Exotic Fermions and Bosons in the Quartification Model

Shao-Long Chen and Ernest Ma

Physics Department, University of California, Riverside, California 92521

Abstract

Exotic fermions of half-integral charges at the TeV energy scale are predicted by the quartification model of Babu, Ma, and Willenbrock. We add to these one copy of their scalar analogs and discuss the ensuing phenomenological implications, i.e. radiative contributions to lepton masses and flavor-changing leptonic decays.
In the recently proposed model of SU(3) quartification, the low-energy gauge group of particle interactions is SU(3)C × SU(2)L × U(1)Y × SU(2)L and the particle content is that of the nonsupersymmetric standard model with two Higgs doublets, and three families of exotic fermions. Whereas the former are singlets under the new gauge symmetry SU(2)L, the latter are doublets, i.e.

\[
\begin{pmatrix} x \\ y \end{pmatrix} \sim (1, 2, 0; 2), \quad x^c \sim (1, 1, -1/2; 2), \quad y^c \sim (1, 1, 1/2; 2).
\]

(1)

Because they have ±1/2 electric charges, they have been named “hemions”.

The SU(2)L gauge symmetry is the unbroken remnant of a leptonic color SU(3)L which combines with the familiar SU(3)C × SU(3)L × SU(3)R to form SU(3)4 at the quartification scale. The exotic hemions are required to be at the TeV energy scale, or else the gauge couplings would not unify. On the other hand, they do not interact directly with quarks or leptons, so they are produced only through the electroweak gauge bosons, i.e. W±, Z, and γ. Since the hemions are SU(2)L doublets, they couple only in pairs, and because they are also fermions, they have Yukawa couplings only to the Higgs scalars. Suppose we now add one set of scalar hemions (\(\tilde{x}, \tilde{y}\)), \(\tilde{x}^c\), and \(\tilde{y}^c\) transforming in the same way as \((x, y)\), \(x^c\), and \(y^c\) of Eq. (1), then Yukawa couplings such as \((xe - y\nu)\tilde{y}^c\), \((\tilde{x}e - \tilde{y}\nu)y^c\), and \(x^c\tilde{x}^ce^c\) become possible. They would induce radiative contributions to lepton masses as well as flavor-changing leptonic decays to one-loop order, which may be observed experimentally.

The evolution of the gauge couplings of SU(3)C, SU(2)L, and U(1)Y are given in one-loop order by the renormalization-group equation

\[
\frac{1}{\alpha_i(M_1)} - \frac{1}{\alpha_i(M_2)} = \frac{b_i}{2\pi} \ln \frac{M_2}{M_1},
\]

(2)

where

\[
SU(3)_C : \quad b_s = -11 + \frac{4}{3}N_f = -7,
\]

(3)
\[ SU(2)_L : \quad b_2 = -\frac{22}{3} + 2N_f + \frac{1}{6}N_H = -1, \quad (4) \]
\[ U(1)_Y : \quad b_Y = \frac{26}{9}N_f + \frac{1}{6}N_H = 9, \quad (5) \]\n
between the unification scale \(M_U\) and the scale \(M_X\) at which the hemions become massive, and

\[ SU(3)_C : \quad b_s = -11 + \frac{4}{3}N_f = -7, \quad (6) \]
\[ SU(2)_L : \quad b_2 = -\frac{22}{3} + \frac{4}{3}N_f + \frac{1}{6}N_H = -3, \quad (7) \]
\[ U(1)_Y : \quad b_Y = \frac{20}{9}N_f + \frac{1}{6}N_H = 7, \quad (8) \]

between \(M_X\) and \(M_Z\). The number of families \(N_f\) is three and the number of Higgs doublets \(N_H\) is assumed to be two. Recognizing that \(\sin^2 \theta_W = 1/3\) at \(M_U\) so that \(\alpha_Y\) is normalized to \(\alpha_U/2\) at \(M_U\), we obtain

\[
\ln \frac{M_X}{M_Z} = \frac{\pi}{17} \left( \frac{6}{\alpha_Y} - \frac{23}{\alpha_2} + \frac{11}{\alpha_s} \right), \quad (9)
\]
\[
\ln \frac{M_U}{M_Z} = \frac{2\pi}{17} \left( \frac{1}{\alpha_Y} - \frac{1}{\alpha_2} - \frac{1}{\alpha_s} \right). \quad (10)
\]

Using the input \[ \]
\[ \alpha_2(M_Z) = (\sqrt{2}/\pi)G_F M_W^2 = 0.0340, \quad (11) \]
\[ \alpha_Y(M_Z) = \alpha_2(M_Z) \tan^2 \theta_W = 0.0102, \quad (12) \]
\[ \alpha_s(M_Z) = 0.1172, \quad (13) \]

we then have

\[ \frac{M_X}{M_Z} = 2.8, \quad \frac{M_U}{M_Z} = 4.4 \times 10^9, \quad (14) \]

which is the case of Ref. [1].

Instead of \(N_H = 2\) assumed in Ref. [1], let us take \(N_H = 1\), but also add one set of the scalar hemions. Equations (9) and (10) are then changed to

\[
\ln \frac{M_X}{M_Z} = \frac{\pi}{119} \left( \frac{37}{\alpha_Y} - \frac{139}{\alpha_2} + \frac{65}{\alpha_s} \right), \quad (15)
\]
\[
\ln \frac{M_U}{M_Z} = \frac{6\pi}{629} \left( \frac{37}{\alpha_Y} - \frac{105}{\alpha_2} + \frac{31}{\alpha_s} \right), \tag{16}
\]
yielding instead
\[
\frac{M_X}{M_Z} = 11.9, \quad \frac{M_U}{M_Z} = 2.9 \times 10^{10}. \tag{17}
\]
Thus \(M_X\) and \(M_U\) have been shifted upward, but the former is still of order 1 TeV and should be accessible to future experimental verification.

In the presence of both hemions and their scalar counterparts (shemions), the following Yukawa couplings are allowed under \(SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(2)_t\):

\[
\mathcal{L}_Y = f_1(\nu y - e x)\tilde{y}^c + f_2(\nu \tilde{y} - e \tilde{x})y^c + f_3 e^c x^c \tilde{x}^c \\
+ f_4 \nu^c(x\tilde{y} - y\tilde{x}) + f_5 \nu^c x^c \tilde{y}^c + f_6 \nu^c y^c \tilde{x}^c + H.c., \tag{18}
\]
where each \(SU(2)_t\) invariant of the form \(x\tilde{y}^c\) means \(x_1\tilde{y}_2^c - x_2\tilde{y}_1^c\). Radiative contributions to various lepton masses become possible, as shown below.

(1) Charged leptons:

![Figure 1: One-loop contributions to the charged-lepton mass matrix.](image)

These contributions are of order
\[
m_e = \frac{f_{1,2}f_3}{16\pi^2} m_{\text{eff}}, \tag{19}
\]
where \(m_{\text{eff}}\) is a function of the hemion and shemion masses and their couplings.
(2) Neutrinos: Contributions to both Dirac and Majorana masses are present. The former comes from the analog of Fig. 1 with couplings $f_1 f_{4,6}$ and $f_2 f_{4,5}$ and the latter in the case of $\nu$ is depicted below.

![Figure 2: One-loop contributions to the Majorana neutrino mass matrix.](image)

This contribution is of order

$$m_\nu = \frac{f_1 f_2}{16\pi^2} m'_{\text{eff}},$$  \hspace{1cm} (20)

where $m'_{\text{eff}}$ is typically smaller than $m_{\text{eff}}$ of Eq. (19) by at least an order of magnitude. Similar contributions exist in the case of $\nu^c$.

These radiative corrections are not directly observable, because they simply add to the already existing tree-level charged-lepton and neutrino masses $[\Pi]$. On the other hand, the same interactions also induce flavor-changing leptonic decays. A typical diagram is shown below.

Using the Lagrangian of Eq. (18), we obtain the amplitude for $l_i \rightarrow l_j + \gamma$:

$$A = \bar{l}_j(p - q)q^\mu \epsilon^\nu i\sigma_{\mu\nu} \left[ \xi \left( \frac{1 + \gamma_5}{2} \right) + \eta \left( \frac{1 - \gamma_5}{2} \right) \right] l_i(p),$$  \hspace{1cm} (21)

where

$$\xi = \frac{ie}{64\pi^2} \left[ \frac{m_i}{M^2} f^i_1 (f^j_1)^* [F(z_1) + z_1^{-1} F(z_1^{-1})] + \frac{m_i}{M^2} f^i_2 (f^j_2)^* [F(z_2) + z_2^{-1} F(z_2^{-1})] + \frac{m_j}{M^2} f^i_3 (f^j_3)^* [F(z_3) + z_3^{-1} F(z_3^{-1})] \right],$$  \hspace{1cm} (22)
Figure 3: Typical diagram for flavor-changing leptonic decay.

and $\eta$ is obtained from $\xi$ with the interchange of $m_i$ and $m_j$, i.e. the masses of $l_i$ and $l_j$ respectively. In the above, $M$ is the invariant fermion mass of the $x$ and $y$ hemions, and $M_c$ that of the $x^c$ and $y^c$ hemions. The function $F(z)$ is given by

$$F(z) = \frac{2 + 5z - z^2}{6(1 - z)^3} + \frac{z \ln z}{(1 - z)^4},$$

(23)

where $z_1 = m_{\tilde{x}}^2/M^2$, $z_2 = m_{\tilde{x}}^2/M_c^2$, and $z_3 = m_{\tilde{x}}^2/M_c^2$. In the limit $z = 1$, $F(z) = 1/12$.

As an illustration, let $z_1 = z_2 = z_3 = 1$, $f_1^{i,j} = f_2^{i,j} = f_3^{i,j} = f$, $M_c = M$, then the $\mu \to e\gamma$ rate is given by

$$\Gamma = \frac{5\alpha |f|^4 m_{\mu}^5}{9(256)^2 \pi^4 M^4},$$

(24)

yielding a branching fraction of

$$B = \frac{5\alpha}{3072\pi} \left( \frac{|f|^2}{G_F M^2} \right)^2.$$

(25)

Using the present experimental upper bound $4$ of $1.2 \times 10^{-11}$, we obtain

$$M > 7|f| \text{ TeV}.$$  

(26)

For $|f|$ of order 0.1, $M$ may thus be of order 1 TeV, in keeping with the expectation of Eq. (17). It also shows that there is a good possibility for a future observation of $\mu \to e\gamma$ just below the present upper bound.
In conclusion, we have shown that the exotic fermions and scalars of half-integral charges, predicted in a natural extension of the Standard Model to include leptonic color and resulting in $SU(3)^4$ quartification, should have masses in the TeV range and may induce an observable $\mu \to e\gamma$ decay rate.

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References


