Probing CP Violation with the Deuteron Electric Dipole Moment

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We present an analysis of the electric dipole moment (EDM) of the deuteron as induced by CP-violating operators of dimension 4, 5 and 6 including $\theta_{QCD}$, the EDMs and color EDMs of quarks, four-quark interactions and the Weinberg operator. We demonstrate that the precision goal of the EDM Collaboration’s proposal to search for the deuteron EDM, $(1 - 3) \times 10^{-27}$ cm, will provide an improvement in sensitivity to these sources of one-two orders of magnitude relative to the existing bounds. We consider in detail the level to which CP-odd phases can be probed within the MSSM.

The most stringent constraints on flavor-diagonal CP violation in the hadronic sector arise from bounds on the EDMs of the neutron [1], mercury [2], and in certain cases thallium [3]. These experiments have important implications for physics beyond the Standard Model, and its supersymmetric extensions in particular (see e.g. [4]). In what follows, we will show that a proposed measurement of the deuteron EDM [5], with projected sensitivity

$$|d_D| < (1 - 3) \times 10^{-27} \text{ cm},$$

(1)

would improve the sensitivity to $\theta_{QCD}$ and SUSY CP-violating phases by one to two orders of magnitude. We find that the dependence of $d_D$ on the underlying QCD-sector CP-odd sources is closest to $d_{\text{Hg}}$ and is complementary to $d_n$. Moreover, in addition to the improvement in precision, $d_D$ has a significant advantage over $d_{\text{Hg}}$ due to the rather transparent nuclear physics in the former and thus smaller theoretical uncertainties. Consequently, the experiment will be able to probe classes of supersymmetric models which escape the current EDM bounds.

We now proceed to analyze the deuteron EDM $d_D$, defined via the interaction of the deuteron spin $\vec{I}$ with an electric field, $\mathcal{H} = -d_D \vec{I} : \vec{E}$, working upwards in energy scale. Starting at the nuclear level, the deuteron EDM receives contributions from a singlet combination of the constituent proton and neutron EDMs, but also arises due to meson (predominantly pion) exchange between the nucleons with CP-odd couplings at one of the meson-nucleon vertices. Thus, we can represent the EDM as

$$d_D = (d_n + d_p) + d_{NN}^{NN},$$

(2)

where the third term includes the meson-exchange contribution and depends on the CP-odd pion nucleon couplings,

$$\mathcal{L}_{QD} = \bar{\psi}_\pi N N \tau^a N \pi^a + \bar{\psi}_\pi N N \tilde{N} N \pi^0.$$  

(3)

In a recent analysis, Khriplovich and Korkin [6] (see also [7]) showed that $d_{NN}^{NN}$ receives a dominant contribution from the isospin-triplet coupling $\tilde{g}(1)$. In a zero-radius approximation for the deuteron wavefunction, the result

$$d_{NN}^{NN} = -\frac{e g_{\pi NN}^{(1)} g_{\pi NN}^{(1)}}{12 \pi m_\pi} \frac{1 + \xi}{(1 + 2\xi)^2},$$

(4)

depends on the parameter $\xi = \sqrt{m_\pi/m_\sigma}$, determined by the deuteron binding energy $\epsilon = 2.23$ MeV. Numerically, this implies

$$d_{NN}^{NN} \simeq -(1.3 \pm 0.3) \times 10^{-27} \text{ cm},$$

(5)

a result that can be improved systematically, and the error correspondingly reduced [6], with the use of more realistic deuteron wave functions.

To make direct contact with models of CP violation, we require the dependence of $d_n$, $d_p$, and $\tilde{g}(1)$ on the parameters in the underlying CP-odd Lagrangian at 1 GeV.

Up to dimension five, the relevant hadronic operators are the $\theta$-term and the EDMs and color EDMs (CEDMs) of quarks

$$\mathcal{L}_{QD} = \frac{\theta}{8\pi} \bar{G} G - i \sum_{q=u,d,s} \left[ \bar{d}_q \bar{q} F \sigma \gamma_5 q + \bar{d}_q \bar{q} g_{\gamma 5} G \sigma \gamma_5 q \right],$$

(6)

where $G G \equiv \epsilon_{\mu\nu\rho\sigma} G^{\mu\nu} G^{\rho\sigma}/2$ and $G \sigma \equiv t^a C^{\mu\nu \sigma} \sigma_{\mu\nu}$. Note that the dimension-six Weinberg operator, $G G$, as well as numerous four-quark operators, may, in certain models, also contribute at a similar level to the quark EDMs and CEDMs.

Models of new CP-violating physics can be cast into two main categories: (i) models that have no Peccei-Quinn (PQ) symmetry [8] and exact CP or P symmetries at high energies and consequently $\theta = 0$ at tree level; and (ii) models that invoke a Peccei-Quinn symmetry to remove any dependence of the observables on $\theta$. In models
of the first type, $\bar{\theta}$ generated by radiative corrections is likely to be the main source of EDMs.

To determine $d_D(\bar{\theta})$, one may first try to make use of the chiral techniques [9] that determine the $\bar{\theta}$-induced pion-nucleon coupling constant, $\tilde{g}_{\pi NN}^{(0)}(\bar{\theta}) = m_s\bar{\theta}f_{\pi}^{-1}\langle N|\bar{dd} - \bar{dd}|N\rangle$ (where $m_s = m_u - m_d/(m_u + m_d)$), and the one loop $O(m_{\pi}^2 \log m_{\pi})$ contribution to $d_n$. It is easy to see, however, that $d_D(\bar{\theta})$ is calculable within this approach because the chiral logarithms exactly cancel in the $d_n + d_p$ combination, and $\tilde{g}^{(1)}(\bar{\theta}) = 0$ unless isospin violating corrections are taken into account.

The cancellation between $d_n(\bar{\theta})$ and $d_p(\bar{\theta})$ does not hold in general. To calculate $d_D(\bar{\theta})$ we use leading order QCD sum-rule estimates which imply [10],

$$d_n(\bar{\theta}) + d_p(\bar{\theta}) = -(2 \pm 0.8)\pi^2 \left(\frac{m_N}{1\text{GeV}}\right)^3 \frac{\langle \bar{q} \bar{q} \rangle}{(1\text{GeV})^3} m_s \chi e\bar{\theta},$$

where $\langle \bar{q} \bar{q} \rangle_F = e_3 \chi F_{\bar{q} \bar{q}}(\bar{q} \bar{q})$ defines the magnetic susceptibility $\chi \sim -(6 - 9)\text{ GeV}^{-2}$ [11] of the vacuum, recently computed to be at the upper end of this range, $\chi = -N_c/(4\pi f_{\pi}^2)$, by Vainshtein [12]. The subleading corrections to the sum rule were computed and are of order 10-15% [10], while the uncertainty in $\chi$ and freedom in the choice of nucleon interpolating current lead to a larger overall uncertainty of 30-40% [10].

It turns out that despite an additional suppression factor, the corresponding contribution to $\tilde{g}^{(1)}(\bar{\theta})$ is not negligible and contributes to $d_D$ at approximately the same level as (7). To take it into account, we note that isospin violation arises predominantly through $\eta - \pi$ mixing as in Fig. 1(a). The inverted diagram of Fig. 1(b) provides at most a 10% correction, due primarily to the small size of $g_{\pi NN}$ and $\langle N|\bar{uu} - \bar{dd}|N\rangle$ relative to $g_{\pi NN}$ and $\langle N|\bar{uu} + \bar{dd} - 2\bar{ss}|N\rangle$. Fig. 1(a) leads to the following result:

$$\tilde{g}_{\pi NN}^{(1)}(\bar{\theta}) = \frac{m_s\bar{\theta} m_u - m_u}{4m_s} \langle N|\bar{dd} - 2\bar{ss}|N\rangle.$$  \hspace{2cm} (8)

Combining (7) and (8), we obtain

$$d_D(\bar{\theta}) = -e\bar{\theta} \left[ \frac{2\pi^2 \chi m_s\langle \bar{q} \bar{q} \rangle}{(1\text{GeV})^3} + \frac{m_u}{m_s} \frac{(m_u - m_d)}{4f_\pi} \langle N|\bar{uu} + \bar{dd} - 2\bar{ss}|N\rangle \right],$$

which numerically takes the form

$$d_D(\bar{\theta}) \simeq -e \left[ (3.5 \pm 1.4) + (1.4 \pm 0.4) \right] \times 10^{-3} \bar{\theta} \text{ [GeV}^{-1}],$$

using standard quark mass ratios [13], and quark condensates over the nucleon (see e.g. [14]). The second term in (10) arises from the CP-odd pion-nucleon interaction.

This result is interesting for several reasons. Firstly, if the projected experimental sensitivity (1) is achieved, a null result for $d_D$ will imply

$$|\bar{\theta}| < 3 \times 10^{-11},$$

which represents an improvement of over an order of magnitude relative to the best current bound arising from the limit on the neutron EDM. We note that the recent inclusion of many-body effects in the nuclear component of the calculation of $d_{\text{Hg}}$ [15] has led to a significant reduction of $d_{\text{Hg}}(\bar{\theta}^{(0)})$, thus relaxing the mercury EDM constraint on $\bar{\theta}$ by an order of magnitude. It is also important to note that the two sources for $\bar{\theta}$ in (9) have quite different origins, and thus a cancellation would be unnatural. Given the relatively good theoretical control over the contribution entering through $\tilde{g}^{(1)}$, the uncertainty in the estimate (7) is of less concern. The bound (11) has important implications for solutions to the strong CP problem within supersymmetry. In particular, the left-right symmetric SUSY models typically predict $\bar{\theta}$ in the range $10^{-8} - 10^{-10}$ [16], allowing a direct probe via the $d_D$ experiment.

Introducing a PQ symmetry allows the axion to relax to its minimum thereby rendering $\bar{\theta}$ unobservable. Adopting this approach, we are left with the dimension five quark EDMs and CEDMs as the leading candidates for the position of dominant CP-odd source. The constituent EDMs of the proton and neutron receive contributions from both of these operators, with the QCD sum-rules result (omitting for now the Weinberg operator) [17]

$$d_n(d_q, d_g) + d_p(d_q, d_g) \simeq (0.5 \pm 0.3)(d_u + d_d) - (0.6 \pm 0.3)e \left[ (\tilde{d}_u - \tilde{d}_d) + 0.3(\tilde{d}_u + \tilde{d}_d) \right],$$

where we have split the CEDM contribution into singlet and triplet combinations. A possible contribution from $\tilde{d}_s$ is removed at this order under PQ relaxation. The quoted errors have the same origin as those in (7) for the dependence of $d_n$ and $d_p$ on $\bar{\theta}$.

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**FIG. 1:** Contributions to $d_D^{\pi NN}(\bar{\theta})$, with isospin violation through $\eta - \pi$ mixing.
The triplet pion nucleon coupling $g^{(1)}_{\pi NN}$ receives a dominant contribution from the triplet combination $(\tilde{d}_u - \tilde{d}_d)$ of CEDMs, and the “best” value for this coupling was recently determined using sum-rules [18],

$$g^{(1)}_{\pi NN} \sim 2^{+4}_{-1} \times 10^{-12} \frac{\tilde{d}_u - \tilde{d}_d}{10^{-26} \text{cm}},$$  \tag{13}$$

with a rather large (overall) uncertainty due to an exact cancellation at the level of vacuum factorization. We quote the non-Gaussian errors determined via parameter variation [18]. Since this result enters without any additional isospin-violating suppression factor, it numerically dominates the CEDM contribution to $d_D$. Combining (12) and (13), we find

$$d_D(d_u, d_q) \simeq -e(\tilde{d}_u - \tilde{d}_d) \left[ 5^{+11}_{-3} + (0.6 \pm 0.3) \right] - (0.2 \pm 0.1)e(\tilde{d}_u + \tilde{d}_d) + (0.5 \pm 0.3)(d_u + d_d),$$  \tag{14}$$

where the constituent nucleon EDMs provide a 10% correction to the triplet CEDM contribution. We conclude from this result that for models with $e\tilde{d}_i \sim d_i$ the deuteron EDM is predominantly sensitive to the triplet combination of CEDMs, as is the mercury EDM. Moreover, if the predicted precision is achieved, its sensitivity to the triplet CEDM combination at the level of a few $10^{-28}$ e cm would represent an improvement on the current mercury EDM bound by two orders of magnitude.

We now turn to an analysis of the predicted sensitivity to new CP-odd sources focusing on the minimal supersymmetric standard model (MSSM) with universal boundary conditions at the GUT scale for all parameters except for those in the Higgs sector. This exception allows us to satisfy all phenomenological and cosmological constraints for a wide range of squark masses while keeping the other parameters fixed [19]. In this case, there are two CP violating phases, identified with the phases of the $\mu$ parameter in the superpotential and the phase of a common trilinear soft-breaking term $A_0$.

In Fig. 2, we plot the EDMs as a function of the left-handed down squark mass by varying $m_0$ from 0.25 - 10 TeV, while keeping $m_{1/2}$ (as well as the other input parameters) fixed. For this choice of parameters, the light Higgs mass is about 120 GeV and the lightest neutralino is a mixed gaugino/Higgsino state. The curves begin at $\tilde{m}_d \sim 1.2$ TeV corresponding to $m_0 = 250$ GeV with $m_{1/2} = 600$ GeV. In this figure, the theoretical average values of the neutron, thallium and mercury EDMs are normalized to their current experimental limits, while $d_D$ is normalized to $3 \times 10^{-27}$ e cm. The theoretical error bands are generally very narrow on these log-scale plots and are not shown. For low $\theta_{\mu(A)}$, the EDMs scale with $\theta$ and therefore the results for other (small) choices of $\theta_{\mu(A)}$ can be deduced from this figure. We immediately see that the predicted sensitivity of $d_D$ to squark masses extends beyond 10 TeV, well beyond that of the existing bounds or the reach of colliders in the foreseeable future. Note that the dips observable in the plot of $d_{Hg}$ for $\theta_\mu \neq 0$ are due primarily to cancellations between quark CEDM and electron EDM contributions.

The $d_D$ experiment will also be able to probe a popular solution to the SUSY CP problem, the “decoupling” scenario. This framework assumes that the sfermions of the first two generations have masses in the multi-TeV range thus suppressing the one-loop EDM contributions to an acceptable level and allowing CP-odd phases to be of order one [20]. To satisfy the cosmological constraints on dark matter abundance [21], and to avoid excessive fine-tuning in the Higgs sector, the masses of the third generation sfermions should be near the electroweak scale. The Weinberg operator is then generated at two-loop order, providing the primary contribution to $d_D$ [22, 23]:

$$d_D \simeq d_n(w) + d_p(w) \sim e w \times 20 \text{ MeV},$$  \tag{15}$$

where $w$ is the coefficient of the Weinberg operator evaluated at 1 GeV. The Weinberg operator provides a neg-
ligible contribution to $d_D^{NN}$ due to additional chiral suppression and isospin violating factors in $\tilde{g}^{(1)}_{eNN}(w)$. Presently, order one CP-violating phases in this framework are barely compatible with the experimental constraint on $d_n$ [24]. Therefore, an improvement in the experimental precision by a factor of 10 or more, to the level of $10^{-27}$ cm, would provide a crucial test for these models. Failure to observe $d_D$ would necessarily imply that the CP-violating phases are small contrary to the primary assumptions of the model.

Next, we analyze constraints on the SUSY CP-violating phases $\theta_A, \theta_\mu$ with the superpartner mass scales fixed as shown in Fig. 3. This is a CMSSM point (the Higgs soft masses are unified with other sfermion masses) with a relatively low Higgs mass of 114 GeV [19]. We observe that $d_D$ combined with the thallium constraint can put tight bounds on both phases including $\theta_A$ that is otherwise poorly constrained. An improvement of the bound on the triplet CEDM combination by a factor of 30 or more would allow one to probe SUSY CP-odd phases of size $10^{-3}$ or below ($10^{-2}$ or so for the $A$-terms). In a number of theoretically motivated scenarios, phases of this size are naturally expected. In particular, if the $A$-terms are hermitian at the GUT scale as happens in the left-right and other models, RG running induces small phases in the diagonal elements. For a variety of textures, the CEDMs of the light quarks are of order $10^{-27}$ cm [25], and thus observable at the upcoming experiment.

Finally, we consider the sensitivity of $d_D$ to the dimension 6 operators, $C_{ij}\bar{q}_i\bar{q}_j\bar{q}_i\gamma_5q_j$, which may be important in two Higgs doublet models, left-right symmetric models, and certain supersymmetric scenarios. Typically, $C_{ij}$ can be parametrized as $C_{ij} = c Y_{ij}^{SM} Y_{SM}^{-2}$, where $Y_{SM}^{(i)}$ are the SM quark Yukawa couplings, $M_h$ is the mass of the (lightest) Higgs boson, and the coefficient $c$ is model dependent. Existing EDM bounds are sensitive to $C_{ij}$ only with the help of an enhancement at large $\tan \beta$, $c \sim \tan^2 \beta$ or $\tan^3 \beta$ [26], or in the top quark sector where $C_{ij}$ induces $\alpha$ and/or light quark (C)EDMs via the Barr-Zee mechanism [27]. The projected sensitivity to $d_D$ would in contrast probe $C_{ij}$ for all quark flavors down to $c \sim 0.01 - 0.1$ for $M_h \sim 100$ GeV, thus providing valuable constraints even for $\tan \beta \sim O(1)$.

In conclusion, we have presented an analysis of the deuterium EDM in terms of the relevant Wilson coefficients and studied the implications of a $d_D$ measurement at the level of a few $10^{-27}$ cm. We have shown that this would lead to a factor of 10 to 100 gain in sensitivity to various CP violating sources of dimension 4, 5 and 6. This has important consequences for supersymmetry and other scenarios for physics beyond the Standard Model.

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