Constraining SUSY Dark Matter with the ATLAS Detector at the LHC

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**Abstract**

In the event that R-Parity conserving supersymmetry (SUSY) is discovered at the LHC, a key issue which will need to be addressed will be the consistency of that signal with astrophysical and non-accelerator constraints on SUSY Dark Matter. This issue is studied for the SPS1a mSUGRA benchmark model by using measurements of end-points and thresholds in the invariant mass spectra of various combinations of leptons and jets in ATLAS to constrain the model parameters. These constraints are then used to assess the statistical accuracy with which quantities such as the Dark Matter relic density and direct detection cross-section can be measured. Systematic effects arising from the use of different mSUGRA RGE codes are also estimated. Results indicate that for SPS1a a statistical/systematic precision on the relic abundance $\sim 2.8\%$ $(3\%)$ can be obtained given $300\,\text{fb}^{-1}$ of data.

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1 Introduction

The complementarity of LHC SUSY measurements and direct and indirect searches for neutralino Dark Matter is an important topic to study given the increasingly strong astrophysical evidence for cold dark matter in the universe [1, 2, 3]. Assuming that R-Parity conserving SUSY is discovered at the LHC, an interesting question will arise regarding the compatibility of that signal with existing relic density constraints (e.g. $0.094 < \Omega h^2 < 0.129$ at 2$\sigma$ from WMAP data [1, 2, 3]), and the implications it has for terrestrial Dark Matter searches.

In this paper we address these issues within the context of the minimal supergravity (mSUGRA) class of SUSY models incorporating gravity-mediated SUSY breaking [4]. A great advantage of mSUGRA models when studying SUSY phenomenology is that they can be described with only five independent parameters, namely the common scalar mass at the GUT scale ($m_0$), the common gaugino mass ($m_{1/2}$), the common trilinear coupling parameter $A_0$, the value of the ratio of the SUSY higgs vacuum expectation values ($\tan \beta$) and the sign of the higgsino mass parameter (sgn($\mu$)). We choose to study one particular model referred to as ‘SPS1a’ ($m_0 = 100$ GeV, $m_{1/2} = 250$ GeV, $A_0 = -100$ GeV, $\tan \beta = 10$, $\mu > 0$), which was defined in Ref. [5]. This model lies in the ‘bulk’ region of the $m_0$--$m_{1/2}$ mSUGRA plane where the relic density is reduced to a small value by $\chi^0_1$ annihilation to leptons or neutrinos via t-channel slepton, stau and sneutrino exchange. This model point was defined prior to the latest WMAP data and consequently possesses a relic density rather larger than that now favoured, however the analysis described in this paper remains valid and provides an interesting case study of the techniques which can be used when bulk region SUSY models are chosen by Nature.

In Section 2 we describe the measurement of mSUGRA parameters at point SPS1a using fits to measurements of end-points and thresholds in the invariant mass spectra of various combinations of leptons and jets observed in ATLAS [6]. In Section 3 we discuss the use of these parameter measurements to calculate both the $\chi^0_1$ relic density and quantities of relevance to direct and indirect search Dark Matter experiments. Finally in Section 4 we review the results and discuss the circumstances under which the measurements can be considered to be model-independent.

2 Extraction of mSUGRA Parameters

The development of techniques for measuring parameters characterising SUSY models has been the subject of much investigation in the last few years, as documented in Ref. [6, 7, 8], and is still a very active field of investigation.

The basic issue is that the presence of two invisible particles in the final state renders the direct measurement of sparticle masses through the detection of invariant mass peaks impossible. Alternative techniques have therefore been developed, based on the exclusive identification of long cascades of two body-decays. It was demonstrated [7] that if a sufficiently long chain can be identified (at least three successive two-body decays), the thresholds and end-points of the various possible invariant mass combinations among the identified products can be used to achieve a model-independent measurement of the masses of the involved sparticles. Once the masses of the lighter sparticles are obtained with this procedure, in particular the mass of the Lightest Supersymmetric Particle (LSP), additional sparticle masses can be measured through the identification of other shorter decay chains. This program has been carried out
recently for SPS1a [9], resulting in a number of measurements of observables which are related to the masses of the sparticles by known algebraic relations (Table 1). The meaning of the different observables, and their expression in terms of sparticle masses is given in Ref. [8]. An

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value (GeV)</th>
<th>Stat+Sys (GeV)</th>
<th>Scale (GeV)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{\ell\ell}^{max}$</td>
<td>83.37</td>
<td>0.03</td>
<td>0.08</td>
<td>0.09</td>
</tr>
<tr>
<td>$m_{q\ell}^{max}$</td>
<td>457.55</td>
<td>1.4</td>
<td>4.6</td>
<td>4.8</td>
</tr>
<tr>
<td>$m_{q\ell}^{low}$</td>
<td>321.28</td>
<td>0.9</td>
<td>3.2</td>
<td>3.3</td>
</tr>
<tr>
<td>$m_{q\ell}^{high}$</td>
<td>400.63</td>
<td>1.0</td>
<td>4.0</td>
<td>4.1</td>
</tr>
<tr>
<td>$m_{lb}^{min}$</td>
<td>220.81</td>
<td>1.6</td>
<td>2.2</td>
<td>2.7</td>
</tr>
<tr>
<td>$m_{lb}^{max}$</td>
<td>199.48</td>
<td>3.6</td>
<td>2.0</td>
<td>4.2</td>
</tr>
<tr>
<td>$m(\tilde{\ell}_{L}) - m(\tilde{\chi}_1^0)$</td>
<td>109.18</td>
<td>1.5</td>
<td>0.1</td>
<td>1.5</td>
</tr>
<tr>
<td>$m_{\ell\ell}^{max}(\tilde{\chi}_1^0)$</td>
<td>279.07</td>
<td>2.3</td>
<td>0.3</td>
<td>2.3</td>
</tr>
<tr>
<td>$m_{\tilde{\tau} \tilde{\tau}}^{max}$</td>
<td>86.03</td>
<td>5.0</td>
<td>0.9</td>
<td>5.1</td>
</tr>
<tr>
<td>$m(\tilde{g}) - 0.99 \times m(\tilde{\chi}_1^0)$</td>
<td>517.22</td>
<td>2.3</td>
<td>5.2</td>
<td>5.7</td>
</tr>
<tr>
<td>$m(\tilde{g}) - m(\tilde{b}_1)$</td>
<td>452.62</td>
<td>10.0</td>
<td>4.5</td>
<td>11.0</td>
</tr>
<tr>
<td>$m(\tilde{g}) - m(\tilde{b}_2)$</td>
<td>96.98</td>
<td>1.5</td>
<td>1.0</td>
<td>1.8</td>
</tr>
<tr>
<td>$m(\tilde{g}) - m(\tilde{b}_2)$</td>
<td>72.75</td>
<td>2.5</td>
<td>0.7</td>
<td>2.6</td>
</tr>
</tbody>
</table>

Table 1: Summary table of the SUSY measurements which can be performed at the LHC with the ATLAS detector. The central values are calculated for ISAJET 7.69. The statistical errors are given for the integrated luminosity of 300 fb$^{-1}$. The uncertainty in the energy scale is taken to result in an error of 1% for measurements including jets, and of 0.1% for purely leptonic measurements. The values quoted here differ somewhat from those quoted in Ref. [9] due to the use of an earlier version of ISASUGRA in that work.

additional measurement used in this analysis is the mass of the light higgs boson from the decay $h \rightarrow \gamma \gamma$ which for 100 fb$^{-1}$, and for $m_h = 114$ GeV, has a statistical uncertainty of $\sim 0.5$ GeV [10].

Two approaches are possible in order to extract model constraints from these measurements:

- Solve the system of measurements for the masses of the involved sparticles, and fit the model parameters to them;
- Perform a direct fit of the parameters of the model to the measured observables.

As discussed in detail in [7, 8], the mass values resulting from solving the experimental constraints are strongly correlated. This is due to the fact that the measured constraints are typically expressed as differences of masses. Therefore the first approach, using uncorrelated mass errors, as is e.g. done in Ref. [11] will result in a very inaccurate estimate of the constraints on the model.

We rely therefore in this work on the direct fit of the observable to the mSUGRA model as embodied in a RGE evolution code. A detailed study of the theoretical uncertainties based on the study of different implementations of the RGE running of the SUSY masses from the GUT
scale to the TeV scale is given in Ref. [12]. Addressing this theoretical uncertainty is outside the scope of this work. We limit ourselves to perform the fit on two different models, namely ISASUGRA v.7.69, and SUSPECT v.2.102, with the aim of studying the impact of the use of different SUSY spectrum generators on the determination of the Dark Matter characteristics.

Two different types of uncertainties are quoted in Table 1: the combined statistical and systematic uncertainties estimated for each measurement, and general errors on the scales of lepton and hadron energy measurement, which affect all the measured quantities in the same way. Since in many cases the scale uncertainties are dominant it is necessary to take into account the correlations between the different measurements when extracting the constraints on the model parameters. A technique often found in the literature (see e.g. Ref. [12]) to estimate the error in parameter determination may be described as follows:

1. The central value \( d(i) \) and measurement error \( \sigma(i) \) is obtained for each of the measured quantities, where \( i \) runs over the number \( N_d \) of available measurements.
2. A scan is performed over a grid of SUSY models, with the nominal value \( dm(i,j) \) of each of the measurements \( i \) being calculated for each model \( j \).
3. The total \( \chi^2 \) is calculated for each point \( j \) on the grid using
   \[
   \chi_j^2 = \sum_i \frac{d^2(i)^2 - dm^2(i,j)}{\sigma^2(i)}
   \]
4. The uncertainty in the hyperspace of SUSY parameters is evaluated using the properties of the \( \chi^2 \) function.

This approach relies on the assumption that the calculated \( \chi^2 \) function is correctly normalised, and the uncertainties in the measurements are independent; however we know that this last assumption is at best very approximate. We therefore use a Monte Carlo technique relying on the generation of simulated experiments sampling the probability density functions of the measured observables. In frequentist statistics, confidence bands describe the probability that an experiment in a set of identical experiments yields a given value for the measured quantities.

We proceed in the following way:

1. An ‘experiment’ is defined as a set of 14 measurements, each of which is generated by picking a value from a gaussian distribution with mean given by the central value given in Table 1. The correlation due to energy scale is taken into account by using a second gaussian distribution for the energy scale, and using the same random number for all the measurements sharing the same scale.
2. For each experiment, the point in mSUGRA parameter space minimising the \( \chi^2 \) is calculated.

We obtain as a result of this calculation a set of mSUGRA models, each of which is the “best” estimate for a Monte Carlo experiment of the mSUGRA model generating the observed measurement pattern. For each of these models the properties of the LSP Dark Matter candidate can then be calculated and the spread of obtained results interpreted as the level of precision with which these properties can be measured by the LHC. For 300 fb\(^{-1}\) of
data the distributions of fitted mSUGRA parameter values are all approximately gaussian in form with widths \( \sim 2\% (m_0), 0.6\% (m_{1/2}), 9\% (\tan\beta) \) and 16\% \((A_0)\) respectively. For 100 fb\(^{-1}\) of data the main differences are that the width of the \(A_0\) distribution increases to 28\% due to the poorer precision on the \(\tilde{\chi}^0\) mass and (more importantly) that the \(\tan\beta\) distribution acquires a significant tail to large values of \(\tan\beta\) (Fig. 1) due to the fact that for 100 fb\(^{-1}\) the masses of the two sbottom squarks can not be measured separately, and are therefore not included in the fit. This tail has a significant impact on the precision with which dark matter properties can be calculated. Therefore most results quoted below assume 300 fb\(^{-1}\) of data.

3 Calculation of Dark Matter Characteristics

Best fit values of \(m_0, m_{1/2}, A_0\) and \(\tan(\beta)\) obtained from each fit to the 14 sampled measurements were passed to dedicated codes in order to calculate the implied characteristics of the \(\tilde{\chi}^0\) Dark Matter candidate. As a result of the generally poor best fit \(\chi^2\) values obtained from the \(\mu < 0\) fits, the correct \(\mu > 0\) assumption was made throughout. The input top mass values used for each model were randomly sampled from a gaussian distribution of mean 175 GeV and (conservative) \(\sigma = 2\) GeV, to represent the uncertainty arising from ATLAS measurements [13].

The \(\tilde{\chi}^0\) relic density \(\Omega \delta h^2\) was calculated using MICROMEGAS v.1.1.1 [14] interfaced to either the ISASUGRA v.7.69 [15] or SUSPECT v.2.102 [16] RGE codes. MICROMEGAS incorporates exact tree-level calculations of all annihilation and co-annihilation processes together.
with one- and two-loop corrections to the Higgs width and mass. MICROMEGAS is hence well suited to this task at SPS1a where stau annihilation processes dominate the relic density calculation. Nevertheless an estimated 2% systematic error must be assigned to the relic density due to uncertainties in solving the Boltzmann equation [17]. An additional systematic uncertainty \( \lesssim 1\% \) is estimated from higher order corrections to the calculated cross-sections. Potential systematic uncertainties arising from differences in the RGE codes were estimated from the mean separation between the predicted \( \Omega h^2 \) distributions obtained from the two codes.

Characteristics of potential Dark Matter detection signatures implied by the ATLAS measurements were assessed using DarkSUSY v.3.14.02 [18, 19], again interfaced to the ISASUGRA v.7.69 or SUSPECT v.2.102 RGE codes. DarkSUSY v.3.14.02 is a relatively mature code which is in general unsuitable for detailed relic density calculations due to its lack of an implementation of e.g. stau co-annihilation processes \(^1\). Nevertheless it is one of only a few publicly available codes which can calculate quantities relevant to direct and indirect detection. Three such quantities were chosen for study in this paper, namely the spin-independent \( \chi^0-p \) scattering cross-section \( \sigma_{sp} \) (equivalent to the scalar scattering cross-section for Majorana neutralinos) governing the rate of elastic nuclear recoils observed in direct search experiments [21],

\(^1\)A new version of DarkSUSY incorporating co-annihilation processes is under development [20].
the flux $\phi_{\text{sun}}$ of high energy neutrinos generated by annihilation of relic $\chi^0_1$ trapped in the solar core, and the flux $\phi_{\text{earth}}$ of neutrinos generated by similar processes occurring within the earth [22]. The last two quantities were calculated by integrating over all neutrino energies for neutrinos emerging within a half-aperture angle of 30° from the centre of sun/earth.

In all of the above cases care was taken to ensure that when using the ISASUGRA RGE code interfaced to a given Dark Matter code, input best fit mSUGRA parameters were obtained using ISASUGRA fits. Similarly all results obtained using SUSPECT made use of SUSPECT-derived input parameters.

4 Results

The distribution of $\Omega_{\chi} h^2$ values obtained from the best-fit mSUGRA models by MICROMEGAS interfaced to ISASUGRA or SUSPECT are shown in Fig. 2. As with all the other quantities investigated here the results obtained with SUSPECT give a somewhat smaller statistical scatter (~ 2.5%) compared with those obtained with ISASUGRA (~ 2.8%). The overall systematic uncertainty arising from the RGE codes is estimated to be ~ 2.4%, giving an overall systematic uncertainty (adding the Boltzmann contribution in quadrature) ~ 3%. In both cases the distributions are reasonably gaussian.

![Figure 3: Values for $\Omega_{\chi} h^2$ calculated with ISASUGRA and MICROMEGAS in the same manner as for Fig. 2(a), except using mSUGRA parameter distributions expected for 100 fb$^{-1}$ of data.](image)

It should be noted that with 100 fb$^{-1}$ of data the significant tail in the $\tan\beta$ distribution arising from the lack of a measurement of the $b_1$ mass leads to significant non-gaussianity in the prediction for $\Omega_{\chi} h^2$ (Fig. 3). The same behaviour is also observed for the other dark matter quantities considered below.
Figure 4: Values for the spin-independent $\tilde{\chi}_1^0$-nucleon elastic scattering cross-section calculated from mSUGRA fits to the SPS1a invariant mass spectrum end-points described in the text. The distribution in Figure (a) was calculated by using results from ISASUGRA v.7.69 fits as input to DarkSUSY v.3.14.02 interfaced to ISASUGRA v.7.69. The distribution in Figure (b) was calculated by using results from SUSPECT v.2.102 fits as input to DarkSUSY v.3.14.02 interfaced to SUSPECT v.2.102.

The distribution of $\log_{10}\sigma_{si}$ values ($\sigma_{si}$ measured in pb) obtained from the best-fit mSUGRA models by DarkSUSY interfaced to ISASUGRA or SUSPECT are shown in Fig. 4. This time the statistical errors are even smaller ($\sim$0.5%) with an RGE systematic error estimated to be $\sim$ 0.4%. These errors are expected to be negligible compared with the intrinsic estimated factor 2 systematic uncertainty arising from uncertainties in the strange quark content of the nucleon and approximations used when calculating higher order gluon loop contributions to the cross-section [23]. Furthermore any experimental measurement of $\sigma_{si}$ is expected to be subject to considerable systematic uncertainty arising from lack of knowledge of the local Dark Matter density and velocity distribution. Consequently such measurements are not expected to agree with ATLAS predictions at the high level of accuracy quoted here.

The distribution of $\log_{10}\phi_{\text{sun}}$ and $\log_{10}\phi_{\text{earth}}$ values ($\phi$ measured in km$^{-2}$ year$^{-1}$) obtained from the best-fit mSUGRA models by DarkSUSY interfaced to ISASUGRA or SUSPECT are shown in Figs. 5 and 6. Statistical errors $\sim$ 0.3% (3%) were obtained for the flux of neutrinos from the sun (earth) (SUSPECT RGE code). RGE code systematic errors were estimated to be $\sim$ 0.5% and 0.6% respectively. Note that for these quantities additional systematic errors in the predictions (at least a factor 2 [23]) due to uncertainties in the $\tilde{\chi}_1^0$ density and $\tilde{\chi}_1^0-\tilde{\chi}_1^0$ annihilation rates in the earth and sun are applicable. Thus the statistical and systematic errors associated with the ATLAS predictions and RGE codes are again effectively negligible.
Figure 5: Values for the flux of high energy neutrinos from \( \tilde{\chi}_1^0 - \tilde{\chi}_1^0 \) annihilation in the sun calculated from mSUGRA fits to the SPS1a invariant mass spectrum end-points described in the text. The distribution in Figure (a) was calculated by using results from ISASUGRA v.7.69 fits as input to DarkSUSY v.3.14.02 interfaced to ISASUGRA v.7.69. The distribution in Figure (b) was calculated by using results from SUSPECT v.2.102 fits as input to DarkSUSY v.3.14.02 interfaced to SUSPECT v.2.102.

5 Discussion and Conclusions

These results demonstrate both the effectiveness of ATLAS mSUGRA parameter measurements for predicting SUSY Dark Matter properties and the excellent agreement which now exists between RGE codes. For SPS1a the statistical and systematic errors arising from these sources are expected to be negligible compared with other sources of systematic error in the estimates of Dark Matter properties. Of course for higher mass scale models in e.g. the focus point, co-annihilation or rapid-annihilation regions of mSUGRA parameter space, the smaller number of mass constraints may well lead to considerably increased statistical errors on mSUGRA parameters and hence the quantities calculated here. The SPS1a benchmark model considered here should therefore probably be considered to be something of a ‘best-case’ scenario. More work is clearly needed to study other more difficult cases.

It is interesting to note that the result of the (mSUGRA) model-dependent calculation of \( \Omega \chi h^2 \) performed here could be assumed to be more model-independent provided certain criteria were met by the SUSY signal observed in ATLAS. This result should be valid for any ‘bulk’ region SUSY model with dominant \( \tilde{\chi}_1^0 \) annihilation into \( \tau \) or slepton/sneutrino pairs.\(^2\)

\(^2\)At SPS1a these processes contribute \( \sim 96\% \) of the total (co)-annihilation cross-section according to MICROMEGAS.
and \(\tilde{\tau}_1\), \(\tilde{l}_R\) and \(\tilde{\chi}_1^0\) masses and \(\tan \beta\) compatible with the observed invariant mass end-point positions and branching fractions. Additional measurements would be needed in order to perform a completely-model independent estimate of the relic density. Examples are the \(\tilde{\chi}_3^0\) mass, which would allow complete determination of the neutralino mixing matrix (if assumed real), additional information on the \(\tilde{\tau}\) sector, and a measurement of the masses of the heavy higgses. Work is in progress in order to evaluate how well the presently available measurements can constrain a generic MSSM, and to identify the crucial measurements needed in order to achieve model-independence.

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References


