Could the next generation of cosmology experiments exclude supergravity?

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Gravitinos are expected to be produced in any local supersymmetric model. Using their abundance prediction as a function of the reheating energy scale, it is argued that the next generation of Cosmic Microwave Background experiments could exclude supergravity or strongly favor "thermal-like" inflation models if \( B \) mode polarized radiation were detected. Galactic cosmic-ray production by evaporating primordial black holes is also investigated as a way of constraining the Hubble mass at the end of inflation. Subsequent limits on the gravitino mass and on the related grand unification parameters are derived.


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I. INTRODUCTION: GRAVITINOS IN THE EARLY UNIVERSE

Although not yet experimentally discovered, supersymmetry (SUSY) is still the best - if not the only - natural extension of the standard model of particle physics. It could provide a general framework to understand the origin of the fundamental difference between fermions and bosons and could help to resolve the difficult problem of mass hierarchies, namely the instability of the electroweak scale with respect to radiative corrections. In global supersymmetry, the generator spinors \( \xi \) are assumed to obey \( \partial_\mu \xi = 0 \). If one wants to deal with local supersymmetry, or supergravity, this condition must be relaxed and \( \xi \) becomes a function of the space coordinates \( x \). New terms, proportional to \( \partial_\mu \xi(x) \), must be canceled by introducing a spin \( 3/2 \) particle, called gravitino, as vector bosons are introduced in gauge theories. The gravitino is part of an \( N=1 \) multiplet which contains the spin 2 graviton (see [2] for an introductory review) and, in the broken phase of supergravity, super-Higgs effects make it massive through the absorption of the Nambu-Goldstone fermion associated with the SUSY breaking sector.

It has long been known that if the gravitino is unstable some severe constraints on its mass must be considered in order to avoid entropy overproduction \( \xi \): \( m_{3/2} \gtrsim 10 \) TeV. On the other hand, if the gravitino is stable, its mass should satisfy \( m_{3/2} \lesssim 1 \) keV \( \xi \) to keep the gravitinos density smaller than the full Universe density (\( \Omega_{3/2} < \Omega_{\text{tot}} \)). In spite of the huge dilution, those constraints are not fully evaded by inflation as gravitinos should be reproduced by scattering processes off the thermal radiation after the Universe has reheated \( \xi \). As the number density of such secondary gravitinos is expected to be proportional to the reheating temperature, it is possible to relate the energy scale of inflation with the requirement that they are not overproduced.

In the first part of this paper, the next generation of cosmic microwave background (CMB) detection experiments is considered as a way of possibly excluding supergravity. It is shown that the energy scale of inflation required to produce an observable amount of tensor mode in the background radiation is not compatible with local supersymmetry in the standard cosmological scenario. In the second part, a new way of constraining the gravitino mass, based on evaporating primordial black holes, is investigated. Taking into account that the black hole masses cannot be much smaller than the Hubble mass at the formation epoch, it is suggested that a detection of cosmic-rays produced by the Hawking mechanism would lead to a lower bound on the reheating scale and, therefore, on the gravitino mass. Links with grand-unified models are given, as an example, in the conclusion. Finally, the basics of the propagation model used to relate the source term to the local spectrum are given in the Appendix A.
II. TENSOR MODE IN THE COSMOLOGICAL BACKGROUND

Observational cosmology has recently entered a new era thanks to several experiments dedicated to the CMB measurements (see [54]), e.g. Maxima, BOOMERanG, ACBAR, DASI, CBI, VSA, ARCHEOPS, and WMAP. They give strong evidences in favor of the inflationary scenario: a density extremely close to the critical value, a nearly scale invariant power spectrum, and a gaussian structure of the perturbations. Furthermore, in addition to the temperature anisotropies, the polarization of the CMB has also been recently observed [12]. For the time being, only the even-parity E mode has been detected and the odd-parity B mode is still to be discovered. The latter is of specific importance as it would probe the primordial gravitational waves through tensor perturbations. Their amplitude can be expressed with the Hubble parameter and the potential of the scalar field driving inflation [14]:

\[ T = \left( \frac{H}{2\pi M_{Pl}} \right) = \frac{2V(\phi)}{3\pi M_{Pl}^4} \]

where \( M_{Pl} = (8\pi G)^{-1/2} = 2.4 \times 10^{18} \text{ GeV} \) is the Planck mass. The important point is that the tensor/scalar ratio \( r = 6.9M_{Pl}^2(V'/V)^2 \) can be related to the energy scale of inflation \( E_{inf}\) [15] by

\[ E_{inf} \approx \left( \frac{r}{0.7} \right)^{1/4} \times 1.8 \times 10^{16} \text{ GeV}. \]

The amplitude of the polarization B mode is therefore directly proportional to \( E_{inf}\).

Figure 1 shows the 1 \( \sigma \) sensitivity of the Planck satellite to polarization, as computed with CMBfast [57]. On the same plot, the B mode polarization in a standard \( \Lambda\)CDM cosmology with an inflation energy scale \( E_{inf} \sim 10^{16} \text{ GeV} \) (dotted line) is also represented. Increasing (resp. lowering) \( E_{inf}\) would result in increasing (lowering) the amplitude of the primordial B mode thus making it easier (more difficult if not impossible) to detect. On the contrary, the level of the expected B mode induced by weak lensing is fixed and rather accurately predicted since it results from lensing effects on the polarization E mode due to scalar perturbations that are now well constrained. The challenge in the detection of the primordial B mode and the estimation of \( E_{inf}\) is then to have a sensitive enough experiment and to avoid contamination by weak lensing. For the Planck experiment, the major hope is the detection at low \( \ell \) thanks to the high reionization optical depth suggested by WMAP [10]. In the case of limited sky coverage experiments, the weak lensing contribution will have to be removed.

With the Planck sensitivity, the B mode should be detected (3\( \sigma \)) if \( E_{inf} > 10^{16} \text{ GeV} \) [17]. This case would be in severe conflict with most supersymmetric models. Indeed, in mSUGRA, the gravitino mass is, by construction, expected to lie around the electroweak scale, \( i.e. \), in the 100 GeV - 1 TeV range [19]. Considering that Deuterium and \(^3\)He should not be overproduced by photodissociation of \(^4\)He below 700 GeV and that Deuterium should not be destroyed beyond the allowed observational values [20] above 700 GeV [21], the reheating temperature must remain lower than \( 2 \times 10^9 \text{ GeV} \) if the branching ratio of gravitinos into photons and photinos is assumed to be unity and lower than \( 5 \times 10^{11} \text{ GeV} \) with a conservative branching ratio of 1/10. The large difference between those limits and the energy scale required to produce a measurable amount B mode polarization makes the exact value of the branching ratio of gravitinos into photons and photinos irrelevant. A detection of the polarization B mode by the Planck satellite would therefore disfavor mSUGRA in standard cosmology.

In gauge-mediated SUSY breaking alternative scenarios, mostly interesting in accounting for a natural suppression of the rate of flavor-changing neutral-current due to the low energy scale, the situation is even more constrained. In this case, gravitinos are the lightest supersymmetric particles and requiring their density not exceed the total density imposes an upper limit on \( T_{RH} \)
between $10^6$ and $10^3$ GeV for masses between 10 MeV and 100 keV \cite{22}. Although some refined models can relax those constraints \cite{23}, local supergravity would, in this case also, be in serious trouble if the reheating temperature was high enough to be probed by the Planck experiment.

A possible way to get around these conclusions is to assume that a substantial amount of entropy was released after the gravitinos and moduli production, that would dilute them according to the entropy conservation ($n/s \approx \text{cte}$). Such a scenario can be realized while keeping the inflationary scale high, e.g., in thermal inflation \cite{24,25}. Some studies \cite{26} even show that a wide modulus mass region ($m_\Phi \approx 10$ eV - $10^4$ GeV) would be allowed but it requires in most cases a very small reheating temperature. Recently, the curvaton scenario \cite{27} has also attracted a considerable interest as it generates a huge amount of entropy through a scalar field that dominates the radiation at a given epoch. One can then argue that a detection of tensor mode polarization, would strongly favor “thermal like” inflation scenarios if supergravity is to remain as the preferred extension of the standard model of particle physics. Interestingly, if evidence in favor of local supersymmetry were obtained either by colliders or by independent astroparticle experiments, this could even be a very promising observational signature for thermal inflation.

Fortunately, the Planck satellite is not expected to be the ultimate experiment to study the CMB polarization and several improvements can be expected in the future. However, as pointed out in Refs. \cite{28,29}, there remains a lower limit to the removal of the polarization $B$ mode foreground induced by gravitational lensing which sets at present time the lower limit on the detectable inflation scale to a few times $10^{15}$ GeV. This scale remains, however, particularly interesting if the fundamental scalars driving the phenomenon are related with grand unification since it lies around the GUT energy (between $10^{15}$ GeV and $3 \times 10^{16}$ GeV depending on whether supersymmetry is considered or not). It therefore makes sense to improve the polarization sensitivity to reach the capability to probe the typical GUT scale where inflation could have occurred if the gravitino limit is ignored.

III. COSMIC-RAYS FROM EVAPORATING BLACK HOLES

Another interesting way to experimentally probe the reheating temperature would be to look for evaporating primordial black holes (PBHs). Such black holes should have formed in the early Universe if the density contrast was high enough on small scales. Many different possible scenarios have been suggested to allow for an important PBH density (see \cite{30} for a review): a dust-like stage

![FIG. 2: Possible reheating temperatures $T_{RH}$ as a function of the PBH density (normalized to the critical density) for different antideuteron flux at 100 MeV: $2 \times 10^{-7}$, $2 \times 10^{-8}$, $2 \times 10^{-9}$, $2 \times 10^{-10}$ from right to left in m$^{-2}$s$^{-1}$sr$^{-1}$GeV$^{-1}$.](image1)

![FIG. 3: Lower limit on the reheating temperature $T_{RH}$ as a function of the 100 MeV antideuteron flux.](image2)
general first order phase transitions, a scale in the power spectrum, to mention only the currently most discussed possibilities. Such PBHs of mass $M$ should evaporate, following a Planck-like spectrum with temperature $T = \hbar c^3/(16\pi kGM)$, which was derived by Hawking using the usual quantum mechanical wave equation for a collapsing object with a postcollapse classical curved metric instead of a precollapse Minkowsky one. If those black holes are present in our galaxy (even with densities as low as $\Omega_{PBH} \sim 10^{-9}$), the emitted quanta should contribute to the observed cosmic-rays. Among them, two kinds of particles are especially interesting: antiprotons and gamma-rays. Antiprotons are useful because the astrophysical background coming from spallation of cosmic-rays on the interstellar medium (so-called secondary particles) is very small (the ratio $p/p$ is smaller than $10^{-4}$ whatever the considered energy) and very well known. A tiny excess due to evaporating black holes could therefore be easily probed in the low energy range since the shape of the PBH spectrum is dominated by fragmentation processes and is then softer than the secondary spectrum. Gamma-rays, coming both from direct emission and from the decay of neutral pions, take advantage of the very small optical depth of the Universe for $\sim 100$ MeV radiation: the source emission can be probed up to redshifts $z \sim 700$. Furthermore, the signal to noise ratio is optimal at this energy as the PBH spectrum becomes softer $(dN/dE \propto E^{-1} \rightarrow dN/dE \propto E^{-3})$ above $100$ MeV (roughly corresponding to the QCD confinement scale) because of partons hadronization and integrated redshift effects.

Using those cosmic-rays, the experiments are currently sensitive to PBHs with masses between $10^{12}$ and $10^{14}$ g. Those values can be intuitively understood as resulting from two opposite effects. On the one hand, the temperature favors the light (i.e. hot) black holes but their number density is very small: by integrating the Hawking flux over energy, it is straightforward to show that the mass spectrum must be proportional to $M^2$ below $M_\star = 5 \times 10^{14}$ g (the initial mass of a black hole whose lifetime is equal to the age of the Universe) whatever the details of the formation mechanism. This is mostly due to the fact that the low-mass behavior is fully governed by the evaporation process, as obtained by writing $dn/dM = (dn/dM_i) \times (dM_i/dM)$ where $M$ stands for the current mass value and $M_i$ for the initial one. The evolution term $dM_i/dM$ is simply determined from $M_i \approx (3\alpha t + M^3)^{1/3}$ where $\alpha \approx \{7.8d_{s=1/2} + 3.1d_{s=1}\} \times 10^{24}$ g$^{-3}$ s$^{-1}$ accounts for the number of available degrees of freedom with $d_{s=1/2} = 90$ and $d_{s=1} = 27$ in the standard model. On the other hand, the "number density" effect favors the heavy black holes but their low temperature makes the emission rate very small, especially when heavy hadrons are considered.

The important point for this study is that only black holes formed after inflation would contribute to the observed phenomena as those formed before were exponentially diluted. Furthermore, whatever the considered formation mechanism, either through the usual collapse of high density-contrast primordial gaussian fluctuations or for near critical phenomena, the PBH mass at the formation epoch is close to the horizon mass at the same time. It cannot be larger as the considered points would not be in causal contact and it cannot be much smaller as they would, in this case, more probably have formed before (as taken into account in the usual Press-Schechter formalism). It means that if the evaporation process was detected, the Hubble mass at the reheating time should be small enough not to induce a cutoff in the PBH mass spectrum which would make the light black holes abundance totally negligible. The best upper limit available on the density of PBHs around $M_* = 5 \times 10^{14}$ g, taking into account both the details of the source term evolution and the background from galaxies and quasars, is currently: $\Omega_{PBH}(M_\star) < 3.3 \times 10^{-9}$.

Fortunately, some hope for future detection is still possible thanks to antideuterons: those nuclei are expected to be very rarely formed by spallation processes below a few GeV for kinematical reasons. The threshold for an antideuteron production is $E = 17 m_p$ (total energy) in the laboratory, 2.4 times higher than for antiproton production. The center of mass is, therefore, moving fast and it is very unlikely to produce an antideuteron nearly at rest (in the 100 MeV - 1 GeV range) in the laboratory. On the other hand, they could be emitted in this energy range by evaporating PBHs and could be probed by the new generation of cosmic-ray detectors: the AMS experiment and the GAPS project. To obtain this result, a coalescence model (see for a review) was used, based mainly on phase space considerations: the antideuteron density in momentum space is proportional to the product of the proton density with the probability of finding a neutron within a small sphere of radius $p_0$ around the proton momentum. Thus:

$$\gamma \frac{d^3 N_d}{dk^3} = \frac{4\pi p_0^3}{3} \left( \gamma \frac{d^3 N_p}{dk^3} \right) \left( \gamma \frac{d^3 N_n}{dk^3} \right)$$

where $p_0$ is the coalescence momentum whose uncertainty window is of the order of 60-280 Mev in extreme cases. The Hawking spectrum has then been convolved with the fragmentation functions, as obtained with the PYTHIA Monte-Carlo simulation of the Lund model:

$$\frac{d^2 N_{\tilde{D}}}{dE dt} = \sum_j \int_0^\infty \frac{\Gamma_{sj}(Q,T)}{h} \left( e^{\frac{Q}{T}} - (-1)^{2s_j} \right)^{-1} \times \frac{dg_{sj\tilde{D}}(Q,E,p_0)}{dE} dQ$$
where \( \frac{dg_jb(Q, E, p_0)}{dE} \) is the number of antideuterons formed with an energy between \( E \) and \( E + dE \) by a partonic jet of type \( j \) and energy \( Q \), evaluated with the coalescence model for a given momentum \( p_0 \), \( \alpha_j \) is the number of degrees of freedom, \( s \) is the spin, and \( \Gamma_s \) is the absorption probability. This coalescence condition (finding an antiproton and an antineutron within the same jet with a momentum difference smaller than \( p_0 \)) was directly tested in the \( \bar{p} - \bar{n} \) center of mass frame as \( p_0 \) is not Lorentz-invariant and implemented within the PYTHIA simulation. This individual flux in then convolved with the PBH mass spectrum. To obtain the top of the atmosphere (experimentally measurable) spectrum, the emitted antideuterons have been propagated within the Galaxy using the diffusion model of Ref. [36], briefly recalled in the appendix at the end of this paper. Finally, the resulting flux were solar-modulated in the force-field approximation.

Figure 2 shows the possible values of the reheating temperature as a function of the density of PBHs at \( 5 \times 10^{14} \) g for different PBH-induced antideuteron flux at 100 MeV (ranging from \( 2 \times 10^{-7} \) m\(^{-2}\)s\(^{-1}\)sr\(^{-1}\)GeV\(^{-1}\)), the maximum value consistent with the gamma-ray upper limit, down to \( 2 \times 10^{-10} \) m\(^{-2}\)s\(^{-1}\)sr\(^{-1}\)GeV\(^{-1}\)). They were obtained with conservative values of all the free parameters entering the model, astrophysical quantities being totally bounded by an exhaustive study of the heavy nuclei data [48]. As expected, there is a degeneracy between the \( D \) flux and \( \Omega_{\text{PBH}} \): the same amount of particles can be produced either by a high normalization of the black hole spectrum and a cutoff in the high mass range (i.e. a low reheating temperature value) of by a low normalization of the black hole spectrum and a cutoff in the low mass range (i.e. a high reheating temperature value). This means that, in the case of detection, it should be possible to give a lower limit on the reheating temperature. Of course, the larger the antideuteron flux, the better the constraint on \( T_{\text{RH}} \). As shown on this figure, for a fixed value of the flux, whatever the value of \( \Omega_{\text{PBH}} \), the reheating temperature value cannot be arbitrarily low since the mass spectrum cannot be cut much above masses roughly corresponding to temperatures of the order of the \( D \) mass (i.e. \( T_{\text{BH}} \sim \) a few GeV and \( T_{\text{RH}} \sim \) a few \( 10^8 \) GeV). The other way round, whatever the value of \( T_{\text{RH}} \), the density of black holes cannot be arbitrarily low since even without any cutoff the source term must remain high enough to account for the considered flux. Naturally, this approach assumes that the measured antideuterons are indeed produced by evaporating black holes. The only other serious candidate as a source of light antimolecules in the low energy range are annihilating supersymmetric particles. It has been demonstrated [49] that only neutralinos with masses around 100-200 GeV could contribute to the observed antideuteron flux. As this mass range will be probed by the Large Hadron Collider, it should be possible to distinguish between antideuterons induced by PBHs and by SUSY particles (some reconstruction problems could occur if the mass spectrum is strongly degenerated, especially between the lightest neutralinos and charginos, but this would hide the lightest supersymmetric particles only for masses in the TeV range).

In the case where they are indeed coming from black holes, Fig. 3 gives the reheating temperature value as a function of the measured \( D \) flux. This result was obtained by varying values of the 100 MeV antideuteron spectrum combined with the upper limit coming from [45] and [57] (\( \Omega_{\text{PBH}} < 3 \times 10^{-9} \)) for the corresponding reheating scale (evaluated by the previously given method). As expected, the limit becomes more stringent when the measured flux is higher and diverges when it goes to the maximum allowed value (otherwise it contradicts previously given limits). When compared with the upper bound coming from Big–Bang nucleosynthesis, this translates into a lower limit on the gravitino mass \( m_{3/2} \). This can be derived by solving the Boltzmann equation for the gravitino number density \( n_{3/2} \) in [21]:

\[
\frac{dn_{3/2}}{dt} + 3Hn_{3/2} = \Sigma \langle \sigma v \rangle n_{\text{rad}} \Gamma_{3/2} \left( \frac{m_{3/2}}{E_{3/2}} \right)^{3/2}
\]

where \( H \) is the Hubble parameter, \( n_{\text{rad}} = \frac{\zeta(3) T^3}{\pi^2} \) is the number density of the scalar bosons in thermal bath, \( \Gamma_{3/2} \) is the relative velocity of the scattering radiation, \( \frac{m_{3/2}}{E_{3/2}} \) is the averaged Lorentz factor, \( \tau_{3/2} \) is the lifetime of the gravitino (computed...
from the supergravity lagrangian [50] and $\Sigma_{\text{tot}}$ is the total cross section (computed in the MSSM framework). Gravitinos are then assumed to decay mostly into photinos and photons, whose pair scattering off the background radiation, photon-photon scattering, pair creation on nuclei, compton scattering, inverse compton scattering of $e^+/e^-$ and induced leptonic cascades are taken into account. Requiring that the subsequent photo-dissociation of light elements does not modify the Big Bang Nucleosynthesis scenario beyond experimental constraints, the upper limit of the reheating temperature can be numerically computed as a function of the gravitino mass [21]. Figure 4 gives this bound as a function of the measured antideuteron flux at 100 MeV for three different branching ratios $B$ of gravitinos into photons and photinos ranging from 0.1 (lowest curve) to 1 (upper curve). As the reheating temperature lower limit is extremely sensitive to the gravitino mass in the 100 GeV - 1 TeV range [21], the curves are quite flat, except when the required value of $T_{RH}$ enters the diverging region. Although the accurate value of $B$ is model dependent, it can safely be taken as lying in the 0.1-1 range, as usually assumed in most studies. Once again, if a "thermal-like" inflation phase occurred, those limits do not stand anymore but could lead to important indications in favor of such a scenario if the gravitino was independently shown to be lighter than those values.

It is important to notice that a great amount of work has also been recently devoted to the non-thermal production of gravitinos and moduli fields (dilaton and modulus fields appearing in the framework of superstring theories which acquire mass through the nonperturbative effects of the supersymmetry breaking). Most papers claim that the upper limit on the reheating temperature must be drastically decreased (by up to 7 orders of magnitude [51]). Those results being still controversial, they were not taken into account in this work but they can only reinforce our conclusions and improve our limits.

IV. PROSPECTS AND CONCLUSION

It must be pointed out that such possible constraints on the gravitino mass can be translated into constraints on more fundamental parameters, making them very valuable in the search for the allowed parameter space in grand unified models. As an example, in models leading naturally to mass scales in the $10^2$-10$^3$ GeV range through a specific dilaton vacuum configuration in supergravity, the gravitino mass can be related with the GUT parameters [52]:

$$m_{3/2} = \left(\frac{5\pi\lambda}{2\pi}\right)^{\sqrt{3}} \left(\frac{M_{\text{GUT}}}{M_{\text{Pl}}}\right)^{3\sqrt{3}} M_{\text{Pl}}.$$ 

With $M_{\text{GUT}} \sim 10^{16}$ GeV and a gauge coupling $\alpha_{\text{GUT}} \sim 1/26$. The superpotential value in the dilaton direction defines the magnitude of the coupling constant $\lambda$ of the self-interacting 24 multiplet. Figure 5 shows how the lower value on $\lambda$ evolves as a function of the reheating temperature which could be probed by the previously given method, for three different branching ratios. Although not very constraining, this lower limit of the order $1.4 \times 10^{-3}$ over the full tested range for $B = 1$ could be one of the first experimental constraints on $\lambda$.

The next generation of CMB experiments will face a new situation. Important efforts are devoted to the search for the polarization $B$ mode [53] and the sensitivity should reach scales of inflation of order $10^{15} - 10^{16}$ GeV. This value is slightly higher than the GUT scale if supersymmetry is ignored ($i.e.$ if gravitinos production is expected not to have occurred), and slightly lower than the GUT scale if supersymmetry is taken into account ($i.e.$ in the case gravitinos are expected to be produced by scattering processes). Considering that the grand unified scale is the highest natural value for the reheating temperature, this means that, if a significant amount of entropy was not released after the moduli production, it should not be possible to detect those tensor modes in both scenarios.

On the other hand, cosmic-ray experiments could be sensitive enough to investigate the allowed reheating temperatures if small black holes were formed at the end of inflation. In this case, important limits could be derived on the gravitino mass and on the related GUT parameters.
V. APPENDIX: ANTIDEUTERON FLUX COMPUTATION

In this two-zone approach, the geometry of the Milky Way is a cylindrical box whose radial extension is \( R = 20 \) kpc from the galactic center, with a matter (stars) disk whose thickness is \( 2h = 200 \) pc and a diffusion halo whose extent is the major source of uncertainty (taken into account in the analysis). The five parameters used are \( K_0, \delta \) (describing the diffusion coefficient \( K(E) = K_0\beta R^\delta \)), the halo half height \( L \), the convective velocity \( V_c \) and the Alfvén velocity \( V_A \). They are varied within a given range determined by an exhaustive and systematic study of cosmic ray nuclei data \[48\]. The same parameters as employed to study the antiproton flux \[37\] are used again in this analysis. The antideuterons density produced by evaporating PBHs per energy bin \( \psi_D \) obeys the following diffusion equation:

\[
\left\{ V_z \frac{\partial}{\partial z} - K \left( \frac{\partial^2}{\partial z^2} \left( r \frac{\partial}{\partial z} \right) \right) \right\} \psi_D(r, z, E) + 2h\delta(z)\Gamma_D \psi_D(r, 0, E) = q^{\text{prim}}(r, z, E)
\]

where \( q^{\text{prim}}(r, z, E) \) corresponds to the source term. The total collision rate is given by \( \Gamma_D = n_H \sigma_{\text{DH}} v_D \) where \( \sigma_{\text{DH}} \) is the total antideuteron cross-section with protons and the hydrogen density, assumed to be constant all over the disk, has been fixed to \( n_H = 1 \text{ cm}^{-3} \).

Performing Bessel transforms, all the quantities can be expanded over the orthogonal set of Bessel functions of zeroth order:

\[
\psi_D = \sum_{i=1}^{\infty} N_i^{D,\text{prim}} J_0(\zeta_i(x))
\]

and the solution of the equation for antideuterons can be written as

\[
N_i^{D,\text{prim}}(0) = \exp \left( -\frac{V_c L}{2K} \right) \frac{y_i(L)}{A_i \sinh(S_i L / 2)}
\]

where

\[
\begin{align*}
\Gamma_D(r, z) &= 2h \int_0^L \left( \frac{V_c(L - z')}{2K} \right) \sinh \left( \frac{S_i(L - z')}{2} \right) q_i^{\text{prim}}(z') dz' \\
S_i &= \left\{ \frac{V_c^2}{K^2} + 4\zeta_i^2 \right\}^{1/2} \\
A_i &= 2h \Gamma^{\text{inc}}_D + V_c + K S_i \coth \left( \frac{S_i L}{2} \right)
\end{align*}
\]

In this model, energy changes (predominantly ionization losses, adiabatic losses and diffusive reacceleration) are taken into account via a second order differential equation for \( N_i^{D,\text{prim}} \). The spatial distribution \( f(r, z) \) of PBHs was assumed to follow

\[
f(r, z) = \frac{R_c^2 + R_z^2}{R_c^2 + r^2 + z^2}
\]

where the core radius \( R_c \) has been fixed to 3.5 kpc and \( R_z = 8 \) kpc. This profile corresponds to the isothermal case with a spherical symmetry, the uncertainties on \( R_c \) and the consequences of a possible flatness have been shown to be irrelevant in \[37\].

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