Quasinormal Modes of Extremal BTZ Black Hole

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Motivated by several pieces of evidence, in order to show that extreme black holes cannot be obtained as limits of non-extremal black holes, in this article we calculate explicitly quasinormal modes for Bañados, Teitelboim and Zanelli (BTZ) extremal black hole and we showed that the imaginary part of the frequency is zero. We obtain exact result for the scalar and fermionic perturbations. We also showed that the frequency is bounded from below for the existence of the normal modes (non-dissipative modes).

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I. INTRODUCTION

There are several pieces of evidence to show that extremal Black Holes (BH) cannot be obtained as limits of non-extremal BH. First of all, these two classes are topologically different. For example, in $2 + 1$ dimensions, the topology of an extremal BH is an annulus, while the non-extremal BH is topologically like a cylinder. Moreover, the extremal BH has zero Hawking temperature ensuring stability against Hawking’s radiation which is not the case for the non-extremal BH. In the language of thermodynamics, many authors have showed, at least semiclassically, that the entropy of an extremal BH is zero in contrast with the non-extremal case where the entropy is proportional to the area. In addition, the classical absorption cross section (or greybody factor) for non extreme BH in three and four dimensions, turns out to be proportional to the area of the horizon while in Refs. it has been shown that the extremal BTZ BH has null greybody factor for scalar and fermion particles. By virtue of all these differences, it is clear that both systems must be treated differently and it is not excepted to pass form one to the other by some procedure limits.

In the past few years there has been a growing interest in studying the QNMs (quasinormal modes) spectrum of BH and its connection to conformal field theory. Unfortunately, finding an exact solution of the QNMs is a very hard task and in some cases, it is almost impossible to get an analytic expression. In spite of the differences mentioned above, studies have focused on non-extremal BH or near extremal BH, aside extremal case as limit of these last ones. In this paper, we give another evidence based on QNMs, specifically that this limit procedure does not work in the case of the extremal BTZ BH. Indeed, we will show that the QNMs of the extremal BTZ BH do not exist for scalar and fermionic perturbations. This conclusion does not occur in the non-extremal case where the QNMs are known. We discuss the boundary conditions of the states at spatial infinity, where there is no plane wave solution. The paper is organized as follows: in the following section we review the Klein Gordon and Dirac equations in a curved spacetime as well as the $2 + 1$-dimensional extremal black hole. In the next section, we study the scalar perturbations and their QNMs. Then, we repeat the same computations for fermionic perturbations. The last section contains our conclusions and remarks.

A. Klein Gordon and Dirac Equations in a Curved Space Time

In order to compute the QNMs we must solve the Klein Gordon as well as the Dirac equations in the three-dimensional extremal black hole background. In this section, we review these equations in order to make the discussion

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The wave equation in the scalar case is given by

\[ \frac{1}{\sqrt{-g}} \partial_{\mu} (\sqrt{-g} g^{\mu \nu} \partial_{\nu}) \psi - m^2 \psi = 0, \]  

(1)

and, assuming spherical or axial symmetry, this equation can be rewritten as

\[ \frac{d^2 R}{dr^2} + (\kappa^2 - V_{eff}(r)) R = 0, \]  

(2)

where \( V_{eff} \) is the effective potential produced by the BH background \[16\].

For the fermionic perturbations, we need to solve the Dirac equation given by \[17\]

\[ \gamma^a E^\mu_a \left( \partial_{\mu} - \frac{1}{8} \omega_{bc\mu} [\gamma^b, \gamma^c] \right) \Psi = m \Psi, \]  

(3)

where \( E^\mu_a \) is the inverse triad which satisfies

\[ E^\mu_a e_b^\mu = \delta^b_a \]

and \( \omega_{\mu} \) is the spin-connection. The set of matrices \( \{\gamma^a\} \) are the Dirac matrices in the tangent space which satisfy the Clifford algebra

\[ \{\gamma^a, \gamma^b\} = 2\eta^{ab}. \]  

(4)

**Extremal 2 + 1 black hole metric.** In a 2 + 1-dimensional spacetime, the Einstein equations with negative cosmological constant \( \Lambda = -\ell^{-2} \) have the following solution \[18\]

\[ ds^2 = -N^2(r) dt^2 + N^{-2}(r) dr^2 + r^2 [d\phi + N^{\phi}(r) dt]^2, \]  

(5)

where the lapse \( N(r) \) and shift \( N^{\phi}(r) \) functions are given by

\[ N(r) = -M + \frac{r^2}{\ell^2} + \frac{J^2}{4r^2}, \]  

(6)

\[ N^{\phi}(r) = -\frac{J}{2r^2}. \]  

(7)

Here \( M \) and \( J \) are the mass and the angular momentum of the black hole, respectively.

The lapse function vanishes when

\[ r_\pm = r_\text{ex} \left[ 1 \pm \sqrt{1 - \frac{J^2}{M^2 \ell^2}} \right]^{\frac{1}{2}}, \]  

(8)

and therefore, the solution (5) is defined for \( r_+ < r < \infty, -\pi < \phi < \pi \) and \( -\infty < t < \infty \). The extremal solution corresponds to \( J^2 = M^2 \ell^2 \), which implies that \( r_\pm = r_\text{ex} = \ell \sqrt{M/2} \). Hence the line element (5) becomes

\[ ds_{\text{ex}}^2 = -\frac{r^2}{\ell^2} - 2 \frac{r_{\text{ex}}^2}{\ell^2} dt^2 + \frac{\ell^2 \omega^2}{(r^2 - r_{\text{ex}}^2)} dr^2 - 2 \frac{r_{\text{ex}}^2}{\ell} dt d\phi + r^2 d\phi^2. \]  

(9)

Let us now study the scalar perturbations for this geometry.

**II. SCALAR PERTURBATIONS**

Equation (1), in the above metric (9) can be solved considering the following Ansatz

\[ \psi = \psi_0 e^{i\omega t} e^{in\phi} R(r), \]  

(10)

where \( R(r) \) is an unknown function and \( \psi_0 \) is a constant. Plugging (10) into the equation (1), the radial function \( R(r) \) satisfies

\[ \left( \frac{(r^2 - r_{\text{ex}}^2)^4}{r^2 \ell^4} \right) R''(r) + \left( \frac{(r^2 - r_{\text{ex}}^2)^3}{r^3 \ell^4} \right) (3r^2 - r_{\text{ex}}^2) R'(r) + \left[ (\omega + n)(\omega - n)r^2 + 2mr_{\text{ex}}^2 - \frac{m^2}{\ell^2} (r^2 - r_{\text{ex}}^2)^2 \right] R(r) = 0. \]  

(11)
The solution of this equation is given as a linear combination

\[ R(r) = AR(1)(r) + BR(2)(r), \]

where the functions \(R(1)(r)\) and \(R(2)(r)\) read

\[ R(1)(r) = e^{-i\Omega_+ \frac{r^2}{r+c_x}} \left( \frac{r^2_c}{r^2 - r^2_c} \right)^{s_+} F[s_+ + \frac{i}{2}, 2s_+, 2i\Omega_+ \frac{r^2_c}{r^2 - r^2_c}], \]

\[ R(2)(r) = e^{-i\Omega_+ \frac{r^2}{r+c_x}} \left( \frac{r^2_c}{r^2 - r^2_c} \right)^{s_-} F[s_- + \frac{i}{2}, 2s_-, 2i\Omega_+ \frac{r^2_c}{r^2 - r^2_c}]. \]

Here \(s_\pm = \frac{1}{2}(1 \pm \sqrt{1 + m^2})\), \(\Omega_\pm = \frac{1}{\sqrt{2m}}(\omega \pm n)\) (we have put \(l = 1\)) and \(F[a, c, z]\) is the confluent hypergeometric function (known as Kummer’s solution).

As the BTZ BH is asymptotically Anti de Sitter (AdS), the definition of the QNMs is different from the one used in an asymptotically flat space [11] [12]. In the former case, this problem was well described in Ref. [14] where they defined QNMs as solutions which are purely ingoing at the horizon, and vanishing at infinity. The vanishing boundary condition implies that the constant \(B\) in [12] must be zero and in the asymptotic limit the wave function becomes

\[ \psi_\infty = Ae^{i\omega t}e^{in\phi}e^{-i\Omega_+ \frac{r^2}{r+c_x}} \left( \frac{r^2_c}{r^2 - r^2_c} \right)^{s_+}. \]

It is simple to see that the required boundary condition is automatically satisfied, showing therefore the absence of QNMs. This result is in contrast with the non-extremal case where QNMs are proportional to the quantized imaginary part of the frequency [11]. Moreover, this result is in agreement with the null absorption cross section for the scalar case [8]. Now, we turn to confirm again this result by showing that the condition of the vanishing flux at infinity is satisfied. For simplicity, we consider the coordinate \(z = \frac{r^2_c}{r^2 - r^2_c}\) for which the conserved radial current becomes

\[ J_r(z) = R^*(z) \frac{d}{dz} R(z) - R(z) \frac{d}{dz} R^*(z). \]

Due to the regularity condition at infinity, the only contribution to the current \(J_r(z)\) comes from \(R(1)(r)\). Then, the conserved current is expressed as

\[ J_r(z) = -i |A|^2 \Omega_+ z^{2s_+} F[s_+ + \frac{i}{2}, 2s_+, 2i\Omega_+ z] F[s_- - \frac{i}{2}, 2s_-, 2i\Omega_+ z], \]

and the flux is given by

\[ \mathcal{F} = \sqrt{g} \frac{1}{2i} J_r(z). \]

It is straightforward to see that the flux vanishes as \(z\) goes to 0 which confirms the absence of QNMs for extremal BTZ BH under scalar perturbations. We also point out that this result is independent of the value of \(m\).

### III. FERMIONIC PERTURBATIONS

In order to solve Dirac’s equation [15] with the extremal BTZ metric, it is convenient to define a dimensionless set of coordinates \(\{u, v, \rho\}\) as follows (with \(\ell\) restored)

\[ u = \frac{t}{\ell} + \phi, \quad v = \frac{t}{\ell} - \phi, \quad e^{2\rho} = \frac{r^2 - r^2_c}{\ell^2}, \]

where \(u, v\) and \(\rho\) range from \(-\infty\) to \(\infty\). In the space \(\{u \times v\}\), two points \((u_1, v_1)\) and \((u_2, v_2)\) are identified if they satisfy \(u_1 = v_2\) and \(v_1 = u_2\), for any value of \(\rho\).

Choosing the Dirac matrices to be given by

\[ \gamma^1 = -i\sigma^3, \quad \gamma^2 = \sigma^1, \quad \gamma^3 = \sigma^2, \]
where $\sigma^i$ are the Pauli matrices and, taking the following Ansatz for the wave function

$$\Psi(u, v, \rho) = \left( \frac{U(u, v, \rho)}{V(u, v, \rho)} \right),$$

the Dirac equation becomes

$$\begin{align*}
&\left[-i \left( \frac{2r_{ex} e^{-2\rho}}{\ell^2} \partial_u + \frac{\partial_v}{r_{ex}} \right) - (\frac{1}{2\ell} + m) \right] U + \left[ \frac{\partial_v}{r_{ex}} - \frac{i}{\ell} (\partial_\rho - 1) \right] V = 0, \\
&\left[ \frac{\partial_v}{r_{ex}} + \frac{i}{\ell} (\partial_\rho - 1) \right] U + \left[ \frac{2r_{ex} e^{-2\rho}}{\ell^2} \partial_u + \frac{\partial_v}{r_{ex}} \right) - (\frac{1}{2\ell} + m) \right] V = 0.
\end{align*}$$

(21) \hspace{2cm} (22)

Let us now look for solutions of this equation of the form

$$\Psi(u, v, \rho) = e^{i(\alpha u + \beta v)} \left( \frac{F(\rho)}{G(\rho)} \right),$$

(23)

where $\alpha$ and $\beta$ are two constants related to the angular and temporal eigenvalues of the solution of (3) in the coordinates $\{t, \phi, r\}$. This means that if the solution behaves like $e^{i(n\phi + \omega t)}$, then

$$\alpha = \frac{1}{2} (\omega \ell + n), \hspace{1cm} \beta = \frac{1}{2} (\omega \ell - n).$$

(24)

For the $\rho$-dependent part of the equation it is useful to define $z = e^{-2\rho}$ and thus, the functions $F(z)$ and $G(z)$ satisfy the following system

$$\begin{align*}
&\left[ \frac{2\alpha r_{ex}}{\ell^2} z + \frac{\beta}{r_{ex}} - \frac{1}{2\ell} - m \right] F(z) + i \left[ \frac{\beta}{r_{ex}} + \frac{1}{\ell} \left( 2z \frac{d}{dz} + 1 \right) \right] G(z) = 0, \\
&i \left[ \frac{\beta}{r_{ex}} - \frac{1}{\ell} \left( 2z \frac{d}{dz} + 1 \right) \right] F(z) - \left[ \frac{2\alpha r_{ex}}{\ell^2} z + \frac{\beta}{r_{ex}} + \frac{1}{2\ell} + m \right] G(z) = 0.
\end{align*}$$

(25) \hspace{2cm} (26)

This system of first order coupled differential equations can be transformed into a second order system. Indeed, using the equation (25) to express $F(z)$ as a function of $G(z)$ and its first derivative and then defining a new variable $x = \alpha (r_{ex}/\ell) z$, one finds that $G(x)$ satisfies the following second order equation

$$A(x)G''(x) + B(x)G'(x) + C(x)G(x) = 0.$$

(27)

The function $A, B$ and $C$ are given by

$$\begin{align*}
A(x) &= (\delta - x)x^2, \\
B(x) &= (2\delta - x)x, \\
C(x) &= -x^3 + (\delta - \tilde{\beta})x^2 + \frac{1}{4} \left( (\tilde{\beta} + 1)^2 + 4\delta(2\tilde{\beta} + \delta) \right) x - \frac{\delta}{4} \left( (2\delta + \tilde{\beta})^2 - 1 \right),
\end{align*}$$

(28) \hspace{2cm} (29) \hspace{2cm} (30)

where the constants $\delta$ and $\tilde{\beta}$ are

$$\delta = \frac{1}{2} \left( \ell m_{\text{eff}} - \tilde{\beta} \right), \hspace{1cm} \tilde{\beta} = \frac{\ell \beta}{r_{ex}}. $$

(31)

and the effective mass is $m_{\text{eff}} = m + \frac{1}{\ell}$. The same procedure can be done for the function $F(\rho)$ which yields to a similar result (for details, see Ref. [7]). In order to solve the equation expressed in (27), we need to distinguish between two cases according to a vanishing or non-vanishing delta.

$\delta = 0$. For a vanishing $\delta$ (i. e. $\beta = r_{ex}(m + 1/2\ell)$, the equation (27) reduces to

$$x^2 G''(x) + x G'(x) + (x^2 + \tilde{\beta} x - \frac{1}{4} (\tilde{\beta} + 1)^2) G(x) = 0,$$

(32)

whose solution is given by

$$G(x) = e^{-ix} \left( P x^s F \left[ s + 1/2 + i(s - 1/2), 1 + 2s, 2ix \right] + Q x^{-s} F \left[ -s + 1/2 + i(s - 1/2), 1 - 2s, 2ix \right] \right).$$

(33)
for the fermionic fields in an extremal BTZ background \cite{7}.

\[ \rho \] and hence the vanishing boundary condition at infinity (\( \rho \rightarrow \infty \) i. e. \( x \rightarrow 0 \)) implies that \( Q \) must be zero. Thus, as we found in the scalar case, we have proved the absence of the QNMs for the fermionic case. We want to note that the absence of QNMs is different from the non-extremal case where the QNMs are proportional to the quantized imaginary part of the frequency \cite{11}.

Another interesting feature of the solution \( G \) (with \( Q = 0 \)) is that, for \( x \approx \infty \), \( G \) behaves like

\[ G(x) \sim e^{-ik(Px^s + Qx^{-s})}, \]  

(34)

and hence the vanishing boundary condition at infinity (\( \rho \rightarrow \infty \) i. e. \( x \rightarrow 0 \)) implies that \( Q \) must be zero. Thus, as we found in the scalar case, we have proved the absence of the QNMs for the fermionic case. We want to note that the absence of QNMs is different from the non-extremal case where the QNMs are proportional to the quantized imaginary part of the frequency \cite{11}.

Another interesting feature of the solution \( G \) (with \( Q = 0 \)) is that, for \( x \approx \infty \), \( G \) behaves like

\[ G(x) \sim \frac{P}{\sqrt{x}} e^{\pm \beta \ln(x)/2} \rightarrow 0. \]  

(35)

This also shows that the function \( G(x) \) is vanishing at the horizon, in agreement with a null absorption cross section for the fermionic fields in an extremal BTZ background \cite{7}.

\[ \delta \neq 0. \] For a non-vanishing \( \delta \), it is more convenient to define a new variable \( y = x/\delta \) and a function \( G(y) \) as

\[ G(y) = \frac{1-y}{\sqrt{y}} D(y). \]  

(36)

In terms of \( D(y) \), the equation \cite{24} turns out to be

\[ y^2(1-y)^2D'' + y(1-y)(1-2y)D' + \left[ \delta^2y^4 + \delta \left( \tilde{\beta} - 2\delta \right) y^3 - \frac{\tilde{\beta}}{4} \left( 2 + \tilde{\beta} + 12\delta \right) y^2 + \right. \]

\[ \left. \frac{1}{4} \left( 5(\tilde{\beta} + \delta)^2 + 2\tilde{\beta}(1 + \delta) - 4 \right) y - \frac{1}{4}(2\tilde{\beta} + \delta)^2 \right] D = 0. \]  

(37)

As in the previous cases, we need to study the asymptotic behavior of the solution of this equation to explore the possibility of obtaining QNMs. Keeping terms up to first order, this limit \( y \rightarrow 0 \) yields to

\[ y^2D_0' + (1-y)yD_0' + \left( -\frac{1}{4}(2\tilde{\beta} + \delta)^2 + \frac{1}{4} \left( \tilde{\beta}(2 + 4\delta) - 3(\tilde{\beta}^2 - \delta^2 - 4) \right) y \right) D_0 = 0, \]  

(38)

whose solution is

\[ D_0(y) = P_0 \ y^{-\frac{\delta}{2}} F \left[ \frac{1}{2} \left( a + b \right), 1 - b, y \right] + Q_0 \ y^{\frac{\delta}{2}} F \left[ \frac{1}{2} \left( a - b \right), 1 + b, y \right]. \]  

(39)

Here \( P_0 \) and \( Q_0 \) are two complex constants, \( a = 2 + \frac{3}{2}(\tilde{\beta}^2 - \delta^2) - \tilde{\beta}(1 + 2\delta) \) and \( b = 2\tilde{\beta} + \delta \) and \( F[c, d; x] \) is the confluent hypergeometric function. Finally the solution reads,

\[ G_0(y) = P_0 \ y^{\frac{\tilde{\beta} + 1}{2}} + Q_0 \ y^{\frac{\tilde{\beta} - 1}{2}}. \]  

(40)

In order to satisfy the adequate boundary condition \cite{14}, we require \( P_0 = 0 \) and \( b > 1 \). With these two conditions, the vanishing boundary condition at infinity again is automatically satisfied by \cite{10} and this fact clearly shows the absence of QNMs in this case.

On the other hand, it is straightforward to prove that the wave function vanishes at horizon. Therefore the fields behavior is as particles confined in a box. Allowing the presence of the normal modes of vibration (non-dissipative modes).

At this point, let us remarking that the condition \( b > 1 \) implies that the frequency is bounded from below for the existence of the normal modes, i. e.

\[ \frac{\omega}{\Omega_{ex}} > n + \sqrt{\frac{M}{2}} - \frac{2}{3}mr_{ex}, \]  

(41)

where \( \Omega_{ex} = t^{-1} \) is the angular velocity of the extremal black hole. This result can be interpreted as that the fundamental frequency is bounded by below. In order to excite the normal modes of the space, frequency should be more bigger this limit. Then in this case extreme BTZ BH behavior as non-dissipative system. On the other hand, we conjectured that this inferior limit is related with the quantization of the black hole horizon area. \cite{15}
However, as noted in Ref. \[11\], the Dirichlet condition is not adequate for the BTZ BH for some values of the mass parameter. The authors in Ref. \[11\] consider this fact as another motivation for imposing the vanishing flux at infinity rather than the Dirichlet condition for asymptotically AdS space-time. Then in the case of Dirac modes, one can also impose vanishing flux at infinity (generally much weaker than the Dirichlet condition), see Refs. \[11\] and \[12\]. From Eqs. (23) and (25) we obtain that

\[ j^\rho(x) = A \left( \frac{x}{\delta - x} \right) \Re \left[ iG(x) \frac{d}{dx} G^*(x) \right] , \]  

where \( A \) is a constant and

\[ F = A \sqrt{r_{ex}^2 + \frac{\alpha \ell}{x}} \left( \frac{x}{\delta - x} \right) \Re \left[ iG(x) \frac{d}{dx} G^*(x) \right] . \]  

In the present case, \( \delta = 0 \), and so the function \( G(x) \) is given by

\[ G(x) = e^{-ix} \left( P x^s F[s + 1/2 + i(s - 1/2), 1 + 2s, 2ix] + Q x^{-s} F[-s + 1/2 + i(s - 1/2), 1 - 2s, 2ix] \right) , \]  

while the flux reads

\[ F = -A \sqrt{r_{ex}^2 + \frac{\alpha \ell}{x}} x^{2s} F[s + 1/2 + i(s - 1/2), 1 + 2s, 2ix] F[s + 1/2 - i(s - 1/2), 1 + 2s, -2ix] . \]  

In the limit \( x \) goes to 0, one has

\[ F \to x^{2s - \frac{1}{2}} , \]

and thus, in order to satisfy the appropriate boundary condition, we require that \( Q = 0 \) together with \( 2s - \frac{1}{2} > 0 \). Under these two conditions, the absence of QNMs is ensured.

Let us end this section with a remark concerning the frequency. Indeed, the condition \( 2s - \frac{1}{2} > 0 \) again implies that the frequency is bounded from below for the existence of the normal modes, i. e.

\[ \omega > \frac{n}{l} - \frac{r_{ex}}{l^2} \]

For \( \delta \neq 0 \), it is possible to prove that the condition \( \[11\] is also satisfied.

IV. CONCLUSION AND REMARKS

Through of the exact solutions found in Refs. \[6\] and \[7\] for the Klein Gordon and the Dirac equations in an extremal BTZ background, we have explicitly discussed the absence of QNMs in BTZ black holes.

In the scalar and fermionic cases we have shown that the vanishing boundary conditions at infinity are automatically satisfied for the exact solutions. This fact implies the absence of the QNMs in the extremal BTZ BH against the non-extremal cases. Which is agrees with several pieces of evidence that show that extremal BH cannot be obtained as limits of non-extremal BH. In order to apply adequate boundary condition, consistent with the presence of QNMs in the AdS space, we have obtained an inferior limit for the existence of normal modes. This last result is remarkable because in general the perturbation equation provides only information about the QNMs. We can observe that the extremal BTZ BH, behaves as a non-dissipative system.

By other way, according to the AdS/CFT correspondence, the black holes correspond to thermal states in the conformal field theory. Due to the fact that the extremal black hole have a zero Hawking temperature, it is possible to compute the zero temperature 2-point functions in an unique form (up to a normalization) from conformal field theory \[20\] \[21\] \[22\]. In this case, the retarded Green’s function will generally have poles and cuts on the real axis, corresponding to stable states and multi-particles, respectively. On the other hand, there also can be poles in the lower half \( \omega \)-plane. The distance of the poles from the real line then determines the decay time of such a resonance, this being relating with the QNM’s \[12\]. In our case, the extremal black hole correspond to a CFT in a cylinder of \( (1 + 1) \) dimensions at zero temperature. The imaginary pole part is null, showing that the extremal BTZ black hole does not exhibit QNM’s. On the other way, the real pole part is different from zero and is quantized, showing the presence of normal modes.
Then, from the CFT point of view, we can obtain another piece of evidence about the distinction between the extremal and the non-extremal cases, related to the different topologies (annulus and cylinder respectively). On the other hand, the AdS$_3$/CFT$_2$ correspondence contains information about normal and quasinormal modes, and in the extremal case it is possible to show the absence of QNM’s according to the perturbation equations developed in this article.

Finally, we hypothesize that these results are related with the interpretation of the extremal black holes as fundamental particles [3].

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[16] e.g. see N. D. Birrel and P.C.W. Davies, Quantum Fields in Curved Space times, Cambridge University Press (1982).