Trugenberger Replies: I answer the criticism [1] of my model of quantum associative memory [2,3].

The main idea behind the concept of associative memories [4] is to get away from classical databases by storing (usually binary) patterns in a dynamics that matches an input onto a memory pattern close to it (in Hamming distance). The key feature of such memories is their ability to correct noisy inputs by *dynamic association*. Patterns are typically stored as local minima of an energy functional which drives a Glauber-type dynamics transforming an input into the closest minimum of the energy. Unfortunately, these models suffer from a capacity shortage limiting the number of patterns that can be stored and recalled on n bits to a linear bound in n .

My contribution consists of a proposal for a quantum version of dynamic association. The idea is to store the given patterns in a unitary operator M , rather than in a classical energy functional. This generates from $|0\rangle$ a uniform superposition of the given patterns. It can be shown [3] that M defines a Hamiltonian which is diagonal in the computational basis with exact global minima at the stored patterns. Quantum dynamic association is then a quantum dynamical evolution which transforms the initial uniform superposition of patterns into an output superposition which is peaked around an input pattern.

This program can be realized either with a decision measurement and postselection of the result [2] or with a direct amplification via the technique of amplitude amplification [3]. There is no limitation on the number of patterns that can be stored and recalled. Admittedly, the direct amplification technique can be implemented efficiently only for a number of stored patterns which is polynomial in n . This however is still a large improvement upon classical models with limitations linear in n .

The authors of the Comment [1] dismiss this model on the grounds that the same result could be obtained by a classical probabilistic database search scheme. I do not dispute this fact. However, the correct version of such a classical search requires multiple random choices of candidate patterns if the accuracy of pattern recall should be tunable as in the proposed quantum algorithm (for example, the direct amplification technique can be tuned to always recognize one of the stored patterns with a Hamming distance expectation value as close to the minimum as required). Therefore, contrary to the authors' statement, all the classical database of pattern must be stored, and not just one pattern. This has nothing to do with associative memories as they are generally understood [4].

The interesting question is to compare the quantum mechanism I have proposed with its own classical coun-

terpart, in which patterns are not stored in a database but rather in an energy functional, and retrieved by a classical dynamics driven by this functional. Note, in this respect, that "a retrieval mechanism which has to know about the data it is supposed to retrieve" not only does not contradict the idea of a memory but could actually be taken as part of the definition of associative memories.

My statement in this regard is correct: quantum associative memories are better than classical associative memories (in their usual definition) since quantum dynamic association can be realized efficiently for at least a polynomial number of patterns without the appearance of the spurious memories that causes the transition to disorder affecting the classical models. Note that this improvement does not violate the Holevo bound.

As far as repetitions of the basic retrieval steps are concerned, I admittedly did not compute the efficiency of the needed probabilistic cloners. The direct amplification technique does indeed involve a reparation of $|M\rangle$ since the memory is completely encoded in the corresponding unitary operator M , or equivalently, in the Hamiltonian (quantum energy operator) $i\log M$. This is however efficient for a polynomial number of patterns and can be "hard wired" in a sequence of standard unitary elementary gates, involving at most two qubits. These are a direct quantum generalization of the two-body interactions characterizing the energy functional of classical neural networks. As I already pointed out, this represents a large advantage over corresponding classical (Hopfield-type) models with linear limitations.

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- [1] T. Brun, H. Klauck, A. Nayak, M. Rötteler, and Ch. Zalka, preceding Comment, Phys. Rev. Lett. **91**, 209801 (2003).
- [2] C. A. Trugenberger, Phys. Rev. Lett. **87**, 067901 (2001); **89**, 277903 (2002).
- [3] C. A. Trugenberger, Quantum Information Processing **1**, 471 (2002).
- [4] For a review, see, e.g., B. Müller and J. Reinhardt, *Neural Networks* (Springer-Verlag, Berlin, 1990).