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Cross section calculations in Glauber model: I. Core plus one-nucleon case

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Abstract

This paper presents a Fortran program to calculate cross sections for various reactions between composite particles in the framework of Glauber theory. The projectile nucleus is described as a core plus one valence-nucleon system. The input data needed for the calculation are the core and target densities and the nucleon-nucleon profile function. The single-particle wave function of the valence nucleon may be provided by a user or generated by the code if appropriate quantum numbers of the wave function are specified. Multiple integrations involving the valence-nucleon wave function are performed by a Monte Carlo technique with Metropolis algorithm. The program gives the total reaction cross section, the one-nucleon removal cross section, the elastic differential cross section, and the longitudinal momentum distribution of the core fragment. Sample calculations include the reactions of $^{13,19}\text{C}+^{12}\text{C}$, $^{11}\text{Be}+^{12}\text{C}$, $^{11}\text{Be}+^9\text{Be}$, and $^8\text{B}+^{12}\text{C}$ systems.

Key words: Glauber model, One-nucleon halo, Reaction cross section, Nucleon-removal cross section, Momentum distribution, Elastic differential cross section

PACS: 25.60.-t, 25.60.Gc, 25.60.Bx, 24.10.-i, 25.70.-z

PROGRAM SUMMARY

Title of program: CSC_GM

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Licensing provisions: none

Operating systems under which the program has been tested: UNIX

Programming language used: Fortran 90

No. of bytes in distributed program, including test data, etc.: 98 MB

Keywords: Glauber model, One-nucleon halo, Reaction cross section, Nucleon-removal cross section, Momentum distribution, Elastic differential cross section

Nature of physical problem: This computer code calculates the cross sections of various reactions for a core plus one valence-nucleon system in the framework of the Glauber model. The examples of calculation include the reaction cross sections for $^{13,19}\text{C}+^{12}\text{C}$, the elastic differential cross section for $^{11}\text{Be}+^{12}\text{C}$ and the momentum distribution of the core nucleus for $^{11}\text{Be}+^9\text{Be}$ and $^8\text{B}+^{12}\text{C}$ systems.

Methods of solution: The scattering amplitude is formulated in the Glauber theory. The phase-shift functions between the core and the target and between the valence nucleon and the target are calculated in the optical-limit approximation. The integration over the valence-nucleon coordinates is performed by Monte Carlo integration, while the integration over the impact parameter is done with a trapezoidal rule.

Restrictions on the complexity of the problem: The projectile nucleus is assumed to be a core plus one valence-nucleon system, in which the valence nucleon is described with a pure configuration. The projectile is assumed to have no particle-bound states except for its ground state. The nucleon-nucleon profile function is assumed to have one-range Gaussian form or zero-range delta function. The core and target densities have to be fit in terms of a combination of Gaussians. The breakup due to the Coulomb interaction is neglected.

Typical running time: For the sample calculations shown in text, the computer time is as follows; 5 min for the total reaction cross section, 5 min/point for the elastic differential cross section, and 10 min/point for the momentum distribution.

LONG WRITE-UP

1 Introduction

Experimental study of unstable nuclei has considerably advanced through the technique of using secondary radioactive beams. The quantities measured in the study include various inclusive cross sections, for example, reaction or interaction cross sections, nucleon-removal cross sections, Coulomb breakup cross sections and momentum distributions of a fragment. These quantities have played an important role to reveal the nuclear structure of unstable nuclei, particularly halo structure near the drip line [1,2].

The aim of this paper is to present a computer code to calculate various cross sections for reactions between nuclei in the framework of the Glauber theory [3]. The projectile nucleus is assumed to have structure of a core nucleus plus a valence nucleon. The cross sections which are available in this code are the total reaction cross section, the one-nucleon removal cross section, the elastic differential cross section, and the longitudinal momentum distribution of the core. The measurement of these cross sections is now a standard work for the study of unstable nuclei, so making the code available appears to be useful as a first step for testing nuclear structure models and reaction mechanism.

The Glauber model is a microscopic reaction theory of high-energy collision based on the eikonal approximation and on the bare nucleon-nucleon interaction. It is now a standard tool to calculate the cross sections because it can account for a significant part of breakup effects which play an important role in the reaction of a weakly bound nucleus [4,5]. However, the calculation of the matrix element of multiple scattering operators, which appear in the Glauber theory, includes a very complicated multi-dimensional integration. The integration is usually reduced to a much simpler one by introducing the optical-limit approximation (OLA) [3,6,7]. This approximation, however, washes out nuclear correlations which are present in the wave function. In this code, more elaborated treatment beyond the OLA is worked out for the valence-nucleon part by using a Monte Carlo quadrature with the Metropolis algorithm [8]. This is suitable for reactions involving those nuclei which have spatially extended and low density distribution as in a halo nucleus. A powerful application of the Monte Carlo integration to the Glauber analysis has very recently been demonstrated in Ref. [9].

The construction of the paper is as follows. Formulas for the cross sections treated in this paper are shown in Sec. 2. The Monte Carlo integration with the Metropolis algorithm is discussed in Sec. 3. The Fortran code is described in Sec. 4. The applicability of the code is demonstrated with various examples in Sec. 5. Summary is given in Sec. 6.

2 Cross sections for various reactions

We consider the reaction of a projectile nucleus P with a target nucleus T. At the initial stage of the reaction, the projectile in the ground state described with an intrinsic wave function Ψ_0 impinges with momentum $\hbar\mathbf{K} = (0, 0, \hbar K)$ on the target in its ground state described with an intrinsic wave function Θ_0 . The center-of-mass wave function is removed from Ψ_0 (Θ_0). At the final stage of the reaction, the projectile goes to the state a specified by a wave function Ψ_a and the target goes to the state c specified by a wave function Θ_c . The state a is not necessarily a bound state but may be a continuum state that includes some fragments. The momentum transferred from the target to the projectile is $\hbar\mathbf{q}$. The scattering amplitude for this reaction is written in the Glauber theory as an integral over the impact parameter $\mathbf{b}$ between the projectile and the target [3,7]:

$$F_{ac}(\mathbf{q}) = \frac{iK}{2\pi} \int d\mathbf{b} e^{-i\mathbf{q}\cdot\mathbf{b}} \langle \Psi_a \Theta_c | 1 - \prod_{i \in P} \prod_{j \in T} (1 - \Gamma_{ij}) | \Psi_0 \Theta_0 \rangle. \quad (1)$$

The integrated cross section for this reaction is given by

$$\sigma_{ac} = \int \frac{d\mathbf{q}}{K^2} |F_{ac}(\mathbf{q})|^2. \quad (2)$$

The profile function Γ in Eq. (1) is determined so as to fit empirical nucleon-nucleon scattering amplitudes and here taken as follows:

$$\begin{aligned} \Gamma(\mathbf{b}) &= \frac{1 - i\alpha}{4\pi\beta} \sigma_{NN} e^{-\frac{\mathbf{b}^2}{2\beta}} & (\text{finite range}) \\ \Gamma(\mathbf{b}) &= \frac{1 - i\alpha}{2} \sigma_{NN} \delta(\mathbf{b}) & (\text{zero range}). \end{aligned} \quad (3)$$

The parameters σ_{NN} , α , and β usually depend on either the proton-proton (neutron-neutron) or proton-neutron case, but we assume that some appropriate average values are used in the calculation [6]. The argument of Γ_{ij} in Eq. (1) is $\mathbf{b} + \mathbf{s}_i^P - \mathbf{s}_j^T$, which stands for the impact parameter between i th and j th nucleons. Here $\mathbf{s}_i^P$ ($\mathbf{s}_i^T$) is the two-dimensional coordinates comprising the x and y components of the i th nucleon coordinate in the projectile (target) relative to its center-of-mass coordinate.

2.1 Total reaction cross section

The total reaction cross section is obtained by summing σ_{ac} over all possible final states ac except for the elastic channel:

$$\sigma_{\text{reac}}(\text{P} + \text{T}) = \sum_{ac \neq 00} \sigma_{ac}. \quad (4)$$

With the use of the closure relation, $\sum_{ac} |\Psi_a \Theta_c\rangle \langle \Psi_a \Theta_c| = 1$, and the unitarity condition, $|1 - \Gamma|^2 = 1$, the reaction cross section reduces to the familiar formula:

$$\sigma_{\text{reac}}(\text{P} + \text{T}) = \int d\mathbf{b} (1 - |e^{i\chi_{\text{PT}}(\mathbf{b})}|^2), \quad (5)$$

where the phase-shift function χ_{PT} is defined by

$$e^{i\chi_{\text{PT}}(\mathbf{b})} = \langle \Psi_0 \Theta_0 | \prod_{i \in \text{P}} \prod_{j \in \text{T}} [1 - \Gamma(\mathbf{b} + \mathbf{s}_i^{\text{P}} - \mathbf{s}_j^{\text{T}})] | \Psi_0 \Theta_0 \rangle. \quad (6)$$

The projectile is assumed to be a system of a core nucleus coupled with a valence nucleon. Its ground-state wave function is given by

$$\Psi_0 = \varphi_0 \Phi_0, \quad (7)$$

where Φ_0 is the ground-state wave function of the core, and φ_0 is the valence-nucleon wave function

$$\varphi_{nljm}(\mathbf{r}) = u_{nlj}(r) \sum_{m_l m_s} \langle l m_l \frac{1}{2} m_s | j m \rangle Y_{lm_l}(\hat{\mathbf{r}}) \chi_{\frac{1}{2} m_s}. \quad (8)$$

Here $\mathbf{r} = (\mathbf{s}, z)$ is the coordinate of the valence nucleon from the center-of-mass of the core. The radial node number n is counted as $n = 0, 1, 2, \dots$

Very recently, the phase-shift function of Eq. (6) has been calculated by the Monte Carlo technique [9] for realistic wave functions which contain nuclear correlations. In this code, we use, however, the OLA for the integration involving the coordinates of the core and the target, while we perform the integration for the valence-nucleon coordinate without any approximation [5,10,11]. In this treatment, Eq. (6) is reduced to the following calculation

$$e^{i\chi_{\text{PT}}(\mathbf{b})} \rightarrow \langle \varphi_0 | e^{i\chi_{\text{CT}}(\mathbf{b}_\text{C}) + i\chi_{\text{NT}}(\mathbf{b}_\text{C} + \mathbf{s})} | \varphi_0 \rangle \quad \text{with} \quad \mathbf{b}_\text{C} = \mathbf{b} - \frac{1}{A_\text{P}} \mathbf{s}, \quad (9)$$

where $\mathbf{b}_C$ is the impact parameter between the core and the target. The core-target phase-shift function, χ_{CT} , and the nucleon-target phase-shift function, χ_{NT} , are defined through the relevant densities ρ by

$$i\chi_{CT}(\mathbf{b}) = - \int d\mathbf{r} \int d\mathbf{r}' \rho_C(\mathbf{r}) \rho_T(\mathbf{r}') \Gamma(\mathbf{b} + \mathbf{s} - \mathbf{s}'), \quad (10)$$

$$i\chi_{NT}(\mathbf{b}) = - \int d\mathbf{r} \rho_T(\mathbf{r}) \Gamma(\mathbf{b} - \mathbf{s}). \quad (11)$$

The density ρ is normalized to the mass number of a nucleus, $\int d\mathbf{r} \rho(\mathbf{r}) = A$, and it is assumed to be given by a combination of Gaussians:

$$\rho(r) = \sum_i c_i e^{-a_i r^2}. \quad (12)$$

This Gaussian expansion enables one to calculate χ_{CT} and χ_{NT} analytically.

When the target is a proton, $\rho_T(\mathbf{r})$ may be replaced with the delta function $\delta(\mathbf{r})$ but it is not necessary to use the OLA for the interaction between the valence-nucleon and the proton-target. Thus we use the following modification in this case:

$$i\chi_{CT}(\mathbf{b}) \rightarrow - \int d\mathbf{r} \rho_C(\mathbf{r}) \Gamma(\mathbf{b} + \mathbf{s}), \quad e^{i\chi_{NT}(\mathbf{b})} \rightarrow 1 - \Gamma(\mathbf{b}). \quad (13)$$

For the proton target, the profile function $\Gamma(\mathbf{b})$ must be chosen to be finite-ranged in this code.

The projectile-target total reaction cross section is thus calculated by

$$\sigma_{\text{reac}}(P + T) = \int d\mathbf{b} \left(1 - |\langle \varphi_0 | e^{i\chi_{CT}(\mathbf{b}_C) + i\chi_{NT}(\mathbf{b}_C + \mathbf{s})} | \varphi_0 \rangle|^2 \right). \quad (14)$$

The cross section should be calculated by averaging with respect to the initial azimuthal component of the angular momentum and taking a sum over all possible final z -components of the angular momentum. In what follows, the matrix element of a spin-independent operator $\Omega(\mathbf{b}, \mathbf{s})$, $\langle \varphi_0 | \Omega(\mathbf{b}, \mathbf{s}) | \varphi_0 \rangle$, is, however, approximated by the average over the z -component of the angular momentum, i.e.,

$$\begin{aligned} \langle \varphi_0 | \Omega(\mathbf{b}, \mathbf{s}) | \varphi_0 \rangle &\rightarrow \frac{1}{2j+1} \sum_{m=-j}^j \int d\mathbf{r} \varphi_{nljm}^*(\mathbf{r}) \Omega(\mathbf{b}, \mathbf{s}) \varphi_{nljm}(\mathbf{r}) \\ &= \frac{1}{4\pi} \int d\mathbf{r} |u_{nlj}(r)|^2 \Omega(\mathbf{b}, \mathbf{s}). \end{aligned} \quad (15)$$

The core-target total reaction cross section is given by

$$\sigma_{\text{reac}}(\text{C} + \text{T}) = \int d\mathbf{b} \left(1 - |e^{i\chi_{\text{CT}}(\mathbf{b})}|^2 \right). \quad (16)$$

2.2 One-nucleon removal cross section

The one-nucleon removal cross section, $\sigma_{-\text{N}}$, is obtained by

$$\sigma_{-\text{N}} = \sum_c \int d\mathbf{k} \sigma_{a=(\mathbf{k}, g=0), c}, \quad (17)$$

where the projectile nucleus breaks up into the core with an internal wave function Φ_g and one-nucleon in the continuum state with asymptotic momentum, $\hbar\mathbf{k}$, relative to the core. We assume that the core remains in its ground state, $g = 0$, during the collision, or in other words, the projectile has no particle-bound excited state.

We can separate the one-nucleon removal cross section into the elastic part ($c = 0$) and the inelastic part ($c \neq 0$) [10]; $\sigma_{-\text{N}} = \sigma_{-\text{N}}^{\text{el}} + \sigma_{-\text{N}}^{\text{inel}}$. The cross section due to the elastic breakup process is given by

$$\begin{aligned} \sigma_{-\text{N}}^{\text{el}} = \int d\mathbf{b} \Big\{ & \langle \varphi_0 | e^{-2\text{Im}\chi_{\text{CT}}(\mathbf{b}_\text{C}) - 2\text{Im}\chi_{\text{NT}}(\mathbf{b}_\text{C} + \mathbf{s})} | \varphi_0 \rangle \\ & - | \langle \varphi_0 | e^{i\chi_{\text{CT}}(\mathbf{b}_\text{C}) + i\chi_{\text{NT}}(\mathbf{b}_\text{C} + \mathbf{s})} | \varphi_0 \rangle |^2 \Big\}, \end{aligned} \quad (18)$$

while the cross section from the inelastic breakup process is given by

$$\sigma_{-\text{N}}^{\text{inel}} = \int d\mathbf{b} \langle \varphi_0 | e^{-2\text{Im}\chi_{\text{CT}}(\mathbf{b}_\text{C})} - e^{-2\text{Im}\chi_{\text{CT}}(\mathbf{b}_\text{C}) - 2\text{Im}\chi_{\text{NT}}(\mathbf{b}_\text{C} + \mathbf{s})} | \varphi_0 \rangle. \quad (19)$$

The one-nucleon removal cross section for halo nuclei is approximately equal to the difference between the two reaction cross sections, $\sigma_{\text{reac}}(\text{P} + \text{T}) - \sigma_{\text{reac}}(\text{C} + \text{T})$. A more rigorous relation can be derived by using the interaction cross section [10].

2.3 Elastic differential cross section

The elastic scattering amplitude is written as

$$F(\mathbf{q}) = e^{i\chi_s} \left\{ F_{\text{Coul}}(\mathbf{q}) + \frac{iK}{2\pi} \int d\mathbf{b} e^{-i\mathbf{q} \cdot \mathbf{b} + 2i\eta \ln(Kb)} \right\}$$

$$\times \left(1 - \langle \varphi_0 | e^{i\chi_{CT}(\mathbf{b}_C) + i\chi_{NT}(\mathbf{b}_C + \mathbf{s})} | \varphi_0 \rangle \right) \Big\}, \quad (20)$$

with the Coulomb elastic scattering amplitude

$$F_{\text{Coul}}(\mathbf{q}) = -\frac{2\eta K}{q^2} \exp \left\{ -2i\eta \ln\left(\frac{q}{2K}\right) + 2i\arg\Gamma(1 + i\eta) \right\}. \quad (21)$$

Here $\eta = Z_P Z_T e^2 / \hbar v$ is the Sommerfeld parameter, v is the incident velocity, and $\chi_s = -2\eta \ln(2Ka)$ with a being a screening radius [3]. The elastic differential cross section does not depend on the radius a . The elastic differential cross section,

$$\frac{d\sigma}{d\Omega} = |F(\mathbf{q})|^2, \quad (22)$$

is given in this code by the ratio to the Rutherford cross section $d\sigma_{\text{Ruth}}/d\Omega = |F_{\text{Coul}}(\mathbf{q})|^2$.

The core-target elastic differential cross section is also calculated at the same energy per nucleon as the projectile-target scattering.

2.4 Longitudinal momentum distribution

The one-nucleon removal reaction is contributed by both the elastic and inelastic processes as discussed in Sec. 2.2. The latter process becomes dominant at higher energies beyond a few hundred MeV/nucleon [10,12]. See Fig. 4 later. We calculate the momentum distribution of the core fragment after the inelastic breakup of the projectile. Let the momentum of the core be $\mathbf{P} = (\mathbf{P}_\perp, P_\parallel)$ and that of the nucleon going to the continuum state be $\hbar\mathbf{k}$. Assuming that the core remains in its ground state, we calculate the momentum distribution by the following equation (in the projectile-rest frame):

$$\frac{d\sigma_{-N}^{\text{inel}}}{d\mathbf{P}} = \int \frac{d\mathbf{q}}{K^2} \sum_{c \neq 0} \int d\mathbf{k} \delta(\mathbf{P} - \frac{A_C}{A_P} \hbar\mathbf{q} + \hbar\mathbf{k}) |F_{(\mathbf{k},0)c}(\mathbf{q})|^2. \quad (23)$$

The scattering wave function of the nucleon should in principle be solved through a dynamical equation but here it is approximated by a plane wave. Then the above equation is reduced to

$$\frac{d\sigma_{-N}^{\text{inel}}}{d\mathbf{P}} = \int d\mathbf{b}_N \left(1 - e^{-2\text{Im}\chi_{NT}(\mathbf{b}_N)} \right)$$

$$\times \frac{1}{(2\pi\hbar)^3} \frac{1}{2j+1} \sum_{mm_s} \left| \int d\mathbf{r} e^{\frac{i}{\hbar} \mathbf{P} \cdot \mathbf{r}} \chi_{\frac{1}{2}m_s}^* e^{i\chi_{CT}(\mathbf{b}_N - \mathbf{s})} \varphi_{nljm}(\mathbf{r}) \right|^2, \quad (24)$$

where $\mathbf{b}_N$ stands for the impact parameter of the valence nucleon with respect to the target. Integrating over the transverse components $\mathbf{P}_\perp$ leads to the longitudinal momentum distribution:

$$\begin{aligned} \frac{d\sigma_{-N}^{\text{inel}}}{dP_\parallel} &= \int d\mathbf{P}_\perp \frac{d\sigma_{-N}^{\text{inel}}}{d\mathbf{P}} \\ &= \frac{1}{2\pi\hbar} \int d\mathbf{b}_N \left(1 - e^{-2\text{Im}\chi_{NT}(\mathbf{b}_N)} \right) \int d\mathbf{s} e^{-2\text{Im}\chi_{CT}(\mathbf{b}_N - \mathbf{s})} \\ &\quad \times \int dz \int dz' e^{\frac{i}{\hbar} P_\parallel (z - z')} u_{nlj}^*(r') u_{nlj}(r) \frac{1}{4\pi} P_l(\hat{\mathbf{r}}' \cdot \hat{\mathbf{r}}), \end{aligned} \quad (25)$$

where $\mathbf{r} = (\mathbf{s}, z)$ and $\mathbf{r}' = (\mathbf{s}, z')$ and P_l is the Legendre polynomial. The integration of Eq. (25) over $P_\parallel$ gives $\sigma_{-N}^{\text{inel}}$. It is clear from Eq. (25) that the longitudinal momentum distribution can be expressed as a sum of contributions from the azimuthal components of the valence-nucleon wave function:

$$\frac{d\sigma_{-N}^{\text{inel}}}{dP_\parallel} = \sum_{m_l = -l}^l \left(\frac{d\sigma_{-N}^{\text{inel}}}{dP_\parallel} \right)_{m_l} \quad (26)$$

with

$$\begin{aligned} \left(\frac{d\sigma_{-N}^{\text{inel}}}{dP_\parallel} \right)_{m_l} &= \frac{1}{2\pi\hbar} \int d\mathbf{b}_N \left(1 - e^{-2\text{Im}\chi_{NT}(\mathbf{b}_N)} \right) \int d\mathbf{s} e^{-2\text{Im}\chi_{CT}(\mathbf{b}_N - \mathbf{s})} \\ &\quad \times \frac{1}{2l+1} \left| \int dz e^{\frac{i}{\hbar} P_\parallel z} u_{nlj}(r) Y_{lm_l}(\hat{\mathbf{r}}) \right|^2. \end{aligned} \quad (27)$$

The longitudinal momentum distribution, Eq. (25), may be compared to that resulting from the Fourier transform of the single-particle wave function:

$$\begin{aligned} \rho(P_\parallel) &= \frac{1}{2\pi\hbar} \int d\mathbf{s} \int dz \int dz' e^{\frac{i}{\hbar} P_\parallel (z - z')} u_{nlj}^*(r') u_{nlj}(r) \frac{1}{4\pi} P_l(\hat{\mathbf{r}}' \cdot \hat{\mathbf{r}}) \\ &= \frac{1}{2\pi\hbar} \frac{1}{2l+1} \sum_{m_l = -l}^l \int d\mathbf{s} \left| \int dz e^{\frac{i}{\hbar} P_\parallel z} u_{nlj}(r) Y_{lm_l}(\hat{\mathbf{r}}) \right|^2. \end{aligned} \quad (28)$$

This distribution may be interpreted as that which one gets when the reaction mechanism is ignored. Comparing Eqs. (25) and (28), we notice that the factor, $1 - e^{-2\text{Im}\chi_{NT}(\mathbf{b}_N)}$, represents the excitation probability of the valence-nucleon and the factor, $e^{-2\text{Im}\chi_{CT}(\mathbf{b}_N - \mathbf{s})}$, represents a survival probability of the core after the interaction with the target.

The momentum distribution of the core which comes out along the beam direction can be calculated by setting $\mathbf{P}_\perp = 0$ in Eq. (24):

$$\begin{aligned}
\left. \frac{d\sigma_{-N}^{\text{inel}}}{d\mathbf{P}} \right|_{\mathbf{P}_\perp=0} &= \frac{1}{(2\pi\hbar)^3} \frac{1}{2l+1} \sum_{m_l=-l}^l \int d\mathbf{b}_N \left(1 - e^{-2\text{Im}\chi_{\text{NT}}(\mathbf{b}_N)} \right) \\
&\quad \times \left| \int d\mathbf{r} e^{\frac{i}{\hbar} P_\parallel z + i\chi_{\text{CT}}(\mathbf{b}_N - \mathbf{s})} u_{nlj}(r) Y_{lm_l}(\hat{\mathbf{r}}) \right|^2 \\
&= \frac{1}{(2\pi\hbar)^3} \int d\mathbf{b}_N \left(1 - e^{-2\text{Im}\chi_{\text{NT}}(\mathbf{b}_N)} \right) \\
&\quad \times \int d\mathbf{r}' \int d\mathbf{r} e^{\frac{i}{\hbar} P_\parallel (z-z') + i\chi_{\text{CT}}(\mathbf{b}_N - \mathbf{s}) - i\chi_{\text{CT}}^*(\mathbf{b}_N - \mathbf{s}')} \\
&\quad \times u_{nlj}^*(r') u_{nlj}(r) \frac{1}{4\pi} P_l(\hat{\mathbf{r}}' \cdot \hat{\mathbf{r}}), \tag{29}
\end{aligned}$$

where $\mathbf{r} = (\mathbf{s}, z)$ and $\mathbf{r}' = (\mathbf{s}', z')$.

3 Monte Carlo integration

The cross section for various reactions discussed in the previous section is obtained by the integration over the coordinates of the valence nucleon as well as the impact parameter $\mathbf{b}$. The integration over the impact parameter is reduced to that of one-dimension in the present formulation, and it is done with the trapezoidal rule.

The multi-dimensional integration over the valence-nucleon coordinates is performed with a Monte Carlo technique. In the Monte Carlo integration, a set of configuration or integration points is generated according to a suitably chosen guiding function $w(\mathbf{x})$. The random walk with the Metropolis algorithm is taken to generate such points in the subroutine **gene** using the subroutine **metro**. These points are used to perform the multi-dimensional integration for all b values. Obviously, the accuracy of the Monte Carlo integration depends on the number of points, N_s , and more importantly on how the set of points is distributed.

Fig. 1 shows the probability distribution of $N_s = 500,000$ points obtained by using $|u_{nlj}(r)|^2$ as a guiding function. The function u_{nlj} is calculated for the $0p_{1/2}$ -neutron orbit in ^{13}C . The root-mean-square (rms) radius of the valence-nucleon orbit is 3.1 fm. The value of δ controls the step size in the Metropolis random walk. One can see that the distribution is rather sensitive to δ . An appropriate value of δ can be monitored by comparing the probability distribution and the function $|u_{nlj}(r)|^2$. These values are written on the file **nsdist.out**. When the 500,000 points of Fig. 1 are used in the Monte Carlo

integration, the total reaction cross section of the $^{13}\text{C}+^{12}\text{C}$ reaction at the energy of 800 MeV/nucleon is 925, 920, 882 mb for $\delta=2.5, 0.1, 0.01$ fm, respectively.

To test the Monte Carlo integration, we calculate the “longitudinal momentum distribution” in two ways. Here the momentum distribution is defined by

$$\begin{aligned}\tilde{\rho}(P_{\parallel}) &= \frac{1}{2\pi\hbar} \int d\mathbf{s} \int dz \int dz' e^{\frac{i}{\hbar}P_{\parallel}(z-z')} u_{nlj}(\mathbf{r}') u_{nlj}(\mathbf{r}) \\ &= \int d\mathbf{s} |\hat{u}_{nlj}(\mathbf{s}, P_{\parallel})|^2\end{aligned}\quad (30)$$

with

$$\hat{u}_{nlj}(\mathbf{s}, P_{\parallel}) = \frac{1}{\sqrt{2\pi\hbar}} \int dz e^{\frac{i}{\hbar}P_{\parallel}z} u_{nlj}(\mathbf{r}). \quad (31)$$

Here $\mathbf{r} = (\mathbf{s}, z)$ and $\mathbf{r}' = (\mathbf{s}, z')$. This integral is very similar to that defined by Eq. (28). Only a difference is that the factor $\frac{1}{4\pi} P_l(\hat{\mathbf{r}}' \cdot \hat{\mathbf{r}})$ is not included in the function $\tilde{\rho}$. The function $\tilde{\rho}$ has structure which is very similar to the momentum distribution (see Eqs. (25) and (27)), so it can serve as a test to monitor the accuracy of the momentum distribution calculation. This function is obtained as a four-dimensional integration and its evaluation is fairly involved in a direct numerical integration. Fig. 2 compares the results with the direct numerical integration and the Monte Carlo integration. The valence-nucleon function u_{nlj} is calculated for the last neutron orbit in ^{11}Be with $n = 1, l = 0, j = 0.5$. The rms radius of the orbit is fairly large (6.9 fm) reflecting the small separation energy of the neutron. The number of points is chosen to be $N_s = 500,000$. One can see that a set of the points generated with $\delta = 1$ fm reproduces the direct result reasonably well and the choice of $\delta = 2.5$ or 5 fm gives a very satisfactory agreement. The file **MCtest.out** serves to show the results of both direct numerical and Monte Carlo integrations.

4 Program description

4.1 Fortran code

The main program calls the following input files in order to prepare the data for the calculation.

csc.inp: This input file defines a reaction system, a cross section to be calculated, target and core densities, and some control parameters.

yourwf.inp: This input file must be prepared if you use your own single-particle wave function. See Sec. 4.2.

wf.inp: You must prepare this input file when you do not have a wave function but use a single-particle wave function that is generated by this code. The input file is read by the subroutine **wavefn**. This subroutine calculates the radial part of the single-particle wave function, $R_{nlj}(r) = ru_{nlj}(r)$, by solving a Schrödinger equation with a potential $U(r)$:

$$\frac{d^2 R(r)}{dr^2} + \frac{2\mu}{\hbar^2} \left[E - U(r) - \frac{l(l+1)\hbar^2}{2\mu r^2} \right] R(r) = 0,$$

$$U(r) = -V_0 f(r) + V_{ls}(\mathbf{l} \cdot \mathbf{s}) r_0^2 \frac{1}{r} \frac{d}{dr} f(r) + V_{\text{Coul}}, \quad (32)$$

where $f(r) = [1 + \exp(\frac{r-R}{a})]^{-1}$ with $R = r_0 A_C^{\frac{1}{3}}$ ($V_{ls} = 17$ MeV) [13]. The depth of the potential, V_0 , is adjusted so as to fit the energy eigenvalue specified by the quantum numbers nlj to that requested by you. The output file **ourwf.inp** is created and stores the single-particle wave function $u_{nlj}(r)$. The norm of $u_{nlj}(r)$, $\int_0^\infty dr r^2 |u_{nlj}(r)|^2$, and the rms radius of the single-particle orbit are calculated in the subroutine **const**.

The preparation of the data needed for the calculation is completed at this stage.

The main program next calls subroutine and function subprograms depending on the values of control parameters, *icond2* and *icond3*.

- With the choice of *icond2*=1, the subroutine **rdsn** calculates the projectile-target total reaction cross section $\sigma_{\text{reac}}(\text{P}+\text{T})$, the core-target total reaction cross section $\sigma_{\text{reac}}(\text{C}+\text{T})$, and the one-nucleon removal cross section σ_{-N} , and the contributions to σ_{-N} from the elastic and inelastic breakup processes, respectively.
- With the choice of *icond2*=2, the subroutine **diffcs** calculates the elastic differential cross sections for both projectile-target and core-target scatterings.
- With the choice of *icond2*=3, the parameter *icond3* gives the following three options: With *icond3*=1, the subroutine **ftrans** calculates the momentum distribution of the core fragment without the reaction mechanism (Eq. (28)) as well as Eq. (30) and the function **pdist** calculates the longitudinal momentum distribution, Eq. (25), of the core by including the reaction mechanism. With *icond3*=2, the function **pindiv** calculates Eq. (27), the individual contribution of each azimuthal component of the single-particle wave function to the momentum distribution. With *icond3*=3, the function **pdistz** calculates the momentum distribution, Eq.(29), of the core which comes out along the beam direction.

The phase-shift function of the core-target elastic scattering, $i\chi_{CT}$, is calculated in the function **zchct**, and that of the nucleon-target scattering, $i\chi_{NT}$, is calculated in the function **zhnt**, respectively. The various reaction cross sections are constructed from these phase-shift functions as discussed in Sec. 2.

The multi-dimensional integration over the valence-nucleon coordinates, which are needed in the subprograms **rcsn**, **diffcs**, **ftrans**, **pdist**, **pindiv** and **pdistz**, is performed with the Monte Carlo quadrature. The guiding function $w(\mathbf{x})$ in the random walk with Metropolis algorithm is chosen differently, depending on the type of integration: In the subroutines **rcsn** and **diffcs**, $w(\mathbf{x})$ is chosen to be $|u_{nlj}(r)|^2$ ($x_1 = x$, $x_2 = y$, $x_3 = z$) and calculated in the function **uprd3** by using a linear interpolation from the data tabulated in **yourwf.inp** or **ourwf.inp**. In the subprograms **ftrans**, **pdist** and **pindiv**, $u_{nlj}(\sqrt{s^2 + z'^2})u_{nlj}(\sqrt{s^2 + z^2})$ plays a role of $w(\mathbf{x})$ with $x_1 = x$, $x_2 = y$, $x_3 = z$, $x_4 = z'$ and is calculated in the function **uprd4**. In the function **pdistz**, $u_{nlj}(r')u_{nlj}(r)$ plays a role of $w(\mathbf{x})$ with $x_1 = x$, $x_2 = y$, $x_3 = z$, $x_4 = x'$, $x_5 = y'$, $x_6 = z'$ and is calculated in the function **uprd6**.

4.2 Input data

The input data specify the reaction system, the type of cross section, the target and core densities, and the control parameters of the Metropolis algorithm etc. The input data are read from the file **csc.inp**. See Table 1 later. The first line gives the mass numbers of the target, projectile and core (A_T , A_P , and A_C), the second line gives the charge numbers of those nuclei (Z_T , Z_P , and Z_C). This code assumes $A_P - A_C = 1$. For a proton target, put $A_T = 1$ and $Z_T = 1$. The third line defines the incident energy of the projectile per nucleon (in MeV). The fourth line defines the parameters of the nucleon-nucleon profile function, Eq. (3): σ_{NN} (in fm²), α , and β (in fm²). When you use the zero-range profile function, you put $\beta = 0.0$. The fifth line gives the orbital angular momentum of the valence nucleon. The sixth line specifies the condition for the Monte Carlo quadrature, the number of configuration points (N_s), the step size (δ in fm) for the random walk in the Metropolis algorithm, and the seed number for generating random numbers (**irand**). The seventh line is the control parameter for the single-particle wave function of the valence nucleon. When you use your own wave function, you put **icond1**=1, otherwise you put **icond1**=0 and the wave function can be generated by this code.

The eighth line defines two control parameters, **icond2** and **icond3**. The parameter **icond2** specifies the cross section which you want to calculate: the total reaction cross section and the one-nucleon removal cross section (**icond2**=1), the elastic differential cross section (**icond2**=2), and the momentum distri-

bution of the core fragment ($icond2=3$). In the case of $icond2=1$ or 2 , the parameter $icond3$ is redundant and you may put any integer, say 0 . In the case of $icond2=3$, the parameter $icond3$ selects three options for the momentum distributions: Eqs. (28) and (25) ($icond3=1$), Eq. (27) ($icond3=2$) and Eq. (29) ($icond3=3$). The ninth line determines the data points at which the elastic differential cross section or the longitudinal momentum distribution is calculated. When you want to calculate the elastic differential cross section ($icond2=2$), the maximum angle (in degree) and the number of points to be calculated are given, or when you want to calculate the longitudinal momentum distribution ($icond2=3$), the maximum momentum (in MeV/c) and the number of points to be calculated are given. In the case of $icond2=1$, this line is redundant and you may put arbitrary numbers.

The tenth line gives the number of Gaussians used to fit the target density, and the following lines give the coefficients c_i and the ranges a_i (in fm⁻²) as defined by Eq. (12). In the case of a proton target, these data are not necessary in the calculation but you must put some values, e.g., the number of Gaussians = 1, and $c_1 = 1.0$ and $a_1 = 1.0$. The next line gives the number of Gaussians used to fit the core density, and it is followed by the lines which give the corresponding coefficients c_i and the ranges a_i .

When $icond1$ is set to 1, you have to complete the input file, **yourwf.inp**, for the valence-nucleon wave function $u_{nlj}(r)$. The first line of this file defines the number of data points (nwf) and the interval of the neighboring points, i.e., the mesh size (deltwf). In the lines what follow, you put the data in a tabular form. A recommended value for the mesh size is around 0.01 fm, and not larger than 0.05 fm. You should not start from $r = 0$. When $icond1$ is equal to 0, you have to complete the input file **wf.inp**, and then the wave function $u_{nlj}(r)$ is calculated by the code as explained in Sec. 4.1.

4.3 Output data

According to the values of $icond2$ and $icond3$, the results of calculation are written on different output files.

For $icond2=1$, the results are written on the file **reac.out**.

For $icond2=2$, the results are written on the file **ang.out**.

For $icond2=3$, the results are written on the file
momdist.out (for $icond3=1$),
comp.out (for $icond3=2$),
per0.out (for $icond3=3$).

For the case of $icond2=1$ or 2 , the file **nsdist.out** is automatically created in order to compare the distribution of N_s integration points with $|u_{nlj}(r)|^2$ at the

middle point r of each interval. For the case of $icond2=3$, the file **MCtest.out** is created to compare the quantity, Eq. (30), calculated by the direct numerical integration with that obtained by the Monte Carlo integration. These two files serve to judge the accuracy of the Monte Carlo integration for the cross section calculation.

In the case where you want to know the phase shifts, $i\chi_{CT}$ and $i\chi_{NT}$, you only need to run the code by choosing $icond2=1$, and then you will obtain $b, i\chi_{CT}(b), i\chi_{NT}(b)$ in the file **phaseshift.out**.

5 Sample calculations

The present code has been tested by comparing with the results (total reaction cross section, one-nucleon removal cross section, and longitudinal momentum distribution) produced by another independent code [14]. In the latter code, the valence-nucleon wave function has to be fit in terms of Gaussians and the integration over the nucleon coordinates as well as the impact parameter is done by the Monte Carlo method but without using the Metropolis algorithm. A user is advised to make sure the convergence of the Monte Carlo integration by changing the number of configuration points N_s and the step size δ which control the accuracy of the present method. The computer time varies with the type of cross sections. It also strongly depends on N_s , the number of Gaussians used to fit the densities of the projectile and the target, and the number of points of angle or momentum requested in the cross section calculation.

As a guide to the computer time, we show the case with two-range Gaussians for the core and target densities and with 500,000 configuration points. On a Digital Unix computer at RIKEN (Alpha Server ES40, 2048 MB Memory, True 64 Unix 4.0G), the computer time is

Total reaction cross section: ~ 5 min
 Elastic differential cross section: ~ 5 min/point
 Momentum distribution: ~ 10 min/point.

The densities used here are, unless otherwise mentioned, taken from Ref. [10].

5.1 Reaction cross sections for $^{13}\text{C}+^{12}\text{C}$ and $^{19}\text{C}+^{12}\text{C}$ systems

The ^{13}C nucleus is described with a $^{12}\text{C} + 0p_{1/2}$ -neutron system. The single-particle wave function for the valence neutron is generated as explained in Sec. 4.1. In what follows, the diffuseness parameter a and the radius parameter r_0 of the potential (32) are chosen to be 0.6 and 1.2 fm, respectively. The potential

Table 1
Input files for the $^{13}\text{C}+^{12}\text{C}$ system

csc.inp			
12.0	13.0	12.0	A_T, A_P, A_C
6.0	6.0	6.0	Z_T, Z_P, Z_C
800.0			incident energy per nucleon (in MeV)
4.26	-0.07	0.21008	profile function (σ_{NN} in fm^2 , α, β in fm^2)
1			l (orbital angular momentum)
500000	2.5	-11213	Monte Carlo parameters ($N_s, \delta, \text{irand}$)
0			<i>icond1</i> (1: available, 0: not available)
1	0		<i>icond2, icond3</i>
20.0	10		max. angle (or momentum) and number of points
2			number of Gaussians used to fit the target density
-1.23342874	0.462770142		coefficient, range (in fm^{-2})
1.38536085	0.373622826		
2			number of Gaussians used to fit the core density
-1.23342874	0.462770142		coefficient, range (in fm^{-2})
1.38536085	0.373622826		
wf.inp			
70.0			initial depth of the optical potential (in MeV)
0.6			diffuseness (in fm)
1.2			radius (in fm)
-4.946			energy eigenvalue for the valence nucleon (in MeV)
0.5			j -value of the valence-nucleon orbit
0			node number of the valence-nucleon orbit

depth V_0 is varied so as to reproduce the single-particle energy, $\varepsilon = 4.946$ MeV. The parameters of the profile function are taken from Ref. [15]. Though the parameters in general depend on the isospin of the nucleons (pp, nn, pn), appropriate average values are taken [6]. The input parameters which have to be filled in the files csc.inp and wf.inp are shown in Table 1. The mass of a nucleus with mass number A is set equal to $931.5A$ MeV/ c^2 . Results of calculation at 800 MeV/nucleon are given in the file reac.out, as shown in Table 2.

The cross sections calculated at different incident energies are compared with

Table 2
Results for the $^{13}\text{C} + ^{12}\text{C}$ system

reac.out	
Projectile:	mass = 13.0 charge = 6.0
Target:	mass = 12.0 charge = 6.0
Energy (MeV/nucleon)	800
Proj. reaction cross section (mb)	924.6
Core reaction cross section (mb)	894.7
N-removal cross section (mb)	29.8
N-removal (elastic) (mb)	3.5
N-removal (inelastic) (mb)	26.2

the experiments [16–18] in Fig. 3. The valence-neutron density is not spatially extended in ^{13}C , thus the difference between the reaction cross sections of $^{13}\text{C}+^{12}\text{C}$ and $^{12}\text{C}+^{12}\text{C}$ is not very large. Though the theory reproduces the $^{12}\text{C}+^{12}\text{C}$ data reasonably well at high energies, the discrepancy is conspicuous at lower energies. This is mostly due to that the profile function at such energies is not very appropriate. A better reproduction of the reaction cross section over the wide range of energy has recently been achieved by using effective nucleon-target profile function [19].

The next example is the energy dependence of the one-nucleon removal cross section for the $^{19}\text{C}+^{12}\text{C}$ system. The spin-parity and the energy of the ^{19}C ground state are not settled down experimentally. The ground state of ^{19}C is assumed to be a $^{18}\text{C} + \text{neutron}$ system: The valence neutron is in the $1s_{1/2}$ orbit with the separation energy of $\varepsilon = 0.24$ MeV [20]. Thus ^{19}C is considered to be a good example for a one-neutron halo nucleus. In fact, the rms radius of the neutron orbit becomes very large (8.5 fm). The density of ^{18}C is constructed from the harmonic-oscillator shell model [18]. Fig. 4 shows the energy dependence of the elastic and inelastic one-nucleon removal cross sections. The one-nucleon removal cross section is large, reflecting the halo structure. The contribution from the inelastic breakup process is much larger than that from the elastic process in the energy region beyond a few hundred MeV per nucleon. The elastic breakup cross section is less than 30 % of the total one-neutron removal cross section.

5.2 Elastic differential cross section for $^{11}\text{Be}+^{12}\text{C}$ system

The ^{11}Be nucleus is treated as a $^{10}\text{Be} + \text{neutron}$ system. The neutron is in the $1s_{1/2}$ orbit with the separation energy of $\varepsilon = 0.503$ MeV. The density

of ^{10}Be is taken from Ref. [21]. The elastic differential cross sections for the projectile-target and the core-target scatterings are given in the ratio to the Rutherford cross sections. The result is written on the file, ang.out, as follows:

theta [deg.]	(ds/dR) _{-P}	(ds/dR) _{-C}
0.10	1.002854	1.003104
0.20	0.990132	0.995163
...	...	...

Fig. 5 shows the cross section of the elastic scattering of ^{11}Be on a ^{12}C target at the energy of 49.3 MeV/nucleon [22]. The solid curve is the result obtained by using the formula given in Sec. 2.3. The dotted curve shows the cross section calculated with the folding model [4], that is, the matrix element of Eq. (9) is further simplified, following the OLA, into

$$\langle \varphi_0 | e^{i\chi_{\text{CT}}(\mathbf{b}_C) + i\chi_{\text{NT}}(\mathbf{b}_C + \mathbf{s})} | \varphi_0 \rangle \rightarrow e^{\langle \varphi_0 | i\chi_{\text{CT}}(\mathbf{b}_C) + i\chi_{\text{NT}}(\mathbf{b}_C + \mathbf{s}) | \varphi_0 \rangle}. \quad (33)$$

The parameters of the profile function are taken from Ref. [23]. Though the above approximation (the dotted curve) seems to fit the experiments better than the solid curve, this does not imply that this crude treatment is justifiable, but simply indicates that the profile function used is favorably modified from the free nucleon-nucleon profile function. Since the halo neutron in ^{11}Be is loosely coupled with the core nucleus and extends to large distances, the breakup effect of the neutron becomes very important and most of the nuclear breakup effect is expected to be accounted for in the solid curve. The breakup effect can be expressed as a dynamical polarization potential when the phase-shift function is calculated according to Eq. (9) [4,24].

5.3 Momentum distributions for $^{11}\text{Be} + ^9\text{Be}$ and $^8\text{B} + ^{12}\text{C}$ systems

With the choice of $icond3 = 1$, the momentum distributions expressed in Eqs. (28) and (25) are written on the output file, momdist.out, as follows:

P [MeV/c]	(ds/dpz) from FT [1/(MeV/c)]	(ds/dpz) [mb/(MeV/c)]
0.0	0.1526497E-01	2.321504
10.0	0.1349002E-01	2.014542
...	...	...

Fig. 6 displays the longitudinal momentum distribution of the ^{10}Be for the $^{11}\text{Be} + ^9\text{Be}$ reaction at the energy of 63 MeV/nucleon [25]. Only the inelastic breakup process is taken into account. The solid and dashed curves are the results of Eqs. (25) and (29), respectively. Both of these include the factors responsible for the reaction mechanism through the phase-shift functions. The

difference between them is in the treatment of the transverse momentum $\mathbf{P}_\perp$. Various components with different $\mathbf{P}_\perp$ contribute to the solid curve, while the pure component with $\mathbf{P}_\perp = 0$ only contributes to the dashed curve. It is understandable that the latter distribution leads to a narrower distribution than the former. The difference of these curves serves to show qualitatively the difference of momentum distribution corresponding to different experimental acceptance. The dotted curve in Fig. 6 is just the result of Fourier transform of the neutron orbit, Eq. (28). A recent calculation has included both the elastic and inelastic processes in a few-body eikonal framework [12,26].

The momentum distribution for the $^8\text{B} + ^{12}\text{C}$ system is calculated at the energy of 1.44 GeV/nucleon in a similar way. In this case the ^8B nucleus is treated as a $^7\text{Be} + \text{proton}$ system, where the proton orbits in the $0p_{3/2}$ state with the separation energy of $\varepsilon = 0.137$ MeV. Though the separation energy is small, the rms radius of the proton orbit is not that large (4.3 fm) because of the presence of the Coulomb barrier. The density of ^7Be is taken from Ref. [27]. With the choice of $icond3=2$, the momentum distribution from each m_l state is written on the output file, comp.out, as follows:

P [MeV/c]	(ds/dpz) [mb/(MeV/c)]
m=0	
0.0	0.3715373E-02
10.0	0.2067766E-01
...	...
m=1	
...	...

Fig. 7 compares with experiment [28] the longitudinal momentum distributions with (solid curve) and without (dotted curve) the reaction mechanism. One sees that the effect of the reaction mechanism is very important, making the broad distribution of the Fourier transform narrower. This change is understood by investigating the contribution of each component of the single-particle orbits to the distribution as shown in Fig. 8. It is seen that the $m_l = \pm 1$ orbits have a large contribution to the longitudinal distribution. The different behavior of the magnetic substates combined with the reaction mechanism can explain the basic physics in the ^7Be longitudinal momentum distribution [29–31].

The calculated distribution is still a little broad, compared to experiment. This suggests that the spatial extension of the valence-proton orbit may probably be too small. This expectation could be supported by the fact that the calculated σ_{-N} is only 75 mb, smaller than the observed value 96 ± 6 mb [28]. We change the radius parameter for the ^7Be -p potential to $r_0 = 1.73$ fm so as to reproduce the σ_{-N} value. Though this radius seems to be extraordinarily large, we adopt this value just to see the extent to which the momentum distribution changes. The rms radius of the proton orbit turns out to be 5.07 fm. The dashed curve in

Fig. 7 is obtained in this way. It shows very good agreement with experiment.

6 Summary

A Fortran program was presented in the framework of the Glauber theory to calculate various cross sections for reactions induced by the projectile which has a core plus one valence-nucleon structure. The multi-dimensional integration over the valence-nucleon coordinates was performed by the Monte Carlo quadrature with the Metropolis algorithm. One can use any single-particle wave function for the valence nucleon. We tested the usefulness and applicability of the code by comparing various sample calculations with available data including the total reaction cross section, the one-nucleon removal cross section, the elastic differential cross section, and the momentum distribution of the core fragment in the treatment of the inelastic breakup process.

The Glauber model is obviously suitable for high energy reactions. For reactions at lower energy, say less than a hundred MeV/nucleon, it would be more practical to use some effective interactions between the nucleon and the target like a nucleon-target optical potential. To implement this modification in the code, one only needs to replace χ_{CT} and χ_{NT} with the corresponding phase-shift functions constructed from the optical potential and the core density.

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Figure Captions

Fig. 1 The probability distribution of $N_s = 500,000$ points for different choices of the step size δ used for the Metropolis random walk. The solid curve is the guiding function $|u_{nlj}(r)|^2$ multiplied by an appropriate constant value to fit the peak of the distribution. The function $u_{nlj}(r)$ is generated from the $0p_{\frac{1}{2}}$ orbit in ^{13}C .

Fig. 2 The longitudinal momentum distribution calculated by a direct numerical method and the Monte Carlo integration. The results of the Monte Carlo integration are shown for different choices of the step size δ used for the Metropolis random walk. Two curves with $\delta = 2.5$ and 5 fm are indistinguishable in the figure.

Fig. 3 The total reaction cross sections for $^{13}\text{C} + ^{12}\text{C}$ and $^{12}\text{C} + ^{12}\text{C}$ systems. The experimental data are taken from Ref. [16,17] for black circles and from Ref. [18] for open circles.

Fig. 4 The energy dependence of the neutron-removal cross section for $^{19}\text{C} + ^{12}\text{C}$ system. Both contributions of the elastic and inelastic processes are shown.

Fig. 5 The elastic differential cross section for $^{11}\text{Be} + ^{12}\text{C}$ scatterings at the energy of 49.3 MeV/nucleon. The solid curve shows the result of the ^{10}Be core + one-neutron model calculation, while the dotted curve is the folding model calculation. The experimental data are taken from Ref. [22].

Fig. 6 The longitudinal momentum distribution of ^{10}Be from the $^{11}\text{Be} + ^9\text{Be}$ reaction at the energy of 63 MeV/nucleon. The solid and dashed curves denote the results calculated from Eqs. (25) and (29), respectively. The dotted curve is the momentum distribution with the reaction mechanism turned off. The experimental data are taken from Ref. [25].

Fig. 7 The longitudinal momentum distribution of ^7Be from the $^8\text{B} + ^{12}\text{C}$ reaction at the energy of 1.44 GeV/nucleon. The solid (dotted) curve denotes the result with (without) the reaction mechanism. See text for the dashed curve. The experimental data are taken from Ref. [28].

Fig. 8 The contribution of the single-particle orbits with different m_l values to the momentum distribution for the $^8\text{B} + ^{12}\text{C}$ reaction.

Fig. (1)

Fig. (2)

Fig. (3)

Fig. (4)

Fig. (5)

Fig. (6)

Fig. (7)

Fig. (8)