Statistical Physics in Meteorology

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Abstract

Various aspects of modern statistical physics and meteorology can be tied together. The historical importance of the University of Wroclaw in the field of meteorology is first pointed out. Next, some basic difference about time and space scales between meteorology and climatology is outlined. The nature and role of clouds both from a geometric and thermal point of view are recalled. Recent studies of scaling laws for atmospheric variables are mentioned, like studies on cirrus ice content, brightness temperature, liquid water path fluctuations, cloud base height fluctuations, .... Technical time series analysis approaches based on modern statistical physics considerations are outlined.

I. INTRODUCTION AND FOREWORD

This contribution to the 18th Max Born Symposium Proceedings, cannot be seen as an extensive review of the connection between meteorology and various aspects of modern statistical physics. Space and time (and weather)

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limit its content. Much of what is found here can rather be considered to result from a biased view point or limited understanding of a frustrated new researcher unsatisfied by the present status of the field. Yet only to be found is a set of basic considerations and reflections expecting to give lines for various investigations, in the spirit of modern statistical physics ideas.

The author came into this subject starting from previous work in econophysics, when he observed that some "weather derivatives" were in use, and some sort of game initiated by the Frankfurt Deutsche Börse in order to attract customers which could predict the temperature in various cities within a certain lapse of time, and win some prize thereafter. This subject was similar to predicting the S&P500 or other financial index values at a certain future time. Whence various techniques which were used in econophysics, like the detrended fluctuation analysis, the multifractals, the moving average crossing techniques, etc. could be attempted from scratch.

Beside the weather (temperature) derivatives other effects are of interest. Much is said and written about e.g. the ozone layer and the Kyoto "agreement". The El Niño system is a great challenge to scientists. Since there is some data available under the form of time series, like the Southern Oscillation Index, it is of interest to look for trends, coherent structures, periods, correlations in noise, etc. in order to bring some knowledge, if possible basic parameters, to this meteorological field and expect to import some modern statistical physics ideas into such climatological phenomena. It appeared that other data are also available, like those obtained under various experiments, put into force by various agencies, like the Atlantic Stratocumulus Transition Experiment (ASTEX) for ocean surfaces or those of the Atmospheric Radiation Measurement Program (ARM), among others.

However it appeared that the data is sometimes of rather limited value because of the lack of precision, or are biased because the raw data is already
transformed through models, and arbitrarily averaged ("filtered") whence
even sometimes lacking the meaning it should contain. Therefore a great
challenge comes through in order to sort out the wheat from the chaff in or-
der to develop meaningful studies. I will mention most of the work to which
I have contributed, being aware that I am failing to acknowledge many more
important reports than those, - for what I truly apologize. There are very in-
teresting lecture notes on the web for basic modules on meteorological training
courses, e.g. one available through ECMWF website\(^4\).

In Sect.2, I will briefly comment on the history of meteorology. The notion
of clouds, in Sect. 3, allows for bringing up the geometrical notion of fractals
for meteorology work, thus scaling laws, and modern data analysis techniques.
Simple technical and useful approaches, based on standard statistical physics
techniques and ideas, in particular based on the scaling hypothesis for phase
transitions and percolation theory features will be found in Sect. 4.

**II. HISTORICAL INTRODUCTION**

From the beginning of times, the earth, sky, weather have been of great
concern. As soon as agriculture, commerce, travelling on land and sea pre-
vailed, men have wished to predict the weather. Later on airborne machines
need atmosphere knowledge and weather predictions for best flying. Nowa-
days there is much money spent on weather predictions for sport activities.
It is known how the knowledge of weather (temperature, wind, humidity, ..)
is relevant, (even *fundamental* !), e.g. in sailing races or in Formula 1 and
car rally races. Let it be recalled the importance of knowing and predicting
the wind (strength and directions), pressure and temperature at high altitude
for the (recent) no-stop balloon round the world trip. The first to draw sea
wind maps was Halley\(^5\), an admirer of Breslau administration. That followed
the "classical" isobaths and isoheights (these are geometrical measures !!!) for sailors needing to go through channels.

I am very pleased to point out that Heinrich Wilhelm Brandes (1777-1834), Professor of Mathematics and Physics at the University of Breslau was the first\(^5\) who had the idea of displaying weather data (temperature, air pressure, a.s.o.) on geographical maps\(^1\). Later von Humboldt (1769-1859) had the idea to connect points in order to draw isotherms\(^5\). It is well known nowadays that various algorithms will give various isotherms, starting from the same temperature data and coordinate table. In fact the maximum or minimum temperature as defined in meteorology\(^6,7\) are far from the ones acceptable in physics laboratories. Note that displayed isotherms connect data points which values are obtained at different times! No need to say that it seems essential to concentrate on predicting the uncertainty in forecast models of weather and climate as emphasized elsewhere\(^8\).

### III. CLIMATE AND WEATHER. THE ROLE OF CLOUDS

Earth’s climate is clearly determined by complex interactions between sun, oceans, atmosphere, land and biosphere\(^9,10\). The composition of the atmosphere is particularly important because certain gases, including water vapor, carbon dioxide, etc., absorb heat radiated from Earth’s surface. As the atmosphere warms up, it in turn radiates heat back to the surface that increases the earth’s "mean surface temperature".

Much attention has been paid recently\(^11,12\) to the importance of the main components of the atmosphere, in particular clouds\(^13\), in the water three forms

\(^1\)It seems that H.W. Brandes left Breslau to get his Ph.D. thesis in Heidelberg in 1826. Alas it seems that the original drawings are not available at this time. Where are they?
— vapor, liquid and solid, for buffering the global temperature against reduced or increased solar heating. This leads to efforts to improve not only models of the earth’s climate but also predictions of climate change, as understood over long time intervals, in contrast to shorter time scales for weather forecast. In fact, with respect to climatology the situation is very complicated because one does not even know what the evolution equations are. Since controlled experiments cannot be performed on the climate system, one relies on using ad hoc models to identify cause-and-effect relationships. Nowadays there are several climate models belonging to many different centers. Their web sites not only carry sometimes the model output used to make images but also provide the source code. It seems relevant to point out here that the stochastic resonance idea was proposed to describe climatology evolution.

It should be remembered that solutions of Navier-Stokes equations forcefully depend on the initial conditions, and steps of integrations. Therefore a great precision on the temperature, wind velocity, etc. cannot be expected and the solution(s) are only looking like a mess after a few numerical steps. The Monte Carlo technique suggests to introduce successively a set of initial conditions, perform the integration of the differential equations and make an average thereafter. It is hereby time to mention Lorenz’s work who simplified Navier-Stokes equations searching for some predictiability. However, predicting the outcome of such a set of equations with complex nonlinear interactions taking place in an open system is a difficult task.

The turbulent character in the atmospheric boundary layer (ABL) is one of its most important features. Turbulence can be caused by a variety of processes, like thermal convection, or mechanically generated by wind shear, or following interactions influenced by the rotation of the Earth. This complexity of physical processes and interactions between them create a variety of atmospheric formations. In particular, in a cloudy ABL the radiative fluxes
produce local sources of heating or cooling within the mixed-layer and therefore can greatly influence its turbulent structure and dynamics, especially in the cloud base. Two practical cases, the marine ABL and the continental ABL have been investigated for their scaling properties

Yet, let it be emphasized that the first modern ideas of statistical physics implemented on cloud studies through fractal geometry are due to Lovejoy who looked at the perimeter-area relationship of rain and cloud areas\textsuperscript{26}, fractal dimension of their shape or ground projection. He discovered the statistical self-similarity of cloud boundaries through area-perimeter analyses of the geometry of satellites, fractal scaling of the cloud perimeter in the horizontal plane. He found the fractal dimension $D_p \simeq 4/3$ over a spectrum of 4 orders of magnitude in size, for small fair weather cumuli ($\sim 1021$ km) up to huge stratus fields ($\sim 103$ km). Cloud size distributions have also been studied from a scaling point of view\textsuperscript{27–30}. Rain has also received much attention\textsuperscript{31–37}.

IV. MODERN STATISTICAL PHYSICS APPROACHES

Due to the nonlinear physics laws governing the phenomena in the atmosphere, the time series of the atmospheric quantities are usually non-stationary\textsuperscript{38,39} as revealed by Fourier spectral analysis, which is usually the first technique to use. Recently, new techniques have been developed that can systematically eliminate trends and cycles in the data and thus reveal intrinsic dynamical properties such as correlations that are very often masked by nonstationarities,\textsuperscript{40,41} Whence many studies reveal long-range power-law correlations in geophysics time series\textsuperscript{39,42} in particular in meteorology\textsuperscript{43–50}. Multi-affine properties\textsuperscript{25,51–59} can also be identified, using singular spectrum or/and wavelets.

There are different levels of essential interest for sorting out correlations
from data, in order to increase the confidence in predictability\textsuperscript{60}. There are investigations based on long-, medium-, and short-range horizons. The $i$-diagram variability ($iVD$) method allows to sort out some short range correlations. The technique has been used on a liquid water cloud content data set taken from the Atlantic Stratocumulus Transition Experiment (ASTEX) 92 field program\textsuperscript{61}. It has also been shown that the random matrix approach can be applied to the empirical correlation matrices obtained from the analysis of the basic atmospheric parameters that characterize the state of atmosphere\textsuperscript{62}. The principal component analysis technique is a standard technique\textsuperscript{63} in meteorology and climate studies. The Fokker-Planck equation for describing the liquid water path\textsuperscript{64} is also of interest. See also some tentative search for power law correlations in the Southern Oscillation Index fluctuations characterizing El Niño\textsuperscript{65}. But there are many other works of interest\textsuperscript{66}.

A. Ice in cirrus clouds

In clouds, ice appears in a variety of forms, shapes, depending on the formation mechanism and the atmospheric conditions\textsuperscript{22,51,67,68}. The cloud inner structure, content, temperature, life time, ... can be studied. In cirrus clouds, at temperatures colder than about $-40^\circ$ C ice crystals form. Because of the vertical extent, ca. from about 4 to 14 km and higher, and the layered structure of such clouds one way of obtaining some information about their properties is mainly by using ground-based remote sensing instruments\textsuperscript{69–72}. Attention can be focussed\textsuperscript{50} on correlations in the fluctuations of radar signals obtained at isodepths of winter and fall cirrus clouds giving (i) the backscattering cross-section, (ii) the Doppler velocity and (iii) the Doppler spectral width of the ice crystals. They correspond to the physical coefficients used in Navier Stokes equations to describe flows, i.e. bulk modulus, viscosity, and
thermal conductivity. It was found that power-law time correlations exist with a crossover between regimes at about 3 to 5 min, but also $1/f$ behavior, characterizing the top and the bottom layers and the bulk of the clouds. The underlying mechanisms for such correlations likely originate in ice nucleation and crystal growth processes.

**B. Stratus clouds**

In stratus clouds, long-range power-law correlations\(^{45,49}\) and multi-affine properties\(^{24,25,57}\) have reported for the liquid water fluctuations, beside the spectral density\(^{73}\). Interestingly, stratus cloud data retrieved from the radiance, recorded as brightness temperature,\(^2\) at the Southern Great Plains central facility and operated in the vertically pointing mode\(^{74}\) indicated a Fourier spectrum, $S(f) \sim f^{-\beta}$, $\beta$ exponent equal to $1.56 \pm 0.03$ pointing to a nonstationary time series. The detrended fluctuation analysis (DFA) method applied on the stratus cloud brightness microwave recording\(^{45,75}\) indicates the existence of long-range power-law correlations over a two hour time.

Contrasts in behaviors, depending on seasons can be pointed out. The DFA analysis of liquid water path data measured in April 1998 gives a scaling exponent $\alpha = 0.34 \pm 0.01$ holding from 3 to 60 minutes. This scaling range is shorter than the 150 min scaling range\(^{45}\) for a stratus cloud in January 1998 at the same site. For longer correlation times a crossover to $\alpha = 0.50 \pm 0.01$ is seen up to about 2 h, after which the statistics of the DFA function is not reliable.

However a change in regime from Gaussian to non-Gaussian fluctuation regimes has been clearly defined for the cloud structure changes using a finite

\(^2\)http://www.phys.unm.edu/ duric/phy423/l1/node3.html
size (time) interval window. It has been shown that the DFA exponent turns from a low value (about 0.3) to 0.5 before the cloud breaks. This indicates that the stability of the cloud, represented by antipersistent fluctuations is (for some unknown reason at this level) turning into a system for which the fluctuations are similar to a pure random walk. The same type of finding was observed for the so called Liquid Water Path\(^3\).

The value of \(\alpha \approx 0.3\) can be interpreted as the \(H_1\) parameter of the multifractal analysis of liquid water content\(^{24,25,52}\) and of liquid water path\(^{57}\). Whence, the appearance of broken clouds and clear sky following a period of thick stratus can be interpreted as a non equilibrium transition or a sort of fracture process in more conventional physics. The existence of a crossover suggests two types of correlated events as in classical fracture processes: nucleation and growth of diluted droplets. Such a marked change in persistence implies that specific fluctuation correlation dynamics should be usefully inserted as ingredients in \textit{ad hoc} models.

\[ \text{C. Cloud base height} \]

The variations in the local \(\alpha\)-exponent ("multi-affinity") suggest that the nature of the correlations change with time, so called intermittency phenomena. The evolution of the time series can be decomposed into successive persistent and anti-persistent sequences. It should be noted that the intermittency of a signal is related to existence of extreme events, thus a distribution of events away from a Gaussian distribution, in the evolution of the process that has generated the data. If the tails of the distribution function follow a power

\[^3\text{The liquid water path (LWP) is the amount of liquid water in a vertical column of the atmosphere; it is measured in cm}^{-3}\text{; ... sometimes in cm }!!!\]
law, then the scaling exponent defines the critical order value after which the statistical moments of the signal diverge. Therefore it is of interest to probe the distribution of the fluctuations of a time dependent signal $y(t)$ prior investigating its intermittency. Much work has been devoted to the cloud base height\textsuperscript{54–56}, under various ABL conditions, and the LWP\textsuperscript{57,64}. Neither the distribution of the fluctuations of liquid water path signals nor those of the cloud base height appear to be Gaussian. The tails of the distribution follow a power law pointing to "large events" also occurring in the meteorological (space and time) framework. This may suggest routes for other models.

D. Sea Surface Temperature

Other time series analysis have been investigated searching for power law exponents, like in atmospheric\textsuperscript{76} or sea surface temperature (SST) fluctuations\textsuperscript{77}. These are of importance for weighing their impacts on regional climate, whence finally to greatly increase predictability of precipitation during all seasons. Currently, climate patterns derived from global SST are used to forecast precipitation.

Recently we have attempted to observe whether the fluctuations in the Southern Oscillation index (SOI) characterizing El Niño were also prone to a power law analysis. For the SOI monthly averaged data time interval 1866-2000, the tails of the cumulative distribution of the fluctuations of SOI signal it is found that large fluctuations are more likely to occur than the Gaussian distribution would predict. An antipersistent type of correlations exist for a time interval ranging from about 4 months to about 6 years. This leads to favor specific physical models for El Niño description\textsuperscript{65}. 
V. CONCLUSIONS

Modern statistical physics techniques for analyzing atmospheric time series signals indicate scaling laws (exponents and ranges) for correlations. A few examples have been given briefly here above, mainly from contributed papers in which the author has been involved. Work by many other authors have not been included for lack of space. This brief set of comments is only intended for indicating how meteorology and climate problems can be tied to scaling laws and inherent time series data analysis techniques. Those ideas/theories have allowed me to reduce the list of quoted references, though even like this I might have been unfair. One example can be recalled in this conclusion to make the point: the stratus clouds break when the molecule density fluctuations become Gaussian, i.e. when the molecular motion becomes Brownian-like. This should lead to better predictability on the cloud evolution and enormously extend the predictability range in weather forecast along the lines of nonlinear dynamics\textsuperscript{78}.

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2 http://www.arm.gov.


4 http://www.ecmwf.int/newsevents/training/rcourse_notes/index.html.


6 http://www.maa.org/features/mathchat/mathchat_4_20_00.html.


16 http://stommel.tamu.edu/baum/climate_modeling.html.


72 http://www.arm.gov/docs/instruments/static/blc.html.


