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ABSTRACT

We estimate the rate at which the proposed space gravitational-wave interferometer LISA could detect intermediate-mass black-hole binaries, that is, binaries containing a black hole in the mass range $10 - 100 \, M_\odot$ orbiting a black hole in the mass range $100 - 1000 \, M_\odot$. For one-year integrations leading up to the innermost stable orbit, and a signal-to-noise ratio of 10, we estimate a detection rate of only 1 per million years for $10 \, M_\odot/100 \, M_\odot$ binaries. The estimate uses the method of parameter estimation via matched filtering, incorporates a noise curve for LISA established by the LISA Science Team that is available online, and employs an IMBH formation rate model used by Miller (2002). We find that the detectable distance is relatively insensitive to LISA arm lengths or acceleration noise, but is roughly inversely proportional to LISA position errors, and varies roughly as $T^{1/2}$, where $T$ is the integration time in years. We also show that, while all IMBH systems in this mass range may be detected in the Virgo cluster up to 40 years before merger, none can be detected there earlier than 400 years before merger. An extended LISA mission that enabled 10-year integrations could detect IMBH systems at the Virgo cluster 1000 years before merger, and systems in galactic globular clusters a million years before merger. We compare and contrast these estimates with earlier estimates by Miller (2002).

Subject headings: gravitation: gravitational radiation, black holes

1. Introduction and summary

In recent years, intermediate-mass black holes (IMBH), holes with masses between hundreds and thousands of solar masses, have attracted considerable interest, both as relics of the evolution and structure of globular clusters, and as possible sources of gravitational radiation for both ground- and space-based laser interferometers (Miller & Colbert 2004). Miller (2002) has estimated the rate of detectable IMBH binaries by the proposed space antenna LISA to be given by

$$ R \approx 7 \times 10^{-3} \left( \frac{h}{0.7} \right)^3 \left( \frac{f_{\text{tot}}}{0.1} \right) \left( \frac{\mu}{10 M_\odot} \right)^{1/2} \times \left( \frac{M_{\text{max}}}{100 M_\odot} \right)^{3/2} \left( \ln \frac{M_{\text{max}}}{M_{\text{min}}} \right)^{-1} \text{yr}^{-1}, \quad (1) $$

where $h$ is the Hubble parameter in units of $100 \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1}$, $f_{\text{tot}}$ is the total fraction of globular clusters that have IMBH, $\mu$ and $M$ are the reduced mass and total mass of the binary respectively, and $M_{\text{min}}$ and $M_{\text{max}}$ denote the range over which such IMBH may exist in globular clusters.

A key ingredient in calculating this rate is the distance to which LISA could detect binary IMBH inspirals for a given signal-to-noise ratio (SNR). Miller (2002) used estimates of signal-to-noise ratios for binary inspiral derived by Flanagan & Hughes (1998) from semi-analytic LISA noise curves. We have recalculated this distance using standard methods based on matched-filtering...
Fig. 1.— Distances in Mpc reached by LISA for IMBH binary inspiral, for SNR=10 and one year of integration prior to merger.

for binary inspiral, and using an up-to-date LISA noise curve available online. \(^2\) We find significantly smaller distances and rates reached for a given SNR and with a significantly different dependence on mass than those obtained by Miller (2002). Figure 1 shows distances reached for SNR=10 and one year of integration leading up to merger, as a function of total mass \(M\) and reduced mass parameter \(\eta = m_1 m_2 / M^2\) \((0 < \eta \leq 1/4)\). For \(M = 100 M_\odot\) and reduced mass \(\mu = \eta M = 10 M_\odot\), we find a distance \(D_L \approx 18\) Mpc, 11 times smaller than Miller’s equation (19) (we assume that all masses are suitably redshifted). The rate of detection we find is given by

\[
R \approx 1.0 \times 10^{-6} \left( \frac{h}{0.7} \right)^3 \left( \frac{\mu}{0.1} \right) \left( \frac{\mu}{10 M_\odot} \right)^{19/8} \\
\times \left( \frac{M_{\max}}{100 M_\odot} \right)^{13/4} \left( \ln \left( \frac{M_{\max}}{M_{\min}} \right) \right)^{-1} \text{yr}^{-1}. \quad (2)
\]

We also analyse LISA’s reach for IMBH systems at epochs earlier than the year leading up to merger. Figure 2 shows distances as a function of years before merger for various IMBH systems, for SNR=10 and one year of integration. While all systems in this mass range are detectable in the Virgo cluster (at 19 Mpc) within 40 years of merger, none is detectable there earlier than 400 years before merger, and at 1000 years before merger, only systems closer than 7 Mpc are detectable. On the other hand, we find that, for systems in this mass range, the distance reached varies roughly as the square-root of the integration time, so that an extended LISA mission that enabled 10-year integrations, could reach roughly three times farther.

The primary reason for this downward revision in the reach of LISA for IMBH is our use of a more current sensitivity curve for LISA, whose noise level is substantially higher than the older Flanagan-Hughes curve used by Miller. The rest of this paper gives the details to support this conclusion. In §2 we determine the distance reached by LISA using standard SNR calculations, and in §3 we use the method of Miller (2002) to estimate the detection rate of systems in the final year of inspiral. In §4 we discuss the results.

2. Calculation of Distance Reached by LISA

The signal-to-noise ratio \(\rho\) for a gravitational-wave signal whose Fourier transform is \(\tilde{h}(f)\) in a detector of noise spectral density \(S_n(f)\) is given by

\[
\rho^2 |\tilde{h}(f)|^2 = 4 \int_{f_i}^{f_f} \frac{|\tilde{h}(f)|^2}{S_n(f)} df, \quad (3)
\]

where \(f_i\) and \(f_f\) are the initial and final frequencies between which the signal is integrated (see
where \( M \) averaging over all angles, the amplitude \( A \) by \( M \) and \( D \)
where \( \tilde{h} \) is the total mass of the system. After
where \( \rho \) is the luminosity distance of the source, and \( \mathcal{M} = \eta^{3/5}M \) is the “chirp” mass. In general
relativity, the phasing function \( \Psi(f) \) can be given
to high post-Newtonian order, but is not relevant
for our purposes. Combining equations (3), (4) and (6), we
obtain the expression for \( D_L \)
where \( B(7) = \int_{f_i}^{f_f} \chi^{-7/3} \sqrt{S_n(f)} \, df \).
For quasi-circular binary inspiral, the initial and final
frequencies can be related to the integration
time \( T \) using the expression obtained from
gravitational radiation damping of the orbit at
quadrupole order (which is sufficiently accurate for
our purpose),
where \( u = \pi \mathcal{M} f \).

In this paper, we shall adopt sensitivity curves
for LISA developed independently by Larson, Hiscock & Hellings (2001) and Armstrong, Estabrook & Tinto (1999) (see figure 3). The two methods are in substantial agreement, and the former has been summarized in the Sensitivity Curve Generator (SCG), available online. The sensitivity curves incorporate sources of instrumental noise in LISA, such as laser shot noise, acceleration noise, thermal noise, etc., coupled with a LISA transfer function which takes into account the effect of the finite time of propagation of the laser beams during the passage of the gravitational waves. We assume the case where all three arms of LISA are of equal length, and we assume that averages over angles and polarizations have been done. The SCG permits various choices to be made for LISA instrumental parameters, and has an option to include an estimate for confusion noise resulting from a background of galactic white-dwarf binaries (Bender & Hils 1997); this background is included in the analysis, but in fact plays no significant role for the late stage of IMBH inspiral because the signals are predominantly at high frequency, well above the white-dwarf band.

We consider IMBH systems containing one
black hole in the mass range \( 10 - 100 \, M_{\odot} \) and
a companion black hole in the mass range \( 100 - 1000 \, M_{\odot} \).
Figure 1 shows the results for the distance as a function of reduced mass parameter for various total masses, assuming a SNR of 10 with a one-year integration time ending at the ISCO (or at the termination of the LISA sensitivity curve, whichever comes first). The distances, in Mpc,
can be fit by the approximate formula

\[
D_L = 190 \left( \frac{10}{\rho} \right) \left( \frac{M}{100 M_\odot} \right)^{15/8} \\
\times \left( \frac{T}{1 \text{ yr}} \right)^{1/2} \text{ Mpc} \\
= 40 \left( \frac{10}{\rho} \right) (4\eta)^{9/8} \left( \frac{M}{100 M_\odot} \right)^{15/8} \\
\times \left( \frac{T}{1 \text{ yr}} \right)^{1/2} \text{ Mpc}. \tag{10}
\]

This scaling can be understood as follows: the waves from IMBH in the final year of inspiral are mainly at the high frequency end of the LISA sensitive band, where \( S_n(f) \propto f^2 \). Also, for these masses and year-long integrations, the second term in equation (9) dominates, so that \( f_i = (1/8\pi)(5/T)^{3/5}M^{-3/5} \ll f_f \). Consequently, the integral \( B(7) \sim f_i^{-10/3} \sim M^{25/12}T^{5/4} \). Substituting this into equation (7), we obtain the scaling in \( M \) shown in (10). A \( T^{1/2} \) integration-time scaling actually fits the curves somewhat better than a \( T^{5/8} \) scaling inferred from the analytic estimate. Equation (10) underestimates the distances at the low mass end, e.g. for \( \eta \sim 1/10 \) and \( M \sim 100 M_\odot \), by about 20 percent.

It is straightforward to show that the distance reached is independent of LISA acceleration noise, but is inversely proportional to the LISA position noise error budget. Halving the position noise doubles the distance reached. The distance is weakly dependent on LISA arm length, varying by between 10 and 40 percent for factor-of-two changes in arm length; varying arm length does not alter the overall level of the high-frequency end of the noise curve, but instead moves the location of the peaks in the oscillations of the transfer function and raises the floor of the noise curve near \( 10^{-3} \) Hz.

We obtain similar results using a sensitivity curve derived by Finn & Thorne (2000) based on work of the LISA Mission Definition team (see reference 44 of Finn & Thorne (2000)). This curve closely matches the curve from the SCG except for the oscillations in the noise root spectral density at high frequency resulting from the LISA transfer function, an effect of the finite arm length. The oscillations have the effect of reducing somewhat the distances inferred from the SCG for a given SNR compared to those from the Finn-Thorne curve, for sources at high frequency.

We also calculate the distances that can be reached in one year’s integration for IMBH systems much earlier than the ISCO stage (see Figure 2). All systems detected within about 40 years of merger, can be detected at the Virgo cluster (19 Mpc) with SNR=10 and one year of integration. However at earlier epochs the reach decreases dramatically, so that by 400 years no systems can be seen at Virgo, and by 1000 years before merger, the distance reached for SNR=10 is in the range 5.6 to 7 Mpc for all IMBH mass ranges considered. This is well short of the Virgo cluster. For systems detected \( 10^6 \) years before merger, the distance reached is 7 Kpc, independent of mass. This behavior can be understood analytically. For observation epochs long before the ISCO, one can show that the initial and final frequencies observed are related by \( f_i \approx f_f(1 + T/T')^{-3/8} \), where \( T \) is the integration time, and \( T' \) is the epoch before merger, with \( T < T' \). Then, in a region where the noise spectral density is dominated by the behavior \( S_n(f) \approx \alpha f^n \), one can show, using equations (7) and (8) that

\[
D_L \approx \frac{2}{5} \left( \frac{8\pi}{\rho} \right)^{n/2} \left( \frac{T}{\alpha} \right)^{1/2} \left( \frac{T'}{5} \right)^{2n/3} \left( \frac{M}{n} \right)^{\frac{5\alpha}{6n}}. \tag{11}
\]

For epochs earlier than 1000 years before merger, the systems are in the low-frequency regime where \( S_n(f) \propto f^{-4} \) (apart from white-dwarf confusion noise), hence \( D_L \approx (1/32\pi^2)(T/\alpha)^{1/2}(T'\rho)^{-1} \), independent of chirp mass. With \( \alpha = 6.09 \times 10^{-51} \text{ Hz}^2 \) giving a good fit to the low-frequency LISA instrumental noise, we obtain

\[
D_L \approx 7 T^{1/2}(1000/T')(10/\rho) \text{ Mpc}, \tag{12}
\]

which perfectly matches the large-epoch behavior in Figure 2. The \( T^{1/2} \) behavior is expected for what are observations of an essentially CW source. These results also are far more pessimistic than the estimates made by Miller & Colbert (2004), which suggested detecting IMBH binaries in the Virgo cluster 1000 years before merger, and in the local system of globular clusters \( 10^6 \) years before merger. On the other hand, because of the \( T^{1/2} \) behavior, such binaries could be reached in an extended LISA mission that enabled 10-year integration times.
3. Rate of detectable mergers

To calculate the rate of inspirals detectable by LISA in the last year before merger we follow the method outlined by Miller (2002). The rate is given by

\[ R = \frac{4\pi}{3} \int D(M)^3 \nu(M)n_{gc} f(M) dM, \quad (13) \]

where \( D(M) \) is the distance reached for a given SNR and integration time, as a function of total mass (holding reduced mass \( \mu \) fixed); \( \nu(M) = 10^{-10} \mu^{-1} M \text{yr}^{-1} \) is the rate at which smaller black holes merge with black holes of mass \( M \) in a given cluster; \( n_{gc} = 8h^3 \text{Mpc}^{-3} \) is the number density of globular clusters in the local universe; and \( f(M) = [f_{tot}/ \ln(\max M/\min M)] M^{-1} \) is the fraction of globular clusters harboring black holes with mass \( M \) per mass interval \( dM \); in the range \( \min M < M < \max M \), with \( \int f(M) dM = f_{tot} \). We assume \( M_{\max} > M_{\min} \). Substituting these formulae, along with equation (10) into equation (13), we obtain equation (2).

4. Discussion

Both the distance reached by LISA and the estimated rate of inspirals detected differ markedly from the estimates given by Miller (2002), namely \( D_L \approx 200(\mu/10 M_\odot)^{1/2}(M/100 M_\odot)^{1/2} \text{Mpc} \), and equation (1). The distance derived by Miller is larger and the mass dependence is different than in equation (10) because he appears to have adopted an equation \([B7]\) from Flanagan & Hughes (1998) that actually applies only to equal-mass systems. For the low masses relevant to IMBH in the year leading up to the merger, the high-frequency end of the LISA noise spectrum dominates, where the dependence on frequency (and hence on \( M \)) is steeply increasing. But the underlying dependence is on chirp mass, so for a given total mass, there is still a strong dependence on reduced mass parameter, which is not reflected in equation \([B7]\) of Flanagan & Hughes (1998). Also, the noise root spectral density curve modeled by Flanagan & Hughes (1998) is lower by a factor of about 3 – 5 than that given by the SCG (figure 3). Their noise curve was based on an out-of-date description of the LISA mission. Distance estimates using the Flanagan-Hughes curve will therefore automatically be larger and rates will be higher than those using the SCG.

We have assumed throughout that the inspiral orbits are quasi-circular, that is, circular apart from the adiabatic inspiral due to radiation reaction. In reality, IMBH binary orbits are likely to be highly eccentric (Gültekin, Miller & Hamilton 2004). This will increase the average gravitational-wave flux for a given orbital period, and could add harmonics in frequency bands where LISA’s noise may be lower; on the other hand it will increase the number of parameters to be estimated in the matched filter, which raises the threshold needed to achieve detection. It is not clear whether the net effect of these complicated and possibly offsetting effects will improve LISA’s reach for IMBH binaries or not. In any event, they are beyond the scope of this paper.

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