It was recently suggested by A. Kapustin that turning on a $B$-field, and allowing some discrepancy between the left and and right-moving complex structures, must induce an identification of B-branes with holomorphic line bundles on a non-commutative complex torus. We translate the stability condition for the branes into this language and identify the stable topological branes with previously proposed non-commutative instanton equations. This involves certain topological identities whose derivation has become familiar in non-commutative field theory. It is crucial for these identities that the instantons are localized. We therefore explore the case of non-constant field strength, whose non-linearities are dealt with thanks to the rank-one Seiberg–Witten map.

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1 Introduction

Branes of various dimensions may be regarded as submanifolds of a target space, since their position is defined by a set of boundary conditions for open strings. As open strings carry gauge fields, taking those into account promotes branes to bundles on the submanifolds. Background fields from closed strings influence the geometry of these bundles; in particular, non-commutative gauge theory becomes a valid tool for the field theory along the branes, when a $B$-field is turned on. Many couplings between open and closed string modes follow from the relevant star-products [1, 2, 3, 4]. Branes also allow for formulations of important field-theoretic issues, such as the problem of instantons. Investigating supersymmetric $D_p$-branes through an effective action, Mariño, Minasian, Moore and Strominger [5] identified a class of instanton equations, and proposed a non-commutative limit thereof,

$$\hat{F}^{(0,2)} = 0,$$

$$\hat{F} \wedge J^{p-1} = 0.$$  

(1)  

(2)

On the other hand, Kapustin [6] recently used the tools of (generalized) complex geometry [7, 8] to identify the influence of the $B$-field on topological branes. He argued that it endows B-branes on the non-commutative torus with a structure of holomorphic line bundles, the complex structure of which is dictated by the $B$-field and an allowed discrepancy between left and right-moving complex structures. Without using a non-commutative description, Kapustin and Li also obtained [9] a stability condition, in agreement with [5], using the world-sheet viewpoint, which makes branes emerge as boundary conditions that are compatible with a supersymmetry algebra.

The purpose of the present note is two-fold: I shall put together these two viewpoints, and perform checks required by the non-linearities of non-commutative gauge theory with non-constant field strength. I shall adapt the stability condition to the non-commutative set-up, thereby showing that the non-commutative instanton equations of [5] are equivalent to the identification, in the presence of a $B$-field, between supersymmetric D-branes and stable holomorphic line bundles on the non-commutative torus. But both formulations were written down on the basis of explicit formulae in the case of constant field strength; their natural extension to non-constant ones, involving highly non-linear terms in the gauge field, remains conjectural. The checks will use the techniques of non-commutative gauge theory, in order to deal with some of these non-linearities, and to establish the non-commutative instanton equations, that were originally written down without relying on these techniques.

The structure of this paper is as follows. In section 2, I shall review the geometric set-up in which the non-commutative proposal for holomorphicity was formulated, and translate into this language the stability condition of matching of spectral flow operators. In section 3, I shall explain how the holomorphicity condition (1) cancels terms that would not be consistent with the Seiberg–Witten map in flat space. The constant factor appearing in the stability condition
will finally be used to produce a topological identity, whose Seiberg–Witten limit reads as the equation (2).

2 A class of stable line bundles

2.1 Topological branes and generalized complex geometry

Consider a (B)-brane as a line bundle over a complex torus. According to a proposal in [6], turning on a $B$-field, and allowing a discrepancy between the left and right-moving complex structures $I_+$ and $I_-$, can give rise to holomorphic line bundles over a non-commutative complex torus. The way to this proposal goes as follows. If $X$ is a torus, endowed with a metric $G$ and a $B$-field, a pair of complex structures $I_+$ and $I_-$ can be used to define a complex structure on the sum $X \oplus X^*$, suited to the even-dimensional cases, where $X$ is the quotient of a complex vector space by a lattice,

$$
\begin{pmatrix}
I_+ & 0 \\
0 & I_-
\end{pmatrix}.
$$

The $B$-field allows for transformations of complex structures on $X \oplus X^*$ discussed in [7, 8] and related to isometries of $X \oplus X^*$. On the other hand, the presence of a metric yields an isomorphism between tangent and cotangent spaces, and $I_+$ and $I_-$ can be interpreted as complex structures for left and right-movers respectively. Given the fact that T-duality is a parity transformation acting on right-movers only, the most general data (involving different left and right-moving complex structures) are relevant to the study of T-duality. It will be assumed that the following complex structure on the sum of tangent and cotangent spaces

$$
\mathcal{I} = \begin{pmatrix}
\tilde{I} + (\delta P)B & -\delta P \\
(\delta \omega + B(\delta P)B + B\tilde{I} + \tilde{I}B & -\tilde{I} - B\delta P)
\end{pmatrix},
$$

is block-upper-triangular. This assumption is related to the transformation by T-duality of a block-diagonal complex structure with no discrepancy between left and right-moving complex structures. It allows us to investigate how non-commutative deformations and discrepancies between complex structures are entangled. The tensor $\tilde{I}$ is half of the sum of the left and right-moving complex structures; the discrepancies $\delta P$ and $\delta \omega$ are defined through the difference between the associated Kaehler forms $\omega_{\pm} = GI_{\pm}$ through

$$
\delta \omega = \frac{1}{2}(\omega_+ - \omega_-),
$$

$$
\delta P = \frac{1}{2}(\omega_+^{-1} - \omega_-^{-1}),
$$

Boundary conditions for fermions and supersymmetry requirements have been shown [6] to give rise under this assumption to the following equation for the field strength,

$$
FI + I^t F = -F \delta PF,
$$

(3)
so that the failure of the field strength to be of type \((1, 1)\) is directly related to the difference allowed between left and right-moving complex structures. This failure has furthermore been related to non-commutativity by proposing that, given the two relations

\[
I = \tilde{I} + (\delta P)B, \\
\delta P = I\theta + \theta I^t,
\]

the failure disappears, provided one considers the basis as a torus with non-commutativity scale \(\theta = B^{-1}\), and the field strength \(\hat{F}\) associated to \(F\) by the Seiberg–Witten map in flat space:

\[
\hat{F}I + I^t\hat{F} = 0. \tag{4}
\]

This proposal means that the non-commutative counterpart of the field strength is of type \((1, 1)\), and thereby ensures that \(N = 2\) supersymmetry compels the brane to be a holomorphic line bundle on a non-commutative complex torus. A few consistency checks were performed in \([6]\) for constant field strength, by substituting \(\hat{F} = (1 + \theta F)^{-1}F\) in the proposal \((4)\) and expanding it in powers of \(\theta F\). In the present note, I shall adopt the reverse approach, substituting the inverse Seiberg-Witten map in \([3]\), and identifying the condition \((4)\) as the suitable tool to make the expansion consistent. Apart from checking the proposal more precisely, the consideration of varying field strength will prove necessary for the consistency with the stability condition for the bundle, because this condition involves the assumption of localized instantons. Instanton equations derived from non-commutative field theory and supersymmetric branes in \([2, 5]\) rely indeed on the existence of a region with the combination \(B + F\) merely consisting of a constant \(B\)-field.

### 2.2 Stability condition

In order to make contact with (some limit of) the deformed equations proposed in \([5]\) for supersymmetric D-branes, we have to supplement the previous proposal with a stability requirement, because stable topological branes correspond to supersymmetric D-branes. To this end, we are going to rephrase, using the language of left and right-moving complex structures, the stability condition derived by Kapustin and Li \([9]\) using world-sheet arguments. Their work recovers the condition of \([5]\) by considering the holomorphic part of the matching condition for spectral flows. As the complex structure \(I\) in the previous statement has been unambiguously identified, making it appear as the complex structure suited to the stability condition will amount to a consistency check.

In terms of a holomorphic \(n\)-form, where \(n\) is the complex dimension of the ambient manifold, the matching condition for the spectral flows reads in terms of a proportionality factor \(e^{i\alpha}\) as follows:

\[
\Omega_{i_1 \ldots i_n} \bar{\psi}_{+}^{i_1} \cdots \bar{\psi}_{+}^{i_n} = e^{i\alpha} \Omega_{i_1 \ldots i_n} \psi_{-}^{i_1} \cdots \psi_{-}^{i_n}.
\]
In order to make contact with the above discussion, we first have to rewrite the latter equation terms of the linear combinations

\[ \psi^i = \frac{1}{2} (\psi^i_+ + \psi^i_-), \]
\[ \rho_i = \frac{1}{2} G_{ij} (\psi^j_+ - \psi^j_-), \]

for which the boundary conditions read

\[ \rho_i = -(B_{ij} + F_{ij}) \psi^j. \]

Complex structures induce a splitting of boundary conditions. We can consider the holomorphic part, whose determinant was expressed in [9] using the discussion in terms of the variables \((\psi_+, \psi_-)\). If the boundary conditions for these variables are written using some linear transformation \(R\), the stability condition for B-branes is worked out by identifying the constant factor \(e^{i\alpha}\) with the determinant of the holomorphic part \(R_h\) of the transformation:

\[ \psi_+ = R \psi_-; \]
\[ \det R_h = e^{i\alpha}. \]

Let us adapt this statement to the variables \((\psi, \rho)\). As the stability requirement is to be eventually expressed in terms of a Kaehler form, it will involve the holomorphic part of the boundary conditions, and therefore a complex structure. Under the assumption of \(I\) being block-upper-triangular, we have to check that \(\rho\) and \(\psi\) make for a \((1,0)\)-form on the direct sum of the tangent and cotangent spaces, according to a complex structure on the sum of tangent and cotangent spaces written in the basis adapted to \((\psi, \rho)\). The matching condition written in terms of \(\psi\) is still independent from complex structures, and is expressed in terms of the combination \(\mathcal{F} := \mathcal{B} + \mathcal{F}\), as:

\[ \det (\mathcal{G} + \mathcal{F}) = e^{i\alpha} \det (\mathcal{G} - \mathcal{F}). \]

But once the boundary condition has been imposed, linking \(\psi\) and \(\rho\) to each other, it must be compatible with the consideration of the holomorphic part of the transformation of the tangent bundle. This means that, if \(\psi\) is holomorphic with respect to \(I\), the fermion \(\rho\) associated to \(\psi\) by the boundary condition must be such that \((\psi, \rho)\) is holomorphic with respect to the complex structure

\[ \begin{pmatrix} I_+ & 0 \\ 0 & I_- \end{pmatrix}, \]

once written in the new basis that is adapted to sections of tangent and cotangent spaces such as \((\psi, \rho)\), namely

\[ \begin{pmatrix} \bar{I} & -\delta P \\ \delta \omega & -\bar{I}^t \end{pmatrix}. \]

This is seen to be guaranteed by the assumption of \(I\) being block-upper-triangular, since the holomorphicity condition on \(\psi^i\) reads from the blocks on the first line, together with the boundary conditions:

\[ \bar{I} \psi + (\delta P) B \psi = i \psi, \]

4
and is sufficient to ensure the relation corresponding to the blocks on the second line, since the latter reads:

\[-B(\delta P)B - B\tilde{I} - \tilde{I}B\] \(\psi + \tilde{I}B\psi = -iB\psi.\)

The formulation we need for the stability condition is thus in terms of the Kaehler form

\[J := G (\tilde{I} + (\delta P) B) = GI,\]

which is suited to the consideration of the holomorphic part of the matching condition. In order to evaluate the factor in terms of our fields, we use the gauge-freedom exchange between \(B\) and \(F\) to express stability through

\[
\frac{\det(J + iF)}{\det(1 + G^{-1}F)} = \frac{\det(J + iB)}{\det(1 + G^{-1}B)}.
\]

were we assumed that the instanton is localized. We evaluated the constant factor far away from the instanton, where \(F\) reduces to the \(B\)-field. As was announced, the field strength cannot be constant, and the identification between branes and stable holomorphic line bundles therefore requires a treatment of the non-linearities carried by the Seiberg–Witten map of non-commutative gauge theory.

3 Compatibility with the Seiberg–Witten map

3.1 Holomorphic line bundles and the Seiberg–Witten map

I shall now perform a few checks of the proposal (4) for varying field strength. These computations are motivated by the stability condition, as just stated, but will be performed on the holomorphicity condition, thus preparing for the combination of these two as an instanton equation.

In order to obtain constraints on the non-commutative gauge theory from the constraint (3) on the field strength, we must formulate this condition in terms of non-commutative fields in position space. The rank-one Seiberg–Witten map in flat space \([10, 11, 12]\) admits a natural formulation in momentum space, since it involves a straight open Wilson line \(W_k\) whose extension \(\theta^{\mu\nu}_k\) depends on the Fourier mode \(k\) in consideration:

\[F_{ij}(k) = \int dx L_s \left( \sqrt{\det(1 - \theta F)} \right) \tilde{F}_{ik} \left( \frac{1}{1 - \theta F} \right)^k_{jk} W_k(x) \ast e^{ikx}, \]

where \(L_s\) denotes the smearing prescription: it averages over all the possible ways of inserting operators along the Wilson line. Expanding such a smeared expression in powers of the gauge field defines the modified \(*_n\) products, where the integer \(n\) labels the order of the expansion. The LHS of the equation

\[FI + I^iF = -F\delta PF\]
only contains one operator in position space. We may thus rewrite it, using the Seiberg–Witten map, as the Fourier transform of the expression

\[
\int dx\, L^* \left( \sqrt{\det(1 - \theta \hat{F})}, \left( \frac{1}{1 - \theta \hat{F}} I + I^t \hat{F} \frac{1}{1 - \theta \hat{F}} \right), W_k(x) \right) \ast e^{ikx},
\]

since objects carrying no index are transparent to the endomorphism \(I\). The RHS is more involved, since it consists of a pointwise product of two fields of gauge theory in position space. Its Fourier transform at momentum \(k_\mu\) will therefore be a convolution of two Wilson lines, whose extensions sum to \(\theta^{\mu\nu} k_\nu\), each of these Wilson lines having one of the fields attached at its beginning. This is equivalent to the concatenation of two Wilson lines into one, of extension \(\theta^{\mu\nu} k_\nu\), the second observable being smeared along the result. For momentum \(k_\mu\), we obtain in momentum space

\[
\int dx\, L^* \left( \sqrt{\det(1 - \theta \hat{F})}, \hat{F} \frac{1}{1 - \theta \hat{F}}, \left( I \hat{F} \frac{1}{1 - \theta \hat{F}} + \theta I^t \hat{F} \frac{1}{1 - \theta \hat{F}} \right), W_k(x) \right) \ast e^{ikx}.
\]

Since differential operators \(*_n\) enable us to expand both expressions at all orders in derivatives and any finite order in the gauge field, we can read off the conditions for the expansions of the two sides of (3) to match order by order in the gauge field, even for varying fields.

The first order in the field strength is consistent with \(\hat{F}\) being of type \((1, 1)\):

\[\hat{F} I + I^t \hat{F} = o(\hat{F}).\]

The next order makes appear a single term on the RHS, because each of the form indices has to be brought by a field strength, with no contribution either from the determinant or from the denominators. It has a counterpart on the LHS, through the quadratic terms in the following expansion of the Seiberg–Witten map in position space:

\[F_{ij} = \hat{F}_{ij} + \theta^{mn} \langle \hat{F}_{im}, \hat{F}_{nj} \rangle_{*2} - \frac{1}{2} \theta^{mn} \langle \hat{F}_{nm}, \hat{F}_{ij} \rangle_{*2} + \theta^{mn} \partial_n \langle \hat{A}_m, \hat{F}_{ij} \rangle_{*2} + O(\hat{F}^3).\]

Now we can read off the condition to be fulfilled in order for the two sides to match at quadratic order in the gauge field:

\[0 = -\frac{1}{2} \theta^{mn} \langle \hat{F}_{nm}, (\hat{F} I + I^t \hat{F})_{ij} \rangle_{*2} + \theta^{mn} \partial_n \langle \hat{A}_m, (\hat{F} I + I^t \hat{F})_{ij} \rangle_{*2}.\]

We observe that it is identically verified if the holomorphicity condition obtained at the linear step is fulfilled. This is the first test of the non-commutative holomorphicity proposal passed by varying fields.

Expanding further in terms of the gauge field will generate infinitely many more involved contributions, but we may note at once that the condition \(\hat{F}^{(0,2)} = 0\) will ensure the cancellation of an infinite subset, which can only be obtained on the LHS, namely the subset of those terms.
with form indices borne by one field strength, and with an arbitrary number of gauge fields from the expansion of the open Wilson line:

\[
\sum_{p \geq 1} \frac{i^p}{p!} (\theta \partial)^{\lambda_1} \cdots (\theta \partial)^{\lambda_p} \left\langle \sqrt{\det \left( 1 - \theta \hat{F} \right)}, \hat{F}_{ij}, \hat{A}_{\lambda_1}, \ldots, \hat{A}_{\lambda_p} \right\rangle_{*p}.
\]

We have altogether seen that the condition \( \hat{F}^{(0,2)} = 0 \) is exactly what we need to cancel a set of terms (of arbitrarily high degree in the gauge field) that does only appear on the LHS. In order to achieve a more convincing argument, we have to explain inductively why the remaining terms match. Since the influence of the Pfaffian and the Wilson line is the same on both sides, we just have to deal with the terms coming from the expansion of the denominator in the Seiberg–Witten map (5). These correspond to terms with the two form-indices borne by two different field strengths, one being the field strength in the numerator, the other one being the last factor in a term of the series expansion of the denominator. Going from some definite order in the expansion in powers of the gauge field to the next one involves a variation with respect to the discrepancy \( \delta P \), because the latter is quadratic in the field strength. But the form of the terms thereby generated is precisely the one that comes from expansions of denominators. Consider a certain order \( p \) in the expansion of \( \hat{F} (1 - \theta \hat{F})^{-1} \) in powers of the field strength inside the LHS of (3), and suppose we have been able to show that the terms of interest match up to that order. Consider the effect of making the discrepancy act on one of the field strengths, say the \( A \)-th, thus producing, up to a sign, a term with one more field strength. We have to sum over all the ways of making this insertion, so that the monomial of order \( p + 1 \) in the field strength to be smeared on the LHS reads

\[
\sum_{A=1}^{p} (\theta \hat{F})^{A-1} \hat{F} (\delta P) \hat{F} (\theta \hat{F})^{p-A}.
\]

Rewriting its smearing as \( *_{p+1} \), we recognize a term that is produced by taking \( A \) powers of \( \hat{F} \) from the first operator and \( p + 1 - A \) from the second one in the expansion of

\[
L_{s} \left( \frac{\hat{F}}{(1 - \theta \hat{F})}, (\delta P) \frac{\hat{F}}{(1 - \theta \hat{F})}, W_k \right),
\]

which coincides with the list of operators smeared on the RHS, when the Pfaffian is disregarded. Restoring the contributions from the Pfaffian and the Wilson line on both sides ensures consistency between the two expansions of the non-commutative image of (3), up to terms that are cancelled on a holomorphic line bundle through (4).

### 3.2 Stability condition as a non-commutative topological identity

The derivation in the previous section relied on the various operators that arise when agreement is demanded between commutative and non-commutative couplings to Ramond–Ramond
fields, in the case of a single Dp-brane. The Seiberg–Witten map embodies this agreement for couplings to $C^{p-1}$, and more field strengths are inserted along the Wilson line when couplings to Ramond–Ramond fields of lower degree are written. Nevertheless, the coupling to the top-form $C^{p+1}$ yields one more identity, although no field strength can carry form-indices in this coupling. For a flat brane, the commutative coupling is a zero form that does not depend on the field strength. On the other hand, the non-commutative expression still has to be gauge-invariant, and to involve additional gauge fields, even if all indices are contracted. This provides an identity between a gauge-invariant non-commutative expression, and a commutative expression that is actually more than gauge-invariant. The commutative side does not know about the variations of the non-commutative field strength, hence the name topological identity. It comes out as the zero-form part\footnote{Integration is along the $p + 1$-dimensional world-volume, and one only retains the couplings in which the sum of form-degrees from Ramond–Ramond fields and gauge fields equals $p + 1.$} of the identity between Ramond–Ramond couplings

$$
\sum_n C^{(n)}(-k) \wedge \int dx \left( e^{B+F} e^{ikx} \right) = \sum_n C^{(n)}(-k) \wedge \int dx L_s \left( \frac{\text{Pf} Q}{\text{Pf} \theta}, e^{Q^{-1}}; W_k(x) \right) * e^{ikx},
$$

where $Q$ is the (inverse of the) non-commutative counterpart of the symplectic structure $B + F$, in the case of constant field strength:

$$
Q^{ij} = \left( \frac{1}{B + F} \right)^{ij} = \theta^{ij} - \theta^{ik} \hat{F}_{kl} \theta^{lj}.
$$

The topological part is the following, whereas higher-degree contributions yield the Seiberg–Witten map used in the previous checks, and derivative corrections:

$$
\delta(k) = \int dx L_s \left( \sqrt{\det \left( 1 - \theta \hat{F} \right)}, W_k(x) \right) * e^{ikx}.
$$

This situation, where a gauge-invariant quantity is actually a constant, is reminiscent of the presence of a constant factor in the requirement of stability. The bias introduced by the complex structure forces us to adapt the above procedure to the case at hand, endowed with complex geometry. Let us note that this way of producing identities, using the duality between commutative and non-commutative descriptions, has been proven by explicit computations to be sensible. We are allowed to repeat it in our context as soon as we can go through the following steps:

1. consider a gauge-invariant quantity depending on commutative gauge fields and $B$-fields,
2. write it in terms of non-commutative variables in the case of constant field strength,
3. remember that the result must be gauge-invariant, and restore gauge-invariance by the smearing prescription.

In the cases where the quantity is simply a constant, the procedure generates a topological identity.
In the case at hand, the obvious candidate for the gauge-invariant quantity is the constant factor $e^{i\alpha}$. If $J$ is the Kaehler form associated to the complex structure $I$, our topological quantity is expressed as

$$e^{i\alpha} = \frac{\det(J + iQ^{-1})}{\det(G + Q^{-1})},$$

which for constant field strength can be expressed in two different ways, using either the symplectic structure $B$ (away from the instanton), or the inverse of the tensor $Q^{ij}$ defined in non-commutative gauge theory.

Of course, this simple substitution of different expressions for the same tensor will suffer from a lack of gauge-invariance in the case of varying field strength. This is a general problem related to the non-locality of non-commutative gauge theory, solved by the smearing prescription along an open Wilson line [16, 17, 18]. The non-commutative expression for constant field strength leads to the following identity in the Seiberg–Witten limit

$$\frac{(Q^{-1})^{p-1} \wedge J}{\text{Pf}(Q^{-1})} = \frac{B^{p-1} \wedge J}{\text{Pf}(B)},$$

which is exactly the limit worked out in [9], where the identity between the two expressions for $Q$ was used to write down the instanton equations in the non-commutative set-up. These non-commutative conditions are equivalent to $\hat{F}$ being of type $(1, 1)$, together with the condition

$$\hat{F} \wedge J^{p-1} = 0,$$

whose gauge-invariant completion is just the analogous smeared expression. This completes a world-sheet derivation of [11] and [12]. Their space-time derivation in [13] was formal and relied on the explicit Seiberg–Witten map for constant field strength, although the result has obvious extension to more general configurations corresponding to localized instantons. We have justified this extension by mapping the bundles to non-commutative gauge theory, using further properties of star-products, that actually define the Seiberg–Witten image of instantons.

4 Conclusions

We have put together a proposed non-commutative version of the instanton equations obtained from supersymmetric D-branes using effective actions, and the stable holomorphic line bundles that emerge from the study of topological branes on the non-commutative torus. We thereby established more firmly the relevance of non-commutative gauge theory for the study of instantons. On the way we identified one of the equations as a topological identity of non-commutative gauge theory, associated to the existence of a constant factor in the stability condition. Complex structures fit into techniques involved in generating topological identities and derivative corrections to effective actions from non-commutative field theory. This was to be awaited since an alternative world-sheet derivation should exist for the results based on the study of branes as supersymmetric solitons.
Nevertheless, the full instanton equations derived in [5] are a two-parameter deformation of usual instanton equations. The limit we investigated corresponds to the Seiberg–Witten limit of non-commutative gauge theory. It is a very peculiar case, just as the derivative corrections to effective actions derived from non-commutativity in the Seiberg–Witten limit are a subset of those that can be derived by going beyond this limit, further deforming the star-product. In a sense, dealing with the non-linearities as we did amounts to taking into account a more precise effective action, involving $\ast_n$-products between fields entering couplings of rank $2n$. Recovering the two-parameter deformation should involve the deformations $\tilde{\ast}_n$ that appeared in [19, 20].

Very roughly, we can note that the first contribution from the metric to these products is quadratic, just as in the deformed instanton equations of [5].

The issue of the inclusion of scalars in stability conditions has been investigated [21], but exhibiting a non-commutative version would require the knowledge of an explicit solution to the Seiberg–Witten equations of non-Abelian gauge theory. On the other hand, describing an Abelian D-brane in terms of lower-dimensional ones in the language of matrix theory allows for identities involving scalars, since the Abelian D-brane does not couple à la Myers [22] to transverse scalars. It should be possible to obtain from a topological identity an instanton equation including scalars, that would show up as infinite matrices.

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