Visible Sector Supersymmetry Breaking
Revisited

Piyush Kumar† and Joseph D. Lykken†,*

†Enrico Fermi Institute and Department of Physics
University of Chicago, Chicago, IL 60637, USA

*Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, IL 60510, USA

Abstract

We revisit the possibility of “visible sector” SUSY models: models which are straightforward renormalizable extensions of the Minimal Supersymmetric Standard Model (MSSM), where SUSY is broken at tree level. Models of this type were abandoned twenty years ago due to phenomenological problems, which we review. We then demonstrate that it is possible to construct simple phenomenologically viable visible sector SUSY models. Such models are indeed very constrained, and have some inelegant features. They also have interesting and distinctive phenomenology. Our models predict light gauginos and very heavy squarks and sleptons. The squarks and sleptons may not be observable at the LHC. The LSP is a stable very light gravitino with a significant Higgsino admixture. The NLSP is mostly Bino. The Higgs boson is naturally heavy. Proton decay is sufficiently and naturally suppressed, even for a cutoff scale as low as $10^8$ GeV. The lightest particle of the O’Raifeartaigh sector (the LOP) is stable, and is an interesting cold dark matter candidate.
1 Introduction

Supersymmetry (SUSY) is a beautiful idea which starts to lose some of its lustre when one tries to build complete models that break SUSY in a realistic way. This lack of felicity has been accentuated in recent years by experimental constraints that seem to force some degree of unexplained tuning in any viable model [1].

In all popular models, supersymmetry breaking occurs in a “hidden sector” and supersymmetry breaking is “mediated” to the visible sector by indirect interactions. The known scenarios for the mediation of SUSY breaking in hidden sector models were classified in [2] as gravity mediation, gauge mediation, and bulk mediation. Simply put, in gravity mediation the soft parameters arise due to couplings which are Planck suppressed, i.e. they vanish as $M_P \to \infty$. In gauge mediation, the soft parameters arise from loop diagrams involving new messenger fields with Standard Model (SM) quantum numbers. In bulk mediation, the hidden and observable sectors reside on different branes separated in extra dimensions, and SUSY breaking is mediated by fields which propagate in the bulk.

In this paper we revisit the possibility of what one might call “visible sector” SUSY models: models which are straightforward renormalizable extensions of the Minimal Supersymmetric Standard Model (MSSM), where SUSY is broken at tree level. Models of this type were abandoned twenty years ago due to phenomenological problems which we will review in the next section. In addition, hidden sector models seemed more attractive theoretically, as they have a natural tie-in to the grand unification and Planck scales, and exhibit radiative electroweak symmetry breaking due to the large top quark Yukawa coupling.

In this paper we demonstrate that it is possible to construct phenomenologically viable visible sector SUSY models. Such models are indeed very constrained, and have some inelegant features. They also have interesting and distinctive phenomenology.

Our model (it is really a class of models) possesses an extra low energy $U(1)$ gauge group, under which the two Higgs fields are charged with the same sign. This implies that the $\mu$ term is forbidden in our model, but an effective $\mu$ term is generated by the spontaneous breaking of the extra $U(1)$. In addition, as will be shown, adding an extra $U(1)$ also helps to sufficiently suppress the $B$ and $L$ violating interactions.

Our model can be considered as a complete effective field theory description of physics below a cutoff scale which can be as low as about $10^8$ GeV. As such, it is impressively simple. It is also similar to the effective models proposed by Banks [3], which are conjectured to mock up the effects of cosmological supersymmetry breaking by string effects.

The paper is organized as follows. In Section 2, earlier models of low energy supersymmetry are reviewed and some of their problems are discussed. In Sections 3 and 4, we introduce the model, outline its main features and calculate its spectrum. In Section 5, we comment on phenomenological implications of the model. Technical details are provided in the Appendix.
2 Earlier Attempts

Any supersymmetric model limited to the Standard Model gauge group has two immediate problems:

- Renormalizable, and thus unsuppressed, $B$ & $L$ nonconserving interactions are present.

- There is a mass sum rule at tree level which is phenomenologically untenable, because it leads to very light superpartners which have not been observed.

In the Minimal Supersymmetric Standard Model (MSSM), these problems are dealt with in the following way [4] :

- A discrete symmetry (R-parity) is imposed on the Lagrangian which forbids all $B$ & $L$ violating renormalizable interactions.

- Supersymmetry is assumed to be explicitly broken by soft (super-renormalizable) terms which allow us to give acceptable values to particle masses. These soft terms are put in by hand.

However, it is still possible to have dimension five R-parity conserving but $B$ & $L$ violating interactions such as $(QQQL)_F$ and $(UUDE)_F$, which among other things seem likely to induce proton decay at a rate greater than the current bounds set by SuperKamiokande [5]. Thus the above remedies are not complete, in addition to being ad hoc.

The approach adopted in this paper is to suppose that the gauge group which survives down to ordinary energies contains an additional factor $G$. As was shown in [6], $G$ must contain a $U(1)$ factor, and so the simplest choice is just $U(1)$. Now, if all the quark and lepton superfields have a $U(1)$ charge with the same sign, then all $d = 4$ R-parity violating and $d = 5$ R-parity conserving interactions involving only quarks and leptons are forbidden.

Giving all the quarks and leptons $U(1)$ charges of the same sign has another advantage. It leads to the possibility that all squarks and sleptons can be sufficiently massive. In addition, the $\mu$ term is forbidden, while an “effective” $\mu$ term is generated when the field coupled to $H_u$ and $H_d$ obtains a vacuum expectation value. This is important for providing sufficient masses to the charginos, as will be seen later.

However, the situation is more subtle. Adding an extra gauge group introduces new anomalies in the MSSM, which was originally anomaly free. So extra fields must be added to cancel these anomalies. Constructing an anomaly free model with a viable phenomenology is not easy. To appreciate these problems better, let us look at them in greater detail.

In any $\mathcal{N} = 1$ supersymmetric gauge theory with gauge group $G = \prod_\alpha G^\alpha$, there is an interaction between vector superfields $V^\alpha$ and chiral superfields $\Phi^a$. The scalar, spinor and vector components of $V^\alpha$ are $D^\alpha$, $\lambda^\alpha$ and $V^\alpha_\mu$ while the scalar and spinor components of $\Phi^a$ are $F_a$, $\phi_a$ and $\psi_a$. The tree level effective potential for the scalar components $\phi_a$ of $\Phi^a$ is given by:
\[
V(\phi) = \frac{1}{2} \sum_{\alpha} (D^\alpha)^2 + \sum_a |F_{ai}|^2
\]
\[
= \sum_{\alpha} \frac{1}{2} g_{a_\alpha}^2 (\sum_a \phi_{ai}^\dagger T_{ij}^{a_\alpha} \phi_{aj} + \xi^\alpha)^2 + \sum_a |\frac{\partial W}{\partial \phi_{ai}}|^2 ,
\]

where \( \alpha \) labels different factors of the gauge group, \( a \) the various chiral superfield representations and \( i \) the components of each representation. The gauge couplings for \( G^\alpha \) are \( g_{a_\alpha} \), and \( T_{ij}^{a_\alpha} \) are the generators of \( G^\alpha \) in the representation of \( \phi_a \). \( W \) denotes the superpotential. The Fayet-Iliopoulos (FI) couplings \( \xi^\alpha \) are only present if there is a U(1) factor [7]. The spontaneous breaking of supersymmetry at tree level leads to a mass relation [8] :

\[
\sum_J (-1)^{2J} (2J + 1) \text{Tr}(m_J^2) = \sum_{\alpha} \frac{1}{2} g_{a_\alpha}^2 \langle D^\alpha \rangle \text{Tr}(t^\alpha) ,
\]

where \( m_J \) is the mass matrix for spin \( J \) fields, \( \alpha \) now runs only over U(1) factors and

\[
\langle D^\alpha \rangle = (\sum_a \langle \phi_{ai} \rangle^\dagger T_{ij}^{a_\alpha} \langle \phi_{aj} \rangle + \xi^\alpha) .
\]

In our model we will introduce a single extra U(1) factor, which we will denote by \( \bar{U}(1) \). Now observe that if \( \langle \bar{D} \rangle \neq 0 \) and the trace over quarks and leptons is separately nonzero, as is the case when all the squarks and sleptons are given \( \bar{U}(1) \) charges of the same sign, then there is a possibility that all the sparticles can be made to receive large masses.

It has proven difficult to construct a renormalizable and anomaly free model of this type. Previous attempts at models along these lines has led to unacceptable features such as the presence of a color breaking minimum, the absence of a \( \bar{D} \) term vev, or both. The best developed earlier models appear in [9], [10]. However in [9], \( \text{Tr}(\bar{U}(1)) \neq 0 \), so there is a \( \bar{U}(1) \)-gravitational anomaly and a quadratically divergent renormalization of the FI term [11], while in [10], the problem of finding the global minimum has not been correctly dealt with.

A more successful recent example are the models of Cheng, Dobrescu and Matchev [12]. These are completely chiral renormalizable models with an anomaly free extra U(1). These models have in addition a hidden sector with extra gauge interactions that generate \( F \) term SUSY breaking dynamically, as in standard gauge mediation, and spontaneously break the extra U(1) at a scale \( \sim 10^3 \) TeV. The dimensionless parameters of the models are tuned at tree level such that the \( D \) term vev ends up of order 100 GeV. These models are phenomenologically viable and have some attractive theoretical features.

Our model also provides a totally anomaly free solution to the problems outlined at the beginning of this section. We have a single input scale of order 20 TeV. Supersymmetry and the extra U(1) are broken spontaneously at tree level. As in the models of [12], a single tree level tuning is necessary.
to generate the electroweak scale from the 20 TeV input scale, but it is not a fine tuning since the radiative corrections are suppressed. $B$ & $L$ violating interactions are greatly suppressed.

3 The Model

Our model is built along the lines of [6], [9] and [10]. A table of all the chiral superfields, together with their quantum numbers, is provided in the Appendix. The model has several exotics with Standard Model quantum numbers: a color octet chiral superfield $K$, an $SU(2)$ triplet superfield $T$, and two vectorlike pairs of hypercharged chiral superfields $J_i, J^c_i$, $i=1,2$. These fields are introduced to cancel the $SU(3)^2\tilde{U}(1)$, $SU(2)^2\tilde{U}(1)$, and $U(1)^2\tilde{U}(1)$ anomalies. To construct a completely anomaly free model, several MSSM singlet fields which are only charged under $\tilde{U}(1)$ are also needed.

Supersymmetry is broken at tree level by an O’Raifeartaigh sector [13], generating $F$ term vevs. The model has Fayet-Iliopoulos terms for both the $\tilde{U}(1)$ and hypercharge, leading to $D$ term vevs. Electroweak symmetry and the $\tilde{U}(1)$ gauge symmetry are broken at tree level. In order to generate one-loop gaugino masses that are large enough to satisfy current experimental bounds, the $F$ and $\tilde{D}$ term vevs must be of order 20 TeV. Thus to obtain the proper electroweak scale, we require a single tree level tuning of the Fayet-Iliopoulos parameters. This is the least attractive property of visible sector SUSY breaking. However the tree level tuning is robust against radiative corrections, i.e. masses which are of order the electroweak scale at tree level remain of order the electroweak scale after radiative corrections.

The superpotential of the model is

$$W = W_{\text{Yukawa}} + W_{\text{O’R}} + W_\mu + \tilde{W} .$$

This superpotential consists of four pieces. The first piece is the superpotential of the MSSM without a $\mu$ term:

$$W_{\text{Yukawa}} = y_E E \bar{E} H + y_D D \bar{D} H + y_U U \bar{U} H'd .$$

The second piece is an O’Raifeartaigh sector which breaks SUSY and the $\tilde{U}(1)$ spontaneously at tree level. Due to this $F$ term breaking, all of the MSSM gauginos, squarks and sleptons receive soft-breaking masses, either at tree level, or at one-loop, or both.

$$W_{\text{O’R}} = \lambda_K K^2 X_1 + \lambda_T T^2 X_1 + \sum_{i=1}^2 \lambda_J J_i J^c_i X_1 + \lambda_R \sum_{i=1}^{11} R^2_i X_1$$

$$+ (f Y + M_2) X_1 X_2 - f \mu^2 Y + M_1 X_2 X_3 .$$

The third piece consists of the MSSM Higgs fields and some MSSM singlets, some of which are charged under $\tilde{U}(1)$. This sector spontaneously also breaks the $\tilde{U}(1)$ gauge symmetry, and simultaneously
generates an effective $\mu$ term.

$$W_\mu = \frac{1}{2}m_\phi^2 + (\mu' + \bar{g}\phi)SN - \kappa^2\phi + \frac{1}{3}\beta\phi^3 + \bar{M}NP - g_H S\phi H'. \quad (7)$$

The last piece couples the hypercharged exotics $J_i$, $J_i'$ to the MSSM singlet field $P$ that appears in $W_\mu$:

$$\tilde{W} = \lambda_P J_i P. \quad (8)$$

This additional coupling is needed to explicitly break an accidental global $U(1)$ symmetry otherwise present in the model, which is spontaneously broken when some of the fields get vevs.

It is important to note that there is some freedom in the choice of $W_\mu$. One could extend the Higgs sector in several ways consistent with cancellation of anomalies. For example, it would be interesting to incorporate the “$\mu$-less” models of Nelson et al [14].

The scalar potential generated is:

$$V = \left|\frac{\partial W}{\partial \phi}\right|^2 + \frac{1}{2}[D_{SU(3)}]^2 + \frac{1}{2}[D_{SU(2)}]^2 + \frac{1}{2}g_H^2 \left[\frac{1}{6}Q^\dagger Q - \frac{2}{3}U^\dagger U + \frac{1}{3}D^\dagger D - \frac{1}{2}L^\dagger L + \bar{E}\dagger\bar{E} + \frac{1}{2}\mathcal{H}^\dagger\mathcal{H}' - \frac{1}{2}\mathcal{H}^\dagger \mathcal{H} + \sum_{i=1}^{2} J_i^\dagger J_i - \sum_{i=1}^{2} J_i'^\dagger J_i' + \xi]^2 + \frac{1}{2}g^2 \left[Q^\dagger Q + \bar{U}\dagger\bar{U} + \bar{D}\dagger\bar{D} + \bar{L}\dagger\bar{L} + \bar{E}\dagger\bar{E} - 2\mathcal{H}^\dagger \mathcal{H}' - 2K^\dagger K - 2T^\dagger T - 2\sum_{i=1}^{2} J_i^\dagger J_i - 2\sum_{i=1}^{2} J_i'^\dagger J_i' + 4X_1^\dagger X_1 - 4X_2^\dagger X_2 + 4X_3^\dagger X_3 + 4\mathcal{S}^\dagger S - 4\mathcal{N}^\dagger N + 4\mathcal{P}^\dagger P - 2\sum_{i=1}^{11} R_i^\dagger R_i + \sum_{i=1}^{3} O_i^\dagger O_i + 4\mathcal{V}^\dagger \mathcal{V} + \bar{\xi}]^2. \quad (9)$$

It is straightforward to show the following features of the above model:

- $Y, \phi, X_1, X_2, S & N$ get vevs of order 20 TeV.
- The exotics $K, T, J_i, J_i'$ all get masses of order 20 TeV.
- For $2g_H^2 < g^2$, the Higgs doublets can be brought down to a form $\mathcal{H} = \begin{pmatrix} h_d \\ 0 \end{pmatrix}$ and $\mathcal{H}' = \begin{pmatrix} 0 \\ h_u \end{pmatrix}$ at the potential minimum, corresponding to the conservation of electric charge.

For a suitable range of parameters, the global minimum can be found by solving the following equations for $h_u$ and $h_d$:

$$\begin{align*}
(g_H^2 + 4g^2 - \frac{g^2 + g_H^2}{4})h_u^2 + (\frac{g^2 + g_H^2}{4} + 4g^2)h_d^2 - 8g^2(V_1^2 - V_2^2 + S^2 - N^2 + \frac{1}{4}\bar{\xi}) + g_H^2 S^2 + \frac{1}{2}g^2\bar{\xi} & = 0, \\
(g_H^2 + 4g^2 - \frac{g^2 + g_H^2}{4})h_u^2 + (\frac{g^2 + g_H^2}{4} + 4g^2)h_d^2 - 8g^2(V_1^2 - V_2^2 + S^2 - N^2 + \frac{1}{4}\bar{\xi}) + g_H^2 S^2 - \frac{1}{2}g^2\bar{\xi} & = 0, \quad (10)
\end{align*}$$
and the following equations for \(X_1, X_2, S\) and \(N\):

\[
M_1^2 V_1 V_2 = f^2 (\mu^2 - V_1 V_2)(V_1^2 + V_2^2),
\]

\[
\frac{M_1^2 V_2^2}{V_1^2 + V_2^2} = [16g^2(V_1^2 - V_2^2 + S^2 - N^2) + 4g^2(\xi^2 - 2h^2)],
\]

\[
\langle \tilde{g}^2 N^2 + 4\tilde{g}^2 \langle \tilde{D} \rangle + g_H^2 h^2 \rangle \langle S \rangle = -\tilde{g} \langle N^* \rangle (\beta \phi^2 + m \phi - \kappa^2),
\]

\[
\langle \tilde{g}^2 S^2 - 4\tilde{g}^2 \langle \tilde{D} \rangle + M^2 \rangle \langle N \rangle = -\tilde{g} \langle S^* \rangle (\beta \phi^2 + m \phi - \kappa^2),
\]

where \(\langle X_1 \rangle = V_1, \langle X_2 \rangle = V_2, \langle S \rangle = S, \langle N \rangle = N, h^2 = h_u^2 + h_d^2 = (174 \text{ GeV})^2\). The vev of \(Y\) is given by \(\langle Y \rangle = -\langle M_2 \rangle / f\) and \(\langle \phi \rangle = -\mu' / \tilde{g}\).

The above vacuum structure ensures that charge and color are not broken. The derivation of these equations and various constraints is carried out in the Appendix. As will be seen later, the qualitative nature of the vacuum will not change even after radiative corrections are taken into account.

As already explained, this model has a single input scale on the order of 20 TeV, and a single tree level tuning to get the proper scale of electroweak symmetry breaking. The precise values of the input parameters are not very much constrained. As an example, an appropriate choice of parameters compatible with all constraints is:

\[
f = 1; \ \lambda_J = \lambda_K = \lambda_T = \lambda_P = 0.75; \ \beta = 0.5; \ \lambda_R = 0.25 , \]

\[
M_1 = M_2 = \tilde{M} = 18 \text{ TeV}; \ \mu = 23.7 \text{ TeV}; \ m \simeq 20 \text{ TeV} ,
\]

\[
V_1 = V_2 = S = \mu' = 20 \text{ TeV}; \ N \simeq 20 \text{ TeV}; \ \kappa \simeq 19 \text{ TeV} ,
\]

\[
\tilde{g}\xi \simeq (20.1 \text{ TeV})^2 \ \tilde{g} = 1; \ g_H = 0.45; \ \tilde{g} = 0.1 .
\]

where we have also shown all of the resulting vevs. The electroweak scale is set by a tree level tuning of \(\xi\). For the above choice of parameters, we have:

\[
h_u^2 = \frac{8g^2(V_1^2 - V_2^2 + S^2 - N^2 + \xi^2 / 4) - g_H^2 S^2}{g_H^2 + 8g^2} - \frac{g_y^2 \xi}{g^2 + g_y^2 - 2g_H^2} , \quad (13)
\]

\[
h_d^2 = \frac{8g^2(V_1^2 - V_2^2 + S^2 - N^2 + \xi^2 / 4) - g_H^2 S^2}{g_H^2 + 8g^2} + \frac{g_y^2 \xi}{g^2 + g_y^2 - 2g_H^2} . \quad (14)
\]

The hypercharge \(F I\) parameter \(\xi\) determines \(\tan(\beta) = h_u / h_d\) and, together with \(V_1, V_2, S & N\), it also determines the the Z boson - \(B'\) boson mixing. For e.g. a \(\tan \beta\) of 2, we need:

\[
g_y \xi \simeq -(0.06 \text{ TeV})^2 . \quad (15)
\]
3.1 $R$ symmetry

One of the problems of earlier models of tree level supersymmetry breaking was the presence of a spontaneously broken continuous $R$ symmetry [15],[16],[6]. This is a consequence of the result shown in [17], that for any generic and calculable theory the existence of an $R$ symmetry is a necessary condition, while the existence of a spontaneously broken $R$ symmetry is a sufficient condition for supersymmetry breaking. A spontaneously broken $R$ symmetry leads to the existence of a massless Goldstone boson, which is undesirable for phenomenology [18].

Our model does not have the above problem because it has no continuous $R$ symmetry. Supersymmetry, however, is still broken spontaneously. This is compatible with the result in [17], because the superpotential of our model is nongeneric, i.e. it does not have all possible terms consistent with the symmetries of the theory. Terms like $K^2 X_3$, $Y^2$, etc. which are otherwise allowed in the superpotential are not present in our model. Their omission is thus not natural in the usual sense. However, non-renormalization theorems assure us that if these terms are not present in the tree level lagrangian, they are not generated to any order in perturbation theory. The model is natural in this weaker sense.

3.2 Discrete symmetries

Our superpotential has several discrete symmetries, a $Z_4$ discrete symmetry which we call O’charge, and several $Z_2$ parities. The O’charges of the various superfields are given in the table in the Appendix. The O’charges $q_i$, where $i$ runs over all the chiral superfields, satisfy the relations

$$\sum q_i^3 = 0 \mod 4,$$

$$\sum q_i = 0 \mod 4.$$  \hspace{1cm} (16)

This means that O’charge can be considered as an anomaly free discrete gauge symmetry [19, 20], which is thus respected also by higher dimension operators that could contribute to the superpotential. We will assume by the same token that the $Z_2$ parities are merely accidental, and thus are not respected by higher dimension operators that contribute to the superpotential.

At the same time that the $\tilde{U}(1)$ symmetry is spontaneously broken, the $Z_4$ O’charge invariance is broken down to a residual $Z_2$ parity. We call the remaining unbroken discrete gauge symmetry O’parity.

4 Superpartner Spectrum

4.1 Fermions

The mass terms for the gauginos, higgsinos and other chiral fermions receive contributions from two sources at tree level – one due to the superpotential, and the other due to trilinear couplings between
the gauginos, fermions and scalars. The Higgsinos, electroweak gauginos and various O’Raifeartaigh sector fields which are only charged under \( \tilde{U}(1) \) mix due to the effects of electroweak and \( \tilde{U}(1) \) symmetry breaking. The charged higgsinos, charged winos and charged O’Raifeartaigh sector fields combine to form mass eigenstates with charge \( \pm 1 \) called charginos. Similarly, the neutral higgsinos, neutral winos and neutral O’Raifeartaigh sector fields mix to form mass eigenstates called neutralinos. The gluinos on the other hand, being color octet fermions, receive no contributions to their masses at tree level. At one loop, diagonal mass parameters are induced for all the gauginos. This is the same as in ordinary models of gauge mediated supersymmetry breaking. The contribution to the masses of the gluinos, winos, bino and b’ino at one loop at the scale \( \mu \) is given by an equation similar to that in [21]:

\[
M_a(\mu) = \frac{\alpha_a(\mu)}{4\pi} \sum_i \Lambda(i) n_a(i),
\]

where \( a \) labels the gauge group \( SU(3), SU(2), U(1) \) and \( \tilde{U}(1) \), \( i \) runs over all chiral fields and \( n_a(i) \) is one-half the Dynkin index of the \( a^{th} \) gauge group. The difference from the equation in [21] is that the mass scale \( \Lambda \) is different for different chiral fields. For our choice of parameters, \( \sum_i \Lambda(i) n_a(i) \approx 3.72 V_1 \) for \( a = SU(3) \), \( \approx 2.48 V_1 \) for \( a = SU(2) \), \( \approx 1.8 V_1 \) for \( a = U(1) \) and \( \approx 40.5 V_1 \) for \( a = \tilde{U}(1) \). This results in a one-loop gluino mass of approximately 600 GeV.

In the gauge eigenstate basis, the chargino mass terms in the Lagrangian (at tree level) are:

\[
-\mathcal{L} = \frac{1}{2} (\Psi^\pm)^T \mathcal{M}_c(\Psi)^\pm
\]

\[
= g_H \langle S \rangle \tilde{H}_u^+ \tilde{H}_d^- + g \langle H_d^{0*} \rangle \tilde{W}^+ \tilde{H}_d^- + g \langle H_d^{0*} \rangle \tilde{W}^- \tilde{H}_u^+ + \sum_{i=1}^{2} \lambda_J \langle X_1 \rangle \psi_j \psi_j^c + 2 \lambda_T \langle X_1 \rangle \psi_T^+ \psi_T^-.
\]

To find the correct chargino content, we also need to add the one-loop mass for the charged wino and diagonalize the resulting matrix. The mass terms for the O’Raifeartaigh sector fields are decoupled from the MSSM fields. This leads to the following mass squared eigenvalues:

\[
M_{J_1}^2 = M_{J_2}^2 = \lambda_J V_1^2 = (15 \text{ TeV})^2; \quad M_{T^+}^2 = M_{T^-}^2 = (2\lambda_T V_1)^2 = (30 \text{ TeV})^2; \\
M_{C_1, C_4}^2 = (9 \text{ TeV})^2; \quad M_{C_2, C_3}^2 = (130 \text{ GeV})^2.
\]
The neutralino mass terms can be analyzed in a similar way. In the gauge eigenstate basis, the neutralino mass terms (at tree level) are:

\[-\Delta \mathcal{L} = \frac{1}{2} (\Psi^0)^T \mathcal{M}_N (\Psi^0)\]

\[
g_H \langle S \rangle \tilde{H}_d^0 \tilde{H}_d^0 + g_H \langle H_u^0 \rangle \tilde{X}_4 \tilde{H}_d^0 + g_H \langle H_d^0 \rangle \tilde{X}_4 \tilde{H}_d^0 + \frac{g_f}{\sqrt{2}} \langle H_u^{0*} \rangle \tilde{B} \tilde{H}_u^0 - \frac{g_f}{\sqrt{2}} \langle H_d^{0*} \rangle \tilde{B} \tilde{H}_d^0 + \frac{g}{\sqrt{2}} \langle H_u^{0*} \rangle \tilde{W} \tilde{H}_u^0 - \frac{g}{\sqrt{2}} \langle H_d^{0*} \rangle \tilde{W} \tilde{H}_d^0 - 2\sqrt{2} g \langle H_u^{0*} \rangle \tilde{B}' \tilde{H}_u^0 - 2\sqrt{2} g \langle H_d^{0*} \rangle \tilde{B}' \tilde{H}_d^0 + 4\sqrt{2} \tilde{g} \langle X_1 \rangle \tilde{B}' \tilde{X}_1 - 4\sqrt{2} \tilde{g} \langle X_2 \rangle \tilde{B}' \tilde{X}_2 + M_1 \tilde{X}_2 \tilde{X}_3 + f \langle X_2 \rangle \tilde{Y} \tilde{X}_2 + f \langle X_1 \rangle \tilde{Y} \tilde{X}_1 + \lambda_R \langle X_1 \rangle \sum_{i=1}^{11} \psi_{R_i} \psi_{R_i} + \tilde{M} \tilde{N} \tilde{P} + (m + 2\beta \langle \phi \rangle) \tilde{\phi} \tilde{\phi} + \tilde{g} \langle P \rangle \tilde{S} \tilde{\phi} + \tilde{g} \langle S \rangle \tilde{N} \tilde{\phi} + 2 \lambda_T \langle X_1 \rangle \psi_{T_0} \psi_{T_0} + 2 \lambda_K \langle X_1 \rangle \psi_{k_0}^{(N)} \psi_{k_0}^{(N)} \] (20)

It is clear that the mass terms for \(\psi_{R_i}, \psi_{T_0}\) and neutral components of \(\psi_K\) are decoupled from other terms. Therefore, it is sufficient to diagonalize the remaining thirteen dimensional mass matrix. There are two zero eigenvalues at tree level. One of them corresponds to the goldstino (\(\tilde{G}\)), which is exactly massless as is expected for a spontaneously broken globally supersymmetric theory. This can be seen easily, because the components of \(\tilde{G}\) are of the form \((\langle D_0 \rangle \sqrt{2}, \langle F_i \rangle)\) and it is also annihilated by \(\mathcal{M}_N\). The other massless field corresponds to the photino. However, it is massless only at tree level. Again, as in the case of charginos, we need to take the mass parameters for the bino, bino and neutral wino induced at one loop into account. Diagonalizing the resulting mass matrix gives the following mass eigenvalues for our choice of parameters:

\[
M^2_{R_i} = (10 \text{ TeV})^2; \quad M^2_{T_0} = M^2_{K^{(N)}} = (30 \text{ TeV})^2; \\
M^2_{\tilde{N}_1, \tilde{N}_{13}} \simeq (30 \text{ TeV})^2; \quad M^2_{\tilde{N}_2, \tilde{N}_{12}} \simeq (30 \text{ TeV})^2; \quad M^2_{\tilde{N}_3, \tilde{N}_{11}} \simeq (25 \text{ TeV})^2; \\
M^2_{\tilde{N}_4, \tilde{N}_{10}} \simeq (10 \text{ TeV})^2; \quad M^2_{\tilde{N}_5, \tilde{N}_9} \simeq (9 \text{ TeV})^2; \quad M^2_{\tilde{N}_6} = (132 \text{ GeV})^2; \\
M^2_{\tilde{N}_7} = (32 \text{ GeV})^2; \quad M^2_{\tilde{N}_8} = 0 . \] (21)

A massless goldstino is expected even after the introduction of a non-gauge invariant mass parameter for the gauginos in the mass matrix, because it is a non-perturbative result. Moreover, since the radiative corrections are small, the field content of the goldstino stays pretty much the same, with approximately the following components:
Thus, the goldstino in our model is mostly made of O’Raifeartaigh sector fields. However, it has a small but interesting higgsino content, which might be relevant for phenomenology [22]. We also expect a light NLSP, which in our case is mostly a Bino. The exact values are not very crucial, and can be made higher by changing the parameters slightly.

4.2 Scalars

When supersymmetry is spontaneously broken at tree level, the scalar mass degeneracy is lifted by $D$ terms. Since charge and color are unbroken, the only non-zero $\langle D \rangle$ terms are for $y$ of $U(1)$, $\tilde{y}$ of $\tilde{U}(1)$ and $t_3$ of $SU(2)$. $\langle \tilde{D} \rangle$ provides a large contribution at tree level, contributing to the mass squared of the squarks and sleptons of the order of the input scale - $(20 \text{ TeV})^2$, as seen in the Appendix. The squarks and sleptons also receive contribution to their masses by radiative corrections. At one loop, the contributions to their masses arise from graphs in Figure 2.

The dominant contribution to both graphs comes only from the $\tilde{U}(1)$ gauge group. The values of the graphs are given by:

\begin{equation}
(\Delta M)^2_a \simeq \frac{g^2}{16\pi^2} \left( \sum_i \tilde{q}_i M^2_{\tilde{q}_i} \right); \quad (\Delta M)^2_b \simeq \frac{g^2}{16\pi^2} \left( h^2 + 4V_1^2 + V_2^2 + S^2 + N^2 \right). \tag{23}
\end{equation}

where $\tilde{q}_i$ are the $\tilde{U}(1)$ charges of the scalar fields and $i$ runs over all the scalar fields in our model. For our model, the first graph gives a negative contribution while the second graph a positive one. For our choice of parameters, as in (12), $(\Delta M)^2_a \simeq -1.5 \times 10^{-3} V_1^2$ and $(\Delta M)^2_b \simeq 8.1 \times 10^{-5} V_1^2$, yielding $(\Delta M)^2_{\text{one \ loop}} \simeq -1.4 \times 10^{-3} V_1^2$.

As can be seen from the scalar potential (9), effective mass parameters for the neutral Higgs fields are generated at tree level. These effective mass parameters also receive the above corrections at one loop. The fact that the one loop contribution is much less than the tree level contribution lends support to
the statement that the tree level equations for $h_u$ and $h_d$ are robust and are not modified qualitatively even after loop corrections are taken into account.

Therefore, the net mass of the squarks and sleptons to one loop is given by:

$$M_{sq/sl}^2 = M_{\text{tree level}}^2 + (\Delta M)_{\text{one loop}}^2$$

$$\simeq g^2 \langle \bar{D} \rangle + (\Delta M)_{\text{one loop}}^2 \simeq 0.09 V_1^2 \simeq (6 \text{ TeV})^2 . \quad (24)$$

We see that the squarks and sleptons in our model are quite heavy, around 6 TeV. There are further corrections to the squark and slepton masses from two loop graphs, which are negligible. In addition, RG evolution has to be used to run these contributions down to the electroweak scale. However, this does not affect the qualitative result: squarks and sleptons in the model are quite heavy. One attractive feature of the above mechanism is that flavor changing (FCNC) processes are naturally suppressed, similar to that in gauge mediated models.

4.3 Gauge Bosons

The $Z$ boson, $W$ Bosons and the $\bar{U}(1)$ gauge boson become massive due to electroweak and $\bar{U}(1)$ symmetry breaking. Since $V_1 = V_2 = 20$ TeV, the $\bar{U}(1)$ gauge boson $B'_\mu$ is much heavier than the $W$ and $Z$ bosons. Apart from the usual mass terms for the $W^+_{\mu}, W^-_{\mu}, Z_{\mu}$ and $B'_\mu$, there is also a $Z_{\mu} - B'_\mu$ mixing term. The $B'_\mu$ mass term and $Z_{\mu} - B'_\mu$ mixing term are given by:

$$M_{B'}^2 = 8g^2(h_u^2 + 4(V_1^2 + V_2^2 + S^2 + N^2)); \quad M_{Z-B'}^2 = 2g\sqrt{g^2 + g'^2(h_u^2 - h_d^2)} . \quad (25)$$

Therefore, the mixing angle defined by

$$\alpha_{Z-B'} = \frac{1}{2} \tan^{-1}\left( \frac{2M_{Z-B'}^2}{M_{B'}^2 - M_Z^2} \right) , \quad (26)$$

is about $5 \times 10^{-6}$, which is well below the experimental upper bound of $\sim 3 \times 10^{-3}$ [23].

5 Phenomenology

5.1 Gravitino (Goldstino) phenomenology

One of the distinctive features of all models of low energy supersymmetry breaking is that the gravitino, the spin 3/2 superpartner of the graviton, is the LSP. This is easy to understand. Supersymmetry has to be promoted to a local symmetry to take gravity into account. So, supersymmetry is now broken by the super-Higgs mechanism, where the gravitino acquires a mass by eating the goldstino. The mass of the gravitino is [24]:

$$\frac{1}{2}$$
Figure 3: Goldstino interactions with superpartner pairs (from [28])

\[ m_{3/2} \approx \frac{\sqrt{\langle F \rangle^2 + \langle D \rangle^2}}{\sqrt{3} M_p} \]  

(27)

where \( \sqrt{\langle F \rangle^2 + \langle D \rangle^2} \) is essentially the supersymmetry breaking scale. Thus, for small \( \sqrt{\langle F \rangle^2 + \langle D \rangle^2} \), as in our model, the gravitino is definitely the LSP. The mass of the gravitino in our model is \( \sim 0.03 \text{ eV} \). The gravitino, by absorbing the goldstino, inherits its non-gravitational interactions and so can play an important role in collider physics. Our gravitinos are sufficiently heavy not to be excluded by current collider limits [25, 26, 27].

Since the gravitino is the LSP, we expect supersymmetric particles, which can be pair produced at \( e^+ e^- \) colliders through tree-level processes, to decay into the NLSP (next-to-lightest supersymmetric particle), which then decays into the gravitino (goldstino). The usual way of analyzing goldstino interactions is by the method of the effective lagrangian. From the supercurrent conservation equation, we get [28]:

\[ \partial_\mu j_\mu^\alpha = i(\sqrt{\langle F \rangle^2 + \langle D \rangle^2})(\sigma_\mu \partial_\mu \tilde{G})_\alpha + \partial_\mu j_\mu^\alpha + ... = 0 \]  

(28)

\[ j_\mu^\alpha \approx (\sigma^\nu \tilde{\sigma}^\mu \psi_\alpha)(\partial_\nu \phi^i) - \frac{1}{2\sqrt{2}} \sigma^\nu \tilde{\sigma}^\rho \sigma_\mu \lambda^\alpha F_{\nu \rho}^a \]  

(29)

Here, the ellipses represent contributions which are unimportant at low energies. Equation (28) can be thought of as the goldstino equation of motion, which can be derived from the following effective lagrangian:

\[ \mathcal{L}_{\text{eff}} = -i \tilde{G} \tilde{\sigma}^\mu \partial_\mu \tilde{G} - \frac{1}{(\sqrt{\langle F \rangle^2 + \langle D \rangle^2})^2}(\tilde{G} \partial_\mu j_\mu^\mu + \text{h.c.}) \]  

(30)

Since the above equation only depends on supercurrent conservation, it does not depend on the details of supersymmetry breaking. From (29), we see that there are scalar-chiral fermion-goldstino and gauge boson-gaugino-goldstino vertices, which can lead to decays to the goldstino (Figure 3).
It is important to note that these goldstino interactions are nonrenormalizable because of \((\sqrt{\langle F \rangle^2 + \langle D \rangle^2})\) in the denominator. At very low energies however, these should give the dominant contribution. The NLSP in our model is mostly a Bino:

\[
\chi^0 (NLSP) \simeq -0.9999 \tilde{B} - 0.003 \tilde{W}^0 + 0.004 \tilde{H}_d^0 - 0.002 \tilde{H}_u^0
\]  

(31)

The dominant decay mode of the NLSP is into the photino and the gravitino. Its decay rate can be calculated as [29]:

\[
\Gamma(\tilde{\gamma} \rightarrow \gamma \tilde{G}) \simeq \frac{m_\gamma^5 \kappa_\gamma}{16\pi(\langle F \rangle^2 + \langle D \rangle^2)}
\]  

(32)

where \(\kappa_\gamma\) is the photino content of the NLSP. In our model, \(\kappa_\gamma \sim 0.15\). The mean decay length of the NLSP with energy \(E\) in the lab frame is therefore,

\[
L \sim \frac{1}{\kappa_\gamma} \left( \frac{100 \text{ GeV}}{m_\gamma} \right)^5 \left( \frac{\langle F \rangle^2 + \langle D \rangle^2}{100 \text{ TeV}} \right)^{1/4} \left( \frac{E^2}{m_\gamma^2} - 1 \right)^{1/2} \times 10^{-2} \text{ cm}
\]  

(33)

which for our model is around 0.1 \(\mu \text{m}\). The experimental signature for a process like \(e^+ e^- \rightarrow \tilde{\gamma} \tilde{\gamma}\) is thus given by missing transverse energy, imbalance in the final-state momenta and a pair of photons. In this case, it is also possible to extend the search to a portion of the parameter space inaccessible in the corresponding gravity mediated scenario, where the LSP is invisible [30].

If gravitinos are in thermal equilibrium at early times and freeze out at temperature \(T_f\), their contribution to the present energy density is [31]:

\[
\Omega_{3/2} h^2 = \frac{m_{3/2}^3}{k eV} \left[ \frac{100}{g_*(T_f)} \right]
\]  

(34)

where \(h\) is the Hubble’s constant in units of \(100 \text{ km sec}^{-1} \text{ Mpc}^{-1}\) and \(g_*(T_f)\) is the effective number of degrees of freedom at \(T_f\). Therefore, for \(m_{3/2} < \text{keV}\), as is the case in our model, the gravitinos do not overclose the universe and late entropy production is not required.

### 5.2 Proton decay and gauge coupling unification

O’charge and \(\tilde{U}(1)\) conservation greatly suppresses \(B \& L\) violating interactions. Proton decay requires dimension 7 operators in the superpotential of the form \(QQQ\chi\phi\phi'\) etc., which are suppressed by the cube of the cutoff. Current lower bounds on the proton lifetime only constrain the cutoff scale to be greater than about \(10^8\) GeV.

The problem of classifying \(\tilde{U}(1)\) extensions of the MSSM which solve the \(\mu\) problem, adequately suppress \(B \& L\) violating operators, and keep gauge coupling unification intact, has been solved by Erler [32]. In addition, the solutions have been constrained to respect chirality, so that fields are
protected from acquiring large masses; as well as SU(5) type charge quantization, so that there are no fractionally charged states. The solutions are based on the assumption there is a mechanism built in to solve the \( \mu \) problem. However, the nature of the mechanism is not specified. The solutions have the feature that at least two MSSM singlets charged only under \( U(1) \) develop a vev.

It can be seen that one of the 33 completely chiral solutions of [32] closely resembles our solution. The slight difference is due to the fact that our model is not chiral. This is clear, however, because in our model, the supersymmetry breaking is effected by an O’Raifeartaigh type model, which is intrinsically non-chiral. Discounting the fields which are present only to make the O’Raifeartaigh model work, our model has the satisfying feature that it also contains two MSSM singlets which develop vevs. Therefore, it is reasonable to expect that our model leads to gauge coupling unification.

5.3 CP Violation

In addition, our model has implications for CP violation. In our model, there is a possibility of solving the strong CP problem through the Nelson-Barr mechanism [33]. If the determinant of the mass matrix of all colored fermions is real, \( \Theta_{QCD} \) vanishes at tree level. In a spontaneously broken supersymmetric theory, \( \Theta_{QCD} \) receives corrections proportional to supersymmetry breaking effects. So, for models of low energy supersymmetry breaking, \( \Theta_{QCD} \) does not receive large corrections [34] and the CP problem can be solved.

5.4 Cold Dark Matter

Our model provides a cold dark matter candidate: \( R \), the lightest particle with odd \( O' \)parity. For our choice of parameters, the mass squared of the fields \( R_i \) is given by:

\[
M_{R_i}^2 \simeq (4 \text{ TeV})^2 ,
\]

These masses are quite adjustable, so we will exploit this freedom to assume that one of the \( R_i \), called \( R_i \), is much lighter than the others. Then the other \( R_i \) will decay to this one via higher dimension terms in the superpotential like \( SSR_iRRR \). Note that \( R \) is the lightest among all the O’Raifeartaigh sector particles. We call it the LOP.

The \( R \) particle is a Standard Model singlet. The heavy exotics \( K, T, J_i, J_i^c \) all decay to \( R \) particles via dimension 7 operators like \( X_1 KRQ\bar{U}H' \), which result in 3-body decays like \( K \rightarrow R + t + \bar{t} \). Even though they are from dimension 7 operators, the decay lifetimes are much less than the age of the universe. This is because the rates are greatly enhanced relative to the proton decay rate, due to phase space and the absence of light Yukawa suppression factors.

In the early universe, \( R \) scalars will annihilate pairwise into pairs of Higgs bosons, via a renormalizable \( \tilde{D} \) term induced quartic coupling. Since the annihilation cross section is rather large, and
since the mass of the $R$ particle is fairly adjustable in the model, the LOP should provide a viable candidate for cold dark matter.

6 Summary and Conclusions

We have presented a class of viable visible sector SUSY models which do not seem to violate any current experimental constraints. Such a model could be a complete and correct description of particle physics below a cutoff scale as low as $10^8$ GeV.

To summarize the phenomenology, these models predict light gauginos and very heavy squarks and sleptons. The squarks and sleptons may not be observable at the LHC. The LSP is a stable very light gravitino with a significant Higgsino admixture. The NLSP is mostly Bino. The Higgs boson is naturally heavy (probably heavier than the MSSM upper bound), but we have not computed it. The Higgs quartic coupling will have “hard” corrections as described in [35]. Proton decay is sufficiently and naturally suppressed, even for a rather low cutoff scale. The lightest particle of the O’Raifeartaigh sector (the LOP) is stable, and is an interesting cold dark matter candidate.

Acknowledgments

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A  Appendix

A.1  Left-chiral superfields with their gauge quantum numbers and O’charge

<table>
<thead>
<tr>
<th>Φ</th>
<th>SU(3)_C</th>
<th>SU(2)_L</th>
<th>U(1)_Y</th>
<th>Û(1)</th>
<th>O’charge</th>
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<td>1/6</td>
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<td>-2/3</td>
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<td>0</td>
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<tr>
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<td>1</td>
<td>1/3</td>
<td>1</td>
<td>0</td>
</tr>
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<td>0</td>
</tr>
<tr>
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<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
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</tr>
<tr>
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<td>1/2</td>
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<tr>
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<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
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<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

where i = 1, 2, 3; j = 1, 2; k = 1, .., 11; l = 1, 2, 3.

A.2  Minimization of the Scalar Potential

The scalar potential consists of two sets of terms - one arising from the superpotential and the other arising from the D terms. Thus the scalar potential V can be written as V = V_W + V_D. From the squark and slepton dependence on the scalar potential, it is straightforward to show that all of them have zero vevs. The expression for V_W, omitting the squarks and sleptons, is given by:

\[
V_W = 4\lambda_K^2 |K|^2 |X_1|^2 + 4\lambda_T^2 |T|^2 |X_1|^2 + 4\lambda_R^2 \sum_{i=1}^{11} |R_i|^2 |X_1|^2 + \sum_{i=1}^{2} |J_i|^2 |\lambda J X_1 + \lambda_P P|^2 +
\]
\[
\sum_{i} |J_{i}^{c}|^2 |\lambda_{i} X_{1} + \lambda_{P} P|^2 + M_{i}^2 |X_{2}|^2 + f^2 |X_{1} X_{2} - \mu^2|^2 + |(\mu' + \tilde{g} \phi) N - g_{H} \mathcal{H}'|^2 + \\
|\lambda_{K} K^2 + \lambda_{T} T^2 + \sum_{i} \lambda_{i} J_{i}^{c} + \lambda_{R} \sum_{i} R_{i}^2 + (f Y + M_{2}) X_{2}|^2 + |MN + \lambda_{P} J_{i}^{c}|^2 + \\
|M_{1} X_{3} + (M_{2} + f Y) X_{1}|^2 + |(\mu' + \tilde{g} \phi) S + \tilde{M} P|^2 + |\beta \phi^2 + m \phi - \kappa^2 + \tilde{g} S N|^2 + \\
g_{\bar{H}}^2 |S|^2 (|H_{u}^+|^2 + |H_{d}^-|^2 + |H_{u}^0|^2 + |H_{d}^0|^2)
\]

The total potential obtained by the sum of \( V_{D} \) and \( V_{W} \) can be minimized by solving the following set of equations:

\[
\frac{\partial V}{\partial K^{*}} = \langle K \rangle |\lambda_{K}^2| |K|^2 + 4 \lambda_{K}^2 |X_{1}|^2 - 2 \tilde{g}^2 \langle \bar{D} \rangle \rangle + \lambda_{K} \langle K^{*} \rangle \lambda_{T} T^2 + \\
\lambda_{J} \sum_{i} \frac{2}{f_{i}} J_{i}^{c} + \lambda_{R} \sum_{i} R_{i}^2 + (f Y + M_{2}) X_{2} = 0
\]

\[
\frac{\partial V}{\partial T^{*}} = \langle T \rangle |\lambda_{T}^2| |T|^2 + 4 \lambda_{T}^2 |X_{1}|^2 - 2 \tilde{g}^2 \langle \bar{D} \rangle + g^2 \langle D_{2} T_{ad}^{(2)} \rangle + \lambda_{T} \langle T^{*} \rangle \lambda_{K} K^2 + \\
\lambda_{J} \sum_{i} \frac{2}{f_{i}} J_{i}^{c} + \lambda_{R} \sum_{i} R_{i}^2 + (f Y + M_{2}) X_{2} = 0
\]

\[
\frac{\partial V}{\partial R_{i}^{*}} = \langle R_{i} \rangle |\lambda_{R}^2 \sum_{i} |R_{i}|^2 + 4 \lambda_{R}^2 |X_{1}|^2 - 2 \tilde{g}^2 \langle \bar{D} \rangle \rangle + \lambda_{R} \langle R_{i}^{*} \rangle \lambda_{T} T^2 + \\
\lambda_{J} \sum_{i} \frac{2}{f_{i}} J_{i}^{c} + \lambda_{K} K^2 + (f Y + M_{2}) X_{2} = 0
\]

\[
\frac{\partial V}{\partial J_{i}^{*}} = \langle J_{i} \rangle \left[ 2 \lambda_{J}^2 |J_{i}^{c}|^2 + \lambda_{J}^2 |X_{1} + P|^2 + g_{\tilde{g}}^2 \langle D_{y} \rangle - 2 \tilde{g}^2 \langle \bar{D} \rangle \rangle + \lambda_{J} \langle J_{i}^{*} \rangle \lambda_{T} T^2 + \\
\lambda_{K} K^2 + \lambda_{R} \sum_{i} R_{i}^2 + (f Y + M_{2}) X_{2} + \tilde{M} N \right] = 0
\]

\[
\frac{\partial V}{\partial J_{i}^{c}} = \langle J_{i}^{c} \rangle \left[ 2 \lambda_{J}^2 |J_{i}^{c}|^2 + \lambda_{J}^2 |X_{1} + P|^2 - g_{\tilde{g}}^2 \langle D_{y} \rangle - 2 \tilde{g}^2 \langle \bar{D} \rangle \rangle + \lambda_{J} \langle J_{i}^{*} \rangle \lambda_{T} T^2 + \\
\lambda_{K} K^2 + \lambda_{J} J_{i} J_{i}^{c} + \lambda_{R} \sum_{i} R_{i}^2 + (f Y + M_{2}) X_{2} + \tilde{M} N \right] = 0
\]

\[
\frac{\partial V}{\partial X_{3}} = \langle X_{3} \rangle \left[ M_{1}^2 + 4 \tilde{g}^2 \langle \bar{D} \rangle \rangle + \langle X_{1} \rangle \left[ M_{1} (M_{2} + f Y) \right] = 0
\]

\[
\frac{\partial V}{\partial Y^{*}} = f (f Y + M_{2}) (|X_{1}|^2 + |X_{2}|^2) + f (X_{1}^{*}) (M_{1} X_{3}) + \\
f (X_{2}^{*}) \left[ \lambda_{T} T^2 + \lambda_{K} K^2 + \lambda_{J} \sum_{i} \frac{2}{f_{i}} J_{i}^{c} + \lambda_{R} \sum_{i=1}^{11} R_{i}^2 \right] = 0
\]

(37)
Expanding $\mathcal{D}$ in terms of fields, and plugging it in the equations for $H_u^+$, $H_d^-$, $H_u^0$ & $H_d^0$, we get:

$$\frac{\partial V}{\partial H_{u+}^+} = \langle H_{u+}^+ | g_H^2 | S^2 \rangle + |H_{u+}^+|^2 (\frac{1}{4} (g^2 + g_0^2) + 4g^2) + |H_{u+}^0|^2 (\frac{1}{4} (g^2 + g_0^2) + 4g^2)$$

$$+ |H_d^0|^2 (\frac{g^2}{2} + 4g^2 - \frac{1}{4} (g^2 + g_0^2)) + |H_d^-|^2 (g_H^2 + 4g^2 - \frac{1}{4} (g^2 + g_0^2)) - 8g^2 (X_1^2 - X_2^2 + S^2 - N^2 + \frac{\bar{\epsilon}}{4} + \frac{g_0^2 \xi}{2}) + \langle H_{d-}^* \rangle [(\frac{g^2}{2} - g_H^2) H_{u+}^0 H_d^0 - g_H (\mu' + \bar{\phi}) N] = 0 \quad (38)$$

$$\frac{\partial V}{\partial H_{d-}^0} = \langle H_{d-}^0 | g_H^2 | S^2 \rangle + |H_{d-}^0|^2 (\frac{1}{4} (g^2 + g_0^2) + 4g^2) + |H_{d+}^0|^2 (\frac{1}{4} (g^2 + g_0^2) + 4g^2)$$

$$+ |H_{d+}^0|^2 (\frac{g^2}{2} + 4g^2 - \frac{1}{4} (g^2 + g_0^2)) + |H_{d-}^0|^2 (g_H^2 + 4g^2 - \frac{1}{4} (g^2 + g_0^2)) -$$

$$8g^2 (X_1^2 - X_2^2 + S^2 - N^2 + \frac{\bar{\epsilon}}{4} + \frac{g_0^2 \xi}{2}) + \langle H_{d+}^* \rangle [(\frac{g^2}{2} - g_H^2) H_{d+}^0 H_{d-}^0 - g_H (\mu' + \bar{\phi}) N] = 0 \quad (39)$$

$$\frac{\partial V}{\partial H_{u+}^0} = \langle H_{u+}^0 | g_H^2 | S^2 \rangle + |H_{u+}^0|^2 (\frac{1}{4} (g^2 + g_0^2) + 4g^2) + |H_{u+}^+|^2 (\frac{1}{4} (g^2 + g_0^2) + 4g^2)$$

$$+ |H_{d+}^0|^2 (\frac{g^2}{2} + 4g^2 - \frac{1}{4} (g^2 + g_0^2)) + |H_{d-}^0|^2 (g_H^2 + 4g^2 - \frac{1}{4} (g^2 + g_0^2)) -$$

$$8g^2 (X_1^2 - X_2^2 + S^2 - N^2 + \frac{\bar{\epsilon}}{4} + \frac{g_0^2 \xi}{2}) + \langle H_{u+}^* \rangle [(\frac{g^2}{2} - g_H^2) H_{d+}^0 H_{d-}^0 - g_H (\mu' + \bar{\phi}) N] = 0 \quad (40)$$
\[ \frac{\partial V}{\partial H_d^0} = (H_d^0)^2 |g_H|^2 |S|^2 + |H_d^-|^2 \left( \frac{1}{4}(g^2 + g_y^2) + 4g^2 \right) + |H_d^0|^2 \left( \frac{1}{4}(g^2 + g_y^2) + 4g^2 \right) \]
\[ + |H_u^0|^2 (g_H^2 + 4g^2 - \frac{1}{4}(g^2 + g_y^2)) + |H_u^+|^2 \left( \frac{g^2}{2} + 4g^2 - \frac{1}{4}(g^2 + g_y^2) \right) - 8g^2 \]
\[ (X_1^2 - X_2^2 + S^2 - N^2 + \frac{\xi}{4} - \frac{g_y^2 \xi}{2}) + \langle H_u^0 \rangle [(\frac{g^2}{2} - g_H^2)(H_u^+)(H_d^-) + g_H(\mu^+ + \tilde{\phi})N] = 0 \quad (41) \]

The solution to the above equations is given by:
\[ \langle K \rangle = \langle T \rangle = \langle R_i \rangle = \langle J_i \rangle = \langle X_3 \rangle = \langle P \rangle = \langle H_u^+ \rangle = \langle H_d^- \rangle = 0; \]
\[ \langle X_1 \rangle \equiv V_1 \neq 0; \langle X_2 \rangle \equiv V_2 \neq 0; \langle S \rangle \equiv S \neq 0; \langle N \rangle \equiv N \neq 0; \langle H_u^0 \rangle \equiv h_u \neq 0; \langle H_d^0 \rangle \equiv h_d \neq 0; \]
\[ \langle Y \rangle = -M_2 \bar{f^2}; \langle \phi \rangle = -\frac{\mu^f}{\bar{g}} \]
provided the following condition is satisfied:
\[ \left( \frac{g^2}{2} - g_H^2 \right) > 0 \quad (42) \]
The equations for \( H_u^0 \) and \( H_d^0 \) boil down to:
\[ (g_H^2 + 4g^2 - \frac{g^2 + g_y^2}{4})h_d^2 + (\frac{g^2}{4} + 4g^2)h_u^2 + \frac{1}{2}g_y^2\xi - 8\tilde{g}^2(V_1^2 - V_2^2 + S^2 - N^2 + \frac{\xi}{4}) = 0 \quad (43) \]
\[ (g_H^2 + 4g^2 - \frac{g^2 + g_y^2}{4})h_u^2 + (\frac{g^2}{4} + 4g^2)h_d^2 - \frac{1}{2}g_y^2\xi - 8\tilde{g}^2(V_1^2 - V_2^2 + S^2 - N^2 + \frac{\xi}{4}) = 0 \quad (44) \]
which is the same as in \((10)\) and \((10)\). The solution is given by:
\[
\begin{align*}
\begin{cases} 
  h_d^2 = \frac{8\tilde{g}^2(V_1^2 - V_2^2 + S^2 - N^2 + \xi/4)}{g_H^2 + 8\tilde{g}^2} - \frac{g_y^2\xi}{g^2 + g_y^2 - 2g_H^2} \\
  h_u^2 = \frac{g_y^2\xi}{[(g^2 + g_y^2)/2 - g_H^2]} 
\end{cases}
\end{align*}
\]
\[ h_d^2 - h_u^2 = h^2 \cos(2\beta) = \frac{g_y^2\xi}{[(g^2 + g_y^2)/2 - g_H^2]} \quad (45) \]
We see that the Fayet-Iliopoulos term for \( U(1)_y - \xi \), determines \( \tan \beta \) and in combination with \( V_1 \), the \( Z \) boson - B' boson mixing \((26)\).
Similarly, with the above vacuum, the equations for \( V_1, V_2, S \& N \) boil down to:
\[ S[\bar{g}^2 N^2 + 4\bar{g}^2 \langle \bar{D} \rangle + g_H^2 h^2] = -\bar{g} N(\beta \phi^2 + m\phi - \kappa^2) \] (46)

\[ N[\bar{g}^2 S^2 - 4\bar{g}^2 \langle \bar{D} \rangle + \bar{M}^2] = -\bar{g} S(\beta \phi^2 + m\phi - \kappa^2) \] (47)

\[ 4\bar{g}^2 \langle \bar{D} \rangle + f^2 V_2^2 = f^2 \mu^2 \left( \frac{V_2}{V_1} \right) \] (48)

\[-4\bar{g}^2 \langle \bar{D} \rangle + f^2 V_1^2 + M_1^2 = f^2 \mu^2 \left( \frac{V_1}{V_2} \right) \] (49)

which is the same as in (11). Now, \( \bar{g}^2 \langle \bar{D} \rangle \) is given in terms of fields by:

\[ \bar{g}^2 \langle \bar{D} \rangle = -2\tilde{g}^2 h^2 + 4\tilde{g}^2 \left( V_1^2 - V_2^2 + S^2 - N^2 + \xi/4 \right) \] (50)

As an illustration, we can fix the vevs \( V_1 = V_2 = S = 20 \text{ TeV} \) and show that all other vevs and dimensionful parameters are also of the same scale.

From (43),(44),(50),(46),(47),(48) and (49), we get:

\[ 4\tilde{g}^2 \langle \bar{D} \rangle = g_H^2 (2S^2 + h^2) = \frac{M_1^2}{2} = \frac{\bar{M}^2 N^2 - \bar{g}^2 h^2 S^2}{S^2 + N^2} \] (51)

Therefore \( N^2 = \frac{2\bar{g}^2 S^2 (S^2 + h^2)}{\bar{M}^2 - 2g_H^2 S^2 - g_H^2 h^2} \). Choosing \( \bar{M}^2 = g_H^2 (4S^2 + h^2) \) for convenience gives us:

\[ N^2 = S^2 + h^2; \mu^2 = S^2(1 + \frac{2\bar{g}^2}{f^2} + \frac{g_H^2 h^2}{f^2 S^2}); M_1^2 = g_H^2 (4S^2 + 2h^2) \] (52)

Also, from (46),(47) and (38), we get:

\[ \beta \langle \phi \rangle^2 + m(\phi) - \kappa^2 + \bar{g} S N = -\frac{SN}{\bar{g}(S^2 + N^2)} [\bar{M}^2 + \bar{g}^2 h^2] \] (53)

\[ \Rightarrow m = \frac{2\beta}{\bar{g}} \mu' + \left( \frac{\bar{g}g_H \sin 2\beta}{2A} \right) \frac{h^2}{S} \] (54)

where \( A = \frac{(M_1^2 + g_H^2 h^2)}{\bar{g}(S^2 + N^2)} \) (55)

Putting in the numbers, we obtain the values of dimensionful parameters, as in (12). Finally, the Fayet-Iliopoulos terms for \( \bar{U}(1) \) are given by:

\[ \tilde{g}^2 \xi = \left( \frac{g_H^2}{4} + 6\bar{g}^2 \right) h^2 + \frac{g_H^2 S^2}{2} \] (56)
References


