Schwarzschild black branes from unstable D-branes

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We study systems with a large number of meta-stable Dp-branes, and show that they describe Schwarzschild and Schwarzschild-like black branes, generalizing the results of Danielsson, Guijosa and Kruczenski [1]. The systems are considered in both the open and closed string pictures. We identify the horizon size and its relation to the physics of open and closed strings. From the closed string perspective the region inside the horizon is where the effects of massive closed strings become important.
1 Introduction

Low energy closed string theory possesses classical solutions describing many types of black branes, both neutral and charged under NSNS or RR fields. From among all these solutions the ones carrying RR charges have received the most attention in the last eight years, starting with [2]. It was shown that D-branes provide a good description of states that evolve into black holes as the number of branes is increased. This lead to the AdS/CFT conjecture [3] and its generalizations [4], which provide a complete description of certain space-times, and in particular black holes in them, by a gauge theory. However this line of thought does not seem to work for uncharged black holes, which do not have a D-brane description. The usual view on Schwarzschild-like black holes is that a massive string state will become a black hole at a high enough mass for fixed coupling. The problem is that the Schwarzschild-like black hole entropy cannot be accounted for by the entropy of a string. Indeed, a black hole with a string size horizon has an entropy \( S \sim l_s M \) which is the maximal entropy that a closed string can have. For a larger black hole the entropy has a different dependence on the mass, and is greater than the entropy of any elementary string. The entropy of a Schwarzschild black \( p \)-brane in \( D = p + d + 3 \) dimensions is given by

\[
S = 2\pi \omega_{d+1}^{-1/d} \left( \frac{2}{d+1} \right)^{(d+1)/d} \kappa^{2/d} V_{p-1/d} M^{(d+1)/d},
\]

(1)

where \( \omega_{d+1} = 2\pi^{1+d/2}/\Gamma(1 + d/2) \) is the volume of a unit \((d + 1)\)-sphere. This lead the authors of [5] to propose that the density of states of the string changes at large density to account for the larger entropy. Another way of forming the black hole is to use a large collection of unstable branes (or branes and anti-branes). One would expect that tachyon condensation will generically produce an uncharged black hole. An example of this is the known solution of Penrose of two colliding plane waves which form a black hole. This is dual to a brane anti-brane system.

Recent studies of the rolling tachyon background [6, 7] have produced new insights into this issue. In particular two results are important. First, it was shown that the rolling tachyon produces mostly massive closed strings which stay very close to the location of the original brane [8, 9]. This provides a dual closed string description of tachyon matter. Second, it was argued that
certain closed string states are just on the verge of forming unstable branes which can come in a variety of dimensions [9].

Consider some massive closed string, and fix the string coupling to some constant value. We can add more energy to the string until its Schwarzschild radius gets to be of the order of the string scale. Now at this point adding more energy will not increase the entropy as much as forming a Schwarzschild black hole, which means that the probability of this string to stay a massive close string state is very small. But the increased entropy seems to indicate that there are new degrees of freedom available for the system. What could they be? The lessons from the rolling tachyon indicate that these degrees of freedom may just be open strings forming on unstable D-branes (or on $D - \bar{D}$ systems, depending on the dimension needed). At some point in the $g_s, M$ plane it becomes more favorable to invest a certain energy in forming the branes, and thus opening up a larger space of degrees of freedom.

Recently it was shown by Danielsson, Guijosa and Kruczenski (DGK) [1] that brane-anti-brane systems based on D3, M2 and M5-branes can describe the thermodynamics of Schwarzschild black holes in certain dimensions. This was generalized to rotating 3-branes in [10].

In this note we will generalize and extend these results, and discuss the emerging picture.

Note added: while this paper was completed a paper [11] with which there is some overlap appeared.

2 Open string picture

We would like to consider how the unstable branes can be used to describe the Schwarzschild black holes in various dimensions. If we take a collection of $N_0$ unstable branes then they will tend to decay. Their decay produces, in one view, very massive closed strings. At some point, if the density of the closed strings is large enough, the probability to emit a closed string may equal the probability to absorb one, and the system can reach a meta-stable configuration (note that this cannot happen if the decay were predominantly
into massless closed strings). In the dual view pairs of open strings are created, and here again if the density of the open strings is large enough then the probability of creation can become the same as the probability of open strings to annihilate back to form the brane. It is the latter, open string, view that we will consider first. Since the specific heat will turn out to be negative it is better to work in the microcanonical ensemble. We will consider the situation in which some of the $N_0$ branes have decayed and have produced open strings on the rest of the $N < N_0$ branes.\footnote{Another possibility is that all the branes decay a bit. It was suggested in \cite{1} that after stabilization there will still be massless open string excitations. One possibility is that since the system is stabilized there are moduli to move the center of mass, which should be just the zero-modes of some massless field on the branes. It is however not clear how this is consistent with observations that the rolling of the tachyon gives mass to all open string excitations \cite{12}. This may be taken into account by changing the tension of the branes to be a parameter as well. We will not consider this possibility here.} As we will see, if $N_0$ is large enough the resulting open strings will tend to be mainly in their massless sector. The energy and entropy of the remaining meta-stable branes are given by

$$M = N\tau_p V + aVN^cT^{\alpha+1}, \quad (2)$$

$$S = \frac{\alpha + 1}{\alpha}aVN^cT^\alpha, \quad (3)$$

where $\tau_p$ is the unstable brane tension and $V$ is its volume. Note that $T$ is the temperature on the unstable brane. Since we want to work in the microcanonical ensemble we will write

$$S = \frac{\alpha + 1}{\alpha}aV\frac{1}{\alpha + 1}N^{\frac{1}{\alpha + 1}}(M - N\tau_p V)^{\frac{1}{\alpha + 1}}. \quad (4)$$

To find the meta-stable configuration we need to maximize the entropy with respect to $N$. This turns out to give

$$N = \frac{cM}{(\alpha + c)\tau_p V} \quad (5)$$

and thus

$$E = \frac{\alpha}{\alpha + c}M, \quad (6)$$

where $E$ is the energy stored in open string excitations. We now plug this back into the entropy expression to get

$$S = (\alpha + 1)a^{\frac{1}{\alpha + 1}}\frac{1}{\alpha + 1}(c + \alpha)^{-\frac{1}{\alpha + 1}}c^{\frac{1}{\alpha + 1}}\tau_p^{-\frac{\alpha}{\alpha + 1}}V^{\frac{1}{\alpha + 1}}M^{\frac{1}{\alpha + 1}}. \quad (7)$$
This does not yet look like the entropy of the Schwarzschild black brane. What values of \( c, a \) and \( \alpha \) should we use? The branes are now meta-stable and we are looking in a regime where \( N \) is large, so this will not be a perturbative regime. We will now make an assumption which we will justify by the results we get. We will assume that the thermodynamics of the meta-stable \( p \)-brane in the large \( N \) regime is the same as the thermodynamics of the stable (i.e. BPS) brane in the large \( N \) regime.

The thermodynamics of the stable branes is given by the formula [4]

\[
S_{\text{stable}} = h_p V g_{YM}^2 \left( \frac{p-3}{5-p} \right) N \frac{\tau_p}{5-p} \frac{9-p}{5-p},
\]

where \( h_p \) is a known constant. Putting this all into the formula in equation (7) we get (dropping numerical constants and remembering that \( g_{ym}^2 \sim g_s l_s^{p-3} \) and \( \tau_p \sim (g_s l_s^{p+1})^{-1} \))

\[
S \sim g_s^{2-p} l_s^{8-p} V \frac{1}{\tau_p} M \frac{8-p}{5-p},
\]

which is exactly the right dependence on these parameters for Schwarzschild black branes, although only up to a numerical factor of order one. The negative specific heat of the Schwarzschild black brane is understood as the statement that extra energy added to the system prefers to go into creating another unstable brane, thereby increasing the number of degrees of freedom, and lowering the temperature of the original excitations [1].

However there is a problem with this interpretation. The unstable branes (or the brane anti-brane system) have a dilaton charge, while the Schwarzschild black branes do not. We will discuss this issue in the last section.

Since the temperature \( T \) was taken to be the temperature of the gas on the brane following the usual field theory relationship between energy of the gas and its entropy, it is not clear that this will also be the temperature of the black hole. So let us compute this. From equation (3) we find

\[
\frac{1}{T} = \frac{\alpha}{\alpha + 1 \frac{S}{M - N \tau_p V}}
\]

Using the solution for \( N \) (5) and the value of \( \alpha \) one finds

\[
\frac{1}{T} = \frac{8-p}{7-p} \frac{S}{M} \sim (g_s N)^{\frac{1}{\tau_p} l_s},
\]
which is exactly the black hole temperature. Notice that since we are in
the regime where \( g_sN \) is large, the temperature is very low, thus justifying
the approximation of taking only the massless open strings into account. At
small horizon sizes of order the string length the massive open strings have
an important contribution to the physics.

2.1 More general brane configurations

We can consider more general brane configurations that arise in the low
energy supergravity theory. We limit ourselves to the simplest case described
by having a gravity theory in \( D \)-dimensions with a dilaton and a single \((p+1)\)-
form potential:

\[
S = -\frac{1}{2\kappa^2} \int d^D x \sqrt{g} \left( R - \frac{1}{2} (\partial \phi)^2 - \frac{1}{2(d+1)!} e^{a\phi} |F_{d+1}|^2 \right) \tag{12}
\]

Black brane solutions for this action were analyzed in [13], and their near-
extremal thermodynamics was extracted. The entropy of the near-extremal
\( p \)-brane solutions and the mass of the extremal solutions are given by \( D = p + d + 3 \)

\[
S(E) = b L^{p(1-\lambda)} \kappa^{\frac{D}{2} - \frac{n}{4}} q_p E^{\lambda} \tag{13}
\]

\[
b = 2^{\frac{1}{2} - \frac{n}{4} + 2} \pi \omega_{d+1} \sqrt{\frac{d}{d+1}} \lambda^{-\lambda n - \frac{n}{4}} \tag{14}
\]

\[
M_{p,0} = \frac{q_p \sqrt{n}}{\sqrt{2\kappa}} L^p \tag{15}
\]

where \( E \) is the energy above extremality, \( L^p \) is the volume of the \( p \)-dimensional
torus which the brane wraps and

\[
\lambda = \frac{d+1}{d} - \frac{n}{2}, \quad n = 4 \left[ a^2 + \frac{2d(p+1)}{D-2} \right]^{-1} \tag{16}
\]

For the extremal solution to be supersymmetric \( n \) has to be an integer, and
it then represents the number of different brane types in the solution. Note
that if we take \( D = 10 \) and \( n = 1 \) this reduces to the example of the previous
section. If we now assume the existence of unstable branes which become
meta-stable as before (or consider systems with branes and anti-branes), with the thermodynamical properties as above, we can again consider systems in which the number of these brane-configurations can vary, i.e. we write

\[ M = f M_{p,0} + E \]
\[ S = S(E) = h S(M - M_{p,0}) . \]

(17)  
(18)

The coefficient \( f \) has to do with the ratio of tension of the unstable branes to the stable ones, and \( h \) is some numerical coefficient that may be present. We look for the number of branes (proportional to \( q_p \)) that maximizes \( S \) for a given \( M \). We find

\[ q_p = \frac{\sqrt{2\kappa}}{f L_p} \frac{\sqrt{n}}{2\lambda + n} M \]
\[ E = \frac{2\lambda}{2\lambda + n} M , \]

(19)  
(20)

and,

\[ S = h f^{-\frac{d+1}{2}} \pi (d + 1)^{-\frac{d+1}{2}} 2^{\frac{d+1}{2} + \frac{d}{2}} \omega_{d+1}^{-\frac{d}{2}} L^p(1 - \frac{d+1}{2}) \kappa^\frac{d}{2} M^{\frac{d+1}{2}} , \]

(21)

which is a factor \( h(2f)^{-\frac{d}{2}} \) times the Schwarzschild black brane entropy.

With this set of branes at hand we can look for non-dilatonic solutions to avoid the issue of dilaton charge. This means putting \( a = 0 \). At least for supersymmetric configurations where \( n \) is an integer, there are only a few possibilities. The M-branes in eleven dimensions and the D3-brane in ten dimensions were considered in [1]. Other possibilities are the self dual string in six dimensions \( (n = 2, p = 1, d = 2) \), which gives the Schwarzschild black string in six dimensions, and \( (n = 3, p = 1, d = 1) \), which is a Schwarzschild black string in five dimensions. Two other non-dilatonic examples, the black hole in four and five dimensions, have \( \lambda = 0 \), and will be discussed at the end of the section.

We see that there is a large class of examples of meta-stable brane configurations that provide a microscopic model for Schwarzschild and Schwarzschild-like black branes.
2.2 Quasi-Particle description

We saw that the black brane is described by the thermodynamics of the meta-stable branes, which we assumed was similar to that of the BPS branes at large $N$. Now the thermodynamics of the BPS branes was shown to have a simple quasi-particle picture [14, 15] (at least for the simple $p$-branes in ten dimensions). This picture accounts in a simple way for the thermodynamical relation, and more importantly for the relation between area and entropy and between horizon size entropy and temperature [16].

It was shown that the identification of the number of quasi-particles with the entropy, together with the decay rates of the quasi-particles, explains the equality of the area in Planck units and the entropy. Since the meta-stable branes follow the same thermodynamical relationship it is reasonable to assume that this description applies to the Schwarzschild black brane case as well, as advocated in [16]. Since some of the mass of the black hole is stored in the brane tension, only a fraction $\frac{g_s}{16-2p}$ of the mass $M$ of the black hole is carried by the quasi-particles’ energy.

From the point of view of the theory on the brane the horizon size is given by a regulated expression for the size of the state [17, 14]. One takes the scalar fields $X_i$ which describe the transverse fluctuations of the brane, and regularizes $\text{Tr}(X^2)$ by integrating out all the very massive states. As shown in [14] the field theory has a double peak distribution which makes this procedure well defined, by including in the trace just the first peak with frequency of order the temperature. Since we assume that the thermodynamics of the unstable branes is very similar to the thermodynamics of the BPS branes, and since the horizon size is related to the spectral density, one can expect that the expression for the size of the state stays the same for the unstable branes. The horizon size for the thermal BPS branes in the low temperature regime is given by

\[(g_sN l_p^{-3})^{1/2}T \sim U_0^{\frac{5-p}{2}} = \left( \frac{r_0}{l_s} \right)^{\frac{5-p}{2}}. \tag{22}\]

Using equation (11) we then get

\[r_0 \sim (g_sN)^{\frac{1}{1+p}} l_s \sim \frac{1}{T}, \tag{23}\]
which is the correct relation for a Schwarzschild black brane. It is remarkable that the relationship between horizon size and temperature for the Schwarzschild black brane follows from the properties of the low temperature field theory on the brane.

The double peaked distribution for the spectral density of the scalar also predicts the relation [14]

\[ U_0^2 \sim \frac{g_s N S}{V N^2 T}. \]  

This has been shown to be correct for the near horizon geometries of [4]. Since the spectral densities should be similar in the meta-stable case, this should be true here as well. Since \( V N / g_s \sim M \) we see that this relation is again just

\[ r_0 \sim 1/T. \]  

One might be worried that we are not in the regime where the thermodynamical expressions are valid (i.e. where in the dual supergravity the curvature is small at the horizon). However this is not the case, the validity of the thermodynamical expressions we are using is in the region [4]

\[ r_0 \ll (g_s N)^{1/(3-p)} l_s, \quad p < 3 \]  
\[ r_0 \gg (g_s N)^{1/(3-p)} l_s, \quad p > 3 \]

which are satisfied for the value from equation (23).

### 2.2.1 General case

All this was true for the simple \( p \)-branes in ten dimensions. For the more general solution there is a similar relationship between temperature and horizon size [13]

\[ E \sim r_h^d \]  
\[ T^{-1} \sim q_p^{n/2} E^{\lambda - 1}, \]

where \( r_h \) is the horizon radius. This implies upon inserting \( q_p \sim E \) that

\[ T^{-1} \sim E^{1/d} \sim r_h, \]

which is the correct relationship for a Schwarzschild black brane.
2.3 \( \lambda = 0 \) case

Two other examples of non-dilatonic black holes are \((n = 3, p = 0, d = 2)\), which is a black hole in five dimensions, and \((n = 4, p = 0, d = 1)\), which is a black hole in four dimensions. For these cases \( \lambda = 0 \) and a more careful analysis is needed.

We start with the solutions from [13],

\[
M = \frac{\sqrt{2} q_p L^p}{\kappa \sqrt{n d}} \left( d + 1 + n d \sinh^2 \gamma \right) \sinh 2\gamma. \tag{31}
\]

Expanding near extremality (large \( \gamma \)) to second order we find

\[
E = M - M_{p,0} = M_{p,0} \left[ \frac{4}{n} \lambda e^{-2\gamma} + 2e^{-4\gamma} \right]. \tag{32}
\]

For \( \lambda = 0 \) we therefore get

\[
E = 2M_{p,0}e^{-4\gamma}. \tag{33}
\]

In this case the entropy near extremality is given by

\[
S = \omega_{d+1}^{-\frac{1}{2}} L^p \left( \frac{2\sqrt{2} \kappa q_p}{\sqrt{n d}} \right)^{\frac{d+1}{4}} \left( 1 + n \sqrt{\frac{E}{2M_{p,0}}} \right), \tag{34}
\]

and the temperature by

\[
\frac{1}{T} = \omega_{d+1}^{-\frac{1}{2}} L^p \left( \frac{2\sqrt{2} \kappa q_p}{\sqrt{n d}} \right)^{\frac{d+1}{4}} \frac{n}{2\sqrt{M_{p,0}}} E^{-\frac{1}{2}}. \tag{35}
\]

We can now do the same calculation as before, fixing the total mass \( M \) and varying the number of branes (proportional to \( q_p \) or \( M_{p,0} \)) to maximize the entropy of the unstable brane (or brane-anti-brane system). As before let us assume that the tension of the unstable brane is given by \( \tau = f \tau_p \) where \( f \) is some unknown constant. We find that the entropy is maximized for

\[
M_{p,0} = \frac{\alpha}{f} M, \tag{36}
\]
where \( \alpha = \frac{13 + \sqrt{7}}{18} \) in the \( n = 4 \) case, and \( \alpha = \frac{7 + \sqrt{5}}{11} \) for the \( n = 3 \) case. Plugging this into the expressions for the entropy and temperature we get

\[
S \sim M^{\frac{d+1}{2}} \quad \text{(37)}
\]
\[
T \sim M^{-\frac{1}{d}}. \quad \text{(38)}
\]

Thus we have a description of the Schwarzschild black hole in four and five dimensions.

### 3 Closed string picture

Now let us see how this is viewed from the closed string perspective. The AdS/CFT and its generalizations have taught us that the density of states of the closed string changes in the presence of BPS branes. The procedure of zooming in on the near horizon geometry is just zooming in on the properties of the closed string theory near the branes. The AdS/CFT claim is that the gauge theory is dual to the closed string theory, i.e. that the number of states in the closed string theory is given by the number of gauge invariant configurations in the gauge theory. This number has the naive Hagedorn behavior only at low enough energy, above which the density of states of the closed string has a different dependence on the parameters (see for example [18]). If we assume that the density of states of the closed string near the unstable branes (after meta-stabilization) is similar to the density of states near BPS branes, then the results of the previous section can be interpreted in terms of closed strings.

We thus view equation (4) as giving the density of states of the closed string at large levels in the presence of a large number \( N \) of \( p \)-dimensional boundaries

\[
\ln \Omega(E) \sim V^{\frac{3-p}{2}} q_s^{\frac{3-2p}{2d}} N^{1/2} E^{\frac{3-p}{2d}}, \quad \text{(39)}
\]

where \( E \) is the energy of the closed string. For the more general configurations one has

\[
\ln \Omega(E) \sim V^{1-\lambda} \kappa^{\frac{\lambda}{2} - \frac{p}{2}} q_p^{n/2} E^{\lambda}. \quad \text{(40)}
\]

While the unstable brane wants to decay into massive closed strings, the massive closed strings produced do not move away from the brane appreciably
[8, 9], thus enabling the decay product to recombine to form the brane. At some point the decay then stops and the brane becomes meta-stable by virtue of the gas of massive closed strings. It is only meta-stable since massless closed strings can be produced by the decay of the massive closed strings and then escape as Hawking radiation.

3.1 The horizon

The appearance of a horizon is one of the puzzles of the black hole issue. While this is well understood in the context of the low energy supergravity, it is not clear what this means in the full string theory. In the GR context the horizon is locally not a special place. As far as we know closed string theory is only defined through its S-matrix elements, which means that the asymptotic coordinate system has a preferred status, unlike in classical GR where one can change to different coordinates without any issue. This is also amplified in the AdS/CFT context in which the asymptotic coordinates, which break down near the horizon according to classical GR, are tied to the gauge theory Hamiltonian one is using. In the AdS/CFT case it was argued that the horizon position can be described as the locus of points where new massless degrees of freedom (that can become tachyonic) appear when a D-brane probe approaches the horizon [19, 20]. This description is through the gauge theory variables, but one can ask what is the dual closed string view.

In other examples where open string tachyons appear, a dual closed string description states that the whole tower of massive closed string modes becomes important, and dominates over the contribution of the massless closed string modes. The appearance of a new massless open string state away from the origin of moduli space signals the breakdown of supergravity and the dominance of massive closed string states. This is consistent with the idea that the black hole states are comprised of many highly excited massive closed strings. This is also the case when one considers the unstable brane description. The horizon is where the low energy supergravity description breaks down due to massive closed string effects, so the horizon is an \( \alpha' \) effect (or \( g_sN \) effect). This is also in line with the analysis of [21, 22], where it was shown that in certain cases the horizon indicates large fluctuations in the metric, and the breakdown of uniqueness of the metric. It is also tempting to
identify the singularity with the existence of the boundaries (branes), being sources in closed string theory rather than on shell states.\(^2\)

4 Dilatonic Schwarzschild branes

We saw that based on all charged branes one can get an object with the correct dependence to be a Schwarzschild black brane. However as mentioned above we expect some non trivial dilaton when the systems are based on dilatonic branes. Now it may be possible that somehow the dilaton is screened, but this seems unlikely. There are however more general solutions to the Einstein-dilaton equations that have a Schwarzschild-like dependence of the entropy on the energy, and that satisfy the relation \(T^{-1} \sim r_H\) [23] (see also [24]). The uncharged solutions are labeled by three parameters \((c_1, c_3, r_0)\). However the requirement that the temperature be finite gives a relationship between \(c_1\) and \(c_3\), restricting them to a two parameter family [25]. There are some indications that the parameter \(c_1 = 0\) (if \(c_3 = 0\)) corresponds to the tachyon at the top of the potential [24], although it is unclear if this is true also when \(c_3 \neq 0\). Confusing the picture a bit more is that the condition for finite temperature when \(c_1 = 0\) cannot be satisfied unless \(p = 0\) or \(p = 3\), but can be satisfied for \(c_1 \neq 0\). So there are at least two possibilities. One is that the top of the tachyon potential is unstable for the other values of \(p\) and some rolling of the tachyon occurs before stabilization, and the other is that the relationship between the parameters and the value of the tachyon is more complicated. In any case it seems likely that the systems based on dilatonic branes describe these kinds of black branes. A better understanding of this is clearly needed.

5 Conclusion and Speculations

We saw that near extremal charged black branes carry the information needed to describe Schwarzschild-like black branes. The generality of this result may suggest some universality in the thermodynamics of the corresponding field

\(^2\)This does not clarify what meaning, if any, the metric inside the horizon has.
theories and in the density of states of closed strings near boundaries. These results suggest that the techniques used to analyze near-extremal charged black branes can be used to study a larger class of systems. In particular this suggests that Schwarzschild black branes (and possibly charged black branes far from extremality) are described by a quasi-particle picture similar to the one describing near extremal black branes. The horizon seems to be the region where massive closed string effects are important, suggesting that at least from this point of view the horizon is a special place. If Schwarzschild black branes can be described by open strings on meta-stable branes this may suggest an extension of the AdS/CFT correspondence to an open-closed duality in which a purely open string theory on any brane is dual to a closed string theory in the background of that brane. This would be a stronger version of the open-closed duality proposed recently by Sen [26].

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