Simultaneous arrival of information in absorbing wave guides

A. Ruschhaupt and J. G. Muga
Departamento de Química-Física, UPV-EHU, Apartado 644, 48080 Bilbao, Spain

We demonstrate that the temporal peak generated by specific electromagnetic pulses may arrive at different positions simultaneously in an absorbing wave guide. The effect can be used for triggering several devices all at once at unknown distances from the sender or generally to transmit information so that it arrives at the same time to receivers at different, unknown locations. This simultaneity cannot be realized by the standard transmission methods.

PACS numbers: 03.65.Xp, 03.65.Ta, 03.65.-w

In a previous paper [1] we have described a surprising effect, namely, that the temporal peak of a quantum wave from a source with a sharp onset may arrive at different locations simultaneously in absorbing media. This “ubiquitous peak” (UP) effect was demonstrated for the Schrödinger equation, where, unlike the Hartman effect, it holds at arbitrarily large distances (see [1] for further differences with the Hartman effect); and also for a relativistic wave equation, but limited to distances where the cut-off but, at variance with other “ultrafast” wave phenomena based on anomalous dispersion in absorbing media [2, 3, 4, 5], which depend on the dominance of the carrier (central) frequency associated with faster than light, infinite or negative group velocities, the ubiquitous peak is, at each position, dominated by the saddle-point contributions above the cut-off frequency. It is thus a fundamentally different phenomenon.

One advantage of the wave guide with respect to the quantum particle described by the Schrödinger equation is that the effect may be implemented more easily and could be measured in a non-invasive way [6]. It also makes possible to trigger several devices at the same time or transmit information so that it arrives simultaneously at unknown locations. This cannot be achieved by standard transmission methods because a receiver could not resend an information bit to the closest one faster than the velocity of light in vacuum c; nor can we design the timing of a series of signals from the source so that they arrive simultaneously at different receivers if their locations are unknown.

We assume a wave guide in z-direction filled with a homogeneous, isotropic, dielectric, dispersive, absorbing medium, i.e. we end up with the following wave equation (see e.g. [7] for details)

$$\left[ \frac{\partial^2}{\partial z^2} + \frac{1}{c^2} \left( i \frac{\partial}{\partial t} \right)^2 \eta^2 \left( i \frac{\partial}{\partial t} \right) - \gamma^2 \right] \phi(z,t) = 0, \quad (1)$$

where η(ω) is the complex refraction index of the medium, and γ is the mode eigenvalue of the waveguide.

We assume that the refraction index is given by the Lorentz model,

$$\eta(\omega) = \sqrt{1 - \omega^2 / \omega_p^2} \approx 1 + i \frac{\omega^2 - \omega_c^2}{4 \delta \omega^2},$$

and consider, for a frequency interval around ω₀, |ω - ω₀| < Δ, the conditions δ ≫ |(ω² - ω_p²)/(2ω)| and δ ≫ ω_p²/(2ω), so that

$$\eta(\omega) \approx \sqrt{1 + \frac{i \omega^2}{2 \delta \omega}} \approx 1 + i \frac{\omega^2}{4 \delta \omega} \equiv 1 + i \frac{\omega}{\omega_1}. \quad (2)$$

(With this choice Eq. (1) is similar to the Klein-Gordon equation discussed in [1].)

We also assume “source” boundary conditions with the value of φ(0, t) = φ₀(t) given for t, and require φ(z, t) = 0 for z > 0, t < 0, which fixes a unique solution φ(z, t). The UP effect was found first for the sharp-onset source function e⁻ⁱω₀ᵗΘ(t) but we shall now examine smoother variants,

$$\phi_m(t) = i \int_{-\infty}^{\infty} d\omega f_m(\omega - \omega_0) e^{-i\omega_0 t} \Theta(t). \quad (3)$$

In the reference case f_m(ω) = f_1(ω) = 1, φ₀(t) = e⁻ⁱω₀ᵗΘ(t). The solution of Eq. (1) with the boundary condition in Eq. (3) fulfilling the demand that φ_m(z, t) = 0 for z > 0, t < 0 is

$$\phi_m(z,t) = i \int_{-\infty}^{\infty} d\omega f_m(\omega - \omega_0) e^{ik(\omega - \omega_0) t} \omega - \omega_0 + i0. \quad (4)$$

with the dispersion relation

$$k(\omega) = \sqrt{\frac{\omega^2}{c^2} \eta^2(\omega) - \gamma^2} = \frac{1}{c} \sqrt{\omega + i n_1}^2 - \omega_c^2,$$

and ω_c = cγ being the cut-off frequency. We may apply the saddle-pole approximation of the integral in Eq. (4) for the case f_m(ω) = 1. The approximation φ ≈ Θ(g)φ₀ + φ⁺ + φ⁻ can be found by following
the steps described e.g. in \[5]. The saddle points are 
\[
\omega_{s\pm} = \pm \beta - in_1
\]
with \( \beta = \omega_c/\sqrt{1 - z^2/(c^2 t^2)} \). The pole contribution, if \( 0 < g(z, t) = (\omega_0 - \beta)(\omega_0 - \omega_c^2) + n_1(\omega_0^2 - \omega_c^2) - 2(\omega_0 - \omega_c^2) \), is \( \phi_p(z, t) = -\exp(izk(\omega_0) - i\omega_0 t) \), whereas the saddle contributions are

\[
\phi_{s\pm}(z, t) = \frac{i}{2\sqrt{2\pi}} \frac{z^{1/2} + i\sqrt{\omega_c}}{c t^{3/2} \sqrt{1 - z^2/c^2 t^2}} \times \exp\left(-in_1 + it\omega_c(\sqrt{1 - z^2/c^2 t^2})\right)
\]

If there is no pole contribution, \( \phi \approx \phi_{s+} + \phi_{s-} \), one may easily obtain lower and upper envelopes for the oscillating signal, \( I_- \leq |\phi_{s+} + \phi_{s-}|^2 \leq I_+ \). Let us now examine the exact solutions for other “window functions” \( f_m \) with a central plateau (see Fig. 1 for examples),

\[
f_2(\omega) = \begin{cases} 
1 & : 0 \leq |\omega| < \Delta\omega \\
1 - \frac{|\omega| - \Delta\omega}{\alpha} & : \Delta\omega \leq |\omega| < \Delta\omega + \alpha \\
0 & : \Delta\omega + \alpha < |\omega|
\end{cases}
\]

\[
f_3(\omega) = \begin{cases} 
1 & : 0 \leq |\omega| < \Delta\omega \\
\exp\left(-\frac{(|\omega| - \Delta\omega)^2}{\alpha^2 + (\Delta\omega - \delta)^2}\right) & : \Delta\omega \leq |\omega| < \Delta\omega + \alpha \\
0 & : \Delta\omega + \alpha < |\omega|
\end{cases}
\]

Since \( f_2 \) and \( f_3 \) are non-zero only in a range around 0 it is enough that the form of the refraction index, Eq. \(2\), is fulfilled in an interval \( 2(\Delta\omega + \alpha) \) around \( \omega_0 \).

Figure 2a shows \( |\phi_m(0, t)|^2 \) for different \( m \) and Fig. 2b shows \( |\phi_m(z, t)|^2 \) for \( z = 150 \text{ m} \). A consequence of the smoothing of the signal onset at the source is the cancellation of the oscillations between the two envelopes, i.e., a much simpler signal structure. Also, the very first sharp causal front for \( f_1 \) is substituted by a smooth increase for the window functions \( f_2, f_3 \), but the maximum around \( t = 2 \mu s \) remains. We are interested in the time \( \tau_T \) of this maximum for \( m = 2, 3 \). The times \( \tau_T \) are depicted in Fig. 3 versus \( z \) for different \( m \) and \( \alpha \): \( \tau_T \) for \( m = 2, 3 \) is nearly independent of \( z \) for intermediate values of \( z \), see also Fig. 4, whereas at large \( z \) the maximum behaves “normally” and grows linearly with \( z \).
UP effect, it follows that the peak is predominantly composed by frequencies above the cut-off $\omega_c$. The group velocity calculated at the positive saddle is always smaller than $c$, but it is unrelated here to the peak’s behaviour.

$\tau_{zs+(z)}$ may be used to estimate a lower value for the start of the effect: $z_{min}$ is defined as the smallest $z$ where $|\frac{T_s}{\tau_{zs+(z)}}| < 1/c$, so that the temporal maximum “moves” beyond $z_{min}$ faster than $c$ without violating causality in any way. Assuming $n_1 \ll \omega_0$ and $1 < \xi \equiv \omega_c/\omega_0 < 3/2^{3/2}$ we can find an analytical formula for the maximal value of $\tau_{zs+}$,

$$\tau_M = \frac{1}{2n_1} \frac{3 - 2\xi^2 - 3(\xi^2 - 1)^{2/3}}{\xi^2} \approx \frac{1}{2n_1},$$

which gives the arrival time of the peak in the region where the UP effect holds. The values $\tau_M$ are shown in Fig. 3 and Fig. 4 with empty symbols.

The approximation $\phi \approx \phi_+$ breaks down when the saddle point reaches the edge of the window function, i.e. $Re(\omega_+) \geq \omega_0 + \Delta \omega$: then the UP effect also breaks down and $\tau_F$ grows linearly with $z$. From the condition $\Re(\omega_+)|_{l=\tau_M} = \omega_0 + \Delta \omega$ the upper boundary for the effect is given by $z_{max} = cT_M \sqrt{1 - \omega_0^2/(\omega_0 + \Delta \omega)^2}$, which increases with $\Delta \omega$. This value of $z_{max}$ coincides with the penetration length of $\omega = \omega_0 + \Delta \omega$ defined by $l = 1/(2\mu_1 k(\omega))$. However, $z_{max}$ cannot be arbitrarily large since the maximum of the saddle eventually vanishes. (The value $z_M$ where this occurs is given by a lengthy expression. In the range of parameters considered in the examples $z_{max} < z_M$ so that $z_{max}$ is the true upper bound.)

Let us examine what happens by changing $\omega_0$ or $n_1$. According to Fig. 4b, the effect exists also for carrier frequencies above the cut-off, $\omega_0 > \omega_c$, but in that case a much greater peak appears at small $z$ which travels with finite velocity, and the simultaneous arrival effect is only seen at larger distances from the source when the main forerunner has not yet arrived and a much smaller peak is formed first. For the applications described below (simultaneous triggering or sending information that arrives simultaneously to different receivers) it is more convenient to use energies just below the cut-off, because the traveling forerunner does not exist, the attenuation is minimal, and the spatial range in which the effect holds becomes maximal. In Fig. 4b we can see that a stronger absorption leads to faster arrival of the peak. The upper and lower limits of the effect diminish when $n_1$ is increased, but detection becomes more difficult because of the attenuation of the signal.

In the following we shall illustrate how this effect can be used to send a triggering signal or information to receivers at unknown distances in such a way that the information arrives at all receivers at nearly the same time.

We shall use a wave function at the source of the form

$$\Phi_0(t) = b_1\phi_{03}(t) + b_2\phi_{03}(t - t_2) + b_3\phi_{03}(t - t_3) + b_4\phi_{03}(t - t_4).$$

As Eq. (1) is linear, $\Phi(z, t)$ can be found by adding the solutions corresponding to each term separately. An example of $|\Phi(z, t)|^2$ is plotted for different $z$ in Fig. 5 where the times of the maxima are nearly independent of $z$. Suppose that the receivers are located at unknown distances from the source and that they do not have synchronized clocks (only their unit time intervals are equal). Each receiver gets $|\Phi(z, t)|^2$ and may use the following operational procedure to find the peaks: $\tau_P$ is the time of finding a peak (the peak is at time $\tau_P - \Delta t$) if $|\Phi(z, \tau_P - \Delta t)|^2 > |\Phi(z, t)|^2$ for $\tau_P - \Delta t > t \geq \tau_P$. Moreover there should be a noise level $l_0$ so that $|\Phi(z, t)|^2$ is assumed as zero if $|\Phi(z, t)|^2 < l_0$. After finding a peak the search for the next peak is started if $|\Phi(z, t)|^2 < l_0$. The noise level may establish an upper limit $z_{noise}$ for the effect more strict than $z_{max}$, beyond which the attenuation makes impossible in practice to distinguish the peak.

The resulting times $\tau_P$ of the different peaks are plotted in Fig. 6. Clearly the receivers find the peaks at nearly the same times $\tau_P$. For comparison, lines with
and partly controllable domain. The task of sending in-
waveguide arrive simultaneously at receivers in a broad
ima generated by specific wave pulses in an absorbing
necessary that the peaks are sent at equal time intervals.
for example, would represent a sequence 1010. It is not
following peaks carry the message. The signal in Fig. 5,
used for calibration, e.g. the higher peak may represent
of the maxima. The height of the first two peaks may be
information coded in the value of
triggering signal. Moreover it is possible to send bits of
known distances if we use only the first maximum as the
effect can be used to trigger receivers at different un-
the peak earlier than if the nearest receiver sends a light
slope 1/c are also plotted. In all cases the receivers get
peak earlier than if the nearest receiver sends a light
signal to them when it finds the peak. This shows that
the effect can be used to trigger receivers at different un-
known distances if we use only the first maximum as the
triggering signal. Moreover it is possible to send bits of
information coded in the value of $|\Phi(z,t)|^2$ at the time
of the maxima. The height of the first two peaks may be
used for calibration, e.g. the higher peak may represent
a logical 1 and the lower peak a logical 0, whereas the
following peaks carry the message. The signal in Fig. 5,
for example, would represent a sequence 1010. It is not
necessary that the peaks are sent at equal time intervals.

Summarizing, we have shown that the temporal max-
ima generated by specific wave pulses in an absorbing
wave guide arrive simultaneously at receivers in a broad
and partly controllable domain. The task of sending in-
formation to arrive at different receivers simultaneously is
different from the question of superluminal velocities be-
cause the information always arrives subluminally [10, 17]
and it will be possible in principle to send information
faster to a single fixed receiver than with the present ef-
flect.

The ubiquitous peak is dominated by above-cut-off fre-
frequencies so it is of a fundamentally different origin from
effects based on superluminal tunneling and on negative
and/or superluminal group velocities [2, 3, 4, 5, 11, 12,
Sons Ltd.
[12] R.Y. Chiao and A.M. Steinberg, Prog. Optics 37, 345
(1997).
(2003).
(2000).
66, 042110 (2002).

We acknowledge support by UPV-EHU (00039.310-
13507/2001), “Ministerio de Ciencia y Tecnolog ´ıa” and
FEDER (BFM2003-01003). AR acknowledges support
by the German Academic Exchange Service (DAAD).