On the Bekenstein-Hawking Entropy, Non-Commutative Branes and Logarithmic Corrections

Axel Krause

Center for String and Particle Theory, Department of Physics, University of Maryland, College Park, MD 20742, USA

Abstract

We extend earlier work on the origin of the Bekenstein-Hawking entropy to higher-dimensional spacetimes. The mechanism of counting states is shown to work universally for all spacetimes arising from a Euclidean doublet $(E_1, M_1) + (E_2, M_2)$ of electric-magnetic dual brane pairs of type II string-theory or M-theory wrapping the spacetime’s event horizon plus the complete internal compactification space. Non-Commutativity on the brane worldvolume fits in naturally in the derivation of the Bekenstein-Hawking entropy plus its logarithmic correction.

1E-mail: krause@physics.umd.edu
1 Introduction

In the past few years string- resp. M-theory made considerable progress towards the stabilization of its multitude of moduli arising from compactification and therefore towards predictivity (see e.g. [1]-[8]). Particularly interesting for addressing the ‘real world’ is heterotic M-theory [9] with its flux compactifications [10] which seem to offer an intriguing rationale for why and how Grand Unified Particle theories have to be combined with gravity. Due to its peculiar blend of classical and quantum physics (see second reference of [9]), it is clear that eventually knowledge of the full quantum heterotic M-theory becomes intimately tied to its promising phenomenology. It is therefore also from a very pragmatic point of view of direct interest to obtain the elusive non-perturbative formulation of M-theory which would require first of all simply to identify its fundamental microscopic states.

Though future collider experiments will probe for the first time energy regimes which are directly relevant to string-/M-theory, we are still far away from testing the theory directly. In order to unravel the mysteries of M-theory we therefore have to look for other, necessarily theoretical, clues. Probably the best guidance in this respect comes from the Bekenstein-Hawking entropy (BH-entropy) [11]. Because of its universal applicability it should clearly reflect some important property of the underlying microscopic theory of Quantum Gravity. Given that the fundamental degrees of freedom of M-theory are still largely obscure in the non-perturbative regime where the string-coupling constant g_s is of order $O(1)$, we would like to propose in this paper, following earlier work [12]-[15], a suitable set of microscopic states for this regime which turn out to be chain-like.

Our argumentation will be based on a general microscopic mechanism to derive the BH-entropy and its leading logarithmic correction based on these chain states. Along the line of reasoning which will involve electric-magnetic dual Euclidean brane pairs, we will see that non-commutativity on the brane worldvolume fits in very naturally. The results presented here will generalize the analysis of [12] in that they apply also to the BH-entropy of higher-dimensional $d > 4$ spacetimes while [12] was devoted to the study of the BH-entropy of $d = 4$ spacetimes.
2 The BH-Entropy in terms of Electric-Magnetic Dual Pairs

2.1 Type II String-Theory Case

Let us start with type II string-theory on a $D=10$ geometric background $\mathcal{M}^{1,d-1} \times \mathcal{M}^{10-d}$ described by $2 \leq d \leq 10$

$$ds^2 = g^{(1,d-1)}_{\mu\nu}(x^\rho)dx^\mu dx^\nu + g^{(10-d)}_{mn}(y^p)dy^m dy^n$$

(1)

with Lorentzian signature. The $d$-dimensional external space $\mathcal{M}^{1,d-1}$ is chosen to be that space whose BH-entropy we are interested in while the internal space is taken to be compact with certain factorization properties which will be pointed out later on. More specifically, because we are interested in an external spacetime with non-zero BH-entropy, let $\mathcal{M}^{1,d-1}$ possess a future event horizon $\mathcal{H}^+$ and let $\mathcal{H}^{d-2}$ denote the $d-2$ dimensional intersection of $\mathcal{H}^+$ with a partial Cauchy surface such that $\mathcal{H}^{d-2}$ represents the boundary of $\mathcal{M}^{1,d-1}$.

Next, let us take a pair of mutually orthogonal (with respect to (1)) Euclidean (with space-like or null-like worldvolume w.r.t. (1)) electric-magnetic dual branes

$$(E_1, M_1) \in \{(Dp, D(6-p)), (F1, NS5)\}.$$  

(2)

Without adding them separately, we understand that (2) includes as well all possible pairs in which any brane (for compactness let us also term the fundamental string $F1$ a brane) is replaced by its anti-brane. As the orthogonal Euclidean $E_1$ and $M_1$ together can cover an 8-dimensional submanifold, they possess the right dimensionality so that we can wrap the pair $(E_1, M_1)$ around the complete $\mathcal{H}^{d-2} \times \mathcal{M}^{10-d}$. In case the metric (induced from (1)) on $\mathcal{H}^{d-2}$ does not factorize, we just have to make sure that either component, $E_1$ or $M_1$, which wraps $\mathcal{H}^{d-2}$ covers it completely. Of course, if the (Euclidean) dimensions of $E_1$ and $M_1$ do not happen to coincide exactly with $d-2$ resp. $10-d$ (in either order) we would require $\mathcal{M}^{10-d}$ to factorize appropriately (e.g. a torus compactification $\mathcal{M}^{10-d} = T^{10-d}$ would factorize completely). Notice that without invoking an appropriate factorization, there is for a $d$-dimensional spacetime $\mathcal{M}^{1,d-1}$ where $d$ ranges over $3, \ldots, 9$ always one dual pair $(D(d-3), D(9-d))$ which could be used. To deal with $d = 2$ or $d = 10$ spacetimes, one would have to extend (1) to include the pair $(D(-1), D7)$ as well.

For reasons which will become clear soon we wrap another pair of mutually orthogonal
Euclidean electric-magnetic dual branes

\[ (E_2, M_2) \in \{(Dq, D(6-q)), (F1, NS5)\} \]  

around \( \mathcal{H}^{d-2} \times \mathcal{M}^{10-d} \) in the same manner as before. Notice that in general \( p \) and \( q \) or \( (E_1, M_1) \) and \( (E_2, M_2) \) need not to coincide in which case the compactification manifold would have to possess a more refined product structure. Let us remark that for the general mechanism to derive the BH-entropy the backreaction of these brane pairs is not essential. We will however explain later at which point the backreaction becomes important. In passing let us also mention that wrapping (some of the) branes around the horizon is very reminiscent of the ‘membrane paradigm’ in which for a \( d = 4 \) black hole the event horizon (or the ‘stretched horizon’) is conceived as a membrane (however not a fundamental one like in string-theory) which enjoys some intriguing non-relativistic properties [16]. One might therefore view part of the current approach also as an embedding of this idea into string-/M-theory where the role of the membrane is played by (part of) the fundamental string-/M-theory branes.

The reason for introducing the two dual pairs is that it will allow for a useful rewriting of the \( d \)-dimensional BH-entropy associated with the boundary \( \mathcal{H}^{d-2} \) as we will now see. The aim is to reexpress the \( d \)-dimensional BH-entropy exclusively in terms of brane notions. Since both pairs \( (E_i, M_i); i = 1, 2 \) cover the manifold \( \mathcal{H}^{d-2} \times \mathcal{M}^{10-d} \), we can write the compactification volume as

\[ \text{vol}(\mathcal{M}^{10-d}) = \frac{\text{vol}(E_i)\text{vol}(M_i)}{\text{vol}(\mathcal{H}^{d-2})}, \quad i = 1, 2 \]  

such that the effective \( d \)-dimensional Newton Constant can be expressed as

\[ G_d = \frac{G_{10}}{\text{vol}(\mathcal{M}^{10-d})} = \frac{(2\pi)^6 \alpha' g_s^2}{8} \times \frac{\text{vol}(\mathcal{H}^{d-2})}{\text{vol}(E_i)\text{vol}(M_i)}, \quad i = 1, 2 \]  

where \( \alpha' \) denotes the Regge slope.

The significance of why we have chosen to use dual branes lies in the fact that the product of their tensions satisfies the generalized Dirac quantization condition

\[ \tau_{E_i} \tau_{M_i} = \frac{1}{(2\pi)^6 \alpha' g_s^2}. \]  

which allows us to express the inverse of the Newton Constant as

\[ \frac{1}{G_d} = 8 \left( \frac{\tau_{E_i} \text{vol}(E_i) \tau_{M_i} \text{vol}(M_i)}{\text{vol}(\mathcal{H}^{d-2})} \right), \quad i = 1, 2. \]
Hence, the BH-entropy associated with the boundary $\mathcal{H}^{d-2}$ of the $d$-dimensional spacetime $\mathcal{M}^{1,d-1}$ can be rewritten purely in terms of the Euclidean Nambu-Goto actions $S_{E_i}, S_{M_i}$ for $E_i, M_i$ as

$$S_{BH} = \frac{\text{vol}(\mathcal{H}^{d-2})}{4G_d} = 2S_{E_i}S_{M_i}, \quad i = 1, 2.$$  \hspace{1cm} (8)

The fact that we considered two dual pairs instead of just one, allows us further to get rid of the prefactor two and write a sum over both dual pairs such that finally the $d$-dimensional BH-entropy becomes

$$S_{BH} = \sum_{i=1,2} S_{E_i}S_{M_i}. \hspace{1cm} (9)$$

For the mechanism given below to microscopically derive $S_{BH}$, it will be crucial that we can replace the prefactor two by the sum at the expense of introducing two dual pairs instead of just one. If we were to use only one dual pair then the derivation of $S_{BH}$ as will be presented below would be off by precisely this factor of two. However, taking two dual pairs instead of one also makes sense from the point of view that an uncharged Schwarzschild black hole cannot be generated from just one dual pair. At least one other anti-brane pair is needed to dispose of the long-range $U(1)$ RR- or NS-fields of the branes.

### 2.2 M-Theory Case

In M-Theory based on its unique dual brane pair $(M2, M5)$, essentially the same analysis goes through. We start from a D=11 geometric background describing a compactification from 11 to $d$ dimensions ($2 \leq d \leq 11$)

$$ds^2 = g^{(1,d-1)}_{\mu\nu}(x^\rho)dx^\mu dx^\nu + g^{(11-d)}_{mn}(y^p)dy^m dy^n$$  \hspace{1cm} (10)

where the external $d$-dimensional spacetime $\mathcal{M}^{1,d-1}$ is the one whose BH-entropy we are interested in. We therefore assume again that it has a non-trivial $d - 2$ dimensional boundary $\mathcal{H}^{d-2}$.

A pair $(M2, M5)$ consisting out of a Euclidean M2 and M5 brane which are mutually orthogonal (w.r.t. the metric (10)) spans a 9-dimensional submanifold. We will let it wrap the complete $\mathcal{H}^{d-2} \times \mathcal{M}^{11-d}$. In case the induced metric on $\mathcal{H}^{d-2}$ does not enjoy a direct product structure, we must cover the full $\mathcal{H}^{d-2}$ by one brane completely. It will be immaterial in the sequel whether this is the M2 or the M5. Except for $d = 5$ and $d = 8$
we would have to take the metric on $\mathcal{M}^{11-d}$ to factorize into a direct product accordingly.

Notice that for $d = 9, 10, 11$ both $M2$ and $M5$ have to wrap the boundary $\mathcal{H}^{d-2}$ which in these cases must therefore exhibit a direct product in order for the following analysis to apply.

For M-theory the compactification volume can then be expressed as

$$\text{vol}(\mathcal{M}^{11-d}) = \frac{\text{vol}(M2)\text{vol}(M5)}{\text{vol}(\mathcal{H}^{d-2})}$$

and the effective $d$-dimensional Newton Constant becomes

$$G_d = \frac{G_{11}}{\text{vol}(\mathcal{M}^{11-d})} = \frac{(2\pi)^7 l_{11}^9}{8} \times \frac{\text{vol}(\mathcal{H}^{d-2})}{\text{vol}(M2)\text{vol}(M5)}$$

where $l_{11}$ denotes the 11-dimensional Planck length. The important property of the dual brane pair, and the reason for having chosen it, is that the product of their tensions satisfies

$$\tau_{M2} \tau_{M5} = \frac{1}{(2\pi)^7 l_{11}^9}$$

which allows to write the inverse of $G_d$ as

$$\frac{1}{G_d} = 8 \frac{(\tau_{M2}\text{vol}(M2))(\tau_{M5}\text{vol}(M5))}{\text{vol}(\mathcal{H}^{d-2})}.$$  

The $d$-dimensional BH-entropy associated with $\mathcal{M}^{1,d-1}$ therefore becomes

$$S_{BH} = \frac{\text{vol}(\mathcal{H}^{d-2})}{4G_d} = 2S_{M2}S_{M5}$$

where $S_{M2}, S_{M5}$ are the respective Euclidean Nambu-Goto actions of the $M2, M5$.

By using a second brane pair ($M2, M5$) wrapped around $\mathcal{H}^{d-2} \times \mathcal{M}^{11-d}$ independently of the first pair (the $M2$ resp. $M5$ of both pairs don’t have to wrap necessarily the same submanifolds) one would arrive at the same conclusion (15). Hence we see that eventually we obtain also in M-theory the result of

$$S_{BH} = \sum_{i=1,2} S_{M2,i}S_{M5,i}$$

which expresses the $d$-dimensional BH-entropy exclusively in terms of brane properties. Moreover, since the Nambu-Goto action does not recognize the difference between a brane and an anti-brane we will understand subsequently that each $M2$ or $M5$ might also be replaced by its anti-brane partner.
3 Derivation of BH-Entropy and Logarithmic Correction

Now that we have reexpressed the BH-entropy in terms of the Nambu-Goto actions of the respective dual branes, our aim will be to find a suitable set of microstates capable of explaining it. For this we will need one more ingredient which will be obtained from revisiting the tension of a Euclidean brane. It is more natural when dealing with a Euclidean brane to treat all its dimensions on an equal basis as there is no longer a distinguished time dimension which in the case of a Lorentzian brane is singled out by the signature of the induced worldvolume’s metric. Consequently, one should interpret the tension of e.g. a $p + 1$ dimensional Euclidean $Dp$-brane not as its ‘mass’ per unit spatial $p$-volume but instead as the inverse of a $p + 1$ dimensional volume unit $v_{Dp}$. In general on any of the Euclidean branes introduced so far we will therefore have some volume unit $v_E, v_M$ given by the inverse of the brane’s tension

$$v_E = \frac{1}{\tau_E}, \quad v_M = \frac{1}{\tau_M}.$$  \hspace{0.5cm} (17)

Such an elementary volume unit on the brane’s worldvolume can be naturally understood if the brane’s worldvolume would be considered being non-commutative. Taking the simplest non-commutativity arising from string-theory

$$[X^i, X^j] = 2i\epsilon^{ij} l^2$$  \hspace{0.5cm} (18)

for the worldvolume coordinates $X^i$ of a Euclidean $Dp$-brane [17] ($p$ odd), one derives the uncertainty relation

$$\Delta X^i \Delta X^j \geq l^2$$  \hspace{0.5cm} (19)

for the ‘coordinates’ $X^i$. From this follows directly the ‘brane worldvolume uncertainty principle’

$$\Delta X^1 \ldots \Delta X^{p+1} \geq l^{p+1}.$$  \hspace{0.5cm} (20)

Indeed in [18] it was argued for such an uncertainty principle for all branes (including $F1, NS5, M2, M5$) by using S- and T-dualities. The universal result was that the smallest allowed volume $l^{p+1}$ in (20) is determined by the tension of the brane through

$$l^{p+1} \simeq \frac{1}{\tau_{Dp}}.$$  \hspace{0.5cm} (21)
for all $Dp$-branes and similarly for the $F1, NS5, M2, M5$ (by substituting $p + 1$ resp. $\tau_{Dp}$ by the Euclidean dimension resp. tension of $F1, NS5, M2, M5$). From the perspective of a brane with non-commutative worldvolume it is therefore clear that $v_E, v_M$ in (17) represent the smallest volume unit which is allowed by the worldvolume uncertainty principle. For the special case of the fundamental string this just states that $2\pi\alpha'$ constitutes a smallest volume resp. that the string-length $l_s$ constitutes a smallest length – a familiar result which has been argued for based on string scattering amplitudes, worldsheet conformal invariance and other arguments [19].

Equipped with this interpretation of the tension of a Euclidean brane its Nambu-Goto action adopts a new meaning. Namely by the above reasoning we are led to conceive a brane’s worldvolume as a lattice arranged out of a number of cells with volume $v_E$ resp. $v_M$. It is then precisely the Nambu-Goto action which tells us how many cells of this size, $N_E$ resp. $N_M$, are contained in the brane

$$N_E = \frac{\text{vol}(E)}{v_E} = \tau_E \text{vol}(E) = S_E$$

and similarly for the magnetic component, $N_M = S_M$. Because of the orthogonality of $E$ and $M$, each dual pair $(E, M)$ contains $N_EN_M$ cells while the doublet $(E_1, M_1) + (E_2, M_2)$ contains

$$N = \sum_{i=1,2} N_{E_i}N_{M_i}$$

(23)
cells. But by virtue of (9) and (22) this implies that the $d$-dimensional BH-entropy of $M^{1,d-1}$ simply becomes an integer

$$S_{BH} = N$$

(24)

where $N$ represents the total number of cells contained in the combined worldvolume of $(E_1, M_1) + (E_2, M_2)$. Notice that $N$ has to be even because of the equality $S_{E_1}S_{M_1} = S_{E_2}S_{M_2}$ which implies $N_{E_1}N_{M_1} = N_{E_2}N_{M_2}$.

One might wonder how the discreteness of a (Euclidean) brane’s worldvolume can be compatible with the notion of a smooth worldvolume known from the weakly coupled regime. Here let us note that for a $Dp$-brane or an $NS5$-brane the volumes $v_{Dp}$ and $v_{NS5}$ are proportional to $g_s$ and $g_s^2$. They therefore become infinitesimally small in the weakly coupled regime where $g_s \to 0$. Therefore, in this limit the brane’s worldvolume indeed becomes quasi-continuous. However, in the non-perturbative regime where $g_s \simeq 1$, $v_{Dp}, v_{NS5}$ will no longer vanish and endow the brane with a discrete worldvolume.
Next, we want to identify a set of ‘microscopic’ states whose entropy matches the BH-entropy and ideally accounts for its logarithmic correction as well. For this purpose let us introduce ‘open’ chains built out of \( N - 1 \) successive links where each link connects two of the cells of the \( N \) cells on the joint \((E_1, M_1) + (E_2, M_2)\) worldvolume (see fig.1). As each link is allowed to start and end on any cell the number of all such ‘classically’ different chain-configurations is \( N^N \). Besides these open chains there is a different class of chains which has the same number of configurations \( N^N \). These are the ‘closed’ chains which are made out of \( N \) links (see fig.2). As far as the state counting is concerned they lead to the same results as the open chains and can therefore be used alternatively in this respect.

We have called the counting of the different chain configurations ‘classical’ because it considered all cells as distinguishable. However, at the quantum level the cells should better be regarded as bosonic degrees of freedom and therefore be considered as indistinguishable. As well-known from standard statistical mechanics, one can easily incorporate this quantum feature by dividing the classical number of configurations through the Gibbs-correction factor \( N! \) which accounts for the indistinguishability of the \( N \) cells. Therefore, quantum-mechanically we obtain a number of

\[
\Omega(N) = \frac{N^N}{N!}
\]

(25)
different open or closed chain states.

By assuming that all these different chain-states are energy-degenerate, i.e. assuming that chains with the same \( N \) will possess the same energy, we can now determine in a
Figure 2: A ‘closed’ chain possesses one more link than an open chain but due to its closedness property (the last link ends on the same cell on which the first link starts) bears the same number of different states.

microcanonical ensemble approach the entropy of the chains. It is given by

$$ S_c = \ln \Omega(N) \tag{26} $$

and can be evaluated in the large $N$ limit (as appropriate for boundaries $\mathcal{H}^{d-2}$ of macroscopical size) by using Stirling’s series

$$ N! = \sqrt{2\pi NN^N} e^{-N} \left(1 + \frac{1}{12N} + \mathcal{O}\left(\frac{1}{N^2}\right)\right) \tag{27} $$

to approximate $\ln(N!)$. The result is

$$ S_c = N - \frac{1}{2} \ln N - \ln \sqrt{2\pi} - \frac{1}{12N} + \mathcal{O}\left(\frac{1}{N^2}\right) \tag{28} $$

which by virtue of (24) becomes

$$ S_c = S_{BH} - \frac{1}{2} \ln S_{BH} - \ln \sqrt{2\pi} - \frac{1}{12S_{BH}} + \mathcal{O}\left(\frac{1}{S_{BH}^2}\right). \tag{29} $$

Thus indeed the entropy of the chain states does match at leading order the BH-entropy plus its known logarithmic correction including the precise numerical prefactor (there was a debate in the literature over whether the prefactor in front of the logarithm has to be $1/2$ or $3/2$; by now there is convincing evidence in favour of $1/2$, see e.g. [20]). We can therefore conclude that the proposed discrete chain states in the non-perturbative $g_s \simeq \mathcal{O}(1)$ regime allow for a universal mechanism of counting states with the correct reproduction of the BH-entropy and its leading logarithmic correction not only for 4-dimensional [12] but also, as demonstrated in this paper, for $d$-dimensional spacetimes with $d - 2$ dimensional boundaries.
4 Final Comments

Let us finally comment on the backreaction of the branes on the spacetime geometry. Ultimately one should think of the 10-dimensional or 11-dimensional background as a result of the brane sources. First steps in this direction for the 4-dimensional Schwarzschild black hole have been undertaken in [13]. From this point of view it is clear that the uncharged d-dimensional hyperspherically symmetric Schwarzschild-Tangherlini black hole has to be associated to branes where the second doublet \((E_2, M_2) = (\overline{E}_1, \overline{M}_1)\) contains the anti-branes of the first doublet in order to be compatible with an uncharged configuration. This also fits because both the Schwarzschild-Tangherlini black hole and the brane anti-brane configuration break all supersymmetry. Moreover, because the Schwarzschild-Tangherlini black hole is a non-dilatonic black hole it seems that for the string-theory case one should employ a self-dual \((D3, D3) + (\overline{D}3, \overline{D}3)\) doublet as the D3-brane is the only non-dilatonic brane. Further work mapping such dual brane pairs in detail to background geometries is underway and we hope to report on this soon.

Let us further mention that the cell-volume \(V_{\text{cell}}\) on each of the dual pairs \((E_i, M_i)\) is given, due to the orthogonality of \(E_i\) and \(M_i\), by the product

\[
V_{\text{cell}} = v_{E_i} v_{M_i} = \frac{1}{\tau_{E_i} \tau_{M_i}} = \begin{cases} 
(2\pi)^{\frac{6}{2}} \alpha' \frac{\sqrt{g_s}}{\sqrt{g_s}} (D = 10) \\
(2\pi)^{\frac{7}{2}} \frac{l_1^9}{l_1^{11}} (D = 11)
\end{cases}.
\]

Restoring the constants \(\hbar\) and \(c\) and noticing that \(G_d \hbar/c^3\) has length-dimension \(L^{d-2}\) this result can be written in a universal way as

\[
V_{\text{cell}} = \begin{cases} 8G_{10} \times \frac{\hbar}{c^3} (D = 10) \\
8G_{11} \times \frac{\hbar}{c^3} (D = 11)
\end{cases}.
\]

It therefore clearly shows first that Newton’s Constant acquires a geometric meaning as the cell volume and second that a non-trivial cell-volume is indeed a quantum effect which vanishes in the classical limit \(\hbar \to 0\). This agrees well with the understanding of the cell-volume through a non-commutative structure on the brane worldvolume which likewise results from a promotion of the classical coordinates \(x^i\) to non-commuting quantum operators \(X^i\).

Furthermore, expressing the effective \(d\)-dimensional Planck-length \(l_d\) in terms of the cell volume \(V_{\text{cell}}\), we obtain

\[
l_d^{d-2} = G_d \frac{\hbar}{c^3} = \frac{G_{10}}{\text{vol}(M^{10-d})} \frac{\hbar}{c^3} = \frac{V_{\text{cell}}}{8\text{vol}(M^{10-d})} (D = 10)
\]
for the 10-dimensional and
\[ l_{d-2}^d = G_d \frac{\hbar}{c^3} = \frac{G_{11}}{\text{vol}(\mathcal{M}^{11-d})} \times \frac{\hbar}{c^3} = \frac{V_{\text{cell}}}{8\text{vol}(\mathcal{M}^{11-d})} \quad (D = 11) \] (33)
for the 11-dimensional M-theory case. The size of the \( d \)-dimensional Planck-length can therefore be understood geometrically as the ratio of the cell volume and the compactification volume.

Finally, with the formulae just given, one can once more derive (24). Starting with
\[ N = \frac{\text{joint worldvolume of } (E_1, M_1) + (E_2, M_2)}{V_{\text{cell}}} \] (34)
as the defining equation for the number of cells, \( N \), one obtains immediately
\[ N = 2\frac{\text{vol}(\mathcal{H}^{d-2})}{V_{\text{cell}}} \times \begin{cases} \text{vol}(\mathcal{M}^{10-d}) & (D = 10) \\ \text{vol}(\mathcal{M}^{11-d}) & (D = 11) \end{cases} \]. (35)
By using (32) resp. (33) to substitute for \( V_{\text{cell}} \) one obtains for both the D=10 and the D=11 case the same result
\[ N = \frac{\text{vol}(\mathcal{H}^{d-2})}{4G_d}, \] (36)
namely the \( d \)-dimensional BH-entropy \( S_{BH} \) of the spacetime \( \mathcal{M}^{1,d-1} \).

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References


[16] see e.g. N. Straumann, astro-ph/9711276;
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