Mass and Isotope Dependence of Limiting Temperatures for Hot Nuclei *

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Abstract

The mass and isotope dependence of limiting temperatures for hot nuclei are investigated. The predicted mass dependence of limiting temperatures is in good agreement with data derived from the caloric curve data. The predicted isotope distribution of limiting temperatures appears to be a parabolic shape and its centroid is not located at the isotope on the $\beta$-stability line ($T=0$) but at neutron-rich side. Our study shows that the mass and isotope dependence of limiting temperatures depend on the interaction and the form of surface tension and its isospin dependence sensitively.

PACS numbers: 24.10.-i,25.70.-z,25.70.Gh

1. INTRODUCTION

It is well known that for nuclear matter, liquid and gas phases can coexist at temperature lower than $T_c$ and only gas phase can exist at or above $T_c$. While for a hot nucleus

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surrounding by vapor, due to the Coulomb instability [1–4], it may not be stable above a temperature which is much lower than the critical temperature $T_c$ of infinite nuclear matter, this temperature is called the limiting temperature $T_{lim}$. Limiting temperatures for nuclei along the $\beta$-stability line were studied by using Skyrme interaction [1,2,5–8] and Furnstahl, Serot and Tang (FST) model [9]. Their studies showed that the limiting temperature was sensitive to the effective nucleon-nucleon interactions. In ref. [10,11] the mass dependence of limiting temperatures for hot nuclei was derived by analyzing the existing caloric data. The establishment of the beam of rare isotopes will provide us the opportunities to study the heavy ion collisions with large isospin asymmetry. The measurements of caloric curves (or other possible observables) can possibly explore the limiting temperatures for hot nuclear systems with large isospin asymmetry. The limiting temperatures for nuclei away from the $\beta$-stability line should depend on the isospin dependent part of the Equation of State (EOS). The newly developed Skyrme forces such as the SLy series are designed to study the properties of nuclei away from the $\beta$-stability line in addition to nuclei along the $\beta$-stability line. It has been shown that these modern Skyrme forces can describe the properties of the nuclei away from $\beta$-stability line much better than the old Skyrme forces [12]. Therefore it seems to us to be worthwhile to apply the modern parametrization of Skyrme force such as the SLy series for studying the limiting temperatures for nuclei both along and away from the $\beta$-stability line.

Additionally, the surface tension is another important factor affecting the limiting temperature. The surface tension represents a work per unit area needed to create the surface, the work is required because the nucleons at the surface are less bounded. Therefore the surface tension is related to the EOS of nuclear matter. The symmetry energy term in the EOS should also contribute to the surface tension. However, the relationship is not available and it is treated independently of the equation of state except the fact that the surface tension vanishes as $T \geq T_c$. In this case, the form of the temperature dependence of the surface tension is not unique in literatures and little attention has been paid to the influence of the different forms of the surface tension on the limiting temperature. The isospin
dependence of the surface tension is even unclear. Therefore it is requisite to investigate the
effect of the different forms of the surface tension, including with and without the isospin
dependence(surface symmetry term), on limiting temperature. By comparing the calculation
results with the experimental data we may obtain the information of the surface tension
and better parametrization of Skyrme interaction.

In this work we follow the approach of Ref. [13,14] to study the mass and isotope depend-
dence of limiting temperatures by using the Skyrme effective force of SLy7 in addition to
SIII and SkM∗. The effect of different forms of the surface tension on the limiting tempera-
ture will also be investigated. In Sec. II, The equation of state by using Sly7 is studied. In
Sec.III we give the coexistence equations and the calculation results of the mass and isotope
dependence of limiting temperatures of hot nuclei. Finally, a summary and discussions is
given in Sec.IV.

2. THE EQUATION OF STATE

The single-particle energy reads

\[ \varepsilon_q = \frac{\hbar^2 k^2}{2m_q^*} + u_q + \varepsilon_{Coul}\delta_{q,p}, \]  

(1)

where \(m_q^*\) and \(u_q\) is the effective mass and the single-particle potential energy of species \(q\), respectively. The third term is the Coulomb energy. The chemical potential \(\mu_q\) of species \(q\) for nuclear system at temperature \(T\) can be written as

\[ \mu_q(T, \rho_n, \rho_p) = u_q(\rho_n, \rho_p) + \varepsilon_{Coul}\delta_{q,p} + T \ln \frac{\lambda_T^3}{g_s} \rho_q + T \sum_{n=1}^{\infty} \frac{n + 1}{n} b_n \left( \frac{\lambda_T^3}{g_s} \rho_q \right)^n. \]  

(2)

Here \(\lambda_T\) is the effective thermal wavelength of the nucleon, which reads

\[ \lambda_T = \left( \frac{2\pi \hbar^2}{m_q^* T} \right)^\frac{1}{2}, \]  

(3)

and \(b_n\)'s are the coefficients of the virial series for ideal Fermi gas. For the most general
form of a Skyrme-type interaction, \(u_q\) can be written as
\[ u_q = A_1 \rho + A_2 \rho_q + A_3 \tau + A_4 \tau_q + A_5 (\alpha + 2) \rho^{\alpha+1} \]

\[ + A_6 \alpha \rho^{\alpha-1} (\rho_n^2 + \rho_p^2) + 2A_6 \rho^\alpha \rho_q. \]  

(4)

\( \rho \) and \( \tau \) are the nuclear density and kinetic energy density, respectively, and \( \rho_q \) and \( \tau_q \) are the density and the kinetic energy density of species \( q \), respectively. \( A_1 \) - \( A_6 \) are

\[ A_1 = t_0 (1 + \frac{1}{2} x_0), \]

\[ A_2 = -t_0 (\frac{1}{2} + x_0), \]

\[ A_3 = \frac{1}{4} [t_1(1 + \frac{1}{2} x_1) + t_2(1 + \frac{1}{2} x_2)], \]

\[ A_4 = \frac{1}{4} [t_2(\frac{1}{2} + x_2) - t_1(\frac{1}{2} + x_1)], \]

\[ A_5 = \frac{1}{12} t_3 (1 + \frac{1}{2} x_3), \]

\[ A_6 = -\frac{1}{12} t_3 (\frac{1}{2} + x_3), \]

(5)

where \( t_0, t_1, t_2, t_3, x_0, x_1, x_2, x_3 \) are the parameters of the Skyrme interaction. The effective mass \( m_q^* \) reads

\[ m_q^* = m [1 + \frac{2m}{\hbar^2} (A_3 \rho + \frac{1}{2} A_4 (1 \pm y) \rho)]^{-1}. \]

(6)

Introducing \( y = \frac{\rho_n - \rho_p}{\rho_n + \rho_p} \) we obtain

\[ \mu_q(T, \rho, y) = u_q(\rho, y) + \varepsilon_{Coul}(\rho) \delta_{q,p} + T \ln \frac{\lambda_T^3}{g_{s,I}} \rho + T \ln (1 \pm y) \]

\[ + T \sum_{n=1}^{\infty} b_n (1 \pm y)^n \left( \frac{\lambda_T^3}{g_{s,I}} \rho \right)^n. \]  

(7)

and

\[ \tau_q = \frac{g_s}{(2\pi)^3} \int d^3 k n_q(k) k^2 \]

\[ = \frac{g_s}{(2\pi)^3} \int d^3 k [1 + \exp((\varepsilon_q - \mu_q)/T)]^{-1} k^2 \]

\[ = \frac{3}{2} T m_q^* \frac{\hbar^2}{\lambda_T^3} \rho \sum_{n=0}^{\infty} b_n (1 \pm y)^n (\frac{\lambda_T^3}{g_{s,I}} \rho)^n, \]

(8)

where the symbol ”+” stands for neutrons and ”−” for protons. Concerning the expansion in the degree of the degeneracy \((\frac{\lambda_T^3}{g_{s,I}} \rho)\) in \( \mu_q \) and \( \tau_q \), in this work, \( n \geq 7 \) term are neglected,
i.e. one more term than in Refs. [13,14] is included. We find that for temperatures and densities relevant to this work (say, \( T \sim 6MeV \) and \( \rho \sim 0.14fm^{-3} \)) at least 6 terms are needed in order to obtain accurate results. \( g_s \) is the spin degeneracy (\( g_s = 2 \)) while \( g_{s,I} \) is the spin-isospin degeneracy (\( g_{s,I} = 4 \)).

Assuming that a hot nucleus is a spherical drop with a sharp edge and uniform distribution of nucleons, the Coulomb energy can be expressed as

\[
\varepsilon_{\text{Coul}} = \frac{6}{5} \frac{Ze^2}{R_L} - \left( \frac{3}{2\pi} \right)^{\frac{2}{3}} \frac{Z^{\frac{2}{3}}e^2}{R_L}. \tag{9}
\]

Here the Coulomb exchange term is included. \( R_L \) is the radius of the nucleus and expressed as

\[
R_L = \left( \frac{3A}{4\pi\rho} \right)^{\frac{1}{3}}. \tag{10}
\]

The symbols \( Z \) and \( A \) are the number of protons and nucleons in the liquid drop, respectively.

The pressure can be obtained through the Gibbs-Duhem relation:

\[
\frac{\partial \tilde{P}}{\partial \rho} = \rho_n \frac{\partial \mu_n}{\partial \rho} + \rho_p \frac{\partial \mu_p}{\partial \rho}. \tag{11}
\]

Thus,

\[
\tilde{P}(T, \rho, y) = \frac{1}{2} \sum_{q=p,n} (1 \pm y) \int_0^\rho \frac{\partial \mu_q}{\partial \rho} d\rho, \tag{12}
\]

where symbol ” + ” stands for neutrons and ” − ” for protons.

The surface tension term has to be introduced in the pressure for a finite system. For the nuclei along the \( \beta \)-stability line we adopt two different forms of the surface tension. The first one is:

\[
\gamma(T) = 1.14MeV \cdot fm^{-2}(1 + \frac{3T}{2T_c})(1 - \frac{T}{T_c})^{\frac{5}{2}} \tag{13}
\]

suggested by Ref. [15] and used by Ref. [6,7,9,13,14] in studying the limiting temperature. Here we call it Surf1. The other one called Surf2 is:

\[
\gamma(T) = \gamma(0)[(T_c^2 - T^2)/(T_c^2 + T^2)]^{\frac{5}{4}} \tag{14}
\]
\[ \gamma(0) \approx 18 \text{MeV/}4\pi r_0^2, \quad r_0 = 1.12 \text{fm}, \]  

which was suggested by [16]. This expression of the surface tension is widely used in SMM calculations for studying multifragmentation process in heavy ion collisions. It was obtained as a parametrization of the calculation of thermodynamic properties of the interface between liquid and gas of symmetrical nuclear matter which were performed with Thomas-Fermi and H-F method by using Skyrme force. \( T_c \) is the critical temperature for infinite nuclear matter. The pressure caused by the surface tension \( (P_{\text{surf}}) \) is given by:

\[ P_{\text{surf}} = -2\gamma(T)/R_L. \]  

Thus, the total pressure of the liquid drop \( (P) \) is written as

\[ P(T, \rho, y) = \tilde{P}(T, \rho, y) + P_{\text{surf}}. \]  

Fig.1 shows the isothermal curves of the chemical potential calculated with SLy7 for:

a) protons with bulk part only, b) neutrons, c) protons with bulk part adding the Coulomb term of a hot \(^{208}\text{Pb}\) nucleus. Fig.2 shows the isothermal curves of the pressure calculated with SLy7, in which the Coulomb and the surface tension term are for a hot \(^{208}\text{Pb}\) nucleus. Fig.3 shows the pressure as a function of density at \( T=5\text{MeV} \) calculated with SLy7, SkM* and SIII in which the Coulomb term and the surface tension term are for a hot \(^{208}\text{Pb}\) nucleus. There are three sets of curves, namely, the bulk, the bulk+Coulomb\((^{208}\text{Pb})\) and the bulk+Coulomb\((^{208}\text{Pb})\)+surface tension\((^{208}\text{Pb})\). In each set, there are three curves corresponding to SLy7, SkM* and SIII, crossing at density around \( \rho \sim 0.13 fm^{-3} \). One sees a considerable negative pressure provided by the surface tension through comparing the sets with and without surface tension. Furthermore, one can find from the figure that the curves for SLy7 and SkM* are very close at the densities relevant to this study, while the curve for SIII is far away from them except at the crossing point. To show the effect of the different forms of the surface tension, in Fig.4 we plot the pressure at \( T=5 \text{ MeV} \) for \(^{208}\text{Pb}\) calculated with the surface tension of Surf1, Surf2, and without a surface tension. One can find that the Surf1 produces a stronger negative pressure than the Surf2 does, which should
have effect on the limiting temperature especially for light nuclear systems.

III. MASS AND ISOTOPE DEPENDENCE OF LIMITING TEMPERATURES

We adopt the the same model used in refs. [13,14] to study the mass and isotope dependence of limiting temperatures for hot nuclei. The model treats the hot nucleus as a uniformly charged drop of nuclear liquid with sharp edge at a given temperature, which is in thermal equilibrium with the surrounding vapor. The equilibrium between the droplet and the vapor surrounding both thermal mechanical and chemical leads to a set of two-phases coexistence equation:

\[ \mu_p(T, \rho_L, y_L) = \mu_p(T, \rho_V, y_V), \]  
\[ \mu_n(T, \rho_L, y_L) = \mu_n(T, \rho_V, y_V), \]  
\[ \tilde{P}(T, \rho_L, y_L) + P_{Surf}(T, \rho_L) = P(T, \rho_V, y_V). \]  

The subscript \( L \) refers to the liquid phase and \( V \) to the vapor phase. For simplification, the Coulomb interaction in vapor is screened in the calculation of the pressure \( P(T, \rho_V, y_V) \) and the chemical potential of protons \( \mu_p(T, \rho_V, y_V) \). These three coexistence equations with three variables can be solved directly to get the coexistence point of the liquid drop and the surrounding vapor. By finding the upper boundary of temperature that the coexistence equations have solution one obtains the limiting temperature.

1. Mass Dependence of Limiting Temperatures

Now let us study the mass dependence of limiting temperatures of nuclei along the \( \beta \)-stability line:

\[ Z = 0.5A - 0.3 \times 10^{-2} A^{7/8}. \]
Fig. 5 - Fig. 7 show the mass dependence of limiting temperatures calculated with surface tension Surf1 and Surf2 and Skyrme interactions SLy7, SkM*, SIII, respectively. The data of [10,11] are also plotted in the figures. From these 3 figures one can see that limiting temperatures decrease with the system size, which is independent of the force used. Furthermore, one finds that the limiting temperatures calculated with SLy7 and SkM* are very close and the largest difference is about or less 0.1 MeV while the limiting temperatures calculated with SIII are much higher. This is because the EOS corresponding to SLy7 and SkM* is of soft one while that for SIII is of stiff one (see Fig. 3). Concerning the surface tension, one can see that the limiting temperatures calculated with Surf2 are higher than that with Surf1. We find, in general, the calculation results with surface tension Surf2 and Skyrme force SLy7 agree with experimental data better than other combinations of surface tension forms and interaction parameter sets. This agreement may illuminate that the surface tension may be better described by Surf2 rather than Surf1 at $T < T_c$. Therefore, in the following calculations we adopt SLy7 and Surf2. In Table 1 we list the values of limiting temperatures for the nuclei along the $\beta$-stability line with mass number ranging from 20 to 250.

2. Isotope Distribution of Limiting Temperatures

Fig. 8 shows the isotope distribution of limiting temperatures for nuclei: a) O, b) Ca, and c) Zr calculated with Skyrme interaction SLy7 and surface tension Surf2, respectively. One can see from these figures that the isotope distribution of limiting temperatures appears to be a parabolic shape. The centroid of the parabolic curve is not located at the isotope on $\beta$-stability line ($T = 0$) but incline to the neutron-rich side. The shift of the centroid of the isotope distribution of limiting temperatures to the neutron-rich side seems to be in consistent with [17] in which it was mentioned that there was evident that neutron rich nuclei had higher limiting temperatures. This effect is due to the fact that the Coulomb term does not depend on the temperature explicitly while the isospin dependent part of the bulk term does depend on the temperature explicitly, which leads the compensation effect
of the Coulomb energy and the symmetry energy with respect to the isospin degree to be different for hot nuclei and cold nuclei.

It has been observed that the isospin fractionation distillation effect on multifragmentation process in neutron-rich heavy ion collisions [18]. We expect that this effect should also be shown up in the equilibrium value of the isospin asymmetry of vapor phase at the limiting temperature. In Fig. 9 we show the isotope distribution of the equilibrium values of the isospin asymmetry of vapor at the limiting temperature, \( y_{\text{v}}^{\text{lim}} \), for O, Ca, and Zr, respectively. One can see from the figure that the equilibrium value of \( y_{\text{v}}^{\text{lim}} \) increases with the increase of the number of neutrons and finally a saturation value might be reached. The isospin distillation effect leads vapor to be even neutron-rich for neutron-rich isotopes and to be even neutron-lack for neutron-lack isotopes which can be seen in the figure clearly. The vanish point of the equilibrium value of \( y_{\text{v}}^{\text{lim}} \), at which the isospin asymmetry of vapor changes from negative to positive value, is shifted a little to the neutron-rich side. The vanish point should be sensitive to the isospin dependent part of both the interaction and the surface tension.

As is already mentioned in the introduction that the \( SLy \) series of Skyrme interactions is designed to describe the properties of isospin asymmetric nuclei, its isospin dependent part is rather different from \( SkM^* \) [12]. We expect that the isospin distribution of limiting temperatures calculated with \( SLy7 \) should be different from that with \( SkM^* \) though the mass distributions of limiting temperatures calculated with \( SLy7 \) and \( SkM^* \) are very close. Our calculation results of the isotope distribution of limiting temperature of Zr shown in Fig. 10 does demonstrate that the isotope distribution of limiting temperatures depends on the isospin dependent part of interaction sensitively. Limiting temperatures calculated with \( SLy7 \) are higher than those with \( SkM^* \) and the difference between them increases as the isospin asymmetry \( |N - Z|/A \) increases. It was demonstrated that the \( SLy \) series of interactions had a better isotope properties than old Skyrme interactions [12], so the isotope distribution predicted with \( SLy7 \) may be more proper.

As pointed out at Sec.1, the surface tension is a work per surface area needed to create
the surface. The symmetry energy term in EOS should also contribute to the surface tension and therefore a symmetry-surface tension term should be introduced in the surface tension. Here we adopt a surface tension including a symmetry-surface term:

\[ \gamma(T) = (\gamma(0) - a_s \sigma^2)\left(\frac{\tau^2 - T^2}{\tau^2 + T^2}\right)^\frac{3}{4}. \]

(22)

to study the effect of the isospin dependence of the surface tension. Here \(a_s\) is taken to be 28.5MeV [12,19]. Fig.11 shows the isotope distribution of limiting temperatures of hot nuclei \(C\) and \(O\) calculated with and without symmetry-surface tension term taken into account, respectively. The symmetry-surface tension term raises limiting temperatures of neutron-rich isotopes but almost does not affect limiting temperatures at the neutron-lack side. Therefore the information of the symmetry-surface tension term can only be obtained by the measurement of limiting temperatures of neutron-rich nuclei.

IV. SUMMARY

In summary, in this work we have studied the mass and isotope dependence of limiting temperatures. The tendency of limiting temperatures decreasing with the increasing of mass number for nuclei along the \(\beta\)-stability line is in agreement with refs. [6,7,9]. The influence of different Skyrme forces and different forms of the surface tension on limiting temperatures of hot nuclei is investigated. We find that the results calculated with newly developed Skyrme force \(SLy7\) and the surface tension of Surf2 are in good agreement with data. The isotope distribution of limiting temperatures appears to be a parabolic shape. The centroid of the parabolic curve is not located at the isotope of \(\beta\)-stability line but inclined to the neutron-rich side. The equilibrium value of the isospin asymmetry of vapor phase at the limiting temperature \(y_v\) shows clearly the isospin fractionation distillation effect. An isospin dependent surface tension is introduced in studying the isotope distribution of limiting temperatures. Our study shows that the neutron-rich side of the isotope distribution of limiting temperatures depends on the isospin dependence of the the surface tension sensitively.

ACKNOWLEDGMENTS
This work is Supported by National Natural Science Foundation of China under Grant Nos. 10175093, 10235030, Science Foundation of Chinese Nuclear Industry and Major State Basic Research Development Program under Contract No. G20000774.
Figures Caption

Fig.1 The isothermal curves of the chemical potential calculated with SLy7 for: a) protons with bulk part only, b) neutrons, c) protons with bulk part adding the Coulomb term of a hot $^{208}\text{Pb}$ nucleus.

Fig.2 The isothermal curves of pressure. The bulk part is calculated with SLy7. The Coulomb and the surface tension part are of a hot $^{208}\text{Pb}$ nucleus.

Fig.3 The isothermal curve of pressure at $T=5\text{MeV}$ calculated with $SLy7$, $SkM^*$, and $SIII$, respectively. The Coulomb and surface tension term is corresponding to a hot $^{208}\text{Pb}$ nucleus.

Fig.4 The comparison of the pressure $P$ at $T = 5\text{MeV}$ for three cases, namely, without surface tension, with Surf1 and Surf2. The surface tension term is corresponding to a hot $^{208}\text{Pb}$ nucleus.

Fig.5 The mass dependence of the limiting temperatures calculated with Skyrme interaction $SLy7$ and surface tension Surf1 and Surf2, respectively.

Fig.6 The same with Fig.6 but with $SKM^*$.

Fig.7 The same with Fig.6 but with $SIII$.

Fig.8 Isotope distributions of the limiting temperatures for $O$, $Ca$, and $Zr$.

Fig.9 Isotope distributions of the equilibrium values of the isospin asymmetry of vapor at limiting temperature for $O, Ca, Zr$, respectively.

Fig.10 Isotope distribution of the limiting temperatures for $Zr$ calculated with $SLy7$ and $SkM^*$, respectively.

Fig.11, the isotope distribution of limiting temperatures of hot nuclei $C$ and $O$ calculated with and without symmetry-surface term taken into account, respectively.

Table.1 The values of limiting temperatures for nuclei along the $\beta$-stability line.
REFERENCES


SLy7

proton bulk

\( \mu_p (\text{MeV}) \)

\( \rho (\text{fm}^{-3}) \)

SLy7

neutron bulk

\( \mu_n (\text{MeV}) \)

\( \rho (\text{fm}^{-3}) \)

SLy7

proton bulk+coul

\( \mu_{p+c} (\text{MeV}) \)

\( \rho (\text{fm}^{-3}) \)
\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure.png}
\caption{Phase diagram for different surface treatments.}
\end{figure}

- Surf1
- Surf2
- without Surf

\textbf{T=5MeV}

\textbf{SLy7}
Para: SLY7

\[ T_{\text{lim}} \text{(MeV)} \]

- exp.
- Surf1
- Surf2

A

\[ 0 \quad 50 \quad 100 \quad 150 \quad 200 \quad 250 \]
\begin{enumerate}
\item \begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure_a}
\caption{$^{11}\text{C} \sim ^{16}\text{C}$}
\end{figure}
\item \begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure_b}
\caption{$^{15}\text{O} \sim ^{21}\text{O}$}
\end{figure}
\end{enumerate}