Y-TYPE FLUX-TUBE FORMATION IN BARYONS

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For more than 300 different patterns of the 3Q systems, the ground-state 3Q potential \( V_{3Q}^{g.s.} \) is investigated using SU(3) lattice QCD with \( 12^3 \times 24 \) at \( \beta = 5.7 \) and \( 16^3 \times 32 \) at \( \beta = 5.8, 6.0 \) at the quenched level. As a result of the detailed analyses, we find that the ground-state potential \( V_{3Q}^{g.s.} \) is well described with so-called Y-ansatz as
\[
V_{3Q} = -A_{3Q} \sum_{i<j} \frac{1}{|r_i - r_j|} + \sigma_{3Q} L_{\text{min}} + C_{3Q},
\]
with the accuracy better than 1%. Here, \( L_{\text{min}} \) denotes the minimal value of total flux-tube length. We also study the excited-state potential \( V_{3Q}^{e.s.} \) using lattice QCD with \( 16^3 \times 32 \) at \( \beta = 5.8, 6.0 \) for more than 100 patterns of the 3Q systems. The energy gap between \( V_{3Q}^{g.s.} \) and \( V_{3Q}^{e.s.} \), which physically means the gluonic excitation energy, is found to be about 1 GeV in the typical hadronic scale. Finally, we suggest a possible scenario which connects the success of the quark model to QCD.

1. Introduction
It is widely accepted that Quantum Chromodynamics (QCD) is the underlying theory of the strong interaction for hadrons and nuclei. In a spirit of the particle physics, it is strongly desired to describe the hadron dynamics directly from QCD. However, due to the strong coupling nature of QCD in the infrared region, it still remains a big challenge to derive physical quantities directly from QCD in an analytic manner.

For the nonperturbative analysis of QCD, lattice QCD Monte Carlo calculations have been established as a reliable method directly based on QCD. Lattice QCD calculations enable us to perform a nonperturbative quantitative analysis in a model-independent way. For instance, lattice QCD calculations have been very successful in reproducing hadron mass spectra. Nevertheless, these results hardly inform us the internal structure...
of hadrons. It would be also indispensable to comprehend hadrons directly from quarks and gluons, the degrees of freedom of QCD.

For this aim, we cast light on the inter-quark potentials, which are directly responsible for the hadron properties and their internal structures at the quark-gluon level.

2. Ground-state three quark potential

For the quark-antiquark (Q\bar{Q}) potential, which is one of the most fundamental quantities in QCD, many lattice QCD studies have been performed. As a result, Q\bar{Q} potential is known to be reproduced by a simple sum as

$$ V_{Q\bar{Q}} = -\frac{A_{Q\bar{Q}}}{r} + \sigma_{Q\bar{Q}} r + C_{Q\bar{Q}} $$

with $r$ the distance between quark and antiquark. Here, $\sigma \simeq 0.89$ GeV/fm denotes the string tension in the flux-tube picture.

However, in contrast with a number of lattice QCD studies for the Q\bar{Q} potential, almost no lattice studies were done before our study\textsuperscript{2} for the three-quark (3Q) potential $V_{3Q}$, which is directly responsible for the baryon properties. We study the ground-state 3Q potential on more than 300 patterns of the spatial quark configurations, using SU(3) lattice QCD at the quenched level with $12^3 \times 24$ at $\beta = 5.7$ and with $16^3 \times 32$ at $\beta = 5.8, 6.0$\textsuperscript{2,3}

As a result, we find that the 3Q potential $V_{3Q}$ can be reproduced by so-called Y-ansatz as\textsuperscript{2,4}

$$ V_{3Q} = -A_{3Q} \sum_{i<j} \frac{1}{|r_i - r_j|} + \sigma_{3Q} L_{\text{min}} + C_{3Q}, \quad (1) $$

with the accuracy better than 1% for all the different patterns of quark configurations. Here, $L_{\text{min}}$ denotes the minimal value of total flux-tube length\textsuperscript{5,6}, which is schematically illustrated in Fig.1.

In the fit analysis with Y-ansatz, we find two remarkable features\textsuperscript{2}; the universality of the string tension as $\sigma_{3Q} \simeq \sigma_{Q\bar{Q}}$ and the one-gluon-exchange consequence as $A_{3Q} \simeq \frac{1}{2} A_{Q\bar{Q}}$.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{The flux-tube configuration in the 3Q system with the minimal length of the Y-type flux-tube, $L_{\text{min}}$.}
\end{figure}
Furthermore, we consider also Y-ansatz with Yukawa-type two-body force as $$V_{3Q}^{\text{Yukawa}} = -A_{3Q}^{\text{Yukawa}} \sum_{i<j} \frac{\exp(-m_B |r_i - r_j|)}{|r_i - r_j|} + \sigma_{3Q}^{\text{Yukawa}} L_{\text{min}} + C_{3Q}^{\text{Yukawa}}$$, which is a conjecture from the dual superconductor scenario on the QCD vacuum. Here, $$m_B$$ denotes the dual gluon mass in the dual superconductor picture. However, we find no evidence of the Yukawa-type two-body force and again confirm the adequacy of the Y-ansatz form in Eq.(1), i.e., $$\sigma_{3Q}^{\text{Yukawa}} \simeq \sigma_{3Q}$$, $$A_{3Q}^{\text{Yukawa}} \simeq A_{3Q}$$, $$m_B \simeq 0$$.

Finally, as a direct evidence of the Y-type flux-tube formation, we show in Fig.2 the flux-tube profile in the spatially-fixed 3Q system obtained using maximally-abelian (MA) projected lattice QCD. The action density in $$R^3$$ is calculated under the presence of spatially-fixed three quarks in a color-singlet state. Here, the distance between the junction and each quark is about 0.5 fm. We thus observe Y-type flux-tube formation in lattice QCD.

![Figure 2](image_url)

Figure 2. The flux-tube profile in the spatially-fixed 3Q system, in the MA projected QCD. The distance between the junction and each quark is about 0.5 fm.

3. Excited-state three quark potential

3.1. Lattice QCD study of gluonic excitations

We also investigate the gluonic excitation in the spatially-fixed 3Q system using SU(3) lattice QCD. In the flux-tube picture, gluonic excitation modes would be regarded as the vibrational modes of a flux-tube. In the valence picture, they correspond to the hybrid hadrons such as $$q\bar{q}G$$ or $$qqqG$$. In particular, hybrid hadrons are probable candidates of the exotic hadrons, which has exotic quantum numbers such as $$J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{++}, ...$$, which cannot be constructed within a simple quark model. It is then worth investigating gluonic excitations directly from QCD, from the experimental viewpoint as well as the theoretical viewpoint.
In Fig. 3 we show the first lattice QCD results for the excited-state 3Q potential obtained with $16^3 \times 32$ lattice at $\beta = 5.8, 6.0$ at the quenched level. Here, the horizontal axis denotes the minimal length of the Y-type flux-tube, $L_{\text{min}}$, as a label to distinguish the three-quark configuration. The vertical axis denotes the energy induced by three static quarks in a color-singlet state. The open symbols are for the ground-state potential $V_{3Q}^{g,s}$ discussed in the previous section, and the filled symbols are for the excited-state potential $V_{3Q}^{e,s}$ in the 3Q system. The energy difference $\Delta E \equiv V_{3Q}^{e,s} - V_{3Q}^{g,s}$ physically means the gluonic excitation energy.

We find that the gluonic excitation energy $\Delta E$ is more than 1 GeV in the typical hadronic scale as $0.5 \text{fm} \leq L_{\text{min}} \leq 1.5 \text{fm}$, which is large in comparison with the excitation energies of quark origin. (For the $Q\bar{Q}$ system, a large gluonic excitation energy is reported in recent lattice studies.) Such a gluonic excitation would contribute significantly in the highly-excited baryons with the excitation energy above 1 GeV. in fact, the lowest hybrid baryon, which is described as $qqqG$ in the valence picture, is expected to have a large mass of about 2 GeV. This large gluonic excitation energy also gives us the reason for the great success of the simple quark model.

3.2. Success of the quark model

The simple quark model contains only constituent quarks as explicit degrees of freedom, and does not have gluonic modes. Nevertheless, the quark model has been very successful in reproducing the low-lying hadron properties and spectra especially for baryons, in spite of the non-relativistic
treatment and the absence of gluonic excitation modes. It is true that the non-relativistic treatment for hadron mass spectra can be justified by the spontaneous chiral symmetry breaking, which gives rise to a large constituent quark mass of about 300 MeV, but we have no reason based on QCD for the absence of the gluonic mode. Now, directly based on QCD, the gluonic excitation energy $\Delta E$ is found to be large, in comparison with quark excitation energy. We propose a possible scenario connecting the success of the quark model to QCD in Fig. 4.

### Quantum Chromodynamics

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**The Quark Model for low-lying hadrons**

Figure 4. A possible scenario connecting the success of the simple quark model to QCD.

### References