Electromagnetic Emission and Energy Loss in the QGP

Guy D. Moore†
† Physics Department, McGill University, 3600 University Street, Montreal QC H3A 2T8, Canada
E-mail: guymoore@physics.mcgill.ca

Abstract. I discuss why photon production from the Quark Gluon Plasma (QGP) presents an interesting problem, both experimentally and theoretically. I show how the photon emission rate can be computed under the simplifying assumption that the QGP fully thermalizes. The theoretical issues are very similar to those for jet energy loss; so it should be possible to treat them in a common formalism and relate the predictions of one phenomenon to those of the other.


1. Photons as a deep probe

The main goals of the RHIC experimental program are, to produce and to characterize the quark-gluon plasma (QGP). The plasma is defined as being a state of matter for which the density of partons (quarks and gluons) is so great that a description in terms of hadronic degrees of freedom is impossible, which is furthermore extensive and relatively close to equilibrium. The problem is that the QGP expands, cools, and hadronizes. Even after hadronization, hadrons continue to interact; much of the information about the initial state is therefore lost. For instance, the spectrum of hadronic final states which we observe at RHIC is well characterized by a thermal model with a temperature of only about 160 MeV, too low for a QGP but reasonable for the freezeout of a hadronic gas [1]. Therefore, the hadronic observables give only rather indirect information about the QGP phase of the collision.

For this reason, it was proposed years ago to study “hard” probes, meaning particles which are produced early in the collision, deep within the QGP, and which then escape with little or no interaction, giving relatively direct information about the early stages of the QGP. Shuryak proposed photons as a promising direct probe [2], and they continue to be actively investigated [3].

The advantage of photons as a probe of the QGP is that any photons produced are almost sure to escape without re-interacting, so they give direct, rather than indirect or processed, information about the early stages of the heavy ion collision. The main disadvantage is that there are several sources of photons, with the concomitant problem of determining which photons arose from which source. Namely, there are

- Prompt photons, those produced by the initial collisions of the partons which constitute the heavy ions being collided;
• QGP photons, those produced during scatterings and interactions within the quark gluon plasma;
• Hadronic photons, those produced after the QGP has changed into a hadron gas but before it has spread out enough that mutual interactions cease; and
• Decay photons, those produced in the decay of certain hadrons, chiefly $\pi^0$ and $\eta$ mesons, “long” after the collision has finished.

Figure 1. Cartoon of the spectrum of each kind of photon produced in a heavy ion collision. Decay photons are expected to dominate at essentially every energy.

Of the “direct” photons (those produced in the collision rather than in much later decays), the QGP and hadronic photons are expected to dominate at lower energies, and the prompt photons at high energies. This is because there are more collisions during the QGP phase than between the initial state partons, so there are more opportunities to produce photons; but the energies of the particles in the QGP are smaller than the energies of the primary hadrons, so the produced photons are softer.

The problem is that decay photons are expected to dominate over direct photons at essentially every energy (see Fig. 1), and they carry no information about the early stages of the collision. Arguably the most interesting photons are the QGP photons; the prompt photons are also interesting, as they give us information about the initial parton content of the nuclei and therefore improve our normalization for jet quenching and other hard probes.

One way to understand the dominance of the decay photons is to note that the fine structure constant $\alpha_{\text{EM}}$ is small, and to ask how the number of each photon type scales with $\alpha_{\text{EM}}$. The prompt, QGP, and hadronic photon numbers all scale as $\alpha_{\text{EM}}$, because the duration of the phases they emerge from, and the energies of the particles involved, are roughly independent of $\alpha_{\text{EM}}$, whereas the processes that produce them (obviously) involve electromagnetic interactions. The number of decay photons, on the other hand, is nearly independent of $\alpha_{\text{EM}}$: if $\alpha_{\text{EM}}$ were smaller, the $\pi^0$ mesons would be longer lived, but would still eventually decay into photons, so the number of produced photons would be unchanged (modulo the few photons from $\eta$ decay).

At this meeting, we heard that prompt photons have now been convincingly detected by the PHENIX collaboration [4]. Fig. 2 shows that the detection only extends from about 4 GeV up, which as we will see means that only prompt photons have been unambiguously observed (so far). The WA98 collaboration at the SPS has claimed an observation of QGP photons as well [5], but the statistical significance is
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Figure 2. PHENIX measurements of photon excess over expected background from $\pi^0$ decay, in central $Pb–Pb$ collisions. (The red curve is the theoretical expectation.) A positive signal is seen from 4 GeV on up.

Figure 3. Left: naively leading order processes for photon production. Right: processes naively higher order, but contributing at the same order.

not that strong. We expect the PHENIX experimental results to improve, certainly tightening our determination of the prompt flux, but also hopefully detecting the QGP photons. This, together with a theoretical analysis of the QGP photon flux, will give us a direct handle on the early stages of heavy ion collisions.

2. Computing the photon emission rate

Computing the photon production by a QGP turns out to be an interesting theoretical problem. Very little is known or understood about the very early stages of the QGP, before it approximately thermalizes, so we will treat only the simpler problem of photon emission from a thermalized QGP, in the approximation that the expansion time of the QGP is long compared to the dynamical timescales associated with the photon production. We will furthermore make an expansion in $\alpha_s \ll 1$, even though it is not obviously justified. This turns out to make the slow expansion rate a consequence of the assumption of near-equilibrium.

This problem was thought solved in 1991, when two groups \cite{6,7} evaluated the leading order diagrams for the process, shown on the left in Fig. 3. This is not quite as easy as it looks, because the intermediate propagator receives plasma corrections, described by Braaten and Pisarski’s Hard Thermal Loops \cite{8}. Their result was,

$$\frac{d\Gamma}{dk} = \frac{2\alpha_s \alpha_{EM} k T^2}{\pi [e^{kT} + 1]} \left[ \ln \left( \frac{3k}{2\pi \alpha_s T} \right) - \frac{1}{2} - \gamma_E + \frac{4 \ln 2}{3} + \frac{\zeta'(2)}{\zeta(2)} + O(T/k) \right] \left[ \sum_q Q_q^2 \right]. \quad (1)$$

Here $\sum_q$ runs over all light ($m \ll T$) quarks, and $Q_q$ is the electric charge of the quark; so the sum is 5/9 if the strange quark is taken to be heavy and 2/3 if it is taken to be light. Note that the result is logarithmically enhanced above the naive $O(\alpha_s \alpha_{EM})$ expected from vertex counting. This occurs from the region where a spacelike propagator becomes soft.
This treatment is incomplete, however, because it misses contributions which are naively suppressed by a further power of $\alpha_s$, but which receive collinear enhancements which render them also $O(\alpha_s\alpha_{\text{EM}})$. This was first noticed by Aurenche, Gelis, Kobes, and Zaraker [9], who were computing the next order corrections to Eq. (1), arising from the diagrams on the right of Fig. 3, and found that they were of the same order.

These diagrams correspond to bremsstrahlung and processes related by crossing symmetry, and they encounter the same collinear enhancements as bremsstrahlung usually encounters. The cross-section for soft $t$-channel scattering is power (Coulomb) divergent in vacuum; this is cut off by plasma effects in the QGP medium, leading to a cross-section which is $O(\alpha_s)$, rather than the naive $O(\alpha_s^2)$. Hard bremsstrahlung of a photon is suppressed by a further power of $\alpha_{\text{EM}}$; even though the scattering is soft and the photon is hard, there is no additional suppression if the photon is emitted collinear to the emitting particle.

One way of understanding this collinear enhancement is to think about the spacetime picture of how the emission occurs. For small angle scattering ($\sim g_s$) and a small photon opening angle ($\sim g_s$), one allow the particles wave packets with a large transverse size ($\sim 1/g_s T$). The combination of small opening angle and large transverse wave packet means that the photon and quark wave packets overlap for a long time ($\sim 1/\alpha_s T$), leading to a large coherent enhancement of the emission. The time during which the wave packets overlap is called the coherence time. Note that it is essential to this argument that the quark move close to the speed of light; heavy quarks to not bremsstrahlung radiate efficiently.

This spacetime picture also shows why treating only these additional diagrams is not sufficient. An additional scattering before the coherence time of the emission will lead to a second photon emission amplitude which overlaps and will interfere with the first: This will happen $O(1)$ of the time, because the mean path between scatterings, $O(1/\alpha_s T)$, coincides with the mean formation time, for thermal photon production. [For production of very soft bremsstrahlung or very hard pair annihilation photons, the interference effects are parametrically large.] A lengthy power counting [10] shows that diagrams with repeated gluon “rungs,” corresponding to interference before and after

Figure 4. Left: spacetime picture of photon emission. Right: interference of photon emissions from sequential scatterings.

Figure 5. Diagrams, and interference effects, requiring resummation

$n$ scattering processes, must be resummed—but that no other processes contribute at leading order in $\alpha_s$. The interference between photon emissions associated with
successive scattering events is destructive, reducing the number of photon emissions from what we would find if each scattering event produced photons independently. This interference effect was first noticed, in a purely QED setting, by Landau, Pomeranchuk, and Migdal [11], and is called the LPM effect. In the QCD context, it has been considered by a few groups starting in the mid-1990’s [12, 13].

Sufficiently infrared modes in the QCD plasma, with wave number \( k \lesssim \alpha_s T \) (the inverse magnetic screening length), are beyond analytic treatment even at weak coupling. Fortunately, the analysis [10] shows that photon emission is not sensitive to this scale at leading order. [Note that the entire treatment we are discussing is framed upon the assumption that a formal expansion in \( \alpha_s \ll 1 \) is justified; we keep all effects which are formally leading in such an expansion, and make all permissible approximations which are valid at leading order. It is not obvious whether the expansion in \( \alpha_s \) is even qualitatively reliable for the environment actually encountered in the QGP, and it will be difficult even to address that question until we are able to perform a calculation beyond leading order.]

Resumming the diagrams for photon emission with multiple scatterings leads to a Boltzmann-like equation, which describes the evolution of the density matrix \(|p_q(t)\rangle\langle(p-k),k,(t)|\), representing the interference between the process where a photon has and has not been emitted, as the quark undergoes scatterings in the medium. The photon emission rate is,

\[
\begin{align*}
\frac{dN_{\gamma}}{dk_3dx} &= \frac{2\alpha_{EM}^2}{4\pi^3k} \int_{-\infty}^{\infty} \frac{dp}{2\pi} \int \frac{d^2p_\perp}{(2\pi)^2} \frac{n_f(p+k)p[1-n_f(p)]}{2p(p+k)^2} \times \\
&\quad \times [p^2+(p+k)^2] \Re \left\{ 2p_\perp \cdot f(p_\perp;p,k) \right\} \\
2p_\perp &= i\delta E \ f(p_\perp;p,k) + \frac{2\pi}{3} g_s^2 \int \frac{d^2q_\perp}{(2\pi)^2} \frac{m_D^2}{q_\perp^2} \frac{T}{m_D^2+q_\perp^2} \times \\
&\quad \times \left[ f(p_\perp;p,k) - f(q+p_\perp;p,k) \right], \\
\delta E &= \left( p_\perp^2 + m_\infty^2 \right) \frac{k}{2p(k+p)}.
\end{align*}
\]

The first expression relates the rate of photon emission to the solution to an integral equation, describing the propagation of the quark or quark plus photon system through the plasma. The second equation is the integral equation; the first term is a “dephasing” term recording how the photon and quark wave packets evolve off from on top of each other, and the second term is a collision term, reminiscent of the collision term in a Boltzmann equation. The third equation expresses \( \delta E \), the energy difference between the system with a quark of momentum \( p \) and a quark plus photon of momentum \( (p-k),k \); this energy difference is responsible for the “dephasing.” Here \( p_\perp \) is the component of \( p \) perpendicular to \( k \), and \( m_\infty^2 = g_s^2 T^2/3 \) is the thermal “mass” of a quark, which parametrizes how forward scattering in the medium changes the dispersion relation of the quark. This is relevant because, as we saw, the coherence length is smaller if the quark moves slower than the speed of light.

This integral equation does not have an analytic solution (that we know of). It can be solved numerically by a variational technique [14], or by Fourier transforming to impact parameter space and evolving an ODE [15]. A few parameter fit of the
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result, is that the bracketed quantity in Eq. (1) should be increased by

\[ \sqrt{1+\frac{1}{6}N_f} \left[ \frac{1.096 \log(12.28 + 1/x)}{x^{3/2}} + \frac{0.266 x}{\sqrt{1 + x/16.27}} \right] \]

where \( N_f \) is the number of light quark flavors (presumably 2 or 3). The result is also displayed, for 2 flavors and \( \alpha_s = 0.2 \), in Fig. 6. Numerically, we find that the result only deviates about 20\% from the answer we would obtain by ignoring the LPM effect and treating the emission from each scattering to be independent (which is equivalent to evaluating Eq. (3) at linear order in the collision term). At very low and very high energies this statement breaks down, but it is valid out past 10\( T \) (say, 3 GeV).

How many photons are produced from the QGP? What we have computed is the production rate from a region of equilibrated QGP at some temperature. To get the total number from the QGP, this must be folded over the hydrodynamical evolution of the plasma. Our results for QGP emission rates have been used by various hydro groups; for instance, the results of Ruuskanen et al. 16 are shown in Fig. 6. Generally, the uncertainties from the hydro evolution (even when constrained to describe correctly the multiplicity and temperature of the final state hadrons) are larger than the uncertainties from the photon production rate 17.

The figure suggests that the photon production by QGP is about a factor of 10 smaller than the background from \( \pi^0 \) decay; above 3 GeV, the \( \pi^0 \) rate dominates even more. That means that the photon production (technically, the ratio of photons to \( \pi^0 \)) must be measured to better than 10\% at about 3 GeV, which should be quite challenging but may be experimentally possible.

3. Jet quenching

Actually, we know that the QGP will not be a fully thermal bath. The particles which take the longest to thermalize are the highest energy ones; therefore one expects a power law, high energy tail of particles moving through the approximately thermalized QGP. This high energy tail is responsible for high energy jets, and their interaction
with the plasma, particularly their energy loss as they traverse the plasma, is the source of jet quenching. The dominant energy loss mechanism is the bremsstrahlung of gluons—a process very similar to the bremsstrahlung of photons, which we have just discussed. This leads to two conclusions:

(i) We should be able to study jet energy loss with the same formalism as photon production:

(ii) High energy particles moving through the QGP should also radiate photons, which will give a power-law, nonthermal extra contribution to the QGP photon production rate. This can be computed in parallel with jet energy loss.

The issue of jet energy loss has been looked at by several authors and has been nicely summarized in these proceedings by Ivan Vitev [18]. Nevertheless, we think it would be beneficial to treat it again, being careful to work systematically to leading order in \( \alpha_s \), not treating the LPM effect as parametrically large (especially as we find that it is usually quite small), and treating the photon production in parallel. This should give a concrete relation between the extent of energy loss and the number of hard photons emitted as hard partons traverse the QGP.

The rate at which a parton of momentum \( p \) emits a gluon of momentum \( k \) in traversing the QGP, per unit time and \( k \), is

\[
\frac{d\Gamma'(p,k)}{dkdt} = \frac{C_s g_s^2}{16\pi p^7} \left[ \frac{1}{1 \pm e^{-k/T}} \right] \left[ \frac{1}{1 \pm e^{-(p-k)/T}} \right] \left\{ \begin{array}{ll}
q \to gg & \frac{1+(1-x)^2}{x^2(1-x)^2} \\
g \to qg & \frac{1}{1-x^2} \\
g \to gg & \frac{1}{x^2(1-x)^2}
\end{array} \right\} \times \\
\times \int \frac{d^2h}{(2\pi)^2} 2h \cdot \text{Re} \, \textbf{F}(h,p,k),
\]

with \( C_s \) the relevant Casimir, \( C_s = (4/3) \) except for \( g \to gg \), for which it is 3. \( \textbf{h} \) is the non-collinearity, \( \textbf{p} \times \textbf{k} \), and \( \textbf{F} \) is the solution to

\[
2\textbf{h} = i\delta E(\textbf{h},p,k)\textbf{F}(\textbf{h}) + \frac{g^2}{2} \int \frac{d^2q_\perp}{(2\pi)^2} C(q_\perp) \left\{ (2C_s - C_A)\textbf{F}(\textbf{h}) - \textbf{F}(\textbf{h} - q_\perp) \right\} \\
+ C_A \left\{ \textbf{F}(\textbf{h}) - \textbf{F}(\textbf{h} + q_\perp) \right\} + C_A \left\{ \textbf{F}(\textbf{h}) - \textbf{F}(\textbf{h} - q_\perp) \right\},
\]

\[
\delta E(\textbf{h},p,k) = \frac{h^2}{2pk(p-k)} + \frac{m_k^2}{2k} + \frac{m_{p-k}^2}{2(p-k)} - \frac{m_p^2}{2p},
\]

\[
C(q_\perp) = \frac{m_D^2}{q_\perp(q_\perp^2 + m_D^2)}, \quad m_D^2 = \frac{g^2T^2}{6}(2N_c+N_f).
\]

Here \( C_A = 3 \) is the adjoint Casimir. The gluon “mass” is the dispersion correction for a hard gluon, which is \( m_D^2/2 \). The production of photons follows the same equation, but with \( C_s \to Q_g^2, C_A \to 1, m^2 = 0, \) and \( g^2 \to e^2 \). These equations then form the scattering kernel for a system of Boltzmann equations describing the hard quark and gluon populations in the plasma [19].

We are in the process of evolving these Boltzmann equations through the hydrodynamically evolving QGP and applying fragmentation functions at the surface of hadronization, to evaluate both the jet quenching and the photon production from the hard secondary partons within a unified framework. Our preliminary results indicate that the production rate of photons from this source will be small, probably negligible compared to the prompt photons.
4. Conclusions

Photons constitute an interesting probe of the Quark Gluon Plasma. Unfortunately, it may be rather difficult to use this probe, because many more photons are expected to be produced from the decay of mesons produced by the fireball, than are produced in the fireball itself. This is an experimental question. The theoretical questions are, to compute the spectrum of produced photons from a piece of QGP, and to fold this over the hydrodynamical evolution (if indeed this is the correct description of the plasma’s evolution) of the plasma produced in a heavy ion collision.

The problem of computing the photon production from a glob of QGP turns out to be more difficult than expected, because collinearly enhanced bremsstrahlung is a leading order process, and it receives a rather complicated partial coherence correction called the LPM correction. This problem is solvable and has now been solved. The tools used are similar to what is required to determine the jet quenching rate, and a unified approach to the two problems is now under way.

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References

[1] See for instance, Gunther Roland, these proceedings.
[16] Sami Rässänен, private communication; see also [14].