3
 \end{aligned} \tag{21}$$

where the leptonic symbols are the same as for Table 2 ; p^μ is the π^+ four momentum ; the form factors $x_{1,2,3,4}$ arise from the interference of the photon with a neutral current of types S, P, A and V respectively at the leptonic vertex. The hadronic vertex in the S, P case is of type 1 and in the V, A case it is of type $(p-P)^\mu$, P^μ being the π^- four momentum. The C properties of the various terms follow because $\vec{p} + \vec{P} = 0$ and the vector $\vec{p}$ therefore only occurs in the C odd combination. If final state $\pi\pi$ interactions were absent then CPT makes the form factors x_1 , x_2 and x_3 real, and the terms associated with the form factors $\text{Im } x_1$, $\text{Im } x_2$, $\text{Im } x_3$ vanish. In the interacting case, below the inelastic threshold, these form factors are proportional to $\tan \delta_1 \text{Re } x_1$, $\tan \delta_2 \text{Re } x_2$, $\tan \delta_3 \text{Re } x_3$ where δ_i are the relevant elastic $\pi^+\pi^-$ phase shift differences at the appropriate energies. Thus the $\text{Im } x_3$ term only contributes above the inelastic threshold as only above the inelastic channel is there a possibility of different final state interactions in the weak and electromagnetic channels for the same $J^P = 1^-$ dipion state. The $\text{Im } x_1$, $\text{Im } x_2$, $\text{Im } x_3$ terms are C even and parity even, C even and parity odd, and C odd and parity odd, respectively.

The pure photon term has the same structure as the $\text{Re } x_4$ term. This is useful to determine which region of phase space is best to look for neutral current effects.

Though all except the $\text{Re } x_4$ term need some sort of lepton polarization this is not necessarily a disadvantage because of the natural polarization found due to synchrotron radiation. Special cases of Eq. (21) have appeared already ^{8),10,14)}. The comparison of Eq. (21) with Table 2 is obvious for the S, P terms. For the V, A terms the relevant terms in Table 2 are $\text{Im}\{g_A W_2\}$, $\text{Re}\{g_V W_2\}$ and $\text{Re}\{g_A W_2\}$ and correspond to $\text{Im } x_3$, $\text{Re } x_4$, and $\text{Re } x_3$, respectively.

4.2. - $e^+e^- \rightarrow 3\pi$

The cross-section for this process is directly obtainable from that for $e^+e^- \rightarrow h_1 h_2 X$ discussed in Subsection 3.3, with some simplifications analogous to those described in the preceding subsections.

The first simplification is the same as in Subsection 4.1. For the second simplification, the distinction between the interacting and non-interacting final state remains unchanged, but the parametrization of the

final state interactions in terms of "phase shifts" is not simple (even below the inelastic threshold) since now one is dealing with a three-particle system.

The third simplification of Subsection 4.1 again applies ; in this case the $3\pi^0$ channel does not couple ¹⁶⁾ in lowest order to the photon, and accordingly we do not consider this channel further. For the $\pi^+\pi^-\pi^0$ channel, if the two independent hadronic momenta are chosen as p^μ and P^μ for π^+ and π^- , respectively, the C properties of the kinematics can again be read directly because $p^\mu \rightleftharpoons P^\mu$ under charge conjugation.

The fourth simplification of Subsection 4.1 is partly modified. Amplitudes employing the hadronic momenta (p and q) or (P and q) and coupling like T again reduce to the S, P coupling types. However, when the hadronic vertex is formed using (p and P), the reduction does not occur because neither p nor P is expressible in terms only of the leptonic momenta. Thus there is now a genuinely new contribution arising from T coupling.

An important distinction relative to $e^+e^- \rightarrow \pi\pi$ arises because the pion is a pseudoscalar. Then the hadronic vertex in the one-photon amplitude has the form $\epsilon_{\mu\nu\alpha\beta} p^\nu q^\alpha P^\beta$, a space-like axial vector of the form $\vec{p} \times \vec{P}$ in the CM frame, to be contrasted with the vector $\vec{p}$ in the $\pi\pi$ case. One consequence of this difference is that analogues of the x_1 and x_2 terms of Eq. (21) now have parity properties opposite to those for the $\pi^+\pi^-$ channels, while the C properties are not changed because $\vec{p} \times \vec{P}$ is C odd as was $\vec{p}$ in the $\pi\pi$ case. Thus the 3π channel usefully supplements the $\pi\pi$ channel.

Another consequence of the one-photon amplitude having the hadronic vertex n_μ only, is a large reduction in the number of independent structure functions for the coupling types V, A, T. For the V, A case there are only three independent structure functions (corresponding to the three weak current vertices n_ν , p_ν and P_ν) as compared to seven expected from the W_2 and W_4 terms of Eq. (A.3).

To provide a comparison with $e^+e^- \rightarrow \pi^+\pi^-$, Eq. (21), we give the structure of the weak electromagnetic interference term for $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ for S, P, V and A coupling types. For the coupling type T see comment 4 at the end of this section.

$$\begin{aligned}
 & \left\{ \vec{n} \cdot [(\vec{s}_+ + \vec{s}_-) \times \vec{p}_-] \Im m X_1 + E \vec{n} \cdot (\vec{s}_+ - \vec{s}_-) \Im m X_2 - E \vec{n} \cdot (h_- \vec{s}_+ \right. \\
 & \left. - h_+ \vec{s}_-) \Re X_1 + \vec{n} \cdot [(h_- \vec{s}_+ + h_+ \vec{s}_-) \times \vec{p}_-] \Re X_2 \right\} \\
 & + \left[E \{ \vec{n} \cdot (\vec{s}_+ \times \vec{p}_-) \vec{n} \cdot \vec{s}_- + \vec{n} \cdot \vec{s}_+ \vec{n} \cdot (\vec{s}_- \times \vec{p}_-) \} \Im m X_3 \right. \\
 & + \left\{ (1 - h_- h_+ + \vec{s}_+ \cdot \vec{s}_-) (\vec{n} \cdot \vec{p}_- \vec{n} \cdot \vec{p}_- - \vec{n} \cdot \vec{n} \vec{p}_- \cdot \vec{p}_-) + 2 \vec{n} \cdot \vec{s}_- \vec{n} \cdot \vec{s}_+ \right. \\
 & \left. E^2 \right\} \Re X_4 + (h_- - h_+) (\vec{n} \cdot \vec{p}_- \vec{n} \cdot \vec{p}_- - \vec{n} \cdot \vec{n} \vec{p}_- \cdot \vec{p}_-) \Re X_3 \left. \right] \\
 & + \left[\left\{ (1 - h_- h_+ + \vec{s}_+ \cdot \vec{s}_-) \vec{n} \cdot \vec{p}_- \vec{Y} \cdot \vec{p}_- + (\vec{n} \cdot \vec{s}_+ \vec{Y} \cdot \vec{s}_- + \vec{n} \cdot \vec{s}_- \vec{Y} \cdot \vec{s}_+) \right. \right. \\
 & \left. E^2 \right\} \Re X_5 + \left\{ -2 (\vec{p}_- \times \vec{Y}) \cdot \vec{n} (1 - h_- h_+) + \vec{n} \cdot (\vec{s}_+ \times \vec{p}_-) \vec{s}_- \cdot \vec{Y} \right. \\
 & + \vec{n} \cdot \vec{s}_- \vec{Y} \cdot (\vec{s}_+ \times \vec{p}_-) + \vec{n} \cdot (\vec{s}_- \times \vec{p}_-) \vec{Y} \cdot \vec{s}_+ \\
 & + \left. \left. \vec{n} \cdot \vec{s}_+ \vec{Y} \cdot (\vec{s}_- \times \vec{p}_-) \right\} E \Im m X_6 + (h_- - h_+) \left\{ 2 \vec{n} \cdot \vec{p}_- \right. \right. \\
 & \left. \left. \vec{p}_- \cdot \vec{Y} \Re X_6 - E \vec{n} \cdot (\vec{p}_- \times \vec{Y}) \Im m X_5 \right\} \right]
 \end{aligned}
 \tag{22}$$

where $Y_\mu = u_1 p_\mu + v_1 P_\mu$, u_1 and v_1 being form factors depending on the available hadronic Lorentz invariants; the terms associated with the form factors $X_{1,2,3,4}$ correspond exactly ^{*)} to the terms associated with the

^{*)} Note that now $\Im m X_3$ can be non-zero even below the inelastic threshold because of the non-unique composition of the relevant 3π state; the same n results from $(\bar{A}p) \times ((1/\bar{A})\bar{p})$ for arbitrary $\bar{A}$.

form factors $x_{1,2,3,4}$ of Eq. (21) of $e^+e^- \rightarrow \pi^+\pi^-$ if there one replaces $(p-P)_\mu$ by n_μ for the forms of both the weak and electromagnetic vertices. The terms $X_{5,6}$ correspond to the weak current having the form Y_μ while the electromagnetic form is n_μ . In writing Y_μ as the sum of only two independent forms (p_μ and P_μ), we have used energy momentum conservation to eliminate the third pion momentum. The types $\vec{Y}$ and $\vec{n}$ for the weak hadronic current correspond respectively to the 3π system in a $J^P = 1^+$ and 1^- state. The expression (22) can be simplified by using identities like $(\vec{p} \times \vec{P}) \cdot (\vec{a} \times \vec{k}) = -\vec{p} \cdot \vec{k} \vec{a} \cdot \vec{P} + \vec{P} \cdot \vec{k} \vec{a} \cdot \vec{p}$, but we have not done so in order to make the comparison with Eq. (21) of $e^+e^- \rightarrow \pi^+\pi^-$ more transparent. We note the following points about Eq. (22).

- (1) As mentioned above, the kinematic parts of the terms with $X_{1,2}$ have the same C properties but the opposite parity behaviour as the terms with $x_{1,2}$ in (21), thus giving information on neutral current components complementary to those contributing in $e^+e^- \rightarrow \pi^+\pi^-$. An important distinction with respect to $e^+e^- \rightarrow \pi^+\pi^-$ is that one of the hadronic invariants on which the various form factors can depend is odd under $p_\mu \rightleftharpoons P_\mu$, namely $q \cdot (P-p)$. The C properties of a given term in the cross-section are thus not determined uniquely by the kinematical part. However, the parity properties are still given by the kinematical part, there being no purely hadronic Lorentz pseudoscalar. Experimentally, of course, the C properties of a given term can be observed and this should reveal whether even or odd powers of $q \cdot (P-p)$ in the relevant form factor are present.
- (2) With unpolarized e^+e^- beams a parity violating effect is now possible through the term $\vec{n} \cdot \vec{p}_- \vec{Y} \cdot \vec{p}_-$ associated with $\text{Re } X_5$. This, of course ⁶⁾, is not possible for the S, P and T types. Here it arises from the interference between the photon amplitude and the parity violating weak amplitude which is of the type V at the leptonic end and $J^P = 1^+$ at the hadronic end. The simplest exclusive channel where such an effect is possible is $e^+e^- \rightarrow 3\pi$ [cf. the inclusive case $e^+e^- \rightarrow h_1 h_2 X$, Section 3.3], since it requires at least three independent hadronic four vectors. The observation of this term would establish the presence of the relevant V coupling at the leptonic end, but not give any information on coupling types A, S, P and T . In this context one may repeat that this parity-odd correlation can arise also from the electromagnetic production and weak decay of the "new" states.

(3) There are the new polarization dependent terms associated with $X_{5,6}$.

The $X_5(X_6)$ terms are $V(A)$ at the leptonic end and 1^+ at the hadronic end, so that they are parity odd (parity even). The regions of phase space populated by the $X_{5,6}$ terms differ from the regions for $X_{3,4}$ which in turn differ from the $x_{3,4}$ terms of the case $e^+e^- \rightarrow \pi\pi$. This should help to isolate the various effects if present. The pure photon term has the structure of the $\text{Re } X_4$ term, in analogy with $e^+e^- \rightarrow \pi^+\pi^-$.

(4) The "mixed-index" structure functions W_1 and W_3 of Eq. (A.3) do not contribute in the 3π channel. As a consequence the term $\text{Im}(g_A W_3^{uv})$ of Section 3.3 does not contribute now to the parity odd correlation $(\vec{p}_1 \times \vec{p}_2) \cdot \vec{K}$.

Also the special form (only n_μ) of the photon term in the 3π channel gives another difference with respect to the channel $h_1 h_2 X$. We first consider the (V,A) types. Thus from Eq. (A.3) we see the W_4 term cannot contribute leaving $\text{Re}(g_V W_2^{rt})$, $r=n$, $t=Y$ as the only source (out of the three possibilities of Section 3.3) of the parity odd correlation $(\vec{p}_1 \times \vec{p}_2) \cdot \vec{K}$. Similarly the structure of the $\text{Im } X_{5,6}$ terms can be understood.

For $i=T$ also there are simplifications with respect to the channel $h_1 h_2 X$. The F_2 term of Eq. (A.5) does not contribute and the only F_1 term contributing is F_1^{rnn} . The only contributing PF_i form factor has the associated kinematics $n_\mu (r_\rho v_\tau - r_\tau v_\rho)$, r and v may be a , b or q .

5. - SUMMARY AND DISCUSSION

Annihilation processes should be capable of yielding important information about the nature of the neutral weak current. The fact that we are looking at the interference between the normal electromagnetic current and the weak current makes possible unambiguous tests for the space-time properties of the neutral weak current. Also the fact that the nature of the interacting leptons is known is an important advantage compared to neutrino induced processes.

In order to look for S, P and T type couplings it is necessary, if one does not detect the polarization of the final muons in the process $e^+e^- \rightarrow \mu^+\mu^-$, to study alternative processes. The simplest configuration allowing us to do this is $e^+e^- \rightarrow h(p)X$ or $e^+e^- \rightarrow \pi^+\pi^-$.

Due to the expected pion form factor effects the latter process has a decreasing cross-section at high energies and will probably only be useful if neutral currents couple more strongly in this process than they do in neutrino induced processes. The inclusive process $e^+e^- \rightarrow h(p)X$ should, however, be capable of studying neutral currents at levels comparable to those appearing in neutrino induced processes. It has the advantage of being a relatively straightforward process to study and yet can give information on all the coupling types S, P, V, A and T. The charge conjugation, parity and time reversal behaviour of these currents may be analyzed and components with many possible types of neutral current couplings at both the lepton and hadron vertices contribute.

In order to study the components having the complementary transformation properties it is necessary to consider a more extended final state configuration. The processes $e^+e^- \rightarrow h(p,s)X$ and $e^+e^- \rightarrow h_1(p_1)h_2(p_2)X$ are capable of yielding the appropriate information.

Our semi-quantitative estimates of Section 2.1 showed that exclusive channels may be capable of giving observable effects if the strength of the weak neutral currents here is sufficiently stronger than that observed in neutrino induced reactions. We have analyzed in detail the structure of the weak electromagnetic interference term for the two simplest exclusive channels $e^+e^- \rightarrow \pi^+\pi^-$ and $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ and we pointed out how the analysis and the effects differ from the corresponding inclusive processes $e^+e^- \rightarrow h(p)X$ and $e^+e^- \rightarrow h_1(p_1)h_2(p_2)X$, respectively.

It is interesting that for polarized e^+e^- beams the coupling types S, P and T produce effects that are complementary to those produced by the more conventional types V and A. For S, P, T contributions only terms linear in the e^+e^- transverse polarization (with or without a linear multiplicative longitudinal $e^\pm$ polarization) appear in the interference term. For the types V, A only terms independent of (with or without a linear or quadratic multiplicative longitudinal polarization) or quadratically dependent on the transverse polarization appear. This is helpful ⁶⁾ to distinguish the various coupling types. For the identification of the types S, P or T no observation of a characteristic weak effect (parity violation) is therefore necessary.

We should emphasize that our analysis is quite general ; we did not assume scaling nor did we restrict ourselves to non-derivative couplings at the hadronic end. The existence of non-electromagnetic neutral currents in e^+e^- annihilation has not yet been experimentally established, and it seems too early to make dynamical assumptions, like scaling, for their behaviour. Assuming purely non-derivative hadronic couplings, some limits on the strength of those components of neutral currents which are simultaneously parity odd and time reversal odd have been obtained ²⁰⁾ from a consideration of electric dipole moments of atoms and molecules, i.e., from very low energy interactions of electrons with nucleons. Similarly another recent attempt ²¹⁾ to determine the space-time structure of neutrino induced hadronic neutral current reactions relies on the non-derivative quark parton model. Hadronic neutral currents may have no similarity to the non-derivative charged weak currents of the quark parton model. In fact, even within the framework of such constituent models there may be contributions ⁴⁾ which are present in the case of neutral currents, even if they do not appear in the quark parton model parametrizations of the charged current data. Model independent determinations of the space-time structure of neutral currents have not yet been attempted.

Recent astrophysical limitations on possible tensor contributions to weak neutral current interactions indicate ²²⁾ that $15G_T \lesssim G_F$ where G_T occurs in the effective interaction Hamiltonian

$$\sqrt{2} G_T (\bar{\nu} \sigma_{\alpha\beta} \nu) (\bar{\psi}_e \sigma^{\alpha\beta} \psi_e)$$

of neutrinos with the leptons ℓ . This does not make our tensor contributions any less interesting for several reasons : (i) the smallness of a purely leptonic interaction does not necessarily imply the smallness of the corresponding semi-leptonic interactions, a point we have already discussed ; (ii) the possible uncertainty about the neutrino identity applies also in the tensor case ¹⁹⁾ ; if the final neutrino in neutral current interactions is different from the initial one and is sufficiently massive ^{5),19)}, the plasma neutrino process discussed in Ref. (22) may be simply forbidden by phase space ; (iii) since the calculation of Ref. (22) rests on a divergent integral, it is possible that the "reasonable" cut-off prescriptions used are wildly wrong, and thus the corresponding limits on G_T must be viewed with caution.

After this paper was completed, we saw a recent analysis ²³⁾ of weak asymmetries in the inclusive process $e^+e^- \rightarrow \pi X$, corresponding to $h=\pi$ in our Section 3.1. Some of their conclusions differ from ours either because of the extra assumptions they make or because of our disagreement ^{*)} with some of their calculations.

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A P P E N D I X

In this Appendix we construct the structure functions needed to describe the hadronic "vertex" formed by the product of the matrix elements of the weak and the electromagnetic currents. We also compute the cross-sections in terms of these structure functions, the leptonic "vertex" being fully known.

We are interested in the reactions $e^+e^- \rightarrow h(p,s)X$, in which the polarization of the hadron h is measured, and $e^+e^- \rightarrow h_1(p_1)h_2(p_2)X$ in which the momenta p_1, p_2 of the two final state hadrons are measured. The Appendix applies directly to $e^+e^- \rightarrow h_1h_2X$, the modifications for $e^+e^- \rightarrow h(p,s)X$ are given in Section 3. Then at the hadronic end there are three independent four vectors which we denote by a, b and the total centre-of-mass momentum q . The simpler case $e^+e^- \rightarrow h_1(p_1)X$ can, of course, also be deduced from the results for $e^+e^- \rightarrow h_1h_2X$.

The hadronic vertex $H_{\alpha\mu}^i(a,b,q)$ formed from the product of the weak and electromagnetic contributions is

$$H_{\alpha\mu}^i(a,b,q) = \sum_j (2\pi)^4 \delta^4(q - \sum_\delta p_\delta - \sum_{\beta=1}^j k_\beta) \quad (A.1)$$

$$\langle 0 | J_\alpha^{i2}(0) | k_1 \dots k_j ; a, b, q \rangle$$

$$\langle k_1 \dots k_j ; a, b, q | J_\mu^{EM}(0) | 0 \rangle$$

Here k_β , $\beta=1, \dots, j$, refer to the (unmeasured) momenta in the final state j and p_γ refer to the measured momenta in the final state. The sum is over all final states and implies integration over their phase space with appropriate phase space factors included. For exclusive processes of course there would be no such sum. i runs over the possible hadronic currents S, P, V, A or T, as defined in Eq. (8), and α represents the corresponding Lorentz indices. As in (A.1), the index μ on the structure functions will always be associated with the electromagnetic current.

The decomposition of $H_{\alpha\mu}^i$ into Lorentz invariant structure functions is given below; for clarity we suppress the dependence of the structure functions on the possible Lorentz invariants formed from a, b and q .

(1) i = S, P

$$H_{\mu}^i(a, b, q) = (\tau_{\mu} - q_{\mu} \frac{r \cdot q}{q^2}) f^r \quad (A.2)$$

where r is a, b or n ($n_{\rho} \equiv \epsilon_{\rho\alpha\beta\gamma} a^{\alpha} b^{\beta} q^{\gamma}$).

(2) i = V, A

$$H_{\mu\nu}^i(a, b, q) = -g_{\mu\nu} W_1 + \frac{1}{2} (r_{\mu} t_{\nu} + r_{\nu} t_{\mu}) W_2^{rt} \\ + \epsilon_{\mu\nu\alpha\beta} u^{\alpha} v^{\beta} W_3^{uv} + \frac{1}{2} (a_{\mu} b_{\nu} - a_{\nu} b_{\mu}) W_4 \quad (A.3)$$

where r and t are to be chosen from a, b or n . u and v are to be chosen from a, b and q . The identity

$$g^{\alpha\beta} \epsilon^{\mu\nu\rho\sigma} - g^{\alpha\mu} \epsilon^{\beta\nu\rho\sigma} = g^{\alpha\nu} \epsilon^{\mu\beta\rho\sigma} + g^{\alpha\rho} \epsilon^{\mu\beta\nu\sigma} + g^{\alpha\sigma} \epsilon^{\mu\beta\nu\rho} \quad (A.4)$$

has been used to show that a term like W_4 but with a or b replaced by n is expressible in terms of W_3 terms and thus not independent of the terms written.

There are additional terms in general, proportional to q_{μ} or q_{ν} , in Eq. (A.3) which we have omitted since in the cross-section they appear as terms multiplied by factors of order m_e/E , where m_e is the electron mass and $2E$ is the total centre-of-mass energy. By the addition of such terms we may make all the terms in Eq. (A.3) conserved with respect to the indices μ and/or ν . For example, the replacement

$$\epsilon_{\mu\nu\alpha\beta} u^{\alpha} v^{\beta} \rightarrow (g_{\mu\mu'} - \frac{q_{\mu} q_{\mu'}}{q^2}) (g_{\nu\nu'} - \frac{q_{\nu} q_{\nu'}}{q^2}) \epsilon^{\mu'\nu'}{}_{\alpha\beta} u^{\alpha} v^{\beta}$$

would make the W_3 term conserved. Since these additional terms contribute only at order m_e/E away from $q^2=0$, the difference between a conserved and non-conserved term is negligible. Thus we do not expect to be able to test conservation for the weak vector and axial vector currents in high energy annihilation processes. For clarity we do not display all the terms necessary to make even the electromagnetic current conserved.

(3) $i = T$

The hadronic vertex may be either a 3-index tensor or a 3-index pseudotensor, according to its parity properties. The tensor part we write as

$$H_{\mu\rho\tau}^i \Big|_{\text{Tensor}} = (\tau_\rho u_\nu - \tau_\nu u_\rho) t_\mu F_1^{\tau ut} + (g_{\rho\mu} \tau_\nu - g_{\nu\mu} \tau_\rho) F_2^\tau \quad (\text{A.5})$$

Here t may be a, b or n ; u may be a, b, q or n ; r may be q, a or b . Since we are writing the tensor part n may only appear quadratically in the F_1 term. We have used Eq. (A.4) to show that terms proportional to

$$n_\nu \epsilon_{\rho\mu\alpha\beta} v^\alpha \tau^\rho - n_\rho \epsilon_{\nu\mu\alpha\beta} v^\alpha \tau^\beta$$

(where v may be a, q or b) are not independent of the terms given above. Similarly, terms of the type $n_\mu \epsilon_{\rho\tau\alpha\beta} r^\alpha v^\beta$ can be shown to be included in the F_1 term. Equation (A.5) is thus quite general.

There are a further set of structure functions for the case $H_{\mu\rho\tau}^i$ is a 3-index pseudotensor. We may conveniently define them by relating the kinematic forms of the pseudotensor case to those of the tensor case.

$$H_{\mu\rho\tau}^i \Big|_{\text{Pseudotensor}} = \frac{1}{2} \epsilon_{\rho\tau}^{\alpha\beta} H_{\mu\alpha\beta}^i \Big|_{\text{Tensor}} \quad (\text{A.6})$$

The structure functions associated with the pseudotensors we denote by PF_1 and PF_2 corresponding to the terms F_1 and F_2 in Eq. (A.5), respectively. The structure functions PF_1 and PF_2 can be shown to cover all the possible terms.

Thus for $e^+e^- \rightarrow h_1(p_1)h_2(p_2)X$ there are, in general, three (complex) structure functions for $i=S,P$, eleven for $i=V,A$, and twenty-two for $i=T$, neglecting terms of order (m_e/E) . For a particular reaction not all these terms need contribute; on the other hand, there may be several contributions with a given type of kinematic structure, as considered in the adaptation for $e^+e^- \rightarrow h(p,s)X$ in Section 3.2. We discuss the simplifications for particular reactions in the main text, but Table 1 and the decomposition into structure functions given in this Appendix adequately provides the basic material for all the channels we consider.

Using the form of the leptonic vertex given in Eq. (9) we may compute the cross-section for the weak electromagnetic interference in terms of the above structure functions. We write the differential cross-section $d\sigma$ in the form

$$d\sigma \left[\frac{E^2 q^2}{\pi \alpha} \right] = \sum_i C_i K_i^2(a,b,q) D p_\gamma \quad (A.7)$$

where C_i are the structure function combinations multiplying the kinematical terms $K_i^2(a,b,q)$. α is the fine structure constant. The C_i and K_i are given in Table 1 for $i=S,P,V,A$ and T for $e^+e^- \rightarrow h_1 h_2 X$ and in Table 2 for the simpler process $e^+e^- \rightarrow h(p)X$. $D p_\rho$ runs over the measured particles ρ in the final state and includes a phase space factor, i.e., for $e^+e^- \rightarrow h(p)X$,

$$D p_\rho = \frac{d^3 p_\rho}{2\omega_\rho (2\pi)^3}$$

where ω_ρ is the energy, if ρ is a boson, and

$$D p_\rho = \frac{d^3 p_\rho}{(2\pi)^3} \frac{m_\rho}{\omega_\rho}$$

if ρ is a fermion with mass m_ρ . In Eq. (A.7) the pointlike form [cf., Eq. (8)] of the weak interaction has been employed. This form arises in the $M_Z \rightarrow \infty$ limit of an interaction due to the exchange of a neutral weak boson Z with mass M_Z . The changes in going to M_Z finite are simple and we do not write them down.

Table 1(a)

i = S, P

C_i	K^i
$\text{Im} [g_{pf}^i]$	$-2E^2 (s_+ - s_-) \cdot r$
$\text{Im} [g_{sf}^i]$	$2E[K \times (s_+ + s_-)] \cdot r$
$\text{Re} [g_{sf}^i]$	$2E^2 (h_+ s_- - h_- s_+) \cdot r$
$\text{Re} [g_{pf}^i]$	$-2E[K \times (h_- s_+ + h_+ s_-)] \cdot r$

Table 1(b)

i = V, A

C_i	K^i
$\text{Re} [g_{V1}^W]$	$4E^2 (1 - h_+ h_-)$
$\text{Re} [g_{A1}^W]$	$-4E^2 (h_+ - h_-)$
$\text{Re} [g_{V2}^{W^{rt}}]$	$2E^2 r^T \cdot t^T (1 - h_+ h_- + s_+ \cdot s_-) - 2E^2 (r \cdot s_- t \cdot s_+ + r \cdot s_+ t \cdot s_-)$
$\text{Re} [g_{A2}^{W^{rt}}]$	$-2E^2 (h_+ - h_-) r^T \cdot t^T$
$\text{Im} [g_{A2}^{W^{rt}}]$	$E[(s_- \cdot t) r \cdot (K \times s_+) + (s_+ \cdot t) r \cdot (K \times s_-) + (s_- \cdot r) t \cdot (K \times s_+) + (s_+ \cdot r) t \cdot (K \times s_-)]$
$\text{Im} [g_{V3}^{W^{uv}}]$	$4E^2 (-v^0 u^3 + v^3 u^0) (h_+ - h_-)$
$\text{Im} [g_{V4}^W]$	$2E(a \times b) \cdot p_+(h_+ - h_-)$
$\text{Im} [g_{A3}^{W^{uv}}]$	$-4E^2 (-v^0 u^3 + v^3 u^0) (1 - h_+ h_-)$
$\text{Im} [g_{A4}^W]$	$-2E(a \times b) \cdot p_+(1 - h_+ h_-)$

Table 1(c)

i = T

C_i	K^i
$\text{Im} \left\{ g_{T1}^{\text{F}^{\text{rut}}} \right\}$	$-4E^2[(h_{-s_+} + h_{+s_-}) \cdot [(r^T \cdot t^T)u - (u^T \cdot t^T)r] + (h_{-s_+} - h_{+s_-}) \cdot t(r^0 u^3 - r^3 u^0)]$
$\text{Im} \left\{ g_{T2}^{\text{F}^{\text{r}}} \right\}$	$4E^2(h_{-s_+} + h_{+s_-}) \cdot r$
$\text{Im} \left\{ g_{T1}^{\text{PF}^{\text{rut}}} \right\}$	$-4E^2[r^0(u \times t^T) - u^0(r \times t^T)] \cdot (h_{-s_+} + h_{+s_-}) - 4E[(r \times u) \cdot p_+ t \cdot (h_{-s_+} - h_{+s_-})]$
$\text{Im} \left\{ g_{T2}^{\text{PF}^{\text{rut}}} \right\}$	$-4E[(h_{-s_+} - h_{+s_-}) \times K] \cdot r$
$\text{Re} \left\{ g_{T1}^{\text{F}^{\text{rut}}} \right\}$	$-4E^2[r^0(u \times t^T) - u^0(r \times t^T)] \cdot (s_+ + s_-) + 4E[(r \times u) \cdot p_+ t \cdot (s_+ - s_-)]$
$\text{Re} \left\{ g_{T2}^{\text{F}^{\text{r}}} \right\}$	$-4E[(s_+ - s_-) \times K] \cdot r$
$\text{Re} \left\{ g_{T1}^{\text{PF}^{\text{rut}}} \right\}$	$+4E^2[(s_+ + s_-)] \cdot [(r^T \cdot t^T)u - (u^T \cdot t^T)r] + (s_+ - s_-) \cdot t(r^0 u^3 - r^3 u^0)$
$\text{Re} \left\{ g_{T2}^{\text{PF}^{\text{r}}} \right\}$	$-4E^2(s_+ + s_-) \cdot r$

Table 2(a)

i = S, P

C_i	K^i	R	C
Im (g_p^f)	$-2E^2 (s_+ - s_-) \cdot p$	-	-
Im (g_S^f)	$2E[K \times (s_+ + s_-)] \cdot p$	+	-
Re (g_S^f)	$2E^2 (h_+ s_- - h_- s_+) \cdot p$	+	-
Re (g_p^f)	$-2E[K \times (h_- s_+ + h_+ s_-)] \cdot p$	-	-

Table 2(b)

i = V, A

C_i	K^i	R	C
$\text{Re} [g_{V1}^W]$	$4E^2(1-h_+h_-)$	+	+
$\text{Re} [g_{A1}^W]$	$-4E^2(h_+ - h_-)$	-	-
$\text{Re} [g_{V2}^W]$	$2[(1-h_+h_- + s_+ \cdot s_-)(p \cdot p K \cdot K - p \cdot K p \cdot K) - 2E^2 p \cdot s_- p \cdot s_+]$	+	+
$\text{Re} [g_{A2}^W]$	$2(h_- - h_+)(p \cdot p K \cdot K - p \cdot K p \cdot K)$	-	-
$\text{Im} [g_{V3}^W]$	$8E^2(h_- - h_+)p \cdot K$	-	+
$\text{Im} [g_{A3}^W]$	$8E^2(1-h_+h_-)p \cdot K$	+	-
$\text{Im} [g_{A2}^W]$	$-2E[p \cdot s_+ p \cdot (K \times s_-) + p \cdot s_- p \cdot (K \times s_+)]$	-	-

Table 2 (c)

i = T

C_i	K^i	R	C
$\text{Im} \left\{ g_{T1}^F \right\}$	$8E^2 p \cdot K p \cdot (h_- s_+ - h_+ s_-)$	+	+
$\text{Im} \left\{ g_{T2}^F \right\}$	$4E^2 (h_- s_+ + h_+ s_-) \cdot p$	+	+
$\text{Im} \left\{ g_{T1}^{PF} \right\}$	$+8E(p \cdot K)(K \times p) \cdot (h_- s_+ + h_+ s_-)$	-	+
$\text{Im} \left\{ g_{T2}^{PF} \right\}$	$-4E[(h_- s_+ - h_+ s_-) \times K] \cdot p$	-	+
$\text{Re} \left\{ g_{T1}^F \right\}$	$-8E(p \cdot K)(K \times p) \cdot (s_+ + s_-)$	+	+
$\text{Re} \left\{ g_{T2}^F \right\}$	$-4E[(s_+ - s_-) \times K] \cdot p$	+	+
$\text{Re} \left\{ g_{T1}^{PF} \right\}$	$+8E^2 p \cdot K p \cdot (s_+ - s_-)$	-	+
$\text{Re} \left\{ g_{T2}^{PF} \right\}$	$-4E^2 (s_+ + s_-) \cdot p$	-	+

Table 3

Space reflection	Charge conjugation	Time reversal	Rank	Quark structure
Even	Even	Even	First	Diagonal
Even	Odd	Odd	Second	Non-diagonal
Odd	Even	Odd	Second	Diagonal
Odd	Odd	Even	Second	Non-diagonal

TABLE CAPTIONS

TABLE 1 (a,b,c)

Contributions of the various structure functions [defined in Eqs. (A.2), (A.3), (A.5) and (A.6)], along with their appropriate multiplicative kinematic structures, to Eq. (A.7). Apart from over-all factors, this is the electromagnetic weak interference term in the differential cross-section. The couplings $g_{S,P,A,V,T}$ are defined in Eq. (8). An appropriate sum over the various possible choices of the available vectors indicated in the superscripts on the structure functions is implied. The table as such applies to the reaction $e^+e^- \rightarrow h_1 h_2 X$, but the complete contributions for $e^+e^- \rightarrow h(p,s)X$ can also be fully deduced from the table by taking into account the systematic modifications discussed in Section 3.2. The symbols $r, t, u, v, s_+, s_-, a, b, p_+, p_-$ in the table are all three-vectors; $2K = p_- - p_+$; A^T means the component of $\vec{A}$ transverse to $\vec{K} = E\hat{z}$, i.e. $A^T = \vec{A} - \vec{A} \cdot \hat{K} \hat{K}$. The symbols x^0 and x^3 , where x may be any of the above variables, refer to the time component and the component in the e^+ direction, respectively. Tables 1 (a,b,c) refer to $i = (S,P)$, $i = (V,A)$ and $i = T$, respectively.

TABLE 2 (a,b,c)

Similar to Table 1, but for the process $e^+e^- \rightarrow h(p)X$ with contributions of all the relevant structure functions explicitly displayed, see Eqs. (11), (12) and (13). p is a three vector as are the other variables (cf. Table 1). The last two columns give the behaviour under parity inversion (P) and charge conjugation (C); odd and even are denoted - and +, respectively. The column for C refers to the case $h = \bar{h}$, or to σ_+ of Eq. (14); for σ_- the behaviour is opposite to that for σ_+ . [This table differs from that in Ref. 6) only in the column specifying the contributing combinations C_i of the appropriate structure functions; some over-all normalization factors have now been supplied and a misprint in the fourth row (the $\text{Re}(g_V W_2)$ term) of Table 2 (b) now corrected.] Tables 2 (a,b,c) refer to $i = (S,P)$, $i = (V,A)$ and $i = T$, respectively.

TABLE 3

Discrete symmetry properties of the various possible contributions in a non-derivative neutrino quark Lagrangian ; various types of quarks (but not neutrinos ^{5),19)} are allowed. The diagonal vs. non-diagonal quark structure and the rank classification ³⁾ are also indicated.

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